Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/24-1.1.2.8-P-x-c-x-^m-a+b-x^2-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [174]. This is test number [24].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (174)	0.00 (0)
Mathematica	100.00 (174)	0.00 (0)
Fricas	97.70 (170)	2.30 (4)
Maple	97.70 (170)	2.30 (4)
Giac	97.70 (170)	2.30 (4)
Maxima	97.70 (170)	2.30 (4)
Sympy	89.08 (155)	10.92 (19)
Mupad	74.14 (129)	25.86 (45)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

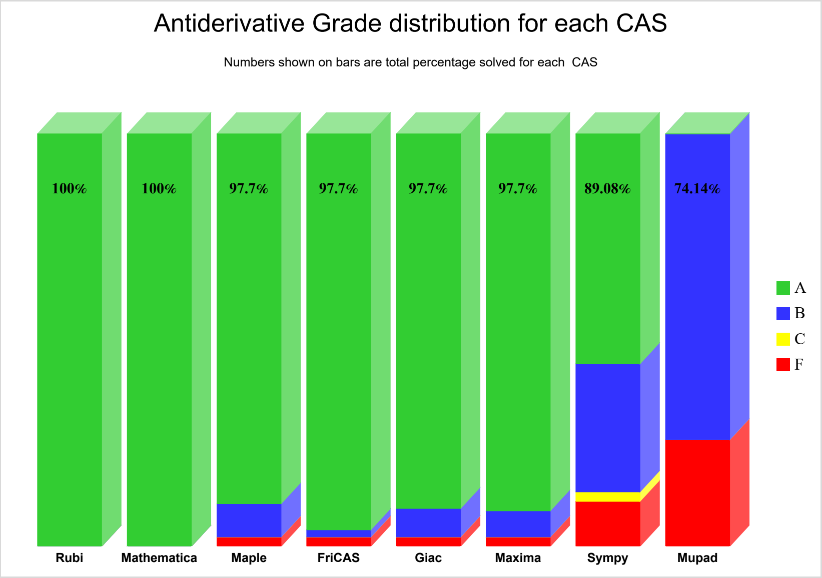
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

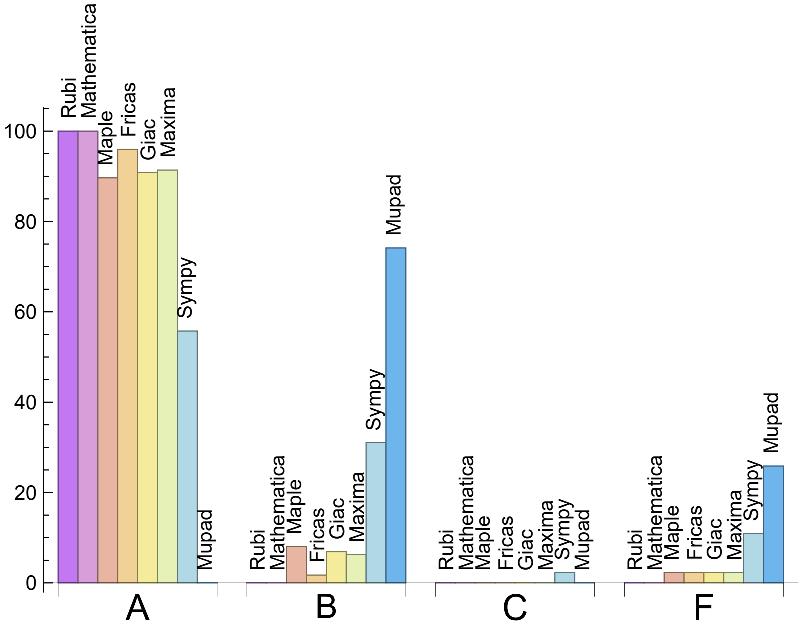
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	100.00	0.00	0.00	0.00
Fricas	95.98	1.72	0.00	2.30
Maxima	91.38	6.32	0.00	2.30
Giac	90.80	6.90	0.00	2.30
Maple	89.66	8.05	0.00	2.30
Sympy	55.75	31.03	2.30	10.92
Mupad	N/A	74.14	0.00	25.86

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	4	100.00 %	0.00 %	0.00 %
Fricas	4	100.00 %	0.00 %	0.00 %
Giac	4	100.00 %	0.00 %	0.00 %
Maxima	4	100.00 %	0.00 %	0.00 %
Sympy	19	0.00 %	100.00 %	0.00 %
Mupad	45	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

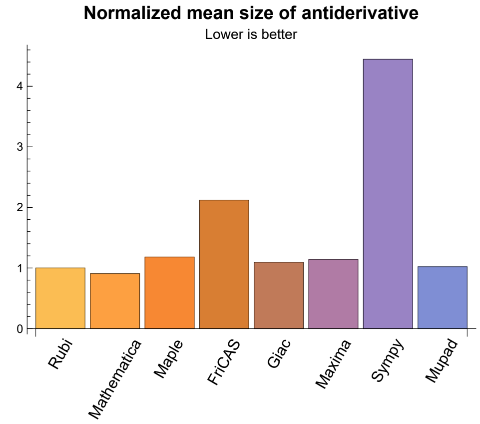
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

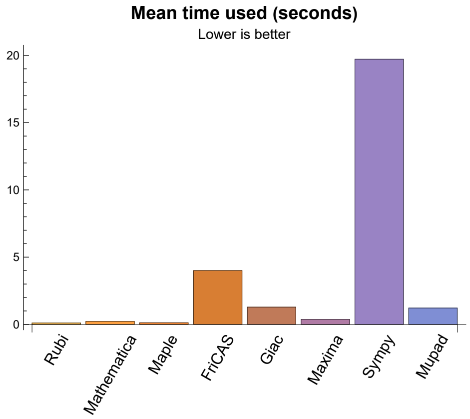
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.11	129.82	1.00	121.00	1.00
Mathematica	0.22	112.94	0.91	103.00	0.91
Maple	0.12	164.12	1.18	128.50	1.00
Maxima	0.37	165.99	1.14	122.50	1.01
Fricas	4.00	285.56	2.12	230.00	2.11
Sympy	19.71	734.59	4.45	219.00	1.87
Giac	1.29	151.56	1.10	125.50	0.99
Mupad	1.22	129.83	1.02	108.00	0.98

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
5
10
15
20
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

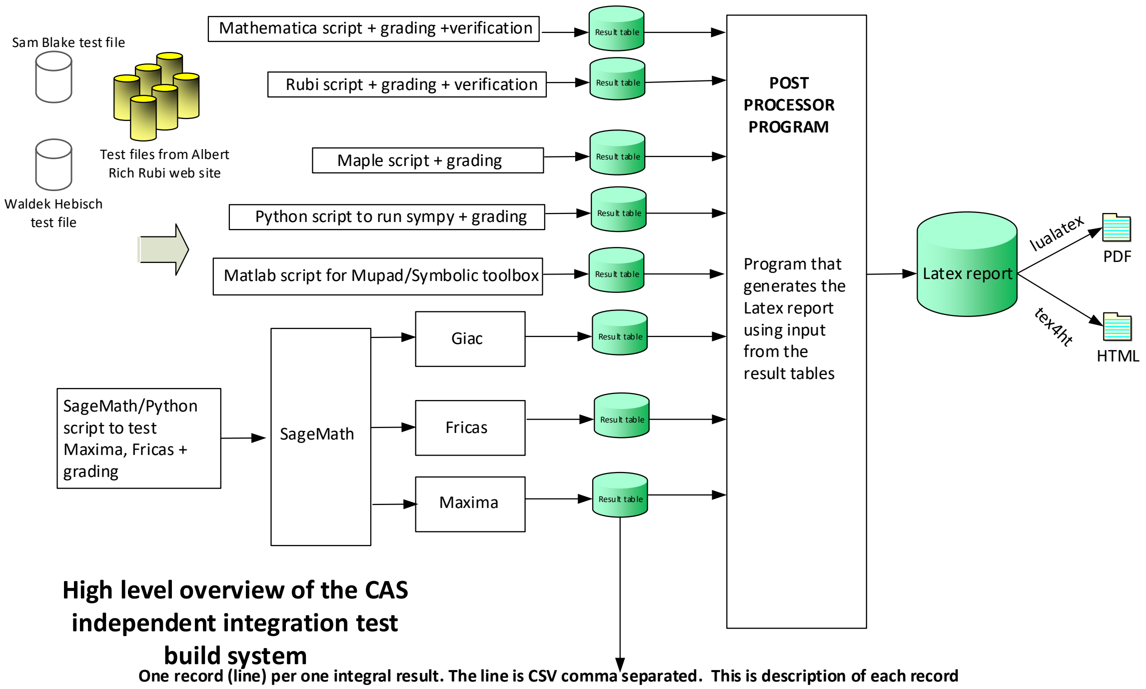
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

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August 26, 2022

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade: { }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 51, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 165, 166, 167, 168, 169, 170, 171 }

B grade: { 7, 37, 48, 49, 50, 52, 160, 161, 162, 163, 164, 172, 173, 174 }

C grade: { }

F grade: { 58, 59, 60, 61 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 164, 165, 166, 167, 168, 169, 170, 171 }

B grade: { 47, 48, 49, 159, 160, 161, 162, 163, 172, 173, 174 }

C grade: { }

F grade: { 58, 59, 60, 61 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade: { 107, 108, 109 }

C grade: { }

F grade: { 58, 59, 60, 61 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 32, 35, 36, 38, 43, 44, 45, 46, 47, 50, 51, 53, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 105, 106, 110, 111, 112, 113, 114, 116, 117, 120, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 135, 136, 137, 138, 146, 147, 148, 153, 154, 155, 156 }

B grade: { 25, 33, 34, 37, 39, 40, 41, 42, 48, 49, 52, 54, 55, 56, 57, 86, 87, 88, 89, 90, 94, 95, 96, 97, 98, 102, 103, 104, 115, 118, 119, 121, 122, 143, 144, 145, 149, 150, 152, 157, 158, 160, 161, 162, 163, 164, 165, 166, 169, 170, 171, 172, 173, 174 }

C grade: { 58, 59, 60, 61 }

F grade: { 91, 92, 93, 99, 100, 101, 107, 108, 109, 131, 132, 139, 140, 141, 142, 151, 159, 167, 168 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 169, 170, 171, 172, 173, 174 }

B grade: { 7, 14, 28, 35, 45, 150, 156, 157, 158, 166, 167, 168 }

C grade: { }

F grade: { 58, 59, 60, 61 }

2.1.8 Mupad

A grade: { }

B grade: { 4, 5, 6, 7, 11, 12, 13, 14, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 90, 92, 93, 96, 98, 100, 101, 102, 104, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 155, 156, 157, 158, 166, 167, 169, 170, 171 }

C grade: { }

F grade: { 1, 2, 3, 8, 9, 10, 15, 16, 17, 22, 29, 36, 47, 48, 58, 59, 60, 61, 86, 87, 88, 89, 91, 94, 95, 97, 99, 103, 105, 107, 151, 152, 153, 154, 159, 160, 161, 162, 163, 164, 165, 168, 172, 173, 174 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	101	120	107	206	192	93	-1
N.S.	1	1.00	0.80	0.94	0.84	1.62	1.51	0.73	-0.01
time (sec)	N/A	0.059	0.179	0.109	0.281	3.704	5.638	1.351	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	87	96	86	175	165	81	-1
N.S.	1	1.00	0.84	0.92	0.83	1.68	1.59	0.78	-0.01
time (sec)	N/A	0.034	0.158	0.108	0.280	5.303	2.671	0.960	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	77	76	67	157	124	68	-1
N.S.	1	1.00	0.96	0.95	0.84	1.96	1.55	0.85	-0.01
time (sec)	N/A	0.018	0.143	0.104	0.282	6.871	2.707	0.802	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	68	54	45	128	70	55	52
N.S.	1	1.00	1.01	0.81	0.67	1.91	1.04	0.82	0.78
time (sec)	N/A	0.013	0.157	0.109	0.265	3.332	1.555	0.798	1.162

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	90	79	59	341	107	78	68
N.S.	1	1.00	1.14	1.00	0.75	4.32	1.35	0.99	0.86
time (sec)	N/A	0.043	0.190	0.106	0.277	4.603	3.001	0.818	1.236

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	88	103	59	333	124	102	89
N.S.	1	1.00	1.17	1.37	0.79	4.44	1.65	1.36	1.19
time (sec)	N/A	0.040	0.173	0.117	0.269	4.963	2.055	0.998	1.688

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	90	127	83	377	107	163	94
N.S.	1	1.00	1.12	1.59	1.04	4.71	1.34	2.04	1.18
time (sec)	N/A	0.043	0.266	0.119	0.275	4.333	2.108	1.718	1.795

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	118	136	126	254	318	115	-1
N.S.	1	1.00	0.79	0.91	0.84	1.69	2.12	0.77	-0.01
time (sec)	N/A	0.065	0.243	0.110	0.327	3.721	25.123	1.628	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	107	112	105	223	287	103	-1
N.S.	1	1.00	0.84	0.88	0.83	1.76	2.26	0.81	-0.01
time (sec)	N/A	0.041	0.272	0.108	0.279	4.505	7.869	0.889	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	101	92	86	205	223	89	-1
N.S.	1	1.00	0.98	0.89	0.83	1.99	2.17	0.86	-0.01
time (sec)	N/A	0.022	0.230	0.115	0.301	3.576	7.829	0.916	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	70	61	176	219	76	54
N.S.	1	1.00	1.00	0.80	0.70	2.02	2.52	0.87	0.62
time (sec)	N/A	0.018	0.257	0.111	0.277	1.983	3.683	1.130	1.182

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	110	109	88	439	218	100	83
N.S.	1	1.00	1.04	1.03	0.83	4.14	2.06	0.94	0.78
time (sec)	N/A	0.064	0.311	0.105	0.273	1.258	8.228	1.061	1.312

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	111	133	88	411	184	124	86
N.S.	1	1.00	1.03	1.23	0.81	3.81	1.70	1.15	0.80
time (sec)	N/A	0.059	0.298	0.132	0.290	1.312	3.249	0.879	1.877

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	109	157	112	425	182	191	91
N.S.	1	1.00	0.98	1.41	1.01	3.83	1.64	1.72	0.82
time (sec)	N/A	0.058	0.364	0.122	0.271	1.152	3.777	1.583	2.120

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	137	152	145	302	469	140	-1
N.S.	1	1.00	0.79	0.88	0.84	1.75	2.71	0.81	-0.01
time (sec)	N/A	0.075	0.331	0.112	0.283	1.260	136.142	1.472	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	125	128	124	271	442	128	-1
N.S.	1	1.00	0.83	0.85	0.83	1.81	2.95	0.85	-0.01
time (sec)	N/A	0.049	0.322	0.111	0.277	1.939	27.288	1.229	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	118	108	105	253	354	114	-1
N.S.	1	1.00	0.94	0.86	0.83	2.01	2.81	0.90	-0.01
time (sec)	N/A	0.031	0.313	0.122	0.280	1.802	27.214	0.750	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	107	86	77	224	348	101	54
N.S.	1	1.00	1.00	0.80	0.72	2.09	3.25	0.94	0.50
time (sec)	N/A	0.026	0.356	0.108	0.285	2.501	8.804	0.784	1.163

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	130	139	119	539	323	125	101
N.S.	1	1.00	0.98	1.05	0.90	4.08	2.45	0.95	0.77
time (sec)	N/A	0.085	0.437	0.108	0.273	1.948	15.362	0.643	1.247

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	133	163	120	519	318	150	104
N.S.	1	1.00	0.98	1.20	0.88	3.82	2.34	1.10	0.76
time (sec)	N/A	0.085	0.422	0.126	0.270	3.334	5.595	0.606	2.164

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	132	187	143	535	279	219	111
N.S.	1	1.00	0.94	1.33	1.01	3.79	1.98	1.55	0.79
time (sec)	N/A	0.081	0.502	0.127	0.288	4.246	6.212	0.756	2.591

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	77	101	88	158	150	74	-1
N.S.	1	1.00	0.74	0.97	0.85	1.52	1.44	0.71	-0.01
time (sec)	N/A	0.049	0.171	0.109	0.280	7.742	3.321	0.852	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	68	77	67	127	94	61	93
N.S.	1	1.00	0.84	0.95	0.83	1.57	1.16	0.75	1.15
time (sec)	N/A	0.028	0.181	0.115	0.276	10.872	1.923	0.961	1.467

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	58	56	47	109	70	50	82
N.S.	1	1.00	1.04	1.00	0.84	1.95	1.25	0.89	1.46
time (sec)	N/A	0.014	0.151	0.114	0.272	9.064	1.891	0.870	1.237

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	37	29	92	102	39	36
N.S.	1	1.00	1.07	0.86	0.67	2.14	2.37	0.91	0.84
time (sec)	N/A	0.009	0.141	0.111	0.285	6.043	0.569	1.214	1.144

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	66	52	33	273	99	58	42
N.S.	1	1.00	1.25	0.98	0.62	5.15	1.87	1.09	0.79
time (sec)	N/A	0.028	0.116	0.111	0.269	3.860	1.390	0.951	1.299

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	57	49	37	101	41	65	39
N.S.	1	1.00	1.21	1.04	0.79	2.15	0.87	1.38	0.83
time (sec)	N/A	0.024	0.126	0.116	0.274	3.902	1.171	1.031	1.201

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	65	69	56	123	66	146	58
N.S.	1	1.00	0.90	0.96	0.78	1.71	0.92	2.03	0.81
time (sec)	N/A	0.035	0.223	0.115	0.275	4.602	1.763	0.971	1.350

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	74	98	85	197	117	70	-1
N.S.	1	1.00	0.91	1.21	1.05	2.43	1.44	0.86	-0.01
time (sec)	N/A	0.030	0.261	0.125	0.286	7.083	4.512	1.392	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	61	74	64	164	83	58	61
N.S.	1	1.00	0.92	1.12	0.97	2.48	1.26	0.88	0.92
time (sec)	N/A	0.025	0.273	0.127	0.271	2.737	3.572	1.181	1.342

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	53	55	46	147	66	48	53
N.S.	1	1.00	1.10	1.15	0.96	3.06	1.38	1.00	1.10
time (sec)	N/A	0.015	0.253	0.113	0.280	1.017	3.068	0.682	1.055

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	32	31	35	46	23	24
N.S.	1	1.00	0.96	1.14	1.11	1.25	1.64	0.82	0.86
time (sec)	N/A	0.005	0.226	0.114	0.274	2.076	2.610	0.600	0.909

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	57	61	48	146	206	59	50
N.S.	1	1.00	1.21	1.30	1.02	3.11	4.38	1.26	1.06
time (sec)	N/A	0.028	0.216	0.122	0.269	1.169	3.584	0.683	1.291

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	82	68	169	235	96	70
N.S.	1	1.00	1.01	1.17	0.97	2.41	3.36	1.37	1.00
time (sec)	N/A	0.038	0.226	0.132	0.292	2.520	5.224	0.706	1.448

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	83	106	89	211	124	171	94
N.S.	1	1.00	0.87	1.12	0.94	2.22	1.31	1.80	0.99
time (sec)	N/A	0.058	0.336	0.128	0.321	2.397	4.250	0.783	1.592

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	72	97	102	239	400	70	-1
N.S.	1	1.00	0.91	1.23	1.29	3.03	5.06	0.89	-0.01
time (sec)	N/A	0.031	0.373	0.119	0.304	2.573	6.298	1.011	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	92	70	63	141	36	51
N.S.	1	1.00	0.83	1.74	1.32	1.19	2.66	0.68	0.96
time (sec)	N/A	0.014	0.385	0.119	0.288	3.135	5.195	1.034	0.967

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	47	32	72	51	49	95	26	34
N.S.	1	0.94	0.64	1.44	1.02	0.98	1.90	0.52	0.68
time (sec)	N/A	0.011	0.361	0.108	0.279	3.130	4.689	1.465	0.924

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	43	50	48	62	146	37	41
N.S.	1	1.00	0.84	0.98	0.94	1.22	2.86	0.73	0.80
time (sec)	N/A	0.007	0.362	0.113	0.277	3.529	4.400	0.978	0.929

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	79	98	80	239	840	82	80
N.S.	1	1.00	1.04	1.29	1.05	3.14	11.05	1.08	1.05
time (sec)	N/A	0.044	0.357	0.116	0.283	4.708	9.765	1.489	1.375

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	94	122	100	264	910	119	96
N.S.	1	1.00	0.90	1.17	0.96	2.54	8.75	1.14	0.92
time (sec)	N/A	0.060	0.400	0.153	0.282	6.191	8.491	1.163	1.579

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	102	146	122	307	1034	197	123
N.S.	1	1.00	0.79	1.13	0.95	2.38	8.02	1.53	0.95
time (sec)	N/A	0.079	0.474	0.152	0.281	9.968	8.430	0.932	1.621

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	39	29	28	31	24	19	20
N.S.	1	1.00	1.44	1.07	1.04	1.15	0.89	0.70	0.74
time (sec)	N/A	0.006	0.078	0.147	0.493	9.001	0.068	0.883	0.040

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	39	29	28	31	24	19	20
N.S.	1	1.00	1.44	1.07	1.04	1.15	0.89	0.70	0.74
time (sec)	N/A	0.012	0.003	0.118	0.507	4.146	0.068	0.708	0.033

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	33	15	22	26	27	30	14
N.S.	1	1.00	1.94	0.88	1.29	1.53	1.59	1.76	0.82
time (sec)	N/A	0.004	0.007	0.135	0.507	3.288	0.024	0.822	0.919

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.002	0.004	0.119	0.490	4.416	0.023	0.982	0.039

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	156	295	435	522	3806	204	-1
N.S.	1	1.00	0.73	1.38	2.04	2.45	17.87	0.96	-0.00
time (sec)	N/A	0.226	1.152	0.167	0.303	3.529	103.215	0.734	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	127	334	447	467	3448	138	-1
N.S.	1	1.00	0.85	2.23	2.98	3.11	22.99	0.92	-0.01
time (sec)	N/A	0.113	0.928	0.119	0.287	4.617	77.576	1.592	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	89	289	240	137	740	112	196
N.S.	1	1.00	0.67	2.19	1.82	1.04	5.61	0.85	1.48
time (sec)	N/A	0.115	0.962	0.119	0.282	2.926	82.037	1.050	1.274

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	79	327	253	122	575	81	186
N.S.	1	1.00	0.53	2.19	1.70	0.82	3.86	0.54	1.25
time (sec)	N/A	0.121	0.754	0.115	0.279	4.446	61.132	1.235	1.192

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	84	217	179	131	660	95	133
N.S.	1	1.00	0.60	1.56	1.29	0.94	4.75	0.68	0.96
time (sec)	N/A	0.102	0.903	0.118	0.298	3.045	50.278	1.095	1.138

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	87	255	197	134	904	94	133
N.S.	1	1.00	0.63	1.83	1.42	0.96	6.50	0.68	0.96
time (sec)	N/A	0.088	0.708	0.117	0.297	3.737	33.533	1.053	1.094

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	76	149	123	119	796	82	99
N.S.	1	1.00	0.64	1.25	1.03	1.00	6.69	0.69	0.83
time (sec)	N/A	0.057	0.823	0.115	0.284	3.221	22.654	0.935	1.049

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	92	189	153	137	1880	112	115
N.S.	1	1.00	0.72	1.49	1.20	1.08	14.80	0.88	0.91
time (sec)	N/A	0.047	0.641	0.115	0.282	3.196	26.836	0.989	1.033

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	130	193	157	465	6613	152	159
N.S.	1	1.00	0.94	1.40	1.14	3.37	47.92	1.10	1.15
time (sec)	N/A	0.105	0.967	0.122	0.321	2.301	36.946	1.678	1.620

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	168	277	228	525	6922	239	225
N.S.	1	1.00	0.89	1.47	1.21	2.79	36.82	1.27	1.20
time (sec)	N/A	0.253	0.954	0.164	0.313	1.398	47.110	1.468	2.095

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	173	327	265	688	11198	325	279
N.S.	1	1.00	0.79	1.49	1.21	3.14	51.13	1.48	1.27
time (sec)	N/A	0.327	1.227	0.160	0.281	1.617	66.598	1.383	2.518

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	43	0	0	0	97	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	2.16	0.00	-0.02
time (sec)	N/A	0.013	0.008	0.025	0.000	0.000	0.543	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	0	0	0	192	0	-1
N.S.	1	1.00	0.90	0.00	0.00	0.00	2.11	0.00	-0.01
time (sec)	N/A	0.029	0.055	0.023	0.000	0.000	1.834	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	56	0	0	0	204	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	2.68	0.00	-0.01
time (sec)	N/A	0.026	0.078	0.023	0.000	0.000	1.951	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	99	0	0	0	298	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	2.46	0.00	-0.01
time (sec)	N/A	0.082	0.123	0.024	0.000	0.000	2.588	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	53	60	57	57
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.92	0.88	0.88
time (sec)	N/A	0.052	0.012	0.054	0.267	3.349	0.009	1.200	1.205

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	53	60	57	57
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.92	0.88	0.88
time (sec)	N/A	0.053	0.012	0.059	0.266	3.533	0.009	1.118	1.184

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	53	60	57	57
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.92	0.88	0.88
time (sec)	N/A	0.044	0.007	0.053	0.276	5.125	0.009	1.554	1.185

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	56	54	54
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.93	0.90	0.90
time (sec)	N/A	0.029	0.007	0.048	0.274	4.940	0.008	1.198	1.164

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	48	48	54	53	52
N.S.	1	1.00	1.00	0.95	0.86	0.86	0.96	0.95	0.93
time (sec)	N/A	0.026	0.011	0.013	0.265	3.974	0.048	1.512	1.170

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	50	48	55	49	50	49
N.S.	1	1.00	1.00	0.93	0.89	1.02	0.91	0.93	0.91
time (sec)	N/A	0.034	0.024	0.015	0.274	3.616	0.059	0.963	1.145

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	51	48	48	55	51	48	47
N.S.	1	1.00	0.94	0.89	0.89	1.02	0.94	0.89	0.87
time (sec)	N/A	0.036	0.019	0.014	0.268	8.285	0.132	1.022	1.138

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	55	49	49	55	54	50	50
N.S.	1	1.00	1.02	0.91	0.91	1.02	1.00	0.93	0.93
time (sec)	N/A	0.034	0.013	0.014	0.268	6.770	0.383	1.201	1.148

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	98	102	101	101	110	105	108
N.S.	1	1.00	0.90	0.94	0.93	0.93	1.01	0.96	0.99
time (sec)	N/A	0.089	0.027	0.102	0.268	3.352	0.015	0.909	1.130

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	92	102	101	101	110	105	108
N.S.	1	1.00	0.84	0.94	0.93	0.93	1.01	0.96	0.99
time (sec)	N/A	0.077	0.036	0.102	0.277	6.000	0.015	0.874	1.111

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	92	102	101	101	110	105	107
N.S.	1	1.00	0.88	0.98	0.97	0.97	1.06	1.01	1.03
time (sec)	N/A	0.051	0.028	0.102	0.304	3.194	0.016	1.160	1.111

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	88	99	98	98	107	102	105
N.S.	1	1.00	0.89	1.00	0.99	0.99	1.08	1.03	1.06
time (sec)	N/A	0.051	0.027	0.105	0.281	5.332	0.015	0.944	1.106

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	88	100	96	96	104	100	103
N.S.	1	1.00	0.96	1.09	1.04	1.04	1.13	1.09	1.12
time (sec)	N/A	0.047	0.031	0.105	0.271	6.106	0.083	1.014	1.106

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	88	98	96	103	99	98	92
N.S.	1	1.00	0.98	1.09	1.07	1.14	1.10	1.09	1.02
time (sec)	N/A	0.057	0.037	0.108	0.267	4.418	0.089	1.125	1.107

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	87	95	96	103	100	97	103
N.S.	1	1.00	0.89	0.97	0.98	1.05	1.02	0.99	1.05
time (sec)	N/A	0.059	0.026	0.112	0.270	2.305	0.173	1.539	1.106

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	83	92	97	103	100	97	106
N.S.	1	1.00	0.85	0.94	0.99	1.05	1.02	0.99	1.08
time (sec)	N/A	0.060	0.036	0.112	0.277	1.035	0.461	1.212	1.275

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	149	150	145	145	163	153	153
N.S.	1	1.00	1.00	1.01	0.97	0.97	1.09	1.03	1.03
time (sec)	N/A	0.127	0.019	0.104	0.268	1.423	0.019	1.634	1.299

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	125	150	145	145	165	153	153
N.S.	1	1.00	0.84	1.01	0.97	0.97	1.11	1.03	1.03
time (sec)	N/A	0.094	0.043	0.115	0.276	3.228	0.018	1.384	1.275

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	124	150	145	145	163	153	153
N.S.	1	1.00	0.90	1.09	1.05	1.05	1.18	1.11	1.11
time (sec)	N/A	0.069	0.037	0.101	0.315	2.799	0.019	0.947	1.278

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	121	147	142	142	158	149	149
N.S.	1	1.00	0.91	1.11	1.07	1.07	1.19	1.12	1.12
time (sec)	N/A	0.067	0.034	0.100	0.276	3.080	0.018	0.999	1.259

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	121	148	140	140	158	148	147
N.S.	1	1.00	0.94	1.15	1.09	1.09	1.22	1.15	1.14
time (sec)	N/A	0.063	0.044	0.103	0.270	2.454	0.112	0.660	1.264

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	123	145	139	147	150	145	121
N.S.	1	1.00	0.99	1.17	1.12	1.19	1.21	1.17	0.98
time (sec)	N/A	0.077	0.055	0.108	0.282	4.506	0.121	0.949	1.184

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	124	142	139	147	151	144	143
N.S.	1	1.00	0.92	1.05	1.03	1.09	1.12	1.07	1.06
time (sec)	N/A	0.075	0.043	0.099	0.273	3.622	0.206	0.682	1.257

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	124	141	142	147	155	146	148
N.S.	1	1.00	0.89	1.01	1.02	1.06	1.12	1.05	1.06
time (sec)	N/A	0.082	0.035	0.100	0.267	4.221	0.498	0.682	1.368

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	130	141	145	332	316	161	-1
N.S.	1	1.00	0.86	0.93	0.96	2.20	2.09	1.07	-0.01
time (sec)	N/A	0.102	0.052	0.103	0.509	3.151	0.542	1.704	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	114	128	127	270	274	137	-1
N.S.	1	1.00	0.88	0.98	0.98	2.08	2.11	1.05	-0.01
time (sec)	N/A	0.087	0.069	0.100	0.501	7.658	0.531	0.766	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	95	95	98	238	245	112	-1
N.S.	1	1.00	0.86	0.86	0.88	2.14	2.21	1.01	-0.01
time (sec)	N/A	0.077	0.040	0.100	0.499	8.502	0.478	1.140	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	81	85	82	180	211	88	-1
N.S.	1	1.00	0.88	0.92	0.89	1.96	2.29	0.96	-0.01
time (sec)	N/A	0.057	0.049	0.103	0.515	5.069	0.460	1.799	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	68	65	64	157	219	66	79
N.S.	1	1.00	0.93	0.89	0.88	2.15	3.00	0.90	1.08
time (sec)	N/A	0.047	0.030	0.097	0.501	6.743	0.423	1.021	1.425

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	73	73	65	158	0	66	-1
N.S.	1	1.00	1.01	1.01	0.90	2.19	0.00	0.92	-0.01
time (sec)	N/A	0.068	0.038	0.102	0.485	6.232	0.000	0.796	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	75	67	67	165	0	68	78
N.S.	1	1.00	0.99	0.88	0.88	2.17	0.00	0.89	1.03
time (sec)	N/A	0.067	0.034	0.098	0.482	9.496	0.000	0.694	1.212

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	84	89	76	205	0	80	97
N.S.	1	1.00	0.91	0.97	0.83	2.23	0.00	0.87	1.05
time (sec)	N/A	0.074	0.057	0.107	0.478	3.186	0.000	0.918	1.304

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	139	134	150	468	335	159	-1
N.S.	1	1.00	0.79	0.76	0.85	2.66	1.90	0.90	-0.01
time (sec)	N/A	0.188	0.090	0.109	0.503	2.386	1.925	0.868	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	128	124	127	372	289	131	-1
N.S.	1	1.00	0.83	0.81	0.82	2.42	1.88	0.85	-0.01
time (sec)	N/A	0.170	0.057	0.106	0.513	3.627	1.823	0.765	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	100	103	108	357	284	111	152
N.S.	1	1.00	0.75	0.77	0.81	2.66	2.12	0.83	1.13
time (sec)	N/A	0.159	0.056	0.099	0.510	4.275	1.703	0.762	1.292

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	92	78	84	287	212	81	-1
N.S.	1	1.00	0.91	0.77	0.83	2.84	2.10	0.80	-0.01
time (sec)	N/A	0.080	0.036	0.104	0.499	2.155	1.358	0.629	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	83	88	89	257	233	88	110
N.S.	1	1.00	0.89	0.95	0.96	2.76	2.51	0.95	1.18
time (sec)	N/A	0.042	0.057	0.103	0.556	3.571	1.062	1.669	1.316

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	85	99	87	296	0	93	-1
N.S.	1	1.00	0.89	1.04	0.92	3.12	0.00	0.98	-0.01
time (sec)	N/A	0.081	0.051	0.108	0.552	5.369	0.000	1.357	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	110	96	105	336	0	103	133
N.S.	1	1.00	1.00	0.87	0.95	3.05	0.00	0.94	1.21
time (sec)	N/A	0.100	0.048	0.104	0.523	6.390	0.000	1.170	1.410

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	112	127	117	441	0	126	158
N.S.	1	1.00	0.83	0.94	0.87	3.27	0.00	0.93	1.17
time (sec)	N/A	0.141	0.071	0.105	0.564	7.440	0.000	1.531	1.348

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	139	140	165	574	357	157	232
N.S.	1	1.00	0.75	0.76	0.89	3.10	1.93	0.85	1.25
time (sec)	N/A	0.240	0.079	0.098	0.591	16.492	97.099	1.258	1.556

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	126	115	136	480	282	122	-1
N.S.	1	1.00	0.81	0.74	0.88	3.10	1.82	0.79	-0.01
time (sec)	N/A	0.162	0.054	0.109	0.538	8.010	86.672	1.354	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	122	123	146	447	304	128	195
N.S.	1	1.00	0.90	0.90	1.07	3.29	2.24	0.94	1.43
time (sec)	N/A	0.114	0.066	0.108	0.527	4.874	74.147	1.126	1.391

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	99	97	111	357	178	97	-1
N.S.	1	1.00	0.83	0.82	0.93	3.00	1.50	0.82	-0.01
time (sec)	N/A	0.080	0.079	0.103	0.545	2.519	8.488	1.881	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	104	98	122	346	184	106	163
N.S.	1	1.00	0.90	0.84	1.05	2.98	1.59	0.91	1.41
time (sec)	N/A	0.050	0.079	0.135	0.519	5.312	3.771	1.596	1.327

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	117	130	133	488	0	128	-1
N.S.	1	1.00	0.90	1.00	1.02	3.75	0.00	0.98	-0.01
time (sec)	N/A	0.093	0.075	0.102	0.504	3.159	0.000	1.052	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	141	125	152	524	0	141	202
N.S.	1	1.00	0.98	0.87	1.06	3.64	0.00	0.98	1.40
time (sec)	N/A	0.154	0.068	0.108	0.512	5.993	0.000	1.691	1.400

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	147	169	172	696	0	162	229
N.S.	1	1.00	0.84	0.97	0.99	4.00	0.00	0.93	1.32
time (sec)	N/A	0.212	0.107	0.103	0.505	3.871	0.000	1.134	1.456

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	18	24	15	25	20
N.S.	1	1.00	1.00	0.95	0.90	1.20	0.75	1.25	1.00
time (sec)	N/A	0.015	0.007	0.122	0.290	2.038	0.027	1.612	0.912

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	18	23	22	14	22	17	14
N.S.	1	1.00	0.78	1.00	0.96	0.61	0.96	0.74	0.61
time (sec)	N/A	0.014	0.014	0.135	0.293	1.357	0.097	2.831	0.102

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	20	20	20	20	20
N.S.	1	1.00	1.00	0.84	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.015	0.010	0.123	0.484	1.780	0.026	2.238	0.038

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	22	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.73	0.80	0.80
time (sec)	N/A	0.017	0.005	0.125	0.496	1.254	0.029	1.959	0.035

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	210	233	219	482	384	250	289
N.S.	1	1.00	1.00	1.11	1.04	2.30	1.83	1.19	1.38
time (sec)	N/A	0.104	0.103	0.153	0.507	3.800	0.462	1.071	0.929

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	162	185	177	392	337	200	243
N.S.	1	1.00	0.94	1.08	1.03	2.28	1.96	1.16	1.41
time (sec)	N/A	0.082	0.080	0.147	0.510	2.869	0.428	0.958	0.944

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	128	137	137	304	185	152	193
N.S.	1	1.00	0.94	1.01	1.01	2.24	1.36	1.12	1.42
time (sec)	N/A	0.071	0.065	0.138	0.530	3.652	0.402	1.546	0.912

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	98	94	97	246	160	106	96
N.S.	1	1.00	0.98	0.94	0.97	2.46	1.60	1.06	0.96
time (sec)	N/A	0.041	0.057	0.142	0.520	4.571	0.362	1.187	0.936

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	83	79	82	223	150	86	76
N.S.	1	1.00	0.99	0.94	0.98	2.65	1.79	1.02	0.90
time (sec)	N/A	0.064	0.046	0.139	0.523	3.529	0.511	0.770	1.067

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	83	79	80	228	151	81	80
N.S.	1	1.00	1.01	0.96	0.98	2.78	1.84	0.99	0.98
time (sec)	N/A	0.061	0.059	0.128	0.497	6.338	1.052	1.064	0.111

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	103	94	99	268	167	105	94
N.S.	1	1.00	0.99	0.90	0.95	2.58	1.61	1.01	0.90
time (sec)	N/A	0.069	0.060	0.155	0.510	9.331	2.525	1.306	1.203

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	139	129	137	326	301	151	127
N.S.	1	1.00	1.01	0.94	1.00	2.38	2.20	1.10	0.93
time (sec)	N/A	0.086	0.083	0.130	0.505	5.801	6.658	1.746	0.981

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	174	163	179	414	354	201	161
N.S.	1	1.00	0.99	0.93	1.02	2.37	2.02	1.15	0.92
time (sec)	N/A	0.097	0.101	0.130	0.492	6.661	25.299	1.551	1.018

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	211	201	219	504	398	249	197
N.S.	1	1.00	1.00	0.95	1.04	2.39	1.89	1.18	0.93
time (sec)	N/A	0.117	0.118	0.127	0.499	4.181	40.337	3.081	0.988

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	227	229	234	600	444	252	413
N.S.	1	1.00	0.95	0.95	0.98	2.50	1.85	1.05	1.72
time (sec)	N/A	0.194	0.087	0.148	0.519	5.446	1.206	1.670	0.099

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	187	182	189	502	257	201	288
N.S.	1	1.00	0.93	0.90	0.94	2.49	1.27	1.00	1.43
time (sec)	N/A	0.162	0.071	0.151	0.520	4.249	1.138	3.475	0.966

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	148	139	145	438	221	152	153
N.S.	1	1.00	0.91	0.85	0.89	2.69	1.36	0.93	0.94
time (sec)	N/A	0.155	0.059	0.145	0.520	3.544	1.051	1.633	1.002

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	122	114	121	380	201	126	113
N.S.	1	1.00	1.03	0.97	1.03	3.22	1.70	1.07	0.96
time (sec)	N/A	0.084	0.061	0.164	0.521	1.769	0.853	2.605	0.100

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	115	107	119	373	197	122	112
N.S.	1	1.00	1.03	0.96	1.06	3.33	1.76	1.09	1.00
time (sec)	N/A	0.093	0.044	0.142	0.493	1.512	2.255	1.782	0.996

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	125	116	132	404	212	123	119
N.S.	1	1.00	1.03	0.96	1.09	3.34	1.75	1.02	0.98
time (sec)	N/A	0.106	0.052	0.132	0.512	1.452	5.869	3.140	0.131

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	151	137	154	470	226	151	145
N.S.	1	1.00	0.99	0.90	1.01	3.09	1.49	0.99	0.95
time (sec)	N/A	0.146	0.057	0.124	0.491	2.664	20.555	1.942	0.999

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	190	174	198	524	0	201	181
N.S.	1	1.00	1.01	0.92	1.05	2.77	0.00	1.06	0.96
time (sec)	N/A	0.203	0.071	0.132	0.527	2.617	0.000	1.414	0.987

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	230	210	243	622	0	252	219
N.S.	1	1.00	1.00	0.91	1.06	2.70	0.00	1.10	0.95
time (sec)	N/A	0.249	0.081	0.156	0.516	3.863	0.000	1.260	1.010

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	272	268	289	798	503	301	506
N.S.	1	1.00	0.95	0.93	1.01	2.78	1.75	1.05	1.76
time (sec)	N/A	0.332	0.108	0.155	0.532	4.228	19.714	2.178	0.999

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	232	219	244	700	316	250	348
N.S.	1	1.00	0.94	0.89	0.99	2.83	1.28	1.01	1.41
time (sec)	N/A	0.266	0.099	0.158	0.519	1.717	18.402	1.987	0.107

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	176	177	199	636	280	200	206
N.S.	1	1.00	0.85	0.86	0.96	3.07	1.35	0.97	1.00
time (sec)	N/A	0.222	0.115	0.142	0.496	2.850	16.436	1.531	0.954

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	156	151	174	579	260	173	163
N.S.	1	1.00	0.93	0.90	1.04	3.47	1.56	1.04	0.98
time (sec)	N/A	0.173	0.086	0.147	0.517	2.114	6.286	2.067	1.018

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	141	139	157	531	243	149	148
N.S.	1	1.00	0.96	0.95	1.07	3.61	1.65	1.01	1.01
time (sec)	N/A	0.103	0.081	0.144	0.523	0.806	3.827	1.485	1.047

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	155	140	164	553	250	153	149
N.S.	1	1.00	1.01	0.92	1.07	3.61	1.63	1.00	0.97
time (sec)	N/A	0.121	0.086	0.154	0.519	1.624	12.305	1.135	1.094

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	169	152	184	606	0	170	166
N.S.	1	1.00	1.01	0.90	1.10	3.61	0.00	1.01	0.99
time (sec)	N/A	0.161	0.098	0.157	0.498	3.828	0.000	1.679	1.027

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	196	176	206	668	0	198	192
N.S.	1	1.00	1.00	0.90	1.05	3.41	0.00	1.01	0.98
time (sec)	N/A	0.238	0.079	0.159	0.554	5.346	0.000	1.412	1.043

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	234	212	252	726	0	250	230
N.S.	1	1.00	1.00	0.91	1.08	3.10	0.00	1.07	0.98
time (sec)	N/A	0.326	0.095	0.145	0.532	4.153	0.000	1.800	1.046

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	276	248	297	820	0	301	268
N.S.	1	1.00	1.00	0.90	1.07	2.96	0.00	1.09	0.97
time (sec)	N/A	0.412	0.105	0.170	0.496	6.084	0.000	2.045	1.070

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	158	382	352	186	442	264	186
N.S.	1	1.00	0.74	1.79	1.64	0.87	2.07	1.23	0.87
time (sec)	N/A	0.175	0.135	0.118	0.288	2.983	0.564	1.605	1.191

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	122	286	267	141	340	197	146
N.S.	1	1.00	0.73	1.71	1.60	0.84	2.04	1.18	0.87
time (sec)	N/A	0.132	0.104	0.113	0.268	5.722	0.454	1.764	1.107

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	89	193	183	99	238	130	103
N.S.	1	1.00	0.74	1.60	1.51	0.82	1.97	1.07	0.85
time (sec)	N/A	0.094	0.067	0.114	0.298	4.030	0.356	1.519	1.064

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	86	139	124	211	102	127	99
N.S.	1	1.00	0.83	1.35	1.20	2.05	0.99	1.23	0.96
time (sec)	N/A	0.095	0.116	0.110	0.303	2.820	15.195	1.744	1.810

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	92	129	105	212	138	114	99
N.S.	1	1.00	0.92	1.29	1.05	2.12	1.38	1.14	0.99
time (sec)	N/A	0.137	0.195	0.129	0.314	2.482	29.206	1.794	1.947

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	102	167	129	233	194	141	133
N.S.	1	1.00	0.89	1.46	1.13	2.04	1.70	1.24	1.17
time (sec)	N/A	0.165	0.212	0.131	0.289	1.242	60.983	2.098	2.191

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	126	250	195	281	303	232	199
N.S.	1	1.00	0.86	1.71	1.34	1.92	2.08	1.59	1.36
time (sec)	N/A	0.199	0.239	0.129	0.290	1.153	85.984	1.081	2.543

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	159	342	278	371	444	361	277
N.S.	1	1.00	0.82	1.75	1.43	1.90	2.28	1.85	1.42
time (sec)	N/A	0.240	0.381	0.135	0.285	1.078	166.551	1.108	2.913

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	186	402	344	432	0	224	-1
N.S.	1	1.00	0.76	1.64	1.40	1.76	0.00	0.91	-0.00
time (sec)	N/A	0.178	0.341	0.120	0.300	1.201	0.000	0.867	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	151	306	259	343	444	175	-1
N.S.	1	1.00	0.78	1.58	1.34	1.77	2.29	0.90	-0.01
time (sec)	N/A	0.147	0.275	0.118	0.293	2.732	47.777	1.405	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	121	215	177	260	362	129	-1
N.S.	1	1.00	0.83	1.48	1.22	1.79	2.50	0.89	-0.01
time (sec)	N/A	0.084	0.158	0.120	0.270	2.436	11.338	1.254	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	105	145	120	229	250	121	-1
N.S.	1	1.00	0.90	1.24	1.03	1.96	2.14	1.03	-0.01
time (sec)	N/A	0.096	0.208	0.131	0.272	3.301	4.230	1.274	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	95	119	103	215	197	176	143
N.S.	1	1.00	0.86	1.08	0.94	1.95	1.79	1.60	1.30
time (sec)	N/A	0.088	0.205	0.125	0.280	4.367	2.228	1.210	2.199

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	98	141	129	231	456	324	105
N.S.	1	1.00	0.83	1.19	1.09	1.96	3.86	2.75	0.89
time (sec)	N/A	0.092	0.238	0.124	0.264	2.531	1.696	1.112	1.724

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	103	206	195	109	891	554	124
N.S.	1	1.00	0.74	1.47	1.39	0.78	6.36	3.96	0.89
time (sec)	N/A	0.126	0.221	0.131	0.280	3.303	2.306	1.004	1.280

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	134	298	278	152	1642	667	171
N.S.	1	1.00	0.71	1.58	1.47	0.80	8.69	3.53	0.90
time (sec)	N/A	0.175	0.304	0.125	0.274	7.509	3.067	1.217	1.282

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	251	562	1221	987	0	342	-1
N.S.	1	1.00	0.66	1.48	3.20	2.59	0.00	0.90	-0.00
time (sec)	N/A	0.460	1.084	0.390	0.317	6.517	0.000	1.172	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	207	531	986	816	9649	265	-1
N.S.	1	1.00	0.74	1.90	3.53	2.92	34.58	0.95	-0.00
time (sec)	N/A	0.311	0.889	0.115	0.312	5.323	211.751	1.200	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	173	500	753	653	6467	203	-1
N.S.	1	1.00	0.82	2.38	3.59	3.11	30.80	0.97	-0.00
time (sec)	N/A	0.271	0.660	0.106	0.319	5.553	98.692	2.449	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	192	147	469	533	491	3803	160	-1
N.S.	1	1.07	0.82	2.62	2.98	2.74	21.25	0.89	-0.01
time (sec)	N/A	0.219	0.614	0.106	0.305	4.313	69.638	1.459	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	98	440	335	141	2088	131	-1
N.S.	1	1.00	0.73	3.28	2.50	1.05	15.58	0.98	-0.01
time (sec)	N/A	0.146	0.425	0.109	0.309	4.098	51.257	0.895	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	179	133	394	313	182	2392	211	-1
N.S.	1	0.97	0.72	2.13	1.69	0.98	12.93	1.14	-0.01
time (sec)	N/A	0.174	0.487	0.107	0.314	4.234	82.954	1.070	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	165	396	337	225	2861	349	-1
N.S.	1	1.00	0.68	1.64	1.39	0.93	11.82	1.44	-0.00
time (sec)	N/A	0.220	0.528	0.191	0.289	4.312	128.926	1.407	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	275	202	446	398	270	3313	592	405
N.S.	1	0.98	0.72	1.59	1.42	0.96	11.79	2.11	1.44
time (sec)	N/A	0.282	0.587	0.268	0.293	5.528	202.100	2.563	2.397

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	234	542	489	311	0	938	421
N.S.	1	1.00	0.70	1.62	1.46	0.93	0.00	2.81	1.26
time (sec)	N/A	0.323	0.646	0.291	0.294	4.650	0.000	0.980	2.836

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	380	270	638	579	354	0	1162	-1
N.S.	1	0.97	0.69	1.63	1.48	0.90	0.00	2.96	-0.00
time (sec)	N/A	0.369	0.740	0.359	0.305	3.527	0.000	0.994	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	158	382	352	186	442	264	186
N.S.	1	1.00	0.74	1.79	1.64	0.87	2.07	1.23	0.87
time (sec)	N/A	0.152	0.034	0.145	0.380	1.302	0.563	0.936	1.203

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	122	286	267	141	340	197	146
N.S.	1	1.00	0.73	1.71	1.60	0.84	2.04	1.18	0.87
time (sec)	N/A	0.125	0.024	0.108	0.376	1.763	0.444	1.363	1.140

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	89	193	183	99	238	130	103
N.S.	1	1.00	0.74	1.60	1.51	0.82	1.97	1.07	0.85
time (sec)	N/A	0.097	0.019	0.116	0.386	0.804	0.361	1.335	1.084

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	257	200	597	826	705	6987	224	-1
N.S.	1	0.98	0.77	2.29	3.16	2.70	26.77	0.86	-0.00
time (sec)	N/A	0.501	0.713	0.105	0.472	2.814	113.642	1.395	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	250	176	544	597	567	5071	204	-1
N.S.	1	1.17	0.82	2.54	2.79	2.65	23.70	0.95	-0.00
time (sec)	N/A	0.290	0.674	0.113	0.448	3.576	78.314	1.743	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	138	539	421	187	2490	220	-1
N.S.	1	1.00	0.72	2.79	2.18	0.97	12.90	1.14	-0.01
time (sec)	N/A	0.240	0.524	0.101	0.430	3.319	106.661	1.901	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [172] had the largest ratio of [37]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.00	20	0.250
2	A	5	5	1.00	20	0.250
3	A	4	4	1.00	18	0.222
4	A	4	4	1.00	17	0.235
5	A	7	7	1.00	20	0.350
6	A	7	7	1.00	20	0.350
7	A	7	7	1.00	20	0.350
8	A	7	5	1.00	20	0.250
9	A	6	5	1.00	20	0.250
10	A	5	4	1.00	18	0.222
11	A	5	4	1.00	17	0.235
12	A	8	7	1.00	20	0.350
13	A	8	8	1.00	20	0.400
14	A	8	7	1.00	20	0.350
15	A	8	5	1.00	20	0.250
16	A	7	5	1.00	20	0.250
17	A	6	4	1.00	18	0.222
18	A	6	4	1.00	17	0.235
19	A	9	7	1.00	20	0.350
20	A	9	8	1.00	20	0.400
21	A	9	8	1.00	20	0.400
22	A	5	4	1.00	20	0.200
23	A	4	4	1.00	20	0.200
24	A	3	3	1.00	18	0.167
25	A	3	3	1.00	17	0.176

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	6	6	1.00	20	0.300
27	A	4	4	1.00	20	0.200
28	A	5	5	1.00	20	0.250
29	A	4	4	1.00	20	0.200
30	A	4	4	1.00	20	0.200
31	A	3	3	1.00	18	0.167
32	A	1	1	1.00	17	0.059
33	A	5	5	1.00	20	0.250
34	A	5	5	1.00	20	0.250
35	A	6	6	1.00	20	0.300
36	A	4	4	1.00	20	0.200
37	A	2	2	1.00	20	0.100
38	A	2	2	0.94	18	0.111
39	A	2	2	1.00	17	0.118
40	A	6	5	1.00	20	0.250
41	A	6	5	1.00	20	0.250
42	A	7	6	1.00	20	0.300
43	A	2	2	1.00	18	0.111
44	A	3	3	1.00	19	0.158
45	A	2	2	1.00	13	0.154
46	A	2	2	1.00	13	0.154
47	A	7	5	1.00	25	0.200
48	A	6	5	1.00	25	0.200
49	A	5	4	1.00	25	0.160
50	A	4	4	1.00	25	0.160
51	A	4	4	1.00	25	0.160
52	A	4	4	1.00	25	0.160
53	A	4	4	1.00	23	0.174
54	A	5	4	1.00	22	0.182
55	A	8	6	1.00	25	0.240
56	A	8	5	1.00	25	0.200
57	A	9	6	1.00	25	0.240
58	A	2	2	1.00	16	0.125
59	A	3	2	1.00	20	0.100
60	A	2	2	1.00	22	0.091

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	5	3	1.00	25	0.120
62	A	2	1	1.00	26	0.038
63	A	2	1	1.00	26	0.038
64	A	2	1	1.00	24	0.042
65	A	2	1	1.00	23	0.043
66	A	2	1	1.00	26	0.038
67	A	2	1	1.00	26	0.038
68	A	2	1	1.00	26	0.038
69	A	2	1	1.00	26	0.038
70	A	2	1	1.00	28	0.036
71	A	2	1	1.00	28	0.036
72	A	3	2	1.00	26	0.077
73	A	3	2	1.00	25	0.080
74	A	3	2	1.00	28	0.071
75	A	3	2	1.00	28	0.071
76	A	2	1	1.00	28	0.036
77	A	2	1	1.00	28	0.036
78	A	2	1	1.00	28	0.036
79	A	2	1	1.00	28	0.036
80	A	3	2	1.00	26	0.077
81	A	3	2	1.00	25	0.080
82	A	3	2	1.00	28	0.071
83	A	3	2	1.00	28	0.071
84	A	2	1	1.00	28	0.036
85	A	2	1	1.00	28	0.036
86	A	5	4	1.00	28	0.143
87	A	5	4	1.00	28	0.143
88	A	5	4	1.00	28	0.143
89	A	5	4	1.00	26	0.154
90	A	5	4	1.00	25	0.160
91	A	5	4	1.00	28	0.143
92	A	5	4	1.00	28	0.143
93	A	5	4	1.00	28	0.143
94	A	6	5	1.00	28	0.179
95	A	6	5	1.00	28	0.179

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	6	5	1.00	28	0.179
97	A	6	5	1.00	26	0.192
98	A	4	4	1.00	25	0.160
99	A	6	5	1.00	28	0.179
100	A	6	5	1.00	28	0.179
101	A	6	5	1.00	28	0.179
102	A	7	5	1.00	28	0.179
103	A	6	5	1.00	28	0.179
104	A	5	4	1.00	28	0.143
105	A	4	4	1.00	26	0.154
106	A	3	3	1.00	25	0.120
107	A	7	6	1.00	28	0.214
108	A	7	5	1.00	28	0.179
109	A	7	5	1.00	28	0.179
110	A	4	3	1.00	17	0.176
111	A	4	3	1.00	17	0.176
112	A	4	4	1.00	21	0.190
113	A	6	5	1.00	15	0.333
114	A	3	2	1.00	30	0.067
115	A	3	2	1.00	30	0.067
116	A	3	2	1.00	30	0.067
117	A	3	2	1.00	27	0.074
118	A	3	2	1.00	30	0.067
119	A	3	2	1.00	30	0.067
120	A	3	2	1.00	30	0.067
121	A	3	2	1.00	30	0.067
122	A	3	2	1.00	30	0.067
123	A	3	2	1.00	30	0.067
124	A	5	4	1.00	30	0.133
125	A	5	4	1.00	30	0.133
126	A	5	4	1.00	30	0.133
127	A	4	3	1.00	27	0.111
128	A	4	3	1.00	30	0.100
129	A	4	3	1.00	30	0.100
130	A	4	3	1.00	30	0.100

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	4	3	1.00	30	0.100
132	A	4	3	1.00	30	0.100
133	A	6	5	1.00	30	0.167
134	A	6	5	1.00	30	0.167
135	A	6	5	1.00	30	0.167
136	A	6	5	1.00	30	0.167
137	A	4	4	1.00	27	0.148
138	A	4	4	1.00	30	0.133
139	A	5	4	1.00	30	0.133
140	A	5	3	1.00	30	0.100
141	A	5	3	1.00	30	0.100
142	A	5	3	1.00	30	0.100
143	A	3	2	1.00	32	0.062
144	A	3	2	1.00	32	0.062
145	A	3	2	1.00	30	0.067
146	A	5	4	1.00	32	0.125
147	A	6	5	1.00	32	0.156
148	A	6	6	1.00	32	0.188
149	A	6	6	1.00	32	0.188
150	A	7	7	1.00	32	0.219
151	A	7	6	1.00	32	0.188
152	A	6	6	1.00	32	0.188
153	A	5	5	1.00	29	0.172
154	A	6	6	1.00	32	0.188
155	A	6	6	1.00	32	0.188
156	A	6	6	1.00	32	0.188
157	A	5	3	1.00	32	0.094
158	A	6	4	1.00	32	0.125
159	A	11	9	1.00	32	0.281
160	A	10	9	1.00	32	0.281
161	A	9	9	1.00	32	0.281
162	A	8	8	1.07	32	0.250
163	A	5	4	1.00	29	0.138
164	A	6	5	0.97	32	0.156
165	A	7	5	1.00	32	0.156

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	8	4	0.98	32	0.125
167	A	9	5	1.00	32	0.156
168	A	10	5	0.97	32	0.156
169	A	4	3	1.00	33	0.091
170	A	4	3	1.00	33	0.091
171	A	4	3	1.00	31	0.097
172	A	10	9	0.98	37	0.243
173	A	6	5	1.17	34	0.147
174	A	6	4	1.00	37	0.108

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^3(A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=127

$$\frac{a^2Bx\sqrt{a+bx^2}}{16b^2}+\frac{Ax^2(a+bx^2)^{3/2}}{5b}+\frac{Bx^3(a+bx^2)^{3/2}}{6b}-\frac{a(16A+15Bx)\,(a+bx^2)^{3/2}}{120b^2}+\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+b}}\right.}{16b^{5/2}}$$

```
[Out] 1/5*A*x^2*(b*x^2+a)^(3/2)/b+1/6*B*x^3*(b*x^2+a)^(3/2)/b-1/120*a*(15*B*x+16*
A)*(b*x^2+a)^(3/2)/b^2+1/16*a^3*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2
)+1/16*a^2*B*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A]
time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {847, 794, 201, 223, 212}

$$\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}+\frac{a^2Bx\sqrt{a+bx^2}}{16b^2}-\frac{a(a+bx^2)^{3/2}\,(16A+15Bx)}{120b^2}+\frac{Ax^2(a+bx^2)^{3/2}}{5b}+\frac{Bx^3(a+bx^2)^{3/2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*B*x*Sqrt[a + b*x^2])/(16*b^2) + (A*x^2*(a + b*x^2)^(3/2))/(5*b) + (B*x
^3*(a + b*x^2)^(3/2))/(6*b) - (a*(16*A + 15*B*x)*(a + b*x^2)^(3/2))/(120*b^
2) + (a^3*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3(A+Bx)\sqrt{a+bx^2}\,dx &= \frac{Bx^3(a+bx^2)^{3/2}}{6b}+\frac{\int x^2(-3aB+6Abx)\sqrt{a+bx^2}\,dx}{6b}\\
&= \frac{Ax^2(a+bx^2)^{3/2}}{5b}+\frac{Bx^3(a+bx^2)^{3/2}}{6b}+\frac{\int x(-12aAb-15abBx)\sqrt{a+bx^2}\,dx}{30b^2}\\
&= \frac{Ax^2(a+bx^2)^{3/2}}{5b}+\frac{Bx^3(a+bx^2)^{3/2}}{6b}-\frac{a(16A+15Bx)\left(a+bx^2\right)^{3/2}}{120b^2}+\frac{(a^2B)}{}\\
&= \frac{a^2Bx\sqrt{a+bx^2}}{16b^2}+\frac{Ax^2(a+bx^2)^{3/2}}{5b}+\frac{Bx^3(a+bx^2)^{3/2}}{6b}-\frac{a(16A+15Bx)\,(a}{120b^2}\\
&= \frac{a^2Bx\sqrt{a+bx^2}}{16b^2}+\frac{Ax^2(a+bx^2)^{3/2}}{5b}+\frac{Bx^3(a+bx^2)^{3/2}}{6b}-\frac{a(16A+15Bx)\,(a}{120b^2}\\
&= \frac{a^2Bx\sqrt{a+bx^2}}{16b^2}+\frac{Ax^2(a+bx^2)^{3/2}}{5b}+\frac{Bx^3(a+bx^2)^{3/2}}{6b}-\frac{a(16A+15Bx)\,(a}{120b^2}
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 101, normalized size = 0.80

$$\frac{\sqrt{a+bx^2}\,(-32a^2A-15a^2Bx+16aAbx^2+10abBx^3+48Ab^2x^4+40b^2Bx^5)}{240b^2}-\frac{a^3B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{16b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-32*a^2*A - 15*a^2*B*x + 16*a*A*b*x^2 + 10*a*b*B*x^3 + 48
*A*b^2*x^4 + 40*b^2*B*x^5))/(240*b^2) - (a^3*B*Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]])/(16*b^(5/2))
```

Maple [A]

time = 0.11, size = 120, normalized size = 0.94

method	result
risch	$-\frac{\left(-40 b^{2} B \,x^{5}-48 A \,b^{2} x^{4}-10 B a b \,x^{3}-16 a A b \,x^{2}+15 a^{2} B x +32 a^{2} A \right) \sqrt{b \,x^{2}+a}}{240 b^{2}}+\frac{B \,a^{3} \ln \left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{16 b^{\frac{5}{2}}}$
default	$B \left(\frac{x^{3} \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{6 b}-\frac{a \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{4 b}-\frac{a \left(\frac{x \sqrt{b \,x^{2}+a}}{2}+\frac{a \ln \left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4 b}\right)}{2 b}\right)+A \left(\frac{x^{2} \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{5 b}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/6*x^3*(b*x^2+a)^(3/2)/b-1/2*a/b*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*
x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+A*(1/5*x^2
*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2))
```

Maxima [A]

time = 0.28, size = 107, normalized size = 0.84

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bx^3}{6\,b}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Ax^2}{5\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bax}{8\,b^2}+\frac{\sqrt{bx^2+a}\,Ba^2x}{16\,b^2}+\frac{Ba^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}-\frac{2\left(bx^2+a\right)^{\frac{3}{2}}Aa}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(3/2)*B*x^3/b + 1/5*(b*x^2 + a)^(3/2)*A*x^2/b - 1/8*(b*x^2
+ a)^(3/2)*B*a*x/b^2 + 1/16*sqrt(b*x^2 + a)*B*a^2*x/b^2 + 1/16*B*a^3*arcsin
h(b*x/sqrt(a*b))/b^(5/2) - 2/15*(b*x^2 + a)^(3/2)*A*a/b^2
```

Fricas [A]

time = 3.70, size = 206, normalized size = 1.62

$$\left[\frac{15\,Ba^3\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(40\,Bb^3x^5+48\,Ab^3x^4+10\,Bab^2x^3+16\,Aab^2x^2-15\,Ba^2bx-32\,Aa^2b)\sqrt{bx^2+a}}{480\,b^3},\,-\frac{15\,Ba^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(40\,Bb^3x^5+48\,Ab^3x^4+10\,Bab^2x^3+16\,Aab^2x^2-15\,Ba^2bx-32\,Aa^2b)\sqrt{bx^2+a}}{240\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(15*B*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(40*B*b^3*x^5 + 48*A*b^3*x^4 + 10*B*a*b^2*x^3 + 16*A*a*b^2*x^2 - 15*B*a^2
*b*x - 32*A*a^2*b)*sqrt(b*x^2 + a))/b^3, -1/240*(15*B*a^3*sqrt(-b)*arctan(s
qrt(-b)*x/sqrt(b*x^2 + a)) - (40*B*b^3*x^5 + 48*A*b^3*x^4 + 10*B*a*b^2*x^3
+ 16*A*a*b^2*x^2 - 15*B*a^2*b*x - 32*A*a^2*b)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A]
time = 5.64, size = 192, normalized size = 1.51

$$A\left(\begin{cases} -\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0 \\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise} \end{cases}\right)-\frac{Ba^{\frac{5}{2}}x}{16b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{3}{2}}x^3}{48b\sqrt{1+\frac{bx^2}{a}}}+\frac{5B\sqrt{a}\,x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{5}{2}}}+\frac{Bbx^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(
15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) - B*a**
(5/2)*x/(16*b**2*sqrt(1 + b*x**2/a)) - B*a**(3/2)*x**3/(48*b*sqrt(1 + b*x**
2/a)) + 5*B*sqrt(a)*x**5/(24*sqrt(1 + b*x**2/a)) + B*a**3*asinh(sqrt(b)*x/s
qrt(a))/(16*b**(5/2)) + B*b*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 1.35, size = 93, normalized size = 0.73

$$-\frac{Ba^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}}+\frac{1}{240}\,\sqrt{bx^2+a}\left(\left(2\left(\left(4\,(5\,Bx+6\,A)x+\frac{5\,Ba}{b}\right)x+\frac{8\,Aa}{b}\right)x-\frac{15\,Ba^2}{b^2}\right)x-\frac{32\,Aa^2}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/16*B*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) + 1/240*sqrt(b*x
^2 + a)*((2*((4*(5*B*x + 6*A)*x + 5*B*a/b)*x + 8*A*a/b)*x - 15*B*a^2/b^2)*x
 - 32*A*a^2/b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\sqrt{b\,x^2+a}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(1/2)*(A + B*x),x)
```

```
[Out] int(x^3*(a + b*x^2)^(1/2)*(A + B*x), x)
```

3.2 $\int x^2(A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=104

$$-\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2(a+bx^2)^{3/2}}{5b}-\frac{(8aB-15Abx)\,(a+bx^2)^{3/2}}{60b^2}-\frac{a^2A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

```
[Out] 1/5*B*x^2*(b*x^2+a)^(3/2)/b-1/60*(-15*A*b*x+8*B*a)*(b*x^2+a)^(3/2)/b^2-1/8*
a^2*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-1/8*a*A*x*(b*x^2+a)^(1/2)/
b
```

Rubi [A]
time = 0.03, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {847, 794, 201, 223, 212}

$$-\frac{a^2A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}-\frac{(a+bx^2)^{3/2}\,(8aB-15Abx)}{60b^2}-\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2(a+bx^2)^{3/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] -1/8*(a*A*x*Sqrt[a + b*x^2])/b + (B*x^2*(a + b*x^2)^(3/2))/(5*b) - ((8*a*B
- 15*A*b*x)*(a + b*x^2)^(3/2))/(60*b^2) - (a^2*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/(8*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2(A+Bx)\sqrt{a+bx^2}\,dx &= \frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}+\frac{\int x(-2aB+5Abx)\sqrt{a+bx^2}\,dx}{5b}\\
&= \frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{(8aB-15Abx)\left(a+bx^2\right)^{3/2}}{60b^2}-\frac{(aA)\int\sqrt{a+bx^2}\,dx}{4b}\\
&= -\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{(8aB-15Abx)\left(a+bx^2\right)^{3/2}}{60b^2}-\frac{\left(a^2A\right)}{}\\
&= -\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{(8aB-15Abx)\left(a+bx^2\right)^{3/2}}{60b^2}-\frac{\left(a^2A\right)}{}\\
&= -\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{(8aB-15Abx)\left(a+bx^2\right)^{3/2}}{60b^2}-\frac{a^2A\,\mathrm{ta}}{}
\end{aligned}
$$

Mathematica [**A**]

time = 0.16, size = 87, normalized size = 0.84

$$
\frac{\sqrt{a+bx^2}\left(-16a^2B+6b^2x^3(5A+4Bx)+abx(15A+8Bx)\right)+15a^2A\sqrt{b}\,\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{120b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-16*a^2*B + 6*b^2*x^3*(5*A + 4*B*x) + a*b*x*(15*A + 8*B*x
)) + 15*a^2*A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(120*b^2)
```

Maple [A]
time = 0.11, size = 96, normalized size = 0.92

method	result	size
risch	$\frac{\left(24 b^{2} B\,x^{4}+30 A\,b^{2} x^{3}+8 B a b\,x^{2}+15 a b A x -16 a^{2} B \right) \sqrt{b\,x^{2}+a}}{120 b^{2}}-\frac{A\,a^{2} \ln \left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{8 b^{\frac{3}{2}}}$	80
default	$B \left(\frac{x^{2} \left(b\,x^{2}+a \right)^{\frac{3}{2}}}{5 b}-\frac{2 a \left(b\,x^{2}+a \right)^{\frac{3}{2}}}{15 b^{2}}\right)+A \left(\frac{x \left(b\,x^{2}+a \right)^{\frac{3}{2}}}{4 b}-\frac{a \left(\frac{x \sqrt{b\,x^{2}+a}}{2}+\frac{a \ln \left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4 b}\right)$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/5*x^2*(b*x^2+a)^(3/2)/b-2/15*a/b^2*(b*x^2+a)^(3/2))+A*(1/4*x*(b*x^2+a)
^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a
)^(1/2))))
```

Maxima [A]
time = 0.28, size = 86, normalized size = 0.83

$$\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bx^2}{5\,b}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Ax}{4\,b}-\frac{\sqrt{bx^2+a}\,Aax}{8\,b}-\frac{Aa^2\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}-\frac{2\left(bx^2+a\right)^{\frac{3}{2}}Ba}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(b*x^2 + a)^(3/2)*B*x^2/b + 1/4*(b*x^2 + a)^(3/2)*A*x/b - 1/8*sqrt(b*x^
2 + a)*A*a*x/b - 1/8*A*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/15*(b*x^2 + a
)^(3/2)*B*a/b^2
```

Fricas [A]
time = 5.30, size = 175, normalized size = 1.68

$$\left[\frac{15\,Aa^2\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,\left(24\,Bb^2x^4+30\,Ab^2x^3+8\,Babx^2+15\,Aabx-16\,Ba^2\right)\sqrt{bx^2+a}}{240\,b^2},\ \frac{15\,Aa^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+\left(24\,Bb^2x^4+30\,Ab^2x^3+8\,Babx^2+15\,Aabx-16\,Ba^2\right)\sqrt{bx^2+a}}{120\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(15*A*a^2*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(24*B*b^2*x^4 + 30*A*b^2*x^3 + 8*B*a*b*x^2 + 15*A*a*b*x - 16*B*a^2)*sqrt(
b*x^2 + a))/b^2, 1/120*(15*A*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) + (24*B*b^2*x^4 + 30*A*b^2*x^3 + 8*B*a*b*x^2 + 15*A*a*b*x - 16*B*a^2)*sqr
t(b*x^2 + a))/b^2]
```

Sympy [**A**]
time = 2.67, size = 165, normalized size = 1.59

$$\frac{Aa^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3A\sqrt{a}\,x^3}{8\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{Abx^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}+B\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*x**3/(8*sqrt(1 + b*x**2
/a)) - A*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + A*b*x**5/(4*sqrt(a)*s
qrt(1 + b*x**2/a)) + B*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x*
*2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x
**4/4, True))
```

Giac [**A**]
time = 0.96, size = 81, normalized size = 0.78

$$\frac{Aa^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}+\frac{1}{120}\,\sqrt{bx^2+a}\,\left(\left(2\left(3\,(4\,Bx+5\,A)x+\frac{4\,Ba}{b}\right)x+\frac{15\,Aa}{b}\right)x-\frac{16\,Ba^2}{b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*A*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/120*sqrt(b*x^2
 + a)*((2*(3*(4*B*x + 5*A)*x + 4*B*a/b)*x + 15*A*a/b)*x - 16*B*a^2/b^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{b\,x^2+a}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(1/2)*(A + B*x),x)
```

```
[Out] int(x^2*(a + b*x^2)^(1/2)*(A + B*x), x)
```

3.3 $\int x(A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=80

$$-\frac{aBx\sqrt{a+bx^2}}{8b}+\frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b}-\frac{a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

```
[Out] 1/12*(3*B*x+4*A)*(b*x^2+a)^(3/2)/b-1/8*a^2*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1
/2))/b^(3/2)-1/8*a*B*x*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {794, 201, 223, 212}

$$-\frac{a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}+\frac{\left(a+bx^2\right)^{3/2}(4A+3Bx)}{12b}-\frac{aBx\sqrt{a+bx^2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] -1/8*(a*B*x*Sqrt[a + b*x^2])/b + ((4*A + 3*B*x)*(a + b*x^2)^(3/2))/(12*b) -
 (a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\begin{aligned}\int x(A+Bx)\sqrt{a+bx^2}\,dx &= \frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b} - \frac{(aB)\int \sqrt{a+bx^2}\,dx}{4b}\\ &= -\frac{aBx\sqrt{a+bx^2}}{8b} + \frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b} - \frac{\left(a^2B\right)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{8b}\\ &= -\frac{aBx\sqrt{a+bx^2}}{8b} + \frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b} - \frac{\left(a^2B\right)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x,\right.}{8b}\\ &= -\frac{aBx\sqrt{a+bx^2}}{8b} + \frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b} - \frac{a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 77, normalized size = 0.96

$$\frac{\sqrt{a+bx^2}\left(8aA+3aBx+8Abx^2+6bBx^3\right)}{24b} + \frac{a^2B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a*A + 3*a*B*x + 8*A*b*x^2 + 6*b*B*x^3))/(24*b) + (a^2*B
*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(3/2))
```

Maple [A]
time = 0.10, size = 76, normalized size = 0.95

method	result	size
risch	$\frac{\left(6bB\,x^3+8Ab\,x^2+3Bax+8Aa\right)\sqrt{b\,x^2+a}}{24b}-\frac{B\,a^2\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{8b^{\frac{3}{2}}}$	65

default	$B\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4b}\right)+\frac{A\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3b}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/4*x*(b*x^2+a)^(3/2)/b-1/4*a/b*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(
x*b^(1/2)+(b*x^2+a)^(1/2))))+1/3*A*(b*x^2+a)^(3/2)/b
```

Maxima [A]
time = 0.28, size = 67, normalized size = 0.84

$$\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Bx}{4\,b}-\frac{\sqrt{bx^{2}+a}\,Bax}{8\,b}-\frac{Ba^{2}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}}+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}A}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*B*x/b - 1/8*sqrt(b*x^2 + a)*B*a*x/b - 1/8*B*a^2*arcsi
nh(b*x/sqrt(a*b))/b^(3/2) + 1/3*(b*x^2 + a)^(3/2)*A/b
```

Fricas [A]
time = 6.87, size = 157, normalized size = 1.96

$$\left[\frac{3\,Ba^{2}\sqrt{b}\,\log\left(-2\,bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,\left(6\,Bb^{2}x^{3}+8\,Ab^{2}x^{2}+3\,Babx+8\,Aab\right)\sqrt{bx^{2}+a}}{48\,b^{2}},\ \frac{3\,Ba^{2}\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+\left(6\,Bb^{2}x^{3}+8\,Ab^{2}x^{2}+3\,Babx+8\,Aab\right)\sqrt{bx^{2}+a}}{24\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*B*a^2*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(6*B*b^2*x^3 + 8*A*b^2*x^2 + 3*B*a*b*x + 8*A*a*b)*sqrt(b*x^2 + a))/b^2, 1/2
4*(3*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (6*B*b^2*x^3 + 8*A
*b^2*x^2 + 3*B*a*b*x + 8*A*a*b)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A]
time = 2.71, size = 124, normalized size = 1.55

$$A\left(\begin{cases}\frac{\sqrt{a}\,x^{2}}{2} & \text{for}\ b=0\\ \frac{\left(a+bx^{2}\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{Ba^{\frac{3}{2}}x}{8b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3B\sqrt{a}\,x^{3}}{8\sqrt{1+\frac{bx^{2}}{a}}}-\frac{Ba^{2}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{3}{2}}}+\frac{Bbx^{5}}{4\sqrt{a}\,\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True))
+ B*a**(3/2)*x/(8*b*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)*x**3/(8*sqrt(1 + b*x*
*2/a)) - B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*b**(3/2)) + B*b*x**5/(4*sqrt(a)
*sqrt(1 + b*x**2/a))
```

Giac [**A**]
time = 0.80, size = 68, normalized size = 0.85

$$\frac{Ba^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}}+\frac{1}{24}\,\sqrt{bx^2+a}\,\left(\left(2\,(3\,Bx+4\,A)x+\frac{3\,Ba}{b}\right)x+\frac{8\,Aa}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*B*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/24*sqrt(b*x^2
+ a)*((2*(3*B*x + 4*A)*x + 3*B*a/b)*x + 8*A*a/b)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{b\,x^2+a}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(1/2)*(A + B*x),x)
```

```
[Out] int(x*(a + b*x^2)^(1/2)*(A + B*x), x)
```

3.4 $\int (A+Bx)\sqrt{a+bx^2}\,dx$

Optimal. Leaf size=67

$$\frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B(a+bx^2)^{3/2}}{3b}+\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

```
[Out] 1/3*B*(b*x^2+a)^(3/2)/b+1/2*a*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+
1/2*A*x*(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {655, 201, 223, 212}

$$\frac{1}{2}Ax\sqrt{a+bx^2}+\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{B(a+bx^2)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)*Sqrt[a + b*x^2],x]
```

```
[Out] (A*x*Sqrt[a + b*x^2])/2 + (B*(a + b*x^2)^(3/2))/(3*b) + (a*A*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int (A+Bx)\sqrt{a+bx^2}\,dx &= \frac{B\left(a+bx^2\right)^{3/2}}{3b}+A\int \sqrt{a+bx^2}\,dx\\
&= \frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}+\frac{1}{2}(aA)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}+\frac{1}{2}(aA)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)\\
&= \frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{3/2}}{3b}+\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}
\end{aligned}$$

Mathematica [A]
time = 0.16, size = 68, normalized size = 1.01

$$\frac{\sqrt{a+bx^2}\,\left(2aB+3Abx+2bBx^2\right)}{6b}-\frac{aA\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)*Sqrt[a + b*x^2],x]
[Out] (Sqrt[a + b*x^2]*(2*a*B + 3*A*b*x + 2*b*B*x^2))/(6*b) - (a*A*Log[-(Sqrt[b]*
x) + Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

Maple [A]
time = 0.11, size = 54, normalized size = 0.81

method	result	size
default	$\frac{B\left(b\,x^2+a\right)^{\frac{3}{2}}}{3b}+A\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)$	54
risch	$\frac{\left(2bB\,x^2+3Abx+2Ba\right)\sqrt{b\,x^2+a}}{6b}+\frac{Aa\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*B*(b*x^2+a)^(3/2)/b+A*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)
+(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.27, size = 45, normalized size = 0.67

$$\frac{1}{2}\,\sqrt{bx^2+a}\,Ax+\frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{b}}+\frac{(bx^2+a)^{\frac{3}{2}}B}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*A*x + 1/2*A*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/3*(b*x
^2 + a)^(3/2)*B/b
```

Fricas [A]
time = 3.33, size = 128, normalized size = 1.91

$$\left[\frac{3\,Aa\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(2\,Bbx^2+3\,Abx+2\,Ba)\sqrt{bx^2+a}}{12\,b},-\frac{3\,Aa\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(2\,Bbx^2+3\,Abx+2\,Ba)\sqrt{bx^2+a}}{6\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*A*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*B*b*x^2 + 3*A*b*x + 2*B*a)*sqrt(b*x^2 + a))/b, -1/6*(3*A*a*sqrt(-b)*arctan
(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*b*x^2 + 3*A*b*x + 2*B*a)*sqrt(b*x^2 + a
))/b]
```

Sympy [A]
time = 1.55, size = 70, normalized size = 1.04

$$\frac{A\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Aa\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}+B\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + A*a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
 + B*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True
))
```

Giac [**A**]
time = 0.80, size = 55, normalized size = 0.82

$$-\frac{Aa\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,\sqrt{b}}+\frac{1}{6}\,\sqrt{bx^2+a}\,\left((2\,Bx+3\,A)x+\frac{2\,Ba}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*A*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/6*sqrt(b*x^2 +
a)*((2*B*x + 3*A)*x + 2*B*a/b)
```

Mupad [**B**]
time = 1.16, size = 52, normalized size = 0.78

$$\frac{B\,{\left(b\,x^2+a\right)}^{3/2}}{3\,b}+\frac{A\,x\,\sqrt{b\,x^2+a}}{2}+\frac{A\,a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(1/2)*(A + B*x),x)
```

```
[Out] (B*(a + b*x^2)^(3/2))/(3*b) + (A*x*(a + b*x^2)^(1/2))/2 + (A*a*log(b^(1/2)*
x + (a + b*x^2)^(1/2)))/(2*b^(1/2))
```

3.5 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx$

Optimal. Leaf size=79

$$\frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] -A*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^(1/2)+1/2*a*B*arctanh(x*b^(1/2)/(b*x^
2+a)^(1/2))/b^(1/2)+1/2*(B*x+2*A)*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {829, 858, 223, 212, 272, 65, 214}

$$\frac{1}{2}\sqrt{a+bx^2}\,(2A+Bx)-\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x^2])/x,x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/2 + (a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
)/(2*Sqrt[b]) - Sqrt[a]*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx &= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{\int \frac{2aAb+abBx}{x\sqrt{a+bx^2}}\,dx}{2b}\\
&= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+(aA)\int \frac{1}{x\sqrt{a+bx^2}}\,dx+\frac{1}{2}(aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{1}{2}(aA)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)+\frac{1}{2}(aB)\text{Sub}\\
&= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{(aA)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,d\right.}{b}\\
&= \frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a}}\right.
\end{aligned}
$$

Mathematica [**A**]
time = 0.19, size = 90, normalized size = 1.14

$$\frac{1}{2}\left((2A+Bx)\sqrt{a+bx^2}+4\sqrt{a}\,A\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{aB\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x^2])/x,x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2] + 4*Sqrt[a]*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*
x^2])/Sqrt[a]] - (a*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/2
```

Maple [**A**]
time = 0.11, size = 79, normalized size = 1.00

method	result
default	$B\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+A\left(\sqrt{b\,x^2+a}-\sqrt{a}\;\ln\left(\frac{2a+2\sqrt{a}\;\sqrt{b\,x^2+a}}{x}\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))+A*((b
*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))
```

Maxima [A]
time = 0.28, size = 59, normalized size = 0.75

$$\frac{1}{2}\sqrt{bx^2+a}\,Bx+\frac{Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}}-A\sqrt{a}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\sqrt{bx^2+a}\,A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x + 1/2*B*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) - A*sqrt(a
)*arcsinh(a/(sqrt(a*b)*abs(x))) + sqrt(b*x^2 + a)*A
```

Fricas [A]
time = 4.60, size = 341, normalized size = 4.32

$$\left[\frac{Ba\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,A\sqrt{a}\,b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(Bbx+2\,Ab)\sqrt{bx^2+a}}{4\,b},\ -\frac{Ba\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-A\sqrt{a}\,b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-(Bbx+2\,Ab)\sqrt{bx^2+a}}{2\,b},\ \frac{4\,A\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+Ba\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(Bbx+2\,Ab)\sqrt{bx^2+a}}{4\,b},\ -\frac{Ba\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-2\,A\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(Bbx+2\,Ab)\sqrt{bx^2+a}}{2\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*A*sqr
t(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(B*b*x + 2*A
*b)*sqrt(b*x^2 + a))/b, -1/2*(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) - A*sqrt(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (B*b
*x + 2*A*b)*sqrt(b*x^2 + a))/b, 1/4*(4*A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*
x^2 + a)) + B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(B*b*x + 2*A*b)*sqrt(b*x^2 + a))/b, -1/2*(B*a*sqrt(-b)*arctan(sqrt(-b)*x/s
qrt(b*x^2 + a)) - 2*A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (B*b*x
+ 2*A*b)*sqrt(b*x^2 + a))/b]
```

Sympy [A]
time = 3.00, size = 107, normalized size = 1.35

$$-A\sqrt{a}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Aa}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{A\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}}+\frac{B\sqrt{a}\,x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Ba\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2)/x,x)
```

```
[Out] -A*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)
) + A*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*sqrt(a)*x*sqrt(1 + b*x**2/a)/2 + B
*a*asinh(sqrt(b)*x/sqrt(a))/(2*sqrt(b))
```

Giac [A]
time = 0.82, size = 78, normalized size = 0.99

$$\frac{2\,Aa\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{Ba\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,\sqrt{b}}+\frac{1}{2}\,\sqrt{bx^2+a}\,(Bx+2\,A)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*A*a*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 1/2*B*a*lo
g(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/2*sqrt(b*x^2 + a)*(B*x + 2
*A)
```

Mupad [B]
time = 1.24, size = 68, normalized size = 0.86

$$A\,\sqrt{b\,x^2+a}-A\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+\frac{B\,x\,\sqrt{b\,x^2+a}}{2}+\frac{B\,a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(1/2)*(A + B*x))/x,x)
```

```
[Out] A*(a + b*x^2)^(1/2) - A*a^(1/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + (B*x*(a
+ b*x^2)^(1/2))/2 + (B*a*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/(2*b^(1/2))
```

3.6 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx$

Optimal. Leaf size=75

$$-\frac{(A-Bx)\sqrt{a+bx^2}}{x}+A\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

```
[Out] -B*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^(1/2)+A*arctanh(x*b^(1/2)/(b*x^2+a)^(
1/2))*b^(1/2)-(-B*x+A)*(b*x^2+a)^(1/2)/x
```

Rubi [A]
time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$,
Rules used = {827, 858, 223, 212, 272, 65, 214}

$$-\frac{\sqrt{a+bx^2}\,(A-Bx)}{x}+A\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x^2])/x^2,x]
```

```
[Out] -(((A - B*x)*Sqrt[a + b*x^2])/x) + A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b
*x^2]] - Sqrt[a]*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx &= -\frac{(A-Bx)\sqrt{a+bx^2}}{x}-\frac{1}{2}\int \frac{-2aB-2Abx}{x\sqrt{a+bx^2}}\,dx\\
&= -\frac{(A-Bx)\sqrt{a+bx^2}}{x}+(Ab)\int \frac{1}{\sqrt{a+bx^2}}\,dx+(aB)\int \frac{1}{x\sqrt{a+bx^2}}\,dx\\
&= -\frac{(A-Bx)\sqrt{a+bx^2}}{x}+(Ab)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)+\frac{1}{2}(aB)\text{S}\\
&= -\frac{(A-Bx)\sqrt{a+bx^2}}{x}+A\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{(aB)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{b}\\
&= -\frac{(A-Bx)\sqrt{a+bx^2}}{x}+A\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{a+}}{\sqrt{}}\right.
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 88, normalized size = 1.17

$$\frac{(-A+Bx)\sqrt{a+bx^2}}{x}+2\sqrt{a}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)-A\sqrt{b}\,\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x^2])/x^2,x]
```

```
[Out] ((-A + B*x)*Sqrt[a + b*x^2])/x + 2*Sqrt[a]*B*ArcTanh[(Sqrt[b]*x - Sqrt[a +
b*x^2])/Sqrt[a]] - A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Maple [A]
time = 0.12, size = 103, normalized size = 1.37

method	result
risch	$-\frac{A\sqrt{b\,x^2+a}}{x}+B\sqrt{b\,x^2+a}+\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)\sqrt{b}\,A-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)$
default	$B\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)+A\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\right.}{2}\right.}{a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] B*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))+A*(-1/a/x
*(b*x^2+a)^(3/2)+2*b/a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b
*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.27, size = 59, normalized size = 0.79

$$A\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-B\sqrt{a}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\sqrt{bx^2+a}\,B-\frac{\sqrt{bx^2+a}\,A}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] A*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - B*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x)))
+ sqrt(b*x^2 + a)*B - sqrt(b*x^2 + a)*A/x
```

Fricas [A]

time = 4.96, size = 333, normalized size = 4.44

$$\left[\frac{A\sqrt{b}x\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}x-a\right)+B\sqrt{a}x\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,\sqrt{bx^{2}+a}\,(Bx-A)}{2\,x},-\frac{2\,A\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^{2}+a}}\right)-B\sqrt{a}x\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)-2\,\sqrt{bx^{2}+a}\,(Bx-A)}{2\,x},\frac{2\,B\sqrt{-a}x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+A\sqrt{b}x\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}x-a\right)+2\,\sqrt{bx^{2}+a}\,(Bx-A)}{2\,x},-\frac{A\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^{2}+a}}\right)-B\sqrt{-a}x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)-\sqrt{bx^{2}+a}\,(Bx-A)}{x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + B*sqrt(
a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*sqrt(b*x^2 + a
)*(B*x - A))/x, -1/2*(2*A*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - B
*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*sqrt(b*x
^2 + a)*(B*x - A))/x, 1/2*(2*B*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 + a))
+ A*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqrt(b*x^
2 + a)*(B*x - A))/x, -(A*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - B*
sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - sqrt(b*x^2 + a)*(B*x - A))/x]
```

Sympy [A]

time = 2.06, size = 124, normalized size = 1.65

$$-\frac{A\sqrt{a}}{x\sqrt{1+\frac{bx^{2}}{a}}}+A\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{Abx}{\sqrt{a}\,\sqrt{1+\frac{bx^{2}}{a}}}-B\sqrt{a}\,\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Ba}{\sqrt{b}\,x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{B\sqrt{b}\,x}{\sqrt{\frac{a}{bx^{2}}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2)/x**2,x)
```

```
[Out] -A*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - A*
b*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + B
*a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)*x/sqrt(a/(b*x**2) + 1)
```

Giac [A]

time = 1.00, size = 102, normalized size = 1.36

$$\frac{2\,Ba\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-A\sqrt{b}\,\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)+\sqrt{bx^{2}+a}\,B+\frac{2\,Aa\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - A*sqrt(b)*
log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + sqrt(b*x^2 + a)*B + 2*A*a*sqrt(b)/
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

Mupad [B]
time = 1.69, size = 89, normalized size = 1.19

$$B\,\sqrt{b\,x^2+a}-\frac{A\,\sqrt{b\,x^2+a}}{x}-B\,\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)-\frac{A\,\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(1/2)*(A + B*x))/x^2,x)
```

```
[Out] B*(a + b*x^2)^(1/2) - (A*(a + b*x^2)^(1/2))/x - B*a^(1/2)*atanh((a + b*x^2)
^(1/2)/a^(1/2)) - (A*b^(1/2)*asin((b^(1/2)*x*1i)/a^(1/2))*(a + b*x^2)^(1/2)
*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

3.7 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx$

Optimal. Leaf size=80

$$-\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2}+\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

```
[Out] -1/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)+B*arctanh(x*b^(1/2)/(b*x^
2+a)^(1/2))*b^(1/2)-1/2*(2*B*x+A)*(b*x^2+a)^(1/2)/x^2
```

Rubi [A]

time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {825, 858, 223, 212, 272, 65, 214}

$$-\frac{\sqrt{a+bx^2}\,(A+2Bx)}{2x^2}-\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}+\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*Sqrt[a + b*x^2])/x^3,x]
```

```
[Out] -1/2*((A + 2*B*x)*Sqrt[a + b*x^2])/x^2 + Sqrt[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]] - (A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*Sqrt[a])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx &= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2}-\frac{\int \frac{-2aAb-4abBx}{x\sqrt{a+bx^2}}\,dx}{4a}\\
&= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2}+\frac{1}{2}(Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx+(bB)\int \frac{1}{\sqrt{a+bx^2}}\,dx\\
&= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2}+\frac{1}{4}(Ab)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)+(bB)\text{Subs}\\
&= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2}+\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{1}{2}A\text{Subst}\Bigg(\int \frac{1}{-\frac{a}{b}+}\\
&= -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2}+\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{Ab\tanh^{-1}\Bigg(\frac{\sqrt{a+b}}{\sqrt{a}}}{2\sqrt{a}}
\end{aligned}
$$

Mathematica [A]
time = 0.27, size = 90, normalized size = 1.12

$$
-\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2}+\frac{Ab\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\sqrt{b}\,B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x^2])/x^3,x]
```

```
[Out] -1/2*((A + 2*B*x)*Sqrt[a + b*x^2])/x^2 + (A*b*ArcTanh[(Sqrt[b]*x - Sqrt[a +
 b*x^2])/Sqrt[a]])/Sqrt[a] - Sqrt[b]*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(62) = 124$.
time = 0.12, size = 127, normalized size = 1.59

method	result
risch	$-\frac{(2Bx+A)\sqrt{b\,x^2+a}}{2x^2}+B\sqrt{b}\,\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)Ab}{2\sqrt{a}}$
default	$A\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)}{2a}\right)+B\Bigg(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{ax}+\frac{2b\Bigg(\frac{x\sqrt{b\,x^2+}}{2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/2/a/x^2*(b*x^2+a)^(3/2)+1/2*b/a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^
(1/2)*(b*x^2+a)^(1/2))/x)))+B*(-1/a/x*(b*x^2+a)^(3/2)+2*b/a*(1/2*x*(b*x^2+a
)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))
```

Maxima [A]

time = 0.27, size = 83, normalized size = 1.04

$$B\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{Ab\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\sqrt{a}}+\frac{\sqrt{bx^2+a}\,Ab}{2\,a}-\frac{\sqrt{bx^2+a}\,B}{x}-\frac{(bx^2+a)^{\frac{3}{2}}A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] B*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - 1/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/sq
rt(a) + 1/2*sqrt(b*x^2 + a)*A*b/a - sqrt(b*x^2 + a)*B/x - 1/2*(b*x^2 + a)^(
3/2)*A/(a*x^2)
```

Fricas [A]

time = 4.33, size = 377, normalized size = 4.71

$$\left[\frac{2\,Ba\sqrt{b}\,x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+A\sqrt{a}\,bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,(2\,Bax+Aa)\sqrt{bx^2+a}}{4\,ax^2},-\frac{4\,Ba\sqrt{-b}\,x^2\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-A\sqrt{a}\,bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(2\,Bax+Aa)\sqrt{bx^2+a}}{4\,ax^2},\frac{A\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+Ba\sqrt{b}\,x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-(2\,Bax+Aa)\sqrt{bx^2+a}}{2\,ax^2},-\frac{2\,Ba\sqrt{-b}\,x^2\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-A\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(2\,Bax+Aa)\sqrt{bx^2+a}}{2\,ax^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + A
*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2*B
*a*x + A*a)*sqrt(b*x^2 + a))/(a*x^2), -1/4*(4*B*a*sqrt(-b)*x^2*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - A*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(a) + 2*a)/x^2) + 2*(2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a*x^2), 1/2*(A*sqrt
(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + B*a*sqrt(b)*x^2*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - (2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a*x^
2), -1/2*(2*B*a*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - A*sqrt(-a
)*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*x + A*a)*sqrt(b*x^2 + a))
/(a*x^2)]
```

Sympy [A]

time = 2.11, size = 107, normalized size = 1.34

$$-\frac{A\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2\sqrt{a}}-\frac{B\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+B\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-\frac{Bbx}{\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(1/2)/x**3,x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*s
qrt(a)) - B*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(b)*asinh(sqrt(b)*x/sqrt
(a)) - B*b*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(62) = 124$.
time = 1.72, size = 163, normalized size = 2.04

$$\frac{Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - B\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3 Ab + 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 Ba\sqrt{b} + \left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab - 2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 - a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - B*sqrt(b)*lo
g(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b
 + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2
+ a))*A*a*b - 2*B*a^2*sqrt(b))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2
```

Mupad [B]
time = 1.79, size = 94, normalized size = 1.18

$$-\frac{A\sqrt{b\,x^2+a}}{2\,x^2} - \frac{B\sqrt{b\,x^2+a}}{x} - \frac{A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,\sqrt{a}} - \frac{B\,\sqrt{b}\;\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\,\sqrt{b\,x^2+a}\;1\mathrm{i}}{\sqrt{a}\,\sqrt{\frac{b\,x^2}{a}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(1/2)*(A + B*x))/x^3,x)
```

```
[Out] - (A*(a + b*x^2)^(1/2))/(2*x^2) - (B*(a + b*x^2)^(1/2))/x - (A*b*atanh((a +
 b*x^2)^(1/2)/a^(1/2)))/(2*a^(1/2)) - (B*b^(1/2)*asin((b^(1/2)*x*1i)/a^(1/2
))*(a + b*x^2)^(1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

3.8 $\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=150

$$\frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx(a+bx^2)^{3/2}}{64b^2}+\frac{Ax^2(a+bx^2)^{5/2}}{7b}+\frac{Bx^3(a+bx^2)^{5/2}}{8b}-\frac{a(32A+35Bx)\left(a+bx^2\right)^{5/2}}{560b^2}+\frac{3a}{}$$

```
[Out] 1/64*a^2*B*x*(b*x^2+a)^(3/2)/b^2+1/7*A*x^2*(b*x^2+a)^(5/2)/b+1/8*B*x^3*(b*x
^2+a)^(5/2)/b-1/560*a*(35*B*x+32*A)*(b*x^2+a)^(5/2)/b^2+3/128*a^4*B*arctanh
(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+3/128*a^3*B*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A]
time = 0.06, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {847, 794, 201, 223, 212}

$$\frac{3a^4B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}+\frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx(a+bx^2)^{3/2}}{64b^2}-\frac{a(a+bx^2)^{5/2}\,(32A+35Bx)}{560b^2}+\frac{Ax^2(a+bx^2)^{5/2}}{7b}+\frac{Bx^3(a+bx^2)^{5/2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a^3*B*x*Sqrt[a + b*x^2])/(128*b^2) + (a^2*B*x*(a + b*x^2)^(3/2))/(64*b^2
) + (A*x^2*(a + b*x^2)^(5/2))/(7*b) + (B*x^3*(a + b*x^2)^(5/2))/(8*b) - (a*
(32*A + 35*B*x)*(a + b*x^2)^(5/2))/(560*b^2) + (3*a^4*B*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(128*b^(5/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx &= \frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}+\frac{\int x^2(-3aB+8Abx)\left(a+bx^2\right)^{3/2}\,dx}{8b}\\
&= \frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}+\frac{\int x(-16aAb-21abBx)\left(a+bx^2\right)}{56b^2}\\
&= \frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{a(32A+35Bx)\left(a+bx^2\right)^{5/2}}{560b^2}+\frac{(a}{}\\
&= \frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{a(32A+35B}{56}\\
&= \frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3(a+bx^2}{8b}\\
&= \frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3(a+bx^2}{8b}\\
&= \frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3(a+bx^2}{8b}
\end{aligned}
$$

Mathematica [A]

time = 0.24, size = 118, normalized size = 0.79

$$\frac{\sqrt{b}\,\sqrt{a+bx^2}\,(80b^3x^6(8A+7Bx)+2a^2bx^2(64A+35Bx)+8ab^2x^4(128A+105Bx)-a^3(256A+105Bx))-105a^4B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{4480b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[a + b*x^2]*(80*b^3*x^6*(8*A + 7*B*x) + 2*a^2*b*x^2*(64*A + 35
*B*x) + 8*a*b^2*x^4*(128*A + 105*B*x) - a^3*(256*A + 105*B*x)) - 105*a^4*B*
Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(4480*b^(5/2))
```

Maple [A]

time = 0.11, size = 136, normalized size = 0.91

method	result
risch	$-\frac{\left(-560B\,b^{3}x^{7}-640A\,b^{3}x^{6}-840Ba\,b^{2}x^{5}-1024aA\,b^{2}x^{4}-70B\,a^{2}b\,x^{3}-128a^{2}Ab\,x^{2}+105B\,a^{3}x+256a^{3}A\right)\sqrt{b\,x^{2}+a}}{4480b^{2}}+\frac{3B\,a^{4}\ln\Big(}{}$
default	$B\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{8b}-\frac{3a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)}{8b}\right)+A\Big(\frac{x^{2}(b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/8*x^3*(b*x^2+a)^(5/2)/b-3/8*a/b*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*
x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(
b*x^2+a)^(1/2))))))+A*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2)
)
```

Maxima [A]

time = 0.33, size = 126, normalized size = 0.84

$$\frac{(bx^2+a)^{\frac{5}{2}}Bx^3}{8\,b}+\frac{(bx^2+a)^{\frac{5}{2}}Ax^2}{7\,b}-\frac{(bx^2+a)^{\frac{5}{2}}Bax}{16\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}Ba^2x}{64\,b^2}+\frac{3\,\sqrt{bx^2+a}\,Ba^3x}{128\,b^2}+\frac{3\,Ba^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}-\frac{2\,(bx^2+a)^{\frac{5}{2}}Aa}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(5/2)*B*x^3/b + 1/7*(b*x^2 + a)^(5/2)*A*x^2/b - 1/16*(b*x^2
 + a)^(5/2)*B*a*x/b^2 + 1/64*(b*x^2 + a)^(3/2)*B*a^2*x/b^2 + 3/128*sqrt(b*x
^2 + a)*B*a^3*x/b^2 + 3/128*B*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 2/35*(b*
x^2 + a)^(5/2)*A*a/b^2
```

Fricas [A]
time = 3.72, size = 254, normalized size = 1.69

$$\left[\frac{105\,Ba^4\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(560\,Bb^4x^7+640\,Ab^4x^6+840\,Bab^3x^5+1024\,Aab^3x^4+70\,Ba^2b^2x^3+128\,Aa^2b^2x^2-105\,Ba^3bx-256\,Aa^3b)\sqrt{bx^2+a}}{8960\,b^3},-\frac{105\,Ba^4\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(560\,Bb^4x^7+640\,Ab^4x^6+840\,Bab^3x^5+1024\,Aab^3x^4+70\,Ba^2b^2x^3+128\,Aa^2b^2x^2-105\,Ba^3bx-256\,Aa^3b)\sqrt{bx^2+a}}{4480\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8960*(105*B*a^4*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(560*B*b^4*x^7 + 640*A*b^4*x^6 + 840*B*a*b^3*x^5 + 1024*A*a*b^3*x^4 + 7
0*B*a^2*b^2*x^3 + 128*A*a^2*b^2*x^2 - 105*B*a^3*b*x - 256*A*a^3*b)*sqrt(b*x
^2 + a))/b^3, -1/4480*(105*B*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)
) - (560*B*b^4*x^7 + 640*A*b^4*x^6 + 840*B*a*b^3*x^5 + 1024*A*a*b^3*x^4 + 7
0*B*a^2*b^2*x^3 + 128*A*a^2*b^2*x^2 - 105*B*a^3*b*x - 256*A*a^3*b)*sqrt(b*x
^2 + a))/b^3]
```

Sympy [A]
time = 25.12, size = 318, normalized size = 2.12

$$Aa\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+Ab\left(\begin{cases}\frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{ax^4\sqrt{a+bx^2}}{35b}+\frac{x^6\sqrt{a+bx^2}}{7} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^6}{6} & \text{otherwise}\end{cases}\right)-\frac{3Ba^{\frac{7}{2}}x}{128b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{5}{2}}x^3}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{13Ba^{\frac{3}{2}}x^5}{64\sqrt{1+\frac{bx^2}{a}}}+\frac{5B\sqrt{a}\,bx^7}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^4\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{5}{2}}}+\frac{Bb^2x^9}{8\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)
/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + A*b
*Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**
2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, N
e(b, 0)), (sqrt(a)*x**6/6, True)) - 3*B*a**(7/2)*x/(128*b**2*sqrt(1 + b*x**
2/a)) - B*a**(5/2)*x**3/(128*b*sqrt(1 + b*x**2/a)) + 13*B*a**(3/2)*x**5/(64
*sqrt(1 + b*x**2/a)) + 5*B*sqrt(a)*b*x**7/(16*sqrt(1 + b*x**2/a)) + 3*B*a**
4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(5/2)) + B*b**2*x**9/(8*sqrt(a)*sqrt(1 +
 b*x**2/a))
```

Giac [A]

time = 1.63, size = 115, normalized size = 0.77

$$-\frac{3\,Ba^4\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\,\right|\right)}{128\,b^{\frac{5}{2}}}-\frac{1}{4480}\,\sqrt{bx^2+a}\,\left(\frac{256\,Aa^3}{b^2}+\left(\frac{105\,Ba^3}{b^2}-2\left(\frac{64\,Aa^2}{b}+\left(\frac{35\,Ba^2}{b}+4\,(128\,Aa+5\,(21\,Ba+2\,(7\,Bbx+8\,Ab)x)x)x\right)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3/128*B*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/4480*sqrt(b
*x^2 + a)*(256*A*a^3/b^2 + (105*B*a^3/b^2 - 2*(64*A*a^2/b + (35*B*a^2/b + 4
*(128*A*a + 5*(21*B*a + 2*(7*B*b*x + 8*A*b)*x)*x)*x)*x)*x)*x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(b\,x^2+a\right)^{3/2}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(3/2)*(A + B*x),x)
```

```
[Out] int(x^3*(a + b*x^2)^(3/2)*(A + B*x), x)
```

3.9 $\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$-\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{aAx(a+bx^2)^{3/2}}{24b}+\frac{Bx^2(a+bx^2)^{5/2}}{7b}-\frac{(12aB-35Abx)\,(a+bx^2)^{5/2}}{210b^2}-\frac{a^3A\tanh^{-1}\Big(\frac{\sqrt{}}{\sqrt{a+}}}{16b^{3/2}}$$

```
[Out] -1/24*a*A*x*(b*x^2+a)^(3/2)/b+1/7*B*x^2*(b*x^2+a)^(5/2)/b-1/210*(-35*A*b*x+
12*B*a)*(b*x^2+a)^(5/2)/b^2-1/16*a^3*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b
^(3/2)-1/16*a^2*A*x*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.04, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {847, 794, 201, 223, 212}

$$-\frac{a^3A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{(a+bx^2)^{5/2}\,(12aB-35Abx)}{210b^2}-\frac{aAx(a+bx^2)^{3/2}}{24b}+\frac{Bx^2(a+bx^2)^{5/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] -1/16*(a^2*A*x*Sqrt[a + b*x^2])/b - (a*A*x*(a + b*x^2)^(3/2))/(24*b) + (B*x
^2*(a + b*x^2)^(5/2))/(7*b) - ((12*a*B - 35*A*b*x)*(a + b*x^2)^(5/2))/(210*
b^2) - (a^3*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx &= \frac{Bx^2(a+bx^2)^{5/2}}{7b}+\frac{\int x(-2aB+7Abx)\left(a+bx^2\right)^{3/2}\,dx}{7b}\\
&= \frac{Bx^2(a+bx^2)^{5/2}}{7b}-\frac{(12aB-35Abx)\left(a+bx^2\right)^{5/2}}{210b^2}-\frac{(aA)\int\left(a+bx^2\right)^{3/2}\,dx}{6b}\\
&= -\frac{aAx(a+bx^2)^{3/2}}{24b}+\frac{Bx^2(a+bx^2)^{5/2}}{7b}-\frac{(12aB-35Abx)\left(a+bx^2\right)^{5/2}}{210b^2}-\\
&= -\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{aAx(a+bx^2)^{3/2}}{24b}+\frac{Bx^2(a+bx^2)^{5/2}}{7b}-\\
&= -\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{aAx(a+bx^2)^{3/2}}{24b}+\frac{Bx^2(a+bx^2)^{5/2}}{7b}-\\
&= -\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{aAx(a+bx^2)^{3/2}}{24b}+\frac{Bx^2(a+bx^2)^{5/2}}{7b}-
\end{aligned}
$$

Mathematica [A]
time = 0.27, size = 107, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\left(-96a^3B+40b^3x^5(7A+6Bx)+3a^2bx(35A+16Bx)+2ab^2x^3(245A+192Bx)\right)+105a^3A\sqrt{b}\,\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{1680b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-96*a^3*B + 40*b^3*x^5*(7*A + 6*B*x) + 3*a^2*b*x*(35*A +
16*B*x) + 2*a*b^2*x^3*(245*A + 192*B*x)) + 105*a^3*A*Sqrt[b]*Log[-(Sqrt[b]*
x) + Sqrt[a + b*x^2]])/(1680*b^2)
```

Maple [A]

time = 0.11, size = 112, normalized size = 0.88

method	result
risch	$\frac{\left(240 b^{3} B x^{6}+280 A b^{3} x^{5}+384 a b^{2} B x^{4}+490 A a b^{2} x^{3}+48 a^{2} b B x^{2}+105 A a^{2} b x-96 B a^{3}\right) \sqrt{b x^{2}+a}}{1680 b^{2}}-\frac{A a^{3} \ln \left(x \sqrt{b}+\sqrt{b x^{2}-}\right.}{16 b^{\frac{3}{2}}}$
default	$B \left(\frac{x^{2} \left(b x^{2}+a\right)^{\frac{5}{2}}}{7 b}-\frac{2 a \left(b x^{2}+a\right)^{\frac{5}{2}}}{35 b^{2}}\right)+A \left(\frac{x \left(b x^{2}+a\right)^{\frac{5}{2}}}{6 b}-\frac{a \left(\frac{x \left(b x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b x^{2}+a}}{2}+\frac{a \ln \left(x \sqrt{b}+\sqrt{b x^{2}+a}\right.}{2 \sqrt{b}}\right.}{4}\right.}{6 b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/7*x^2*(b*x^2+a)^(5/2)/b-2/35*a/b^2*(b*x^2+a)^(5/2))+A*(1/6*x*(b*x^2+a)
^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/
b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))
```

Maxima [A]

time = 0.28, size = 105, normalized size = 0.83

$$\frac{(bx^2+a)^{\frac{5}{2}}Bx^2}{7\,b}+\frac{(bx^2+a)^{\frac{5}{2}}Ax}{6\,b}-\frac{(bx^2+a)^{\frac{3}{2}}Aax}{24\,b}-\frac{\sqrt{bx^2+a}\,Aa^2x}{16\,b}-\frac{Aa^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}-\frac{2\,(bx^2+a)^{\frac{5}{2}}Ba}{35\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/7*(b*x^2 + a)^(5/2)*B*x^2/b + 1/6*(b*x^2 + a)^(5/2)*A*x/b - 1/24*(b*x^2 +
 a)^(3/2)*A*a*x/b - 1/16*sqrt(b*x^2 + a)*A*a^2*x/b - 1/16*A*a^3*arcsinh(b*x
/sqrt(a*b))/b^(3/2) - 2/35*(b*x^2 + a)^(5/2)*B*a/b^2
```

Fricas [A]

time = 4.50, size = 223, normalized size = 1.76

$$\left[\frac{105\,Aa^3\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(240\,Bb^3x^6+280\,Ab^3x^5+384\,Bab^2x^4+490\,Aab^2x^3+48\,Ba^2bx^2+105\,Aa^2bx-96\,Ba^3)\sqrt{bx^2+a}}{3360\,b^2},\ \frac{105\,Aa^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(240\,Bb^3x^6+280\,Ab^3x^5+384\,Bab^2x^4+490\,Aab^2x^3+48\,Ba^2bx^2+105\,Aa^2bx-96\,Ba^3)\sqrt{bx^2+a}}{1680\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/3360*(105*A*a^3*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(240*B*b^3*x^6 + 280*A*b^3*x^5 + 384*B*a*b^2*x^4 + 490*A*a*b^2*x^3 + 48
*B*a^2*b*x^2 + 105*A*a^2*b*x - 96*B*a^3)*sqrt(b*x^2 + a))/b^2, 1/1680*(105*
A*a^3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (240*B*b^3*x^6 + 280*A*
b^3*x^5 + 384*B*a*b^2*x^4 + 490*A*a*b^2*x^3 + 48*B*a^2*b*x^2 + 105*A*a^2*b*
x - 96*B*a^3)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A]
time = 7.87, size = 287, normalized size = 2.26

$$\frac{Aa^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{17Aa^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{11A\sqrt{a}\,bx^5}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{Aa^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{Ab^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+Ba\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+Bb\left(\begin{cases}\frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{ax^4\sqrt{a+bx^2}}{35b}+\frac{x^6\sqrt{a+bx^2}}{7} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^6}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*A*a**(3/2)*x**3/(48*sqrt(1 + b*
x**2/a)) + 11*A*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - A*a**3*asinh(sqrt(
b)*x/sqrt(a))/(16*b**(3/2)) + A*b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) +
B*a*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)
/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + B*b
*Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**
2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, N
e(b, 0)), (sqrt(a)*x**6/6, True))
```

Giac [A]
time = 0.89, size = 103, normalized size = 0.81

$$\frac{Aa^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}-\frac{1}{1680}\,\sqrt{bx^2+a}\left(\frac{96\,Ba^3}{b^2}-\left(\frac{105\,Aa^2}{b}+2\left(\frac{24\,Ba^2}{b}+(245\,Aa+4\,(48\,Ba+5\,(6\,Bbx+7\,Ab)x)x)x\right)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*A*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - 1/1680*sqrt(b*x
^2 + a)*(96*B*a^3/b^2 - (105*A*a^2/b + 2*(24*B*a^2/b + (245*A*a + 4*(48*B*a
 + 5*(6*B*b*x + 7*A*b)*x)*x)*x)*x)*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(b\,x^2+a\right)^{3/2}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(3/2)*(A + B*x),x)
```

```
[Out] int(x^2*(a + b*x^2)^(3/2)*(A + B*x), x)
```

3.10 $\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

Optimal. Leaf size=103

$$-\frac{a^2Bx\sqrt{a+bx^2}}{16b}-\frac{aBx\left(a+bx^2\right)^{3/2}}{24b}+\frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}-\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

```
[Out] -1/24*a*B*x*(b*x^2+a)^(3/2)/b+1/30*(5*B*x+6*A)*(b*x^2+a)^(5/2)/b-1/16*a^3*B
*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-1/16*a^2*B*x*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.02, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {794, 201, 223, 212}

$$-\frac{a^3B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}-\frac{a^2Bx\sqrt{a+bx^2}}{16b}+\frac{\left(a+bx^2\right)^{5/2}(6A+5Bx)}{30b}-\frac{aBx\left(a+bx^2\right)^{3/2}}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] -1/16*(a^2*B*x*Sqrt[a + b*x^2])/b - (a*B*x*(a + b*x^2)^(3/2))/(24*b) + ((6*
A + 5*B*x)*(a + b*x^2)^(5/2))/(30*b) - (a^3*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]])/(16*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx &= \frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}-\frac{(aB)\int\left(a+bx^2\right)^{3/2}\,dx}{6b}\\
&= -\frac{aBx(a+bx^2)^{3/2}}{24b}+\frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}-\frac{\left(a^2B\right)\int\sqrt{a+bx^2}\,dx}{8b}\\
&= -\frac{a^2Bx\sqrt{a+bx^2}}{16b}-\frac{aBx(a+bx^2)^{3/2}}{24b}+\frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}-\frac{\left(a^3B\right)}{}\\
&= -\frac{a^2Bx\sqrt{a+bx^2}}{16b}-\frac{aBx(a+bx^2)^{3/2}}{24b}+\frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}-\frac{\left(a^3B\right)}{}\\
&= -\frac{a^2Bx\sqrt{a+bx^2}}{16b}-\frac{aBx(a+bx^2)^{3/2}}{24b}+\frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}-\frac{a^3B\,\mathrm{ta}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 101, normalized size = 0.98

$$\frac{\sqrt{a+bx^2}\left(48a^2A+15a^2Bx+96aAbx^2+70abBx^3+48Ab^2x^4+40b^2Bx^5\right)}{240b}+\frac{a^3B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{16b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(48*a^2*A + 15*a^2*B*x + 96*a*A*b*x^2 + 70*a*b*B*x^3 + 48*
A*b^2*x^4 + 40*b^2*B*x^5))/(240*b) + (a^3*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x
^2]])/(16*b^(3/2))
```

Maple [A]
time = 0.12, size = 92, normalized size = 0.89

method	result	size

risch	$\frac{\left(40 b^{2} B \,x^{5}+48 A \,b^{2} x^{4}+70 B a b \,x^{3}+96 a A b \,x^{2}+15 a^{2} B x +48 a^{2} A \right) \sqrt{b \,x^{2}+a}}{240 b}-\frac{B \,a^{3} \ln \left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{16 b^{\frac{3}{2}}}$	89
default	$B \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{5}{2}}}{6 b}-\frac{a \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b \,x^{2}+a}}{2}+\frac{a \ln \left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)}{6 b}\right)+\frac{A \left(b \,x^{2}+a \right)^{\frac{5}{2}}}{5 b}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/6*x*(b*x^2+a)^(5/2)/b-1/6*a/b*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x
^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+1/5*A/b*(b*x^2+a
)^(5/2)
```

Maxima [A]
time = 0.30, size = 86, normalized size = 0.83

$$\frac{(bx^2+a)^{\frac{5}{2}}Bx}{6\,b}-\frac{(bx^2+a)^{\frac{3}{2}}Bax}{24\,b}-\frac{\sqrt{bx^2+a}\,Ba^2x}{16\,b}-\frac{Ba^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}+\frac{(bx^2+a)^{\frac{5}{2}}A}{5\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*B*x/b - 1/24*(b*x^2 + a)^(3/2)*B*a*x/b - 1/16*sqrt(b*
x^2 + a)*B*a^2*x/b - 1/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/5*(b*x^2
 + a)^(5/2)*A/b
```

Fricas [A]
time = 3.58, size = 205, normalized size = 1.99

$$\left[\frac{15\,Ba^3\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(40\,Bb^3x^5+48\,Ab^3x^4+70\,Bab^2x^3+96\,Aab^2x^2+15\,Ba^2bx+48\,Aa^2b)\sqrt{bx^2+a}}{480\,b^2},\ \frac{15\,Ba^3\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(40\,Bb^3x^5+48\,Ab^3x^4+70\,Bab^2x^3+96\,Aab^2x^2+15\,Ba^2bx+48\,Aa^2b)\sqrt{bx^2+a}}{240\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/480*(15*B*a^3*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(40*B*b^3*x^5 + 48*A*b^3*x^4 + 70*B*a*b^2*x^3 + 96*A*a*b^2*x^2 + 15*B*a^2
```

```
*b*x + 48*A*a^2*b)*sqrt(b*x^2 + a))/b^2, 1/240*(15*B*a^3*sqrt(-b)*arctan(sq
rt(-b)*x/sqrt(b*x^2 + a)) + (40*B*b^3*x^5 + 48*A*b^3*x^4 + 70*B*a*b^2*x^3 +
 96*A*a*b^2*x^2 + 15*B*a^2*b*x + 48*A*a^2*b)*sqrt(b*x^2 + a))/b^2]
```

Sympy [**A**]
time = 7.83, size = 223, normalized size = 2.17

$$Aa\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Ab\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+\frac{Ba^{\frac{5}{2}}x}{16b\sqrt{1+\frac{bx^2}{a}}}+\frac{17Ba^{\frac{3}{2}}x^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{11B\sqrt{a}\,bx^5}{24\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{3}{2}}}+\frac{Bb^2x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)
) + A*b*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x
**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) +
 B*a**(5/2)*x/(16*b*sqrt(1 + b*x**2/a)) + 17*B*a**(3/2)*x**3/(48*sqrt(1 + b
*x**2/a)) + 11*B*sqrt(a)*b*x**5/(24*sqrt(1 + b*x**2/a)) - B*a**3*asinh(sqrt
(b)*x/sqrt(a))/(16*b**(3/2)) + B*b**2*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [**A**]
time = 0.92, size = 89, normalized size = 0.86

$$\frac{Ba^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}+\frac{1}{240}\,\sqrt{bx^2+a}\,\left(\frac{48\,Aa^2}{b}+\left(\frac{15\,Ba^2}{b}+2\,(48\,Aa+(35\,Ba+4\,(5\,Bbx+6\,Ab)x)x)x\right)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*B*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/240*sqrt(b*x^
2 + a)*(48*A*a^2/b + (15*B*a^2/b + 2*(48*A*a + (35*B*a + 4*(5*B*b*x + 6*A*b
)*x)*x)*x)*x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(b\,x^2+a\right)^{3/2}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(3/2)*(A + B*x),x)
```

```
[Out] int(x*(a + b*x^2)^(3/2)*(A + B*x), x)
```

3.11 $\int (A+Bx)\left(a+bx^2\right)^{3/2} dx$

Optimal. Leaf size=87

$$\frac{3}{8}aAx\sqrt{a+bx^2}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{3a^2A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

```
[Out] 1/4*A*x*(b*x^2+a)^(3/2)+1/5*B*(b*x^2+a)^(5/2)/b+3/8*a^2*A*arctanh(x*b^(1/2)
/(b*x^2+a)^(1/2))/b^(1/2)+3/8*a*A*x*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {655, 201, 223, 212}

$$\frac{3a^2A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{3}{8}aAx\sqrt{a+bx^2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (3*a*A*x*Sqrt[a + b*x^2])/8 + (A*x*(a + b*x^2)^(3/2))/4 + (B*(a + b*x^2)^(5
/2))/(5*b) + (3*a^2*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx &= \frac{B\left(a+bx^2\right)^{5/2}}{5b}+A\int \left(a+bx^2\right)^{3/2}\,dx\\
&= \frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{1}{4}(3aA)\int \sqrt{a+bx^2}\,dx\\
&= \frac{3}{8}aAx\sqrt{a+bx^2}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{1}{8}\left(3a^2A\right)\int \frac{1}{\sqrt{a+bx}}\\
&= \frac{3}{8}aAx\sqrt{a+bx^2}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{1}{8}\left(3a^2A\right)\text{Subst}\left(\int \frac{}{1}\right.\\
&= \frac{3}{8}aAx\sqrt{a+bx^2}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{3a^2A\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+}}\right.}{8\sqrt{b}}
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 87, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}\left(8a^2B+2b^2x^3(5A+4Bx)+abx(25A+16Bx)\right)-15a^2A\sqrt{b}\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{40b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)*(a + b*x^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^2*B + 2*b^2*x^3*(5*A + 4*B*x) + a*b*x*(25*A + 16*B*x)
) - 15*a^2*A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(40*b)
```

Maple [A]
time = 0.11, size = 70, normalized size = 0.80

method	result	size
default	$\frac{B\left(b\,x^2+a\right)^{\frac{5}{2}}}{5b}+A\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)$	70

risch	$\frac{\left(8 b^{2} B \,x^{4}+10 A \,b^{2} x^{3}+16 B a b \,x^{2}+25 a b A x +8 a^{2} B \right) \sqrt{b \,x^{2}+a}}{40 b}+\frac{3 a^{2} A \ln \left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{8 \sqrt{b}}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*B*(b*x^2+a)^(5/2)/b+A*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/
2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.28, size = 61, normalized size = 0.70

$$\frac{1}{4} \, {\left(b x^{2} + a\right)}^{\frac{3}{2}} A x + \frac{3}{8} \, \sqrt{b x^{2} + a} A a x + \frac{3 \, A a^{2} \operatorname{arsinh}\left(\frac{b x}{\sqrt{a b}}\right)}{8 \, \sqrt{b}} + \frac{{\left(b x^{2} + a\right)}^{\frac{5}{2}} B}{5 \, b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2 + a)*A*a*x + 3/8*A*a^2*arcsinh(b
*x/sqrt(a*b))/sqrt(b) + 1/5*(b*x^2 + a)^(5/2)*B/b
```

Fricas [A]
time = 1.98, size = 176, normalized size = 2.02

$$\left[\frac{15 \, A a^{2} \sqrt{b} \log\left(-2 \, b x^{2} - 2 \, \sqrt{b x^{2} + a} \sqrt{b} x - a\right) + 2 \, {\left(8 \, B b^{2} x^{4} + 10 \, A b^{2} x^{3} + 16 \, B a b x^{2} + 25 \, A a b x + 8 \, B a^{2}\right)} \sqrt{b x^{2} + a}}{80 \, b}, -\frac{15 \, A a^{2} \sqrt{-b} \arctan\left(\frac{\sqrt{-b} x}{\sqrt{b x^{2} + a}}\right) - {\left(8 \, B b^{2} x^{4} + 10 \, A b^{2} x^{3} + 16 \, B a b x^{2} + 25 \, A a b x + 8 \, B a^{2}\right)} \sqrt{b x^{2} + a}}{40 \, b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/80*(15*A*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(8*B*b^2*x^4 + 10*A*b^2*x^3 + 16*B*a*b*x^2 + 25*A*a*b*x + 8*B*a^2)*sqrt(b*
x^2 + a))/b, -1/40*(15*A*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) -
(8*B*b^2*x^4 + 10*A*b^2*x^3 + 16*B*a*b*x^2 + 25*A*a*b*x + 8*B*a^2)*sqrt(b*x
^2 + a))/b]
```

Sympy [A]
time = 3.68, size = 219, normalized size = 2.52

$$\frac{A a^{\frac{3}{2}} x \sqrt{1 + \frac{b x^{2}}{a}}}{2} + \frac{A a^{\frac{3}{2}} x}{8 \sqrt{1 + \frac{b x^{2}}{a}}} + \frac{3 A \sqrt{a} b x^{3}}{8 \sqrt{1 + \frac{b x^{2}}{a}}} + \frac{3 A a^{2} \operatorname{asinh}{\left(\frac{\sqrt{b} x}{\sqrt{a}} \right)}}{8 \sqrt{b}} + \frac{A b^{2} x^{5}}{4 \sqrt{a} \sqrt{1 + \frac{b x^{2}}{a}}} + B a \left(\begin{cases} \frac{\sqrt{a} x^{2}}{2} & \text{for}\: b = 0 \\\frac{\left(a + b x^{2}\right)^{\frac{3}{2}}}{3 b} & \text{otherwise} \end{cases}\right) + B b \left(\begin{cases} - \frac{2 a^{2} \sqrt{a + b x^{2}}}{15 b^{2}} + \frac{a x^{2} \sqrt{a + b x^{2}}}{15 b} + \frac{x^{4} \sqrt{a + b x^{2}}}{5} & \text{for}\: b \neq 0 \\\frac{\sqrt{a} x^{4}}{4} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2),x)
```

```
[Out] A*a**(3/2)*x*sqrt(1 + b*x**2/a)/2 + A*a**(3/2)*x/(8*sqrt(1 + b*x**2/a)) + 3
*A*sqrt(a)*b*x**3/(8*sqrt(1 + b*x**2/a)) + 3*A*a**2*asinh(sqrt(b)*x/sqrt(a)
)/(8*sqrt(b)) + A*b**2*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + B*a*Piecewise(
(sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) + B*b*Piecew
ise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) +
x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True))
```

Giac [A]
time = 1.13, size = 76, normalized size = 0.87

$$-\frac{3\,Aa^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}+\frac{1}{40}\,\sqrt{bx^2+a}\left(\frac{8\,Ba^2}{b}+(25\,Aa+2\,(8\,Ba+(4\,Bbx+5\,Ab)x)x)x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -3/8*A*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/40*sqrt(b*x^2
 + a)*(8*B*a^2/b + (25*A*a + 2*(8*B*a + (4*B*b*x + 5*A*b)*x)*x)*x)
```

Mupad [B]
time = 1.18, size = 54, normalized size = 0.62

$$\frac{B\,{\left(b\,x^2+a\right)}^{5/2}}{5\,b}+\frac{A\,x\,{\left(b\,x^2+a\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(3/2)*(A + B*x),x)
```

```
[Out] (B*(a + b*x^2)^(5/2))/(5*b) + (A*x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2],
 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(3/2)
```

3.12 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx$

Optimal. Leaf size=106

$$\frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2}+\frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2}+\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}-a^{3/2}A\tanh^{-1}\Bigg(\frac{\sqrt{a+b}}{\sqrt{a}}$$

```
[Out] 1/12*(3*B*x+4*A)*(b*x^2+a)^(3/2)-a^(3/2)*A*arctanh((b*x^2+a)^(1/2)/a^(1/2))
+3/8*a^2*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+1/8*a*(3*B*x+8*A)*(b*
x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {829, 858, 223, 212, 272, 65, 214}

$$-a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}+\frac{1}{8}a\sqrt{a+bx^2}\,(8A+3Bx)+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4A+3Bx)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(3/2))/x,x]
```

```
[Out] (a*(8*A + 3*B*x)*Sqrt[a + b*x^2])/8 + ((4*A + 3*B*x)*(a + b*x^2)^(3/2))/12
+ (3*a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*Sqrt[b]) - a^(3/2)*A*Ar
cTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx = \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{\int \frac{(4aAb+3abBx)\sqrt{a+bx^2}}{x}\,dx}{4b}$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{\int \frac{8a^2Ab^2+3a^2b^2Bx}{x\sqrt{a+bx^2}}}{8b^2}$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \left(a^2A\right)\int \frac{1}{x\sqrt{a+}}$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{1}{2}\left(a^2A\right)\text{Subst}\Bigg(\int$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{3a^2B\tanh^{-1}\Bigg(}{8\sqrt{b}}$$

$$= \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{3a^2B\tanh^{-1}\Bigg(}{8\sqrt{b}}$$

Mathematica [A]
time = 0.31, size = 110, normalized size = 1.04

$$2a^{3/2}A\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{24}\left(\sqrt{a+bx^2}\left(32aA+15aBx+8Abx^2+6bBx^3\right)-\frac{9a^2B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x,x]
```

```
[Out] 2*a^(3/2)*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + (Sqrt[a + b*x^
2]*(32*a*A + 15*a*B*x + 8*A*b*x^2 + 6*b*B*x^3) - (9*a^2*B*Log[-(Sqrt[b]*x)
+ Sqrt[a + b*x^2]])/Sqrt[b])/24
```

Maple [A]
time = 0.10, size = 109, normalized size = 1.03

method	result
default	$B\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+A\Bigg(\frac{\left(b\,x^2+a\right)^{\frac{3}{2}}}{3}+a\Bigg(\sqrt{b\,x^2+a}-\sqrt{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^
(1/2)+(b*x^2+a)^(1/2))))+A*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*
ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))
```

Maxima [A]
time = 0.27, size = 88, normalized size = 0.83

$$\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Bx+\frac{3}{8}\sqrt{bx^2+a}\,Bax+\frac{3\,Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}-Aa^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}A+\sqrt{bx^2+a}\,Aa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/4*(b*x^2 + a)^(3/2)*B*x + 3/8*sqrt(b*x^2 + a)*B*a*x + 3/8*B*a^2*arcsinh(b
*x/sqrt(a*b))/sqrt(b) - A*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^
2 + a)^(3/2)*A + sqrt(b*x^2 + a)*A*a
```

Fricas [A]
time = 1.26, size = 439, normalized size = 4.14

$$\left[\frac{9\,Ba^2\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+24\,Aa^{\frac{3}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,\left(6\,Bb^2x^3+8\,Ab^2x^2+15\,Babx+32\,Aab\right)\sqrt{bx^2+a}}{48\,b},-\frac{9\,Ba^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-12\,Aa^{\frac{3}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-\left(6\,Bb^2x^3+8\,Ab^2x^2+15\,Babx+32\,Aab\right)\sqrt{bx^2+a}}{24\,b},\frac{48\,A\sqrt{-a}\,ab\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+9\,Ba^2\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,\left(6\,Bb^2x^3+8\,Ab^2x^2+15\,Babx+32\,Aab\right)\sqrt{bx^2+a}}{48\,b},-\frac{9\,Ba^2\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-24\,A\sqrt{-a}\,ab\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(6\,Bb^2x^3+8\,Ab^2x^2+15\,Babx+32\,Aab\right)\sqrt{bx^2+a}}{24\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/48*(9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 24
*A*a^(3/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(6*B*b
^2*x^3 + 8*A*b^2*x^2 + 15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b, -1/24*(9*
B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 12*A*a^(3/2)*b*log(-(b*
x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (6*B*b^2*x^3 + 8*A*b^2*x^2 +
15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b, 1/48*(48*A*sqrt(-a)*a*b*arctan(s
qrt(-a)/sqrt(b*x^2 + a)) + 9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) + 2*(6*B*b^2*x^3 + 8*A*b^2*x^2 + 15*B*a*b*x + 32*A*a*b)*sqr
t(b*x^2 + a))/b, -1/24*(9*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 - 24*A*sqrt(-a)*a*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (6*B*b^2*x^3 + 8*A*
b^2*x^2 + 15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b]
```

Sympy [A]
time = 8.23, size = 218, normalized size = 2.06

$$-Aa^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Aa^2}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{Aa\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}}+Ab\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\: b=0\\\frac{\left(a+bx^2\right)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+\frac{Ba^{\frac{3}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{Ba^{\frac{3}{2}}x}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3B\sqrt{a}\,bx^3}{8\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{b}}+\frac{Bb^2x^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2)/x,x)
```

```
[Out] -A*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a**2/(sqrt(b)*x*sqrt(a/(b*x**2)
+ 1)) + A*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + A*b*Piecewise((sqrt(a)*x**2/2,
 Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) + B*a**(3/2)*x*sqrt(1 + b*x*
*2/a)/2 + B*a**(3/2)*x/(8*sqrt(1 + b*x**2/a)) + 3*B*sqrt(a)*b*x**3/(8*sqrt(
1 + b*x**2/a)) + 3*B*a**2*asinh(sqrt(b)*x/sqrt(a))/(8*sqrt(b)) + B*b**2*x**
5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 1.06, size = 100, normalized size = 0.94

$$\frac{2\,Aa^2\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{3\,Ba^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}} + \frac{1}{24}\,\sqrt{bx^2+a}\,(32\,Aa+(15\,Ba+2\,(3\,Bbx+4\,Ab)x)x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 2*A*a^2*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/8*B*a^
2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/24*sqrt(b*x^2 + a)*(32
*A*a + (15*B*a + 2*(3*B*b*x + 4*A*b)*x)*x)
```

Mupad [B]
time = 1.31, size = 83, normalized size = 0.78

$$\frac{A\,(b\,x^2+a)^{3/2}}{3} - A\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right) + A\,a\,\sqrt{b\,x^2+a} + \frac{B\,x\,(b\,x^2+a)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(3/2)*(A + B*x))/x,x)
```

```
[Out] (A*(a + b*x^2)^(3/2))/3 - A*a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + A*a*
(a + b*x^2)^(1/2) + (B*x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*
x^2)/a))/((b*x^2)/a + 1)^(3/2)
```

3.13 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx$

Optimal. Leaf size=108

$$\frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2}-\frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x}+\frac{3}{2}aA\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-a^{3/2}B\tanh^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a}}\right.$$

```
[Out] -1/3*(-B*x+3*A)*(b*x^2+a)^(3/2)/x-a^(3/2)*B*arctanh((b*x^2+a)^(1/2)/a^(1/2)
)+3/2*a*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)+1/2*(3*A*b*x+2*B*a)*(b
*x^2+a)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$,
Rules used = {827, 829, 858, 223, 212, 272, 65, 214}

$$a^{3/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{\left(a+bx^2\right)^{3/2}(3A-Bx)}{3x}+\frac{1}{2}\sqrt{a+bx^2}\,(2aB+3Abx)+\frac{3}{2}aA\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(3/2))/x^2,x]
```

```
[Out] ((2*a*B + 3*A*b*x)*Sqrt[a + b*x^2])/2 - ((3*A - B*x)*(a + b*x^2)^(3/2))/(3*
x) + (3*a*A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2 - a^(3/2)*B*Arc
Tanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx = -\frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} - \frac{1}{2}\int \frac{(-2aB-6Abx)\sqrt{a+bx^2}}{x}\,dx$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} - \frac{\int \frac{-4a^2bB-6aAb^2x}{x\sqrt{a+bx^2}}\,dx}{4b}$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{1}{2}(3aAb)\int \frac{1}{\sqrt{a+bx^2}}$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{1}{2}(3aAb)\text{Subst}\Bigg(\int \frac{}{1}$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{3}{2}aA\sqrt{b}\,\tanh^{-1}\Bigg(\frac{}{\sqrt{}}$$

$$= \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2} - \frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{3}{2}aA\sqrt{b}\,\tanh^{-1}\Bigg(\frac{}{\sqrt{}}$$

Mathematica [A]
time = 0.30, size = 111, normalized size = 1.03

$$\frac{\sqrt{a+bx^2}\left(bx^2(3A+2Bx)+a(-6A+8Bx)\right)}{6x} + 2a^{3/2}B\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{3}{2}aA\sqrt{b}\,\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(b*x^2*(3*A + 2*B*x) + a*(-6*A + 8*B*x)))/(6*x) + 2*a^(3/2
)*B*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] - (3*a*A*Sqrt[b]*Log[-(S
qrt[b]*x) + Sqrt[a + b*x^2]])/2
```

Maple [A]
time = 0.13, size = 133, normalized size = 1.23

method	result
risch	$-\frac{aA\sqrt{b\,x^2+a}}{x} + \frac{Bb\,x^2\sqrt{b\,x^2+a}}{3} + \frac{4aB\sqrt{b\,x^2+a}}{3} + \frac{bAx\sqrt{b\,x^2+a}}{2} + \frac{3a\sqrt{b}\,A\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right.}{2}$

default	$B\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\;\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)\right)+A\left(-\frac{(b\,x^2+a)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x\left(b\,x^2+a\right.}{4}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+
a)^(1/2))/x)))+A*(-1/a/x*(b*x^2+a)^(5/2)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a
*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))
```

Maxima [**A**]
time = 0.29, size = 88, normalized size = 0.81

$$\frac{3}{2}\sqrt{bx^2+a}\,Abx+\frac{3}{2}Aa\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-Ba^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}B+\sqrt{bx^2+a}\,Ba-\frac{\left(bx^2+a\right)^{\frac{3}{2}}A}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(b*x^2 + a)*A*b*x + 3/2*A*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - B*a^(3
/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2)*B + sqrt(b*x^2 +
a)*B*a - (b*x^2 + a)^(3/2)*A/x
```

Fricas [**A**]
time = 1.31, size = 411, normalized size = 3.81

$$\left[\frac{9\,Aa\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+6\,Ba^{\frac{3}{2}}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(2\,Bbx^3+3\,Abx^2+8\,Bax-6\,Aa)\sqrt{bx^2+a}}{12\,x},\,-\frac{9\,Aa\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-3\,Ba^{\frac{3}{2}}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-(2\,Bbx^3+3\,Abx^2+8\,Bax-6\,Aa)\sqrt{bx^2+a}}{6\,x},\,\frac{12\,B\sqrt{-a}\,ax\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+9\,Aa\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(2\,Bbx^3+3\,Abx^2+8\,Bax-6\,Aa)\sqrt{bx^2+a}}{12\,x},\,-\frac{9\,Aa\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-6\,B\sqrt{-a}\,ax\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(2\,Bbx^3+3\,Abx^2+8\,Bax-6\,Aa)\sqrt{bx^2+a}}{6\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/12*(9*A*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 6*
B*a^(3/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*B*b*
x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 + a))/x, -1/6*(9*A*a*sqrt(-b)
*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*B*a^(3/2)*x*log(-(b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (2*B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*
sqrt(b*x^2 + a))/x, 1/12*(12*B*sqrt(-a)*a*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)
```

```
) + 9*A*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*
B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 + a))/x, -1/6*(9*A*a*sqrt
(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 6*B*sqrt(-a)*a*x*arctan(sqrt(-a
)/sqrt(b*x^2 + a)) - (2*B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 +
 a))/x]
```

Sympy [A]

time = 3.25, size = 184, normalized size = 1.70

$$-\frac{Aa^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{A\sqrt{a}\,bx\sqrt{1+\frac{bx^2}{a}}}{2}-\frac{A\sqrt{a}\,bx}{\sqrt{1+\frac{bx^2}{a}}}+\frac{3Aa\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2}-Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Ba^2}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ba\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}}+Bb\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for } b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2)/x**2,x)
```

```
[Out] -A*a**(3/2)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(a)*b*x*sqrt(1 + b*x**2/a)/2 - A
*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + 3*A*a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/2
- B*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**2/(sqrt(b)*x*sqrt(a/(b*x**2)
 + 1)) + B*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*b*Piecewise((sqrt(a)*x**2/2
, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True))
```

Giac [A]

time = 0.88, size = 124, normalized size = 1.15

$$\frac{2\,Ba^2\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3}{2}\,Aa\sqrt{b}\,\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)+\frac{2\,Aa^2\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}+\frac{1}{6}\,\sqrt{bx^2+a}\,(8\,Ba+(2\,Bbx+3\,Ab)x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a^2*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/2*A*a*
sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 2*A*a^2*sqrt(b)/((sqrt(b)*
x - sqrt(b*x^2 + a))^2 - a) + 1/6*sqrt(b*x^2 + a)*(8*B*a + (2*B*b*x + 3*A*b
)*x)
```

Mupad [B]

time = 1.88, size = 86, normalized size = 0.80

$$\frac{B\,{\left(b\,x^2+a\right)}^{3/2}}{3}-B\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)+B\,a\,\sqrt{b\,x^2+a}-\frac{A\,{\left(b\,x^2+a\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(3/2)*(A + B*x))/x^2,x)
```

```
[Out] (B*(a + b*x^2)^(3/2))/3 - B*a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + B*a*
(a + b*x^2)^(1/2) - (A*(a + b*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(b*x
^2)/a))/(x*((b*x^2)/a + 1)^(3/2))
```

3.14 $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=111

$$-\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x}-\frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2}+\frac{3}{2}a\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{3}{2}\sqrt{a}\,Ab\tanh^{-1}\Bigg(\sqrt{a}$$

```
[Out] -1/2*(-B*x+A)*(b*x^2+a)^(3/2)/x^2-3/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))*
a^(1/2)+3/2*a*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)-3/2*(-A*b*x+B*a)
*(b*x^2+a)^(1/2)/x
```

Rubi [A]

time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {827, 858, 223, 212, 272, 65, 214}

$$-\frac{\left(a+bx^2\right)^{3/2}(A-Bx)}{2x^2}-\frac{3\sqrt{a+bx^2}\,(aB-Abx)}{2x}-\frac{3}{2}\sqrt{a}\,Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{3}{2}a\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(3/2))/x^3,x]
```

```
[Out] (-3*(a*B - A*b*x)*Sqrt[a + b*x^2])/(2*x) - ((A - B*x)*(a + b*x^2)^(3/2))/(2
*x^2) + (3*a*Sqrt[b]*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/2 - (3*Sqrt[a]
*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx &= -\frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2}-\frac{3}{8}\int \frac{\left(-4aB-4Abx\right)\sqrt{a+bx^2}}{x^2}\,dx \\ &= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x}-\frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2}+\frac{3}{16}\int \frac{8aAb+8abBx}{x\sqrt{a+bx^2}} \\ &= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x}-\frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2}+\frac{1}{2}(3aAb)\int \frac{1}{x\sqrt{a+b}} \\ &= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x}-\frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2}+\frac{1}{4}(3aAb)\text{Subst}\Bigg(\int \frac{}{x} \\ &= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x}-\frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2}+\frac{3}{2}a\sqrt{b}\,B\tanh^{-1}\Bigg(\frac{}{\sqrt{}} \\ &= -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x}-\frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2}+\frac{3}{2}a\sqrt{b}\,B\tanh^{-1}\Bigg(\frac{}{\sqrt{}}\end{aligned}$$

Mathematica [A]

time = 0.36, size = 109, normalized size = 0.98

$$\frac{1}{2}\left(\frac{\sqrt{a+bx^2}\,\left(bx^2(2A+Bx)-a(A+2Bx)\right)}{x^2}+6\sqrt{a}\,Ab\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)-3a\sqrt{b}\,B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x^3,x]
```

```
[Out] ((Sqrt[a + b*x^2]*(b*x^2*(2*A + B*x) - a*(A + 2*B*x)))/x^2 + 6*Sqrt[a]*A*b*
ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] - 3*a*Sqrt[b]*B*Log[-(Sqrt[b
]*x) + Sqrt[a + b*x^2]])/2
```

Maple [A]

time = 0.12, size = 157, normalized size = 1.41

method	result
risch	$-\frac{a\sqrt{b\,x^2+a}\,(2Bx+A)}{2x^2}+\frac{bBx\sqrt{b\,x^2+a}}{2}+\frac{3\sqrt{b}\,Ba\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2}+bA\sqrt{b\,x^2+a}-\frac{3bA\sqrt{a}}{}$

default	$A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{2a\,x^{2}}+\frac{3b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)}{2a}\right)+B\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{ax}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/2/a/x^2*(b*x^2+a)^(5/2)+3/2*b/a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/
2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))+B*(-1/a/x*(b*x^2+a)^(5/
2)+4*b/a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*
ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))
```

<u>Maxima</u> [A]
time = 0.27, size = 112, normalized size = 1.01

$$\frac{3}{2}\sqrt{bx^{2}+a}\,Bbx+\frac{3}{2}\,Ba\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{3}{2}\,A\sqrt{a}\,b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{3}{2}\sqrt{bx^{2}+a}\,Ab+\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}Ab}{2\,a}-\frac{\left(bx^{2}+a\right)^{\frac{3}{2}}B}{x}-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}A}{2\,ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(b*x^2 + a)*B*b*x + 3/2*B*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - 3/2*A*
sqrt(a)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 3/2*sqrt(b*x^2 + a)*A*b + 1/2*(b*
x^2 + a)^(3/2)*A*b/a - (b*x^2 + a)^(3/2)*B/x - 1/2*(b*x^2 + a)^(5/2)*A/(a*x
^2)
```

<u>Fricas</u> [A]
time = 1.15, size = 425, normalized size = 3.83

$$\left[\frac{3\,Ba\sqrt{b}\,x^{2}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+3\,A\sqrt{a}\,bx^{2}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,\left(Bbx^{3}+2\,Abx^{2}-2\,Bax-Aa\right)\sqrt{bx^{2}+a}}{4\,x^{2}},\;-\frac{6\,Ba\sqrt{-b}\,x^{2}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-3\,A\sqrt{a}\,bx^{2}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)-2\,\left(Bbx^{3}+2\,Abx^{2}-2\,Bax-Aa\right)\sqrt{bx^{2}+a}}{4\,x^{2}},\;\frac{6\,A\sqrt{-a}\,bx^{2}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+3\,Ba\sqrt{b}\,x^{2}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,\left(Bbx^{3}+2\,Abx^{2}-2\,Bax-Aa\right)\sqrt{bx^{2}+a}}{4\,x^{2}},\;-\frac{3\,Ba\sqrt{-b}\,x^{2}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-3\,A\sqrt{-a}\,bx^{2}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)-\left(Bbx^{3}+2\,Abx^{2}-2\,Bax-Aa\right)\sqrt{bx^{2}+a}}{2\,x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 3
*A*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(B
*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqrt(b*x^2 + a))/x^2, -1/4*(6*B*a*sqrt(
-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*A*sqrt(a)*b*x^2*log(-(b*x^2
```

```
- 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(B*b*x^3 + 2*A*b*x^2 - 2*B*a*x
- A*a)*sqrt(b*x^2 + a))/x^2, 1/4*(6*A*sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b
*x^2 + a)) + 3*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
 a) + 2*(B*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqrt(b*x^2 + a))/x^2, -1/2*(3
*B*a*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*A*sqrt(-a)*b*x^2*a
rctan(sqrt(-a)/sqrt(b*x^2 + a)) - (B*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqr
t(b*x^2 + a))/x^2]
```

Sympy [**A**]
time = 3.78, size = 182, normalized size = 1.64

$$-\frac{3A\sqrt{a}\,b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2}-\frac{Aa\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{2x}+\frac{Aa\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}}+\frac{Ab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}}-\frac{Ba^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}}+\frac{B\sqrt{a}\,bx\sqrt{1+\frac{bx^2}{a}}}{2}-\frac{B\sqrt{a}\,bx}{\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba\sqrt{b}\,\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(3/2)/x**3,x)
```

```
[Out] -3*A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a*sqrt(b)*sqrt(a/(b*x**2) +
 1)/(2*x) + A*a*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + A*b**(3/2)*x/sqrt(a/(b*x
**2) + 1) - B*a**(3/2)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(a)*b*x*sqrt(1 + b*x*
*2/a)/2 - B*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + 3*B*a*sqrt(b)*asinh(sqrt(b)*x/
sqrt(a))/2
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(88) = 176.
time = 1.58, size = 191, normalized size = 1.72

$$\frac{3\,Aab\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{3}{2}\,Ba\sqrt{b}\,\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)+\frac{1}{2}\,(Bbx+2\,Ab)\sqrt{bx^2+a}+\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^3Aab+2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)Aa^2b-2\,Ba^3\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 3*A*a*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/2*B*a*
sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/2*(B*b*x + 2*A*b)*sqrt(b
*x^2 + a) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a*b + 2*(sqrt(b)*x - sqrt(b*
x^2 + a))^2*B*a^2*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a^2*b - 2*B*a^3
*sqrt(b))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2
```

Mupad [**B**]
time = 2.12, size = 91, normalized size = 0.82

$$A\,b\,\sqrt{b\,x^2+a}-\frac{A\,a\,\sqrt{b\,x^2+a}}{2\,x^2}-\frac{3\,A\,\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2}-\frac{B\,{\left(b\,x^2+a\right)}^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(3/2)*(A + B*x))/x^3,x)
```

```
[Out] A*b*(a + b*x^2)^(1/2) - (A*a*(a + b*x^2)^(1/2))/(2*x^2) - (3*A*a^(1/2)*b*at
anh((a + b*x^2)^(1/2)/a^(1/2)))/2 - (B*(a + b*x^2)^(3/2)*hypergeom([-3/2, -
1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a + 1)^(3/2))
```

3.15 $\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=173

$$\frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx(a+bx^2)^{3/2}}{128b^2}+\frac{a^2Bx(a+bx^2)^{5/2}}{160b^2}+\frac{Ax^2(a+bx^2)^{7/2}}{9b}+\frac{Bx^3(a+bx^2)^{7/2}}{10b}-\frac{a(160A+1}{}$$

```
[Out] 1/128*a^3*B*x*(b*x^2+a)^(3/2)/b^2+1/160*a^2*B*x*(b*x^2+a)^(5/2)/b^2+1/9*A*x
^2*(b*x^2+a)^(7/2)/b+1/10*B*x^3*(b*x^2+a)^(7/2)/b-1/5040*a*(189*B*x+160*A)*
(b*x^2+a)^(7/2)/b^2+3/256*a^5*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+
3/256*a^4*B*x*(b*x^2+a)^(1/2)/b^2
```

Rubi [A]
time = 0.07, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {847, 794, 201, 223, 212}

$$\frac{3a^5B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}+\frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx(a+bx^2)^{3/2}}{128b^2}+\frac{a^2Bx(a+bx^2)^{5/2}}{160b^2}-\frac{a(a+bx^2)^{7/2}(160A+189Bx)}{5040b^2}+\frac{Ax^2(a+bx^2)^{7/2}}{9b}+\frac{Bx^3(a+bx^2)^{7/2}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (3*a^4*B*x*Sqrt[a + b*x^2])/(256*b^2) + (a^3*B*x*(a + b*x^2)^(3/2))/(128*b^
2) + (a^2*B*x*(a + b*x^2)^(5/2))/(160*b^2) + (A*x^2*(a + b*x^2)^(7/2))/(9*b
) + (B*x^3*(a + b*x^2)^(7/2))/(10*b) - (a*(160*A + 189*B*x)*(a + b*x^2)^(7/
2))/(5040*b^2) + (3*a^5*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(5/2
))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx &= \frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}+\frac{\int x^2(-3aB+10Abx)\left(a+bx^2\right)^{5/2}\,dx}{10b}\\
&= \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}+\frac{\int x(-20aAb-27abBx)\left(a+bx^2\right)}{90b^2}\\
&= \frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}-\frac{a(160A+189Bx)\left(a+bx^2\right)^{7/2}}{5040b^2}+\\
&= \frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}+\frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b}+\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}-\\
&= \frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}+\frac{Ax^2\left(a+bx^2\right)^{7/2}}{9b}+\frac{Bx^3\left(a+bx^2\right)}{10b}\\
&= \frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}+\frac{Ax^2\left(a+bx^2\right.}{9b}\\
&= \frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}+\frac{Ax^2\left(a+bx^2\right.}{9b}\\
&= \frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx\left(a+bx^2\right)^{3/2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{5/2}}{160b^2}+\frac{Ax^2\left(a+bx^2\right.}{9b}
\end{aligned}$$

Mathematica [A]
time = 0.33, size = 137, normalized size = 0.79

$$\frac{\sqrt{b}\,\sqrt{a+bx^2}\,(896b^4x^8(10A+9Bx)+10a^3bx^2(128A+63Bx)-5a^4(512A+189Bx)+24a^2b^2x^4(800A+651Bx)+16ab^3x^6(1520A+1323Bx))-945a^5B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{80640b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[a + b*x^2]*(896*b^4*x^8*(10*A + 9*B*x) + 10*a^3*b*x^2*(128*A
+ 63*B*x) - 5*a^4*(512*A + 189*B*x) + 24*a^2*b^2*x^4*(800*A + 651*B*x) + 16
*a*b^3*x^6*(1520*A + 1323*B*x)) - 945*a^5*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x
^2]])/(80640*b^(5/2))
```

Maple [A]
time = 0.11, size = 152, normalized size = 0.88

method	result
risch	$-\frac{(-8064B\,b^4x^9-8960A\,b^4x^8-21168Ba\,b^3x^7-24320a\,b^3A\,x^6-15624B\,a^2b^2x^5-19200a^2A\,b^2x^4-630B\,a^3b\,x^3-1280A\,a^3b\,x^2+945}{80640b^2}$

default	$B\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{10b}-\frac{3a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{8b}\right.}{10b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/10*x^3*(b*x^2+a)^(7/2)/b-3/10*a/b*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/
6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/
2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))+A*(1/9*x^2*(b*x^2+a)^(7
/2)/b-2/63*a/b^2*(b*x^2+a)^(7/2))
```

Maxima [A]
time = 0.28, size = 145, normalized size = 0.84

$$\frac{(bx^2+a)^{\frac{7}{2}}Bx^3}{10\,b}+\frac{(bx^2+a)^{\frac{7}{2}}Ax^2}{9\,b}-\frac{3\,(bx^2+a)^{\frac{7}{2}}Bax}{80\,b^2}+\frac{(bx^2+a)^{\frac{5}{2}}Ba^2x}{160\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}Ba^3x}{128\,b^2}+\frac{3\,\sqrt{bx^2+a}\,Ba^4x}{256\,b^2}+\frac{3\,Ba^5\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}}-\frac{2\,(bx^2+a)^{\frac{7}{2}}Aa}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/10*(b*x^2 + a)^(7/2)*B*x^3/b + 1/9*(b*x^2 + a)^(7/2)*A*x^2/b - 3/80*(b*x^
2 + a)^(7/2)*B*a*x/b^2 + 1/160*(b*x^2 + a)^(5/2)*B*a^2*x/b^2 + 1/128*(b*x^2
 + a)^(3/2)*B*a^3*x/b^2 + 3/256*sqrt(b*x^2 + a)*B*a^4*x/b^2 + 3/256*B*a^5*a
rcsinh(b*x/sqrt(a*b))/b^(5/2) - 2/63*(b*x^2 + a)^(7/2)*A*a/b^2
```

Fricas [A]
time = 1.26, size = 302, normalized size = 1.75

$$\left[\frac{945\,Ba^5\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(8064\,Bb^5x^9+8960\,Ab^5x^8+21168\,Bab^4x^7+24320\,Aab^4x^6+15624\,Ba^2b^3x^5+19200\,Aa^2b^3x^4+630\,Ba^3b^2x^3+1280\,Aa^3b^2x^2-945\,Ba^4bx-2560\,Aa^4b)\sqrt{bx^2+a}}{161280\,b^3},-\frac{945\,Ba^5\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(8064\,Bb^5x^9+8960\,Ab^5x^8+21168\,Bab^4x^7+24320\,Aab^4x^6+15624\,Ba^2b^3x^5+19200\,Aa^2b^3x^4+630\,Ba^3b^2x^3+1280\,Aa^3b^2x^2-945\,Ba^4bx-2560\,Aa^4b)\sqrt{bx^2+a}}{80640\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/161280*(945*B*a^5*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) + 2*(8064*B*b^5*x^9 + 8960*A*b^5*x^8 + 21168*B*a*b^4*x^7 + 24320*A*a*b^4*
x^6 + 15624*B*a^2*b^3*x^5 + 19200*A*a^2*b^3*x^4 + 630*B*a^3*b^2*x^3 + 1280*
A*a^3*b^2*x^2 - 945*B*a^4*b*x - 2560*A*a^4*b)*sqrt(b*x^2 + a))/b^3, -1/8064
0*(945*B*a^5*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8064*B*b^5*x^9
+ 8960*A*b^5*x^8 + 21168*B*a*b^4*x^7 + 24320*A*a*b^4*x^6 + 15624*B*a^2*b^3*
x^5 + 19200*A*a^2*b^3*x^4 + 630*B*a^3*b^2*x^3 + 1280*A*a^3*b^2*x^2 - 945*B*
a^4*b*x - 2560*A*a^4*b)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A]
time = 136.14, size = 469, normalized size = 2.71

$$Aa^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+2Aab\left(\begin{cases}\frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{ax^4\sqrt{a+bx^2}}{35b}+\frac{x^6\sqrt{a+bx^2}}{7} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^6}{6} & \text{otherwise}\end{cases}\right)+Ab^2\left(\begin{cases}-\frac{16a^4\sqrt{a+bx^2}}{315b^4}+\frac{8a^3x^2\sqrt{a+bx^2}}{315b^3}-\frac{2a^2x^4\sqrt{a+bx^2}}{105b^2}+\frac{ax^6\sqrt{a+bx^2}}{63b}+\frac{x^8\sqrt{a+bx^2}}{9} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^8}{8} & \text{otherwise}\end{cases}\right)-\frac{3Ba^{\frac{9}{2}}x}{256b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{Ba^{\frac{7}{2}}x^3}{256b\sqrt{1+\frac{bx^2}{a}}}+\frac{129Ba^{\frac{5}{2}}x^5}{640\sqrt{1+\frac{bx^2}{a}}}+\frac{73Ba^{\frac{3}{2}}bx^7}{160\sqrt{1+\frac{bx^2}{a}}}+\frac{29B\sqrt{a}\,b^2x^9}{80\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^5\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{256b^{\frac{5}{2}}}+\frac{Bb^3x^{11}}{10\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] A*a**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x*
*2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) +
```

```
2*A*a*b*Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a
+ b*x**2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**
2)/7, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + A*b**2*Piecewise((-16*a**4*sqrt(
a + b*x**2)/(315*b**4) + 8*a**3*x**2*sqrt(a + b*x**2)/(315*b**3) - 2*a**2*x
**4*sqrt(a + b*x**2)/(105*b**2) + a*x**6*sqrt(a + b*x**2)/(63*b) + x**8*sqr
t(a + b*x**2)/9, Ne(b, 0)), (sqrt(a)*x**8/8, True)) - 3*B*a**(9/2)*x/(256*b
**2*sqrt(1 + b*x**2/a)) - B*a**(7/2)*x**3/(256*b*sqrt(1 + b*x**2/a)) + 129*
B*a**(5/2)*x**5/(640*sqrt(1 + b*x**2/a)) + 73*B*a**(3/2)*b*x**7/(160*sqrt(1
 + b*x**2/a)) + 29*B*sqrt(a)*b**2*x**9/(80*sqrt(1 + b*x**2/a)) + 3*B*a**5*a
sinh(sqrt(b)*x/sqrt(a))/(256*b**(5/2)) + B*b**3*x**11/(10*sqrt(a)*sqrt(1 +
b*x**2/a))
```

Giac [A]
time = 1.47, size = 140, normalized size = 0.81

$$-\frac{3\,Ba^5\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}-\frac{1}{80640}\left(\frac{2560\,Aa^4}{b^2}+\left(\frac{945\,Ba^4}{b^2}-2\left(\frac{640\,Aa^3}{b}+\left(\frac{315\,Ba^3}{b}+4\left(2400\,Aa^2+\left(1953\,Ba^2+2\left(1520\,Aab+7\left(189\,Bab+8\left(9\,Bb^2x+10\,Ab^2\right)x\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -3/256*B*a^5*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/80640*(2560
*A*a^4/b^2 + (945*B*a^4/b^2 - 2*(640*A*a^3/b + (315*B*a^3/b + 4*(2400*A*a^2
 + (1953*B*a^2 + 2*(1520*A*a*b + 7*(189*B*a*b + 8*(9*B*b^2*x + 10*A*b^2)*x)
*x)*x)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\left(b\,x^2+a\right)}^{5/2}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^(5/2)*(A + B*x),x)
```

```
[Out] int(x^3*(a + b*x^2)^(5/2)*(A + B*x), x)
```

3.16 $\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=150

$$-\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax(a+bx^2)^{3/2}}{192b}-\frac{aAx(a+bx^2)^{5/2}}{48b}+\frac{Bx^2(a+bx^2)^{7/2}}{9b}-\frac{(16aB-63Abx)\,(a+bx^2)^{7/}}{504b^2}$$

```
[Out] -5/192*a^2*A*x*(b*x^2+a)^(3/2)/b-1/48*a*A*x*(b*x^2+a)^(5/2)/b+1/9*B*x^2*(b*
x^2+a)^(7/2)/b-1/504*(-63*A*b*x+16*B*a)*(b*x^2+a)^(7/2)/b^2-5/128*a^4*A*arc
tanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-5/128*a^3*A*x*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.05, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {847, 794, 201, 223, 212}

$$-\frac{5a^4A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax(a+bx^2)^{3/2}}{192b}-\frac{(a+bx^2)^{7/2}\,(16aB-63Abx)}{504b^2}-\frac{aAx(a+bx^2)^{5/2}}{48b}+\frac{Bx^2(a+bx^2)^{7/2}}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (-5*a^3*A*x*Sqrt[a + b*x^2])/(128*b) - (5*a^2*A*x*(a + b*x^2)^(3/2))/(192*b
) - (a*A*x*(a + b*x^2)^(5/2))/(48*b) + (B*x^2*(a + b*x^2)^(7/2))/(9*b) - ((
16*a*B - 63*A*b*x)*(a + b*x^2)^(7/2))/(504*b^2) - (5*a^4*A*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx &= \frac{Bx^2(a+bx^2)^{7/2}}{9b}+\frac{\int x(-2aB+9Abx)\left(a+bx^2\right)^{5/2}\,dx}{9b}\\
&= \frac{Bx^2(a+bx^2)^{7/2}}{9b}-\frac{(16aB-63Abx)\left(a+bx^2\right)^{7/2}}{504b^2}-\frac{(aA)\int\left(a+bx^2\right)^{5/2}\,dx}{8b}\\
&= -\frac{aAx(a+bx^2)^{5/2}}{48b}+\frac{Bx^2(a+bx^2)^{7/2}}{9b}-\frac{(16aB-63Abx)\left(a+bx^2\right)^{7/2}}{504b^2}-\\
&= -\frac{5a^2Ax(a+bx^2)^{3/2}}{192b}-\frac{aAx(a+bx^2)^{5/2}}{48b}+\frac{Bx^2(a+bx^2)^{7/2}}{9b}-(16aB-63\\
&= -\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax(a+bx^2)^{3/2}}{192b}-\frac{aAx(a+bx^2)^{5/2}}{48b}+\frac{Bx^2(a+b}{9b}\\
&= -\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax(a+bx^2)^{3/2}}{192b}-\frac{aAx(a+bx^2)^{5/2}}{48b}+\frac{Bx^2(a+b}{9b}\\
&= -\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax(a+bx^2)^{3/2}}{192b}-\frac{aAx(a+bx^2)^{5/2}}{48b}+\frac{Bx^2(a+b}{9b}
\end{aligned}$$

Mathematica [A]

time = 0.32, size = 125, normalized size = 0.83

$$\frac{\sqrt{a+bx^2}\left(-256a^4B+112b^4x^7(9A+8Bx)+a^3bx(315A+128Bx)+8ab^3x^5(357A+304Bx)+6a^2b^2x^3(413A+320Bx)\right)+315a^4A\sqrt{b}\,\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8064b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-256*a^4*B + 112*b^4*x^7*(9*A + 8*B*x) + a^3*b*x*(315*A +
 128*B*x) + 8*a*b^3*x^5*(357*A + 304*B*x) + 6*a^2*b^2*x^3*(413*A + 320*B*x)
) + 315*a^4*A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8064*b^2)
```

Maple [A]

time = 0.11, size = 128, normalized size = 0.85

method	result
default	$B\left(\frac{x^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{9b}-\frac{2a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{63b^{2}}\right)+A\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{}\right.}{}\right.}{4}\right.}{6}\right.}{8b}\right.$
risch	$\frac{\left(896B\,b^{4}x^{8}+1008A\,b^{4}x^{7}+2432a\,b^{3}B\,x^{6}+2856Aa\,b^{3}x^{5}+1920a^{2}B\,b^{2}x^{4}+2478A\,a^{2}b^{2}x^{3}+128B\,a^{3}b\,x^{2}+315A\,a^{3}bx-256B\,a^{4}\right)\sqrt{b\,x^{2}}}{8064b^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/9*x^2*(b*x^2+a)^(7/2)/b-2/63*a/b^2*(b*x^2+a)^(7/2))+A*(1/8*x*(b*x^2+a)
^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*
(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))
```

Maxima [A]

time = 0.28, size = 124, normalized size = 0.83

$$\frac{(bx^2+a)^{\frac{7}{2}}Bx^2}{9\,b}+\frac{(bx^2+a)^{\frac{7}{2}}Ax}{8\,b}-\frac{(bx^2+a)^{\frac{5}{2}}Aax}{48\,b}-\frac{5\,(bx^2+a)^{\frac{3}{2}}Aa^2x}{192\,b}-\frac{5\,\sqrt{bx^2+a}\,Aa^3x}{128\,b}-\frac{5\,Aa^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}-\frac{2\,(bx^2+a)^{\frac{7}{2}}Ba}{63\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/9*(b*x^2 + a)^(7/2)*B*x^2/b + 1/8*(b*x^2 + a)^(7/2)*A*x/b - 1/48*(b*x^2 +
 a)^(5/2)*A*a*x/b - 5/192*(b*x^2 + a)^(3/2)*A*a^2*x/b - 5/128*sqrt(b*x^2 +
a)*A*a^3*x/b - 5/128*A*a^4*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/63*(b*x^2 + a
)^(7/2)*B*a/b^2
```

Fricas [A]
time = 1.94, size = 271, normalized size = 1.81

$$\left[\frac{315\,Aa^4\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(896\,Bb^4x^8+1008\,Ab^4x^7+2432\,Bab^3x^6+2856\,Aab^3x^5+1920\,Ba^2b^2x^4+2478\,Aa^2b^2x^3+128\,Ba^3bx^2+315\,Aa^3bx-256\,Ba^4)\sqrt{bx^2+a}}{16128\,b^2},\ \frac{315\,Aa^4\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(896\,Bb^4x^8+1008\,Ab^4x^7+2432\,Bab^3x^6+2856\,Aab^3x^5+1920\,Ba^2b^2x^4+2478\,Aa^2b^2x^3+128\,Ba^3bx^2+315\,Aa^3bx-256\,Ba^4)\sqrt{bx^2+a}}{8064\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/16128*(315*A*a^4*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 2*(896*B*b^4*x^8 + 1008*A*b^4*x^7 + 2432*B*a*b^3*x^6 + 2856*A*a*b^3*x^5
+ 1920*B*a^2*b^2*x^4 + 2478*A*a^2*b^2*x^3 + 128*B*a^3*b*x^2 + 315*A*a^3*b*x
 - 256*B*a^4)*sqrt(b*x^2 + a))/b^2, 1/8064*(315*A*a^4*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) + (896*B*b^4*x^8 + 1008*A*b^4*x^7 + 2432*B*a*b^3*x^6
 + 2856*A*a*b^3*x^5 + 1920*B*a^2*b^2*x^4 + 2478*A*a^2*b^2*x^3 + 128*B*a^3*b
*x^2 + 315*A*a^3*b*x - 256*B*a^4)*sqrt(b*x^2 + a))/b^2]
```

Sympy [A]
time = 27.29, size = 442, normalized size = 2.95

$$\frac{5Aa^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^2}{a}}}+\frac{133Aa^{\frac{5}{2}}x^3}{384\sqrt{1+\frac{bx^2}{a}}}+\frac{127Aa^{\frac{3}{2}}bx^5}{192\sqrt{1+\frac{bx^2}{a}}}+\frac{23A\sqrt{a}\,b^2x^7}{48\sqrt{1+\frac{bx^2}{a}}}-\frac{5Aa^4\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{Ab^3x^9}{8\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}+Ba^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+2Bab\left(\begin{cases}\frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{ax^4\sqrt{a+bx^2}}{35b}+\frac{x^6\sqrt{a+bx^2}}{7} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^6}{6} & \text{otherwise}\end{cases}\right)+Bb^2\left(\begin{cases}-\frac{16a^4\sqrt{a+bx^2}}{315b^4}+\frac{8a^3x^2\sqrt{a+bx^2}}{315b^3}-\frac{2a^2x^4\sqrt{a+bx^2}}{105b^2}+\frac{ax^6\sqrt{a+bx^2}}{63b}+\frac{x^8\sqrt{a+bx^2}}{9} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^8}{8} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] 5*A*a**(7/2)*x/(128*b*sqrt(1 + b*x**2/a)) + 133*A*a**(5/2)*x**3/(384*sqrt(1
 + b*x**2/a)) + 127*A*a**(3/2)*b*x**5/(192*sqrt(1 + b*x**2/a)) + 23*A*sqrt(
a)*b**2*x**7/(48*sqrt(1 + b*x**2/a)) - 5*A*a**4*asinh(sqrt(b)*x/sqrt(a))/(1
28*b**(3/2)) + A*b**3*x**9/(8*sqrt(a)*sqrt(1 + b*x**2/a)) + B*a**2*Piecewis
e((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x*
*4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + 2*B*a*b*Piecewi
se((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**2)/(105*
b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, Ne(b, 0))
```

```
, (sqrt(a)*x**6/6, True)) + B*b**2*Piecewise((-16*a**4*sqrt(a + b*x**2)/(31
5*b**4) + 8*a**3*x**2*sqrt(a + b*x**2)/(315*b**3) - 2*a**2*x**4*sqrt(a + b*
x**2)/(105*b**2) + a*x**6*sqrt(a + b*x**2)/(63*b) + x**8*sqrt(a + b*x**2)/9
, Ne(b, 0)), (sqrt(a)*x**8/8, True))
```

Giac [A]
time = 1.23, size = 128, normalized size = 0.85

$$\frac{5 \, A a^{4} \log \left({\left| -\sqrt{b} x + \sqrt{b x^{2} + a} \right|}\right)}{128 \, b^{\frac{3}{2}}} - \frac{1}{8064} \, {\left(\frac{256 \, B a^{4}}{b^{2}} - {\left(\frac{315 \, A a^{3}}{b} + 2 \, {\left(\frac{64 \, B a^{3}}{b} + {\left(1239 \, A a^{2} + 4 \, {\left(240 \, B a^{2} + {\left(357 \, A a b + 2 \, {\left(152 \, B a b + 7 \, {\left(8 \, B b^{2} x + 9 \, A b^{2}\right)} x\right)} x\right)} x\right)} x\right)} x\right)} x\right)} x\right)} \sqrt{b x^{2} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 5/128*A*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - 1/8064*(256*B*
a^4/b^2 - (315*A*a^3/b + 2*(64*B*a^3/b + (1239*A*a^2 + 4*(240*B*a^2 + (357*
A*a*b + 2*(152*B*a*b + 7*(8*B*b^2*x + 9*A*b^2)*x)*x)*x)*x)*x)*x)*x)*sqrt(b*
x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,{\left(b\,x^2+a\right)}^{5/2}\,\left(A+B\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^(5/2)*(A + B*x),x)
```

```
[Out] int(x^2*(a + b*x^2)^(5/2)*(A + B*x), x)
```

3.17 $\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=126

$$-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx(a+bx^2)^{3/2}}{192b}-\frac{aBx(a+bx^2)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{5a^4B\tanh^{-1}\left(\frac{\sqrt{b}}{\sqrt{a+}}\right.}{128b^{3/2}}$$

```
[Out] -5/192*a^2*B*x*(b*x^2+a)^(3/2)/b-1/48*a*B*x*(b*x^2+a)^(5/2)/b+1/56*(7*B*x+8
*A)*(b*x^2+a)^(7/2)/b-5/128*a^4*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2
)-5/128*a^3*B*x*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.03, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {794, 201, 223, 212}

$$-\frac{5a^4B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx(a+bx^2)^{3/2}}{192b}+\frac{(a+bx^2)^{7/2}(8A+7Bx)}{56b}-\frac{aBx(a+bx^2)^{5/2}}{48b}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (-5*a^3*B*x*Sqrt[a + b*x^2])/(128*b) - (5*a^2*B*x*(a + b*x^2)^(3/2))/(192*b
) - (a*B*x*(a + b*x^2)^(5/2))/(48*b) + ((8*A + 7*B*x)*(a + b*x^2)^(7/2))/(5
6*b) - (5*a^4*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*b^(3/2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx &= \frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{(aB)\int\left(a+bx^2\right)^{5/2}\,dx}{8b}\\
&= -\frac{aBx(a+bx^2)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{(5a^2B)\int\left(a+bx^2\right)^{3/2}\,d}{48b}\\
&= -\frac{5a^2Bx(a+bx^2)^{3/2}}{192b}-\frac{aBx(a+bx^2)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-(5\\
&= -\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx(a+bx^2)^{3/2}}{192b}-\frac{aBx(a+bx^2)^{5/2}}{48b}+(8A+7B\\
&= -\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx(a+bx^2)^{3/2}}{192b}-\frac{aBx(a+bx^2)^{5/2}}{48b}+(8A+7B\\
&= -\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx(a+bx^2)^{3/2}}{192b}-\frac{aBx(a+bx^2)^{5/2}}{48b}+(8A+7B
\end{aligned}
$$

Mathematica [A]
time = 0.31, size = 118, normalized size = 0.94

$$\frac{\sqrt{b}\sqrt{a+bx^2}\left(48b^3x^6(8A+7Bx)+3a^3(128A+35Bx)+8ab^2x^4(144A+119Bx)+2a^2bx^2(576A+413Bx)\right)+105a^4B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2688b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[b]*Sqrt[a + b*x^2]*(48*b^3*x^6*(8*A + 7*B*x) + 3*a^3*(128*A + 35*B*x)
 + 8*a*b^2*x^4*(144*A + 119*B*x) + 2*a^2*b*x^2*(576*A + 413*B*x)) + 105*a^4
*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2688*b^(3/2))
```

Maple [A]
time = 0.12, size = 108, normalized size = 0.86

method	result
default	$B\left(\frac{x\left(bx^{2}+a\right)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x\left(bx^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(bx^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{8b}\right)+\frac{A\left(bx^{2}+a\right)}{7b}$
risch	$\frac{\left(336Bb^{3}x^{7}+384Ab^{3}x^{6}+952Bab^{2}x^{5}+1152aAb^{2}x^{4}+826Ba^{2}bx^{3}+1152a^{2}Abx^{2}+105Ba^{3}x+384a^{3}A\right)\sqrt{bx^{2}+a}}{2688b}-$ $5Ba^{4}\ln\left(x\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x
^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+
a)^(1/2))))))+1/7*A/b*(b*x^2+a)^(7/2)
```

Maxima [A]

time = 0.28, size = 105, normalized size = 0.83

$$\frac{(bx^2+a)^{\frac{7}{2}}Bx}{8\,b}-\frac{(bx^2+a)^{\frac{5}{2}}Bax}{48\,b}-\frac{5\,(bx^2+a)^{\frac{3}{2}}Ba^2x}{192\,b}-\frac{5\,\sqrt{bx^2+a}\,Ba^3x}{128\,b}-\frac{5\,Ba^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}+\frac{(bx^2+a)^{\frac{7}{2}}A}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/8*(b*x^2 + a)^(7/2)*B*x/b - 1/48*(b*x^2 + a)^(5/2)*B*a*x/b - 5/192*(b*x^2
 + a)^(3/2)*B*a^2*x/b - 5/128*sqrt(b*x^2 + a)*B*a^3*x/b - 5/128*B*a^4*arcsi
nh(b*x/sqrt(a*b))/b^(3/2) + 1/7*(b*x^2 + a)^(7/2)*A/b
```

Fricas [A]
time = 1.80, size = 253, normalized size = 2.01

$$\left[\frac{105\,Ba^{4}\sqrt{b}\,\log\left(-2\,bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,(336\,Bb^{4}x^{7}+384\,Ab^{4}x^{6}+952\,Bab^{3}x^{5}+1152\,Aab^{3}x^{4}+826\,Ba^{2}b^{2}x^{3}+1152\,Aa^{2}b^{2}x^{2}+105\,Ba^{3}bx+384\,Aa^{3}b)\sqrt{bx^{2}+a}}{5376\,b^{2}},\ \frac{105\,Ba^{4}\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+(336\,Bb^{4}x^{7}+384\,Ab^{4}x^{6}+952\,Bab^{3}x^{5}+1152\,Aab^{3}x^{4}+826\,Ba^{2}b^{2}x^{3}+1152\,Aa^{2}b^{2}x^{2}+105\,Ba^{3}bx+384\,Aa^{3}b)\sqrt{bx^{2}+a}}{2688\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/5376*(105*B*a^4*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(336*B*b^4*x^7 + 384*A*b^4*x^6 + 952*B*a*b^3*x^5 + 1152*A*a*b^3*x^4 + 8
26*B*a^2*b^2*x^3 + 1152*A*a^2*b^2*x^2 + 105*B*a^3*b*x + 384*A*a^3*b)*sqrt(b
*x^2 + a))/b^2, 1/2688*(105*B*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) + (336*B*b^4*x^7 + 384*A*b^4*x^6 + 952*B*a*b^3*x^5 + 1152*A*a*b^3*x^4 +
826*B*a^2*b^2*x^3 + 1152*A*a^2*b^2*x^2 + 105*B*a^3*b*x + 384*A*a^3*b)*sqrt(
b*x^2 + a))/b^2]
```

Sympy [A]
time = 27.21, size = 354, normalized size = 2.81

$$Aa^{2}\left(\begin{cases}\frac{\sqrt{a}\,x^{2}}{2} & \text{for}\ b=0\\ \frac{(a+bx^{2})^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+2Aab\left(\begin{cases}-\frac{2a^{2}\sqrt{a+bx^{2}}}{15b^{2}}+\frac{ax^{2}\sqrt{a+bx^{2}}}{15b}+\frac{x^{4}\sqrt{a+bx^{2}}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^{4}}{4} & \text{otherwise}\end{cases}\right)+Ab^{2}\left(\begin{cases}\frac{8a^{3}\sqrt{a+bx^{2}}}{105b^{3}}-\frac{4a^{2}x^{2}\sqrt{a+bx^{2}}}{105b^{2}}+\frac{ax^{4}\sqrt{a+bx^{2}}}{35b}+\frac{x^{6}\sqrt{a+bx^{2}}}{7} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^{6}}{6} & \text{otherwise}\end{cases}\right)+\frac{5Ba^{\frac{7}{2}}x}{128b\sqrt{1+\frac{bx^{2}}{a}}}+\frac{133Ba^{\frac{5}{2}}x^{3}}{384\sqrt{1+\frac{bx^{2}}{a}}}+\frac{127Ba^{\frac{3}{2}}bx^{5}}{192\sqrt{1+\frac{bx^{2}}{a}}}+\frac{23B\sqrt{a}\,b^{2}x^{7}}{48\sqrt{1+\frac{bx^{2}}{a}}}-\frac{5Ba^{4}\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{3}{2}}}+\frac{Bb^{3}x^{9}}{8\sqrt{a}\,\sqrt{1+\frac{bx^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] A*a**2*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), Tr
ue)) + 2*A*a*b*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(
a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, T
rue)) + A*b**2*Piecewise((8*a**3*sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*
sqrt(a + b*x**2)/(105*b**2) + a*x**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a
+ b*x**2)/7, Ne(b, 0)), (sqrt(a)*x**6/6, True)) + 5*B*a**(7/2)*x/(128*b*sqr
t(1 + b*x**2/a)) + 133*B*a**(5/2)*x**3/(384*sqrt(1 + b*x**2/a)) + 127*B*a**
(3/2)*b*x**5/(192*sqrt(1 + b*x**2/a)) + 23*B*sqrt(a)*b**2*x**7/(48*sqrt(1 +
 b*x**2/a)) - 5*B*a**4*asinh(sqrt(b)*x/sqrt(a))/(128*b**(3/2)) + B*b**3*x**
9/(8*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.75, size = 114, normalized size = 0.90

$$\frac{5\,Ba^{4}\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{128\,b^{\frac{3}{2}}}+\frac{1}{2688}\left(\frac{384\,Aa^{3}}{b}+\left(\frac{105\,Ba^{3}}{b}+2\,(576\,Aa^{2}+(413\,Ba^{2}+4\,(144\,Aab+(119\,Bab+6\,(7\,Bb^{2}x+8\,Ab^{2})x)x)x)x)x\right)x\right)\sqrt{bx^{2}+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 5/128*B*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/2688*(384*A*
a^3/b + (105*B*a^3/b + 2*(576*A*a^2 + (413*B*a^2 + 4*(144*A*a*b + (119*B*a*
b + 6*(7*B*b^2*x + 8*A*b^2)*x)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(b\,x^2+a\right)^{5/2}\,\left(A+B\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^(5/2)*(A + B*x),x)
```

```
[Out] int(x*(a + b*x^2)^(5/2)*(A + B*x), x)
```

3.18 $\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

Optimal. Leaf size=107

$$\frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{B(a+bx^2)^{7/2}}{7b}+\frac{5a^3A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}$$

```
[Out] 5/24*a*A*x*(b*x^2+a)^(3/2)+1/6*A*x*(b*x^2+a)^(5/2)+1/7*B*(b*x^2+a)^(7/2)/b+
5/16*a^3*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+5/16*a^2*A*x*(b*x^2+a
)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {655, 201, 223, 212}

$$\frac{5a^3A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{B(a+bx^2)^{7/2}}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (5*a^2*A*x*Sqrt[a + b*x^2])/16 + (5*a*A*x*(a + b*x^2)^(3/2))/24 + (A*x*(a +
 b*x^2)^(5/2))/6 + (B*(a + b*x^2)^(7/2))/(7*b) + (5*a^3*A*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(16*Sqrt[b])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx &= \frac{B(a+bx^2)^{7/2}}{7b}+A\int \left(a+bx^2\right)^{5/2}\,dx\\
&= \frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{B(a+bx^2)^{7/2}}{7b}+\frac{1}{6}(5aA)\int \left(a+bx^2\right)^{3/2}\,dx\\
&= \frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{B(a+bx^2)^{7/2}}{7b}+\frac{1}{8}\left(5a^2A\right)\int \sqrt{a+}\\
&= \frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{B(a+bx^2)^{7/2}}{7b}\\
&= \frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{B(a+bx^2)^{7/2}}{7b}\\
&= \frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{B(a+bx^2)^{7/2}}{7b}
\end{aligned}$$

Mathematica [A]
time = 0.36, size = 107, normalized size = 1.00

$$\frac{\sqrt{a+bx^2}\left(48a^3B+8b^3x^5(7A+6Bx)+3a^2bx(77A+48Bx)+2ab^2x^3(91A+72Bx)\right)-105a^3A\sqrt{b}\,\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{336b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)*(a + b*x^2)^(5/2),x]
```

```
[Out] (Sqrt[a + b*x^2]*(48*a^3*B + 8*b^3*x^5*(7*A + 6*B*x) + 3*a^2*b*x*(77*A + 48
*B*x) + 2*a*b^2*x^3*(91*A + 72*B*x)) - 105*a^3*A*Sqrt[b]*Log[-(Sqrt[b]*x) +
 Sqrt[a + b*x^2]])/(336*b)
```

Maple [A]
time = 0.11, size = 86, normalized size = 0.80

method	result

default	$\frac{B\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{7b}+A\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)$
risch	$\frac{\left(48b^{3}B\,x^{6}+56A\,b^{3}x^{5}+144a\,b^{2}B\,x^{4}+182Aa\,b^{2}x^{3}+144a^{2}bB\,x^{2}+231A\,a^{2}bx+48B\,a^{3}\right)\sqrt{b\,x^{2}+a}}{336b}+\frac{5a^{3}A\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+}\right.}{16\sqrt{b}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*B*(b*x^2+a)^(7/2)/b+A*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/
2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
)))
```

Maxima [A]
time = 0.29, size = 77, normalized size = 0.72

$$\frac{1}{6}\left(bx^{2}+a\right)^{\frac{5}{2}}Ax+\frac{5}{24}\left(bx^{2}+a\right)^{\frac{3}{2}}Aax+\frac{5}{16}\sqrt{bx^{2}+a}\,Aa^{2}x+\frac{5\,Aa^{3}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}}+\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}B}{7\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*A*x + 5/24*(b*x^2 + a)^(3/2)*A*a*x + 5/16*sqrt(b*x^2
+ a)*A*a^2*x + 5/16*A*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/7*(b*x^2 + a)^
(7/2)*B/b
```

Fricas [A]
time = 2.50, size = 224, normalized size = 2.09

$$\left[\frac{105\,Aa^{3}\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,\left(48\,Bb^{3}x^{6}+56\,Ab^{3}x^{5}+144\,Bab^{2}x^{4}+182\,Aab^{2}x^{3}+144\,Ba^{2}bx^{2}+231\,Aa^{2}bx+48\,Ba^{3}\right)\sqrt{bx^{2}+a}}{672\,b},-\frac{105\,Aa^{3}\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-\left(48\,Bb^{3}x^{6}+56\,Ab^{3}x^{5}+144\,Bab^{2}x^{4}+182\,Aab^{2}x^{3}+144\,Ba^{2}bx^{2}+231\,Aa^{2}bx+48\,Ba^{3}\right)\sqrt{bx^{2}+a}}{336\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/672*(105*A*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(48*B*b^3*x^6 + 56*A*b^3*x^5 + 144*B*a*b^2*x^4 + 182*A*a*b^2*x^3 + 144*B
```

```
*a^2*b*x^2 + 231*A*a^2*b*x + 48*B*a^3)*sqrt(b*x^2 + a))/b, -1/336*(105*A*a^
3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (48*B*b^3*x^6 + 56*A*b^3*x^
5 + 144*B*a*b^2*x^4 + 182*A*a*b^2*x^3 + 144*B*a^2*b*x^2 + 231*A*a^2*b*x + 4
8*B*a^3)*sqrt(b*x^2 + a))/b]
```

Sympy [A]
time = 8.80, size = 348, normalized size = 3.25

$$\frac{Aa^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3Aa^{\frac{5}{2}}x}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{35Aa^{\frac{3}{2}}bx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{17A\sqrt{a}\,b^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{5Aa^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{Ab^3x^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+Ba^2\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+2Bab\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+Bb^2\left(\begin{cases}\frac{8a^3\sqrt{a+bx^2}}{105b^3}-\frac{4a^2x^2\sqrt{a+bx^2}}{105b^2}+\frac{ax^4\sqrt{a+bx^2}}{35b}+\frac{x^6\sqrt{a+bx^2}}{7} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^6}{6} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2),x)
```

```
[Out] A*a**(5/2)*x*sqrt(1 + b*x**2/a)/2 + 3*A*a**(5/2)*x/(16*sqrt(1 + b*x**2/a))
+ 35*A*a**(3/2)*b*x**3/(48*sqrt(1 + b*x**2/a)) + 17*A*sqrt(a)*b**2*x**5/(24
*sqrt(1 + b*x**2/a)) + 5*A*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*sqrt(b)) + A*b
**3*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + B*a**2*Piecewise((sqrt(a)*x**2/2,
 Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) + 2*B*a*b*Piecewise((-2*a**2
*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*sqrt(a
+ b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + B*b**2*Piecewise((8*a**3*
sqrt(a + b*x**2)/(105*b**3) - 4*a**2*x**2*sqrt(a + b*x**2)/(105*b**2) + a*x
**4*sqrt(a + b*x**2)/(35*b) + x**6*sqrt(a + b*x**2)/7, Ne(b, 0)), (sqrt(a)*
x**6/6, True))
```

Giac [A]
time = 0.78, size = 101, normalized size = 0.94

$$-\frac{5\,Aa^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{336}\left(\frac{48\,Ba^3}{b}+\left(231\,Aa^2+2\,\left(72\,Ba^2+\left(91\,Aab+4\,\left(18\,Bab+\left(6\,Bb^2x+7\,Ab^2\right)x\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*A*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/336*(48*B*a^
3/b + (231*A*a^2 + 2*(72*B*a^2 + (91*A*a*b + 4*(18*B*a*b + (6*B*b^2*x + 7*A
*b^2)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

Mupad [B]
time = 1.16, size = 54, normalized size = 0.50

$$\frac{B\,{\left(b\,x^2+a\right)}^{7/2}}{7\,b}+\frac{A\,x\,{\left(b\,x^2+a\right)}^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^(5/2)*(A + B*x),x)
```

```
[Out] (B*(a + b*x^2)^(7/2))/(7*b) + (A*x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2],
 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(5/2)
```

3.19 $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx$

Optimal. Leaf size=132

$$\frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}+\frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2}+\frac{5a^3B\tanh^{-1}\left(\frac{\surd}{\sqrt{a}}\right.}{16\sqrt{b}}$$

```
[Out] 1/24*a*(5*B*x+8*A)*(b*x^2+a)^(3/2)+1/30*(5*B*x+6*A)*(b*x^2+a)^(5/2)-a^(5/2)
*A*arctanh((b*x^2+a)^(1/2)/a^(1/2))+5/16*a^3*B*arctanh(x*b^(1/2)/(b*x^2+a)^
(1/2))/b^(1/2)+1/16*a^2*(5*B*x+16*A)*(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {829, 858, 223, 212, 272, 65, 214}

$$-a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{5a^3B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}+\frac{1}{16}a^2\sqrt{a+bx^2}\,(16A+5Bx)+\frac{1}{24}a\left(a+bx^2\right)^{3/2}(8A+5Bx)+\frac{1}{30}\left(a+bx^2\right)^{5/2}(6A+5Bx)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(5/2))/x,x]
```

```
[Out] (a^2*(16*A + 5*B*x)*Sqrt[a + b*x^2])/16 + (a*(8*A + 5*B*x)*(a + b*x^2)^(3/2
))/24 + ((6*A + 5*B*x)*(a + b*x^2)^(5/2))/30 + (5*a^3*B*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(16*Sqrt[b]) - a^(5/2)*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx = \frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2}+\frac{\int \frac{(6aAb+5abBx)\left(a+bx^2\right)^{3/2}}{x}\,dx}{6b}$$

$$= \frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}+\frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2}+ \int (24a^2Ab^2+15a$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}+\frac{1}{30}(6A+5B$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}+\frac{1}{30}(6A+5B$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}+\frac{1}{30}(6A+5B$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}+\frac{1}{30}(6A+5B$$

$$= \frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}+\frac{1}{30}(6A+5B$$

Mathematica [A]
time = 0.44, size = 130, normalized size = 0.98

$$2a^{5/2}A\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{240}\left(\sqrt{a+bx^2}\left(8b^2x^4(6A+5Bx)+2abx^2(88A+65Bx)+a^2(368A+165Bx)\right)-\frac{75a^3B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x,x]
```

```
[Out] 2*a^(5/2)*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + (Sqrt[a + b*x^
2]*(8*b^2*x^4*(6*A + 5*B*x) + 2*a*b*x^2*(88*A + 65*B*x) + a^2*(368*A + 165*
B*x)) - (75*a^3*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/240
```

Maple [A]
time = 0.11, size = 139, normalized size = 1.05

method	result

default	$B\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+A\left(\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{(b\,x^{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(5/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a
)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+A*(1/5*(b*x^2+a)^(5/
2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x
^2+a)^(1/2))/x))))
```

Maxima [**A**]
time = 0.27, size = 119, normalized size = 0.90

$$\frac{1}{6}\left(bx^{2}+a\right)^{\frac{5}{2}}Bx+\frac{5}{24}\left(bx^{2}+a\right)^{\frac{3}{2}}Bax+\frac{5}{16}\sqrt{bx^{2}+a}\,Ba^{2}x+\frac{5\,Ba^{3}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\sqrt{b}}-Aa^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{5}\left(bx^{2}+a\right)^{\frac{5}{2}}A+\frac{1}{3}\left(bx^{2}+a\right)^{\frac{3}{2}}Aa+\sqrt{bx^{2}+a}\,Aa^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 1/6*(b*x^2 + a)^(5/2)*B*x + 5/24*(b*x^2 + a)^(3/2)*B*a*x + 5/16*sqrt(b*x^2
+ a)*B*a^2*x + 5/16*B*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) - A*a^(5/2)*arcsin
h(a/(sqrt(a*b)*abs(x))) + 1/5*(b*x^2 + a)^(5/2)*A + 1/3*(b*x^2 + a)^(3/2)*A
*a + sqrt(b*x^2 + a)*A*a^2
```

Fricas [**A**]
time = 1.95, size = 539, normalized size = 4.08

$$\left[\frac{75\,Ba^{3}\sqrt{b}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}x-a\right)+240\,Aa^{\frac{5}{2}}b\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,\left(40\,Bb^{3}x^{5}+48\,Ab^{3}x^{4}+130\,Bab^{2}x^{3}+176\,Aab^{2}x^{2}+165\,Ba^{2}bx+368\,Aa^{2}b\right)\sqrt{bx^{2}+a}}{480\,b},\ -\frac{75\,Ba^{3}\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^{2}+a}}\right)-120\,Aa^{\frac{5}{2}}b\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)-\left(40\,Bb^{3}x^{5}+48\,Ab^{3}x^{4}+130\,Bab^{2}x^{3}+176\,Aab^{2}x^{2}+165\,Ba^{2}bx+368\,Aa^{2}b\right)\sqrt{bx^{2}+a}}{240\,b},\ \frac{480\,A\sqrt{-a}a^{2}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+75\,Ba^{3}\sqrt{b}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}x-a\right)+2\,\left(40\,Bb^{3}x^{5}+48\,Ab^{3}x^{4}+130\,Bab^{2}x^{3}+176\,Aab^{2}x^{2}+165\,Ba^{2}bx+368\,Aa^{2}b\right)\sqrt{bx^{2}+a}}{480\,b},\ -\frac{75\,Ba^{3}\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^{2}+a}}\right)-240\,A\sqrt{-a}a^{2}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)-\left(40\,Bb^{3}x^{5}+48\,Ab^{3}x^{4}+130\,Bab^{2}x^{3}+176\,Aab^{2}x^{2}+165\,Ba^{2}bx+368\,Aa^{2}b\right)\sqrt{bx^{2}+a}}{240\,b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] [1/480*(75*B*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
240*A*a^(5/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(40
*B*b^3*x^5 + 48*A*b^3*x^4 + 130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^2*b
```

```
*x + 368*A*a^2*b)*sqrt(b*x^2 + a))/b, -1/240*(75*B*a^3*sqrt(-b)*arctan(sqrt
(-b)*x/sqrt(b*x^2 + a)) - 120*A*a^(5/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - (40*B*b^3*x^5 + 48*A*b^3*x^4 + 130*B*a*b^2*x^3 + 176*A
*a*b^2*x^2 + 165*B*a^2*b*x + 368*A*a^2*b)*sqrt(b*x^2 + a))/b, 1/480*(480*A*
sqrt(-a)*a^2*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 75*B*a^3*sqrt(b)*log(-2*b
*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(40*B*b^3*x^5 + 48*A*b^3*x^4 +
130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^2*b*x + 368*A*a^2*b)*sqrt(b*x^2
 + a))/b, -1/240*(75*B*a^3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 24
0*A*sqrt(-a)*a^2*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (40*B*b^3*x^5 + 48*A*
b^3*x^4 + 130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^2*b*x + 368*A*a^2*b)*
sqrt(b*x^2 + a))/b]
```

Sympy [A]

time = 15.36, size = 323, normalized size = 2.45

$$-Aa^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)+\frac{Aa^3}{\sqrt{b}\,x\sqrt{\frac{a}{bx^2}+1}}+\frac{Aa^2\sqrt{b}\,x}{\sqrt{\frac{a}{bx^2}+1}}+2Aab\left(\begin{cases}\frac{\sqrt{a}\,x^2}{2} & \text{for}\ b=0\\ \frac{(a+bx^2)^{\frac{3}{2}}}{3b} & \text{otherwise}\end{cases}\right)+Ab^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for}\ b\neq 0\\ \frac{\sqrt{a}\,x^4}{4} & \text{otherwise}\end{cases}\right)+\frac{Ba^{\frac{5}{2}}x\sqrt{1+\frac{bx^2}{a}}}{2}+\frac{3Ba^{\frac{5}{2}}x}{16\sqrt{1+\frac{bx^2}{a}}}+\frac{35Ba^{\frac{3}{2}}bx^3}{48\sqrt{1+\frac{bx^2}{a}}}+\frac{17B\sqrt{a}\,b^2x^5}{24\sqrt{1+\frac{bx^2}{a}}}+\frac{5Ba^3\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16\sqrt{b}}+\frac{Bb^3x^7}{6\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2)/x,x)
```

```
[Out] -A*a**(5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a**3/(sqrt(b)*x*sqrt(a/(b*x**2)
+ 1)) + A*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + 2*A*a*b*Piecewise((sqrt(a)*
x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b), True)) + A*b**2*Piecewise((-
2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt(a + b*x**2)/(15*b) + x**4*s
qrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4, True)) + B*a**(5/2)*x*sqrt(1
 + b*x**2/a)/2 + 3*B*a**(5/2)*x/(16*sqrt(1 + b*x**2/a)) + 35*B*a**(3/2)*b*x
**3/(48*sqrt(1 + b*x**2/a)) + 17*B*sqrt(a)*b**2*x**5/(24*sqrt(1 + b*x**2/a)
) + 5*B*a**3*asinh(sqrt(b)*x/sqrt(a))/(16*sqrt(b)) + B*b**3*x**7/(6*sqrt(a)
*sqrt(1 + b*x**2/a))
```

Giac [A]

time = 0.64, size = 125, normalized size = 0.95

$$\frac{2\,Aa^3\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{5\,Ba^3\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{240}\,\left(368\,Aa^2+\left(165\,Ba^2+2\,\left(88\,Aab+\left(65\,Bab+4\,\left(5\,Bb^2x+6\,Ab^2\right)x\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 2*A*a^3*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 5/16*B*a
^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/240*(368*A*a^2 + (165
*B*a^2 + 2*(88*A*a*b + (65*B*a*b + 4*(5*B*b^2*x + 6*A*b^2)*x)*x)*x)*x)*sqrt
(b*x^2 + a)
```

Mupad [B]

time = 1.25, size = 101, normalized size = 0.77

$$\frac{A\,{\left(b\,x^2+a\right)}^{5/2}}{5}+A\,a^2\,\sqrt{b\,x^2+a}+\frac{A\,a\,{\left(b\,x^2+a\right)}^{3/2}}{3}+\frac{B\,x\,{\left(b\,x^2+a\right)}^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{{\left(\frac{b\,x^2}{a}+1\right)}^{5/2}}+A\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,1{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(5/2)*(A + B*x))/x,x)
```

```
[Out] (A*(a + b*x^2)^(5/2))/5 + A*a^2*(a + b*x^2)^(1/2) + A*a^(5/2)*atan(((a + b*
x^2)^(1/2)*1i)/a^(1/2))*1i + (A*a*(a + b*x^2)^(3/2))/3 + (B*x*(a + b*x^2)^(
5/2)*hypergeom([-5/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(5/2)
```

3.20 $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx$

Optimal. Leaf size=136

$$\frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2}+\frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2}-\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x}+\frac{15}{8}a^2A\sqrt{b}\,\tanh^{-1}\Bigg($$

```
[Out] 1/12*(15*A*b*x+4*B*a)*(b*x^2+a)^(3/2)-1/5*(-B*x+5*A)*(b*x^2+a)^(5/2)/x-a^(5
/2)*B*arctanh((b*x^2+a)^(1/2)/a^(1/2))+15/8*a^2*A*arctanh(x*b^(1/2)/(b*x^2+
a)^(1/2))*b^(1/2)+1/8*a*(15*A*b*x+8*B*a)*(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {827, 829, 858, 223, 212, 272, 65, 214}

$$a^{5/2}(-B)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{15}{8}a^2A\sqrt{b}\,\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)+\frac{1}{8}a\sqrt{a+bx^2}\,(8aB+15Abx)-\frac{\left(a+bx^2\right)^{5/2}(5A-Bx)}{5x}+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4aB+15Abx)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(5/2))/x^2,x]
```

```
[Out] (a*(8*a*B + 15*A*b*x)*Sqrt[a + b*x^2])/8 + ((4*a*B + 15*A*b*x)*(a + b*x^2)^
(3/2))/12 - ((5*A - B*x)*(a + b*x^2)^(5/2))/(5*x) + (15*a^2*A*Sqrt[b]*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8 - a^(5/2)*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt
[a]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx = -\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x} - \frac{1}{2}\int \frac{(-2aB-10Abx)\left(a+bx^2\right)^{3/2}}{x}\,dx$$

$$= \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x} - \frac{\int \frac{(-8a^2bB-30aAb}{3}}{8}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2} + \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx}{}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2} + \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx}{}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2} + \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx}{}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2} + \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx}{}$$

$$= \frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2} + \frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2} - \frac{(5A-Bx}{}$$

Mathematica [A]
time = 0.42, size = 133, normalized size = 0.98

$$\frac{\sqrt{a+bx^2}\,\left(-8a^2(15A-23Bx)+6b^2x^4(5A+4Bx)+abx^2(135A+88Bx)\right)}{120x} + 2a^{5/2}B\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right) - \frac{15}{8}a^2A\sqrt{b}\,\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x^2,x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2*(15*A - 23*B*x) + 6*b^2*x^4*(5*A + 4*B*x) + a*b*x^
2*(135*A + 88*B*x)))/(120*x) + 2*a^(5/2)*B*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*
x^2])/Sqrt[a]] - (15*a^2*A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/8
```

Maple [A]
time = 0.13, size = 163, normalized size = 1.20

method	result
risch	$-\frac{a^2A\sqrt{b\,x^2+a}}{x}+\frac{b^2B\,x^4\sqrt{b\,x^2+a}}{5}+\frac{11bBa\,x^2\sqrt{b\,x^2+a}}{15}+\frac{23B\,a^2\sqrt{b\,x^2+a}}{15}+\frac{A\,b^2x^3\sqrt{b\,x^2+a}}{4}-$

default	$B\left(\frac{(b x^2+a)^{\frac{5}{2}}}{5}+a\left(\frac{(b x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b x^2+a}}{x}\right)\right)\right)\right)+A\left(-\frac{(b x^2+a)}{ax}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln
((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))+A*(-1/a/x*(b*x^2+a)^(7/2)+6*b/a*(1/6
*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2
)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))
```

Maxima [A]
time = 0.27, size = 120, normalized size = 0.88

$$\frac{5}{4}\left(bx^2+a\right)^{\frac{3}{2}}Abx+\frac{15}{8}\sqrt{bx^2+a}\,Aabx+\frac{15}{8}Aa^2\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-Ba^{\frac{5}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}B+\frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}Ba+\sqrt{bx^2+a}\,Ba^2-\frac{\left(bx^2+a\right)^{\frac{5}{2}}A}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="maxima")
```

```
[Out] 5/4*(b*x^2 + a)^(3/2)*A*b*x + 15/8*sqrt(b*x^2 + a)*A*a*b*x + 15/8*A*a^2*sqr
t(b)*arcsinh(b*x/sqrt(a*b)) - B*a^(5/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/5
*(b*x^2 + a)^(5/2)*B + 1/3*(b*x^2 + a)^(3/2)*B*a + sqrt(b*x^2 + a)*B*a^2 -
(b*x^2 + a)^(5/2)*A/x
```

Fricas [A]
time = 3.33, size = 519, normalized size = 3.82

$$\left[\frac{225\,Aa^2\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+120\,Ba^{\frac{5}{2}}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(24\,Bb^2x^5+30\,Ab^2x^4+88\,Babx^3+135\,Aabx^2+184\,Ba^2x-120\,Aa^2)\sqrt{bx^2+a}}{240\,x},\ -\frac{225\,Aa^2\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-60\,Ba^{\frac{5}{2}}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-(24\,Bb^2x^5+30\,Ab^2x^4+88\,Babx^3+135\,Aabx^2+184\,Ba^2x-120\,Aa^2)\sqrt{bx^2+a}}{120\,x},\ \frac{240\,B\sqrt{-a}\,a^2x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+225\,Aa^2\sqrt{b}\,x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(24\,Bb^2x^5+30\,Ab^2x^4+88\,Babx^3+135\,Aabx^2+184\,Ba^2x-120\,Aa^2)\sqrt{bx^2+a}}{240\,x},\ -\frac{225\,Aa^2\sqrt{-b}\,x\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-120\,B\sqrt{-a}\,a^2x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(24\,Bb^2x^5+30\,Ab^2x^4+88\,Babx^3+135\,Aabx^2+184\,Ba^2x-120\,Aa^2)\sqrt{bx^2+a}}{120\,x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/240*(225*A*a^2*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 120*B*a^(5/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*
(24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x^3 + 135*A*a*b*x^2 + 184*B*a^2*x -
 120*A*a^2)*sqrt(b*x^2 + a))/x, -1/120*(225*A*a^2*sqrt(-b)*x*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - 60*B*a^(5/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
a) + 2*a)/x^2) - (24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x^3 + 135*A*a*b*x^
2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x, 1/240*(240*B*sqrt(-a)*a^2*
x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 225*A*a^2*sqrt(b)*x*log(-2*b*x^2 - 2*s
qrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x
^3 + 135*A*a*b*x^2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x, -1/120*(2
25*A*a^2*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 120*B*sqrt(-a)*a^2
*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a
*b*x^3 + 135*A*a*b*x^2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x]
```

Sympy [A]

time = 5.60, size = 318, normalized size = 2.34

$$- \frac{A a^{\frac{5}{2}}}{x \sqrt{1 + \frac{b x^{2}}{a}}} + A a^{\frac{3}{2}} b x \sqrt{1 + \frac{b x^{2}}{a}} - \frac{7 A a^{\frac{3}{2}} b x}{8 \sqrt{1 + \frac{b x^{2}}{a}}} + \frac{3 A \sqrt{a} b^{2} x^{3}}{8 \sqrt{1 + \frac{b x^{2}}{a}}} + \frac{15 A a^{2} \sqrt{b} \operatorname{asinh}{\left(\frac{\sqrt{b} x}{\sqrt{a}} \right)}}{8} + \frac{A b^{3} x^{5}}{4 \sqrt{a} \sqrt{1 + \frac{b x^{2}}{a}}} - B a^{\frac{5}{2}} \operatorname{asinh}{\left(\frac{\sqrt{a}}{\sqrt{b} x} \right)} + \frac{B a^{3}}{\sqrt{b} x \sqrt{\frac{a}{b x^{2}} + 1}} + \frac{B a^{2} \sqrt{b} x}{\sqrt{\frac{a}{b x^{2}} + 1}} + 2 B a b \left(\begin{cases} \frac{\sqrt{a} x^{2}}{2} & \text{for}\: b = 0 \\\frac{\left(a + b x^{2}\right)^{\frac{3}{2}}}{3 b} & \text{otherwise} \end{cases}\right) + B b^{2} \left(\begin{cases} - \frac{2 a^{2} \sqrt{a + b x^{2}}}{15 b^{2}} + \frac{a x^{2} \sqrt{a + b x^{2}}}{15 b} + \frac{x^{4} \sqrt{a + b x^{2}}}{5} & \text{for}\: b \neq 0 \\\frac{\sqrt{a} x^{4}}{4} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2)/x**2,x)
```

```
[Out] -A*a**(5/2)/(x*sqrt(1 + b*x**2/a)) + A*a**(3/2)*b*x*sqrt(1 + b*x**2/a) - 7*
A*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*A*sqrt(a)*b**2*x**3/(8*sqrt(1 + b
*x**2/a)) + 15*A*a**2*sqrt(b)*asinh(sqrt(b)*x/sqrt(a))/8 + A*b**3*x**5/(4*s
qrt(a)*sqrt(1 + b*x**2/a)) - B*a**(5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**3
/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1) +
 2*B*a*b*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a + b*x**2)**(3/2)/(3*b),
True)) + B*b**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqrt
(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4,
True))
```

Giac [A]

time = 0.61, size = 150, normalized size = 1.10

$$\frac{2 \, B a^{3} \arctan\left(-\frac{\sqrt{b} x - \sqrt{b x^{2} + a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{15}{8} \, A a^{2} \sqrt{b} \log\left({\left| -\sqrt{b} x + \sqrt{b x^{2} + a} \right|}\right) + \frac{2 \, A a^{3} \sqrt{b}}{{\left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)}^{2} - a} + \frac{1}{120} \, {\left(184 \, B a^{2} + {\left(135 \, A a b + 2 \, {\left(44 \, B a b + 3 \, {\left(4 \, B b^{2} x + 5 \, A b^{2}\right)} x\right)} x\right)} x\right)} \sqrt{b x^{2} + a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] 2*B*a^3*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 15/8*A*a
^2*sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 2*A*a^3*sqrt(b)/((sqrt(
```

```
b)*x - sqrt(b*x^2 + a))^2 - a) + 1/120*(184*B*a^2 + (135*A*a*b + 2*(44*B*a*
b + 3*(4*B*b^2*x + 5*A*b^2)*x)*x)*x)*sqrt(b*x^2 + a)
```

Mupad [B]
time = 2.16, size = 104, normalized size = 0.76

$$\frac{B\,(b\,x^2+a)^{5/2}}{5}+B\,a^2\,\sqrt{b\,x^2+a}+\frac{B\,a\,(b\,x^2+a)^{3/2}}{3}-\frac{A\,(b\,x^2+a)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,\left(\frac{b\,x^2}{a}+1\right)^{5/2}}+B\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(5/2)*(A + B*x))/x^2,x)
```

```
[Out] (B*(a + b*x^2)^(5/2))/5 + B*a^2*(a + b*x^2)^(1/2) + B*a^(5/2)*atan(((a + b*
x^2)^(1/2)*1i)/a^(1/2))*1i + (B*a*(a + b*x^2)^(3/2))/3 - (A*(a + b*x^2)^(5/
2)*hypergeom([-5/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a + 1)^(5/2))
```

3.21 $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx$

<u>Optimal</u>. Leaf size=141

$$\frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2}-\frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x}-\frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2}+\frac{15}{8}a^2\sqrt{b}\,B\tanh^{-1}\Bigg($$

```
[Out] -5/12*(-2*A*b*x+3*B*a)*(b*x^2+a)^(3/2)/x-1/4*(-B*x+2*A)*(b*x^2+a)^(5/2)/x^2
-5/2*a^(3/2)*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))+15/8*a^2*B*arctanh(x*b^(1
/2)/(b*x^2+a)^(1/2))*b^(1/2)+5/8*a*b*(3*B*x+4*A)*(b*x^2+a)^(1/2)
```

<u>Rubi</u> [A]
time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {827, 829, 858, 223, 212, 272, 65, 214}

$$-\frac{5}{2}a^{3/2}Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{15}{8}a^2\sqrt{b}\,B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)-\frac{\left(a+bx^2\right)^{5/2}(2A-Bx)}{4x^2}-\frac{5\left(a+bx^2\right)^{3/2}(3aB-2Abx)}{12x}+\frac{5}{8}ab\sqrt{a+bx^2}\,(4A+3Bx)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x)*(a + b*x^2)^(5/2))/x^3,x]
```

```
[Out] (5*a*b*(4*A + 3*B*x)*Sqrt[a + b*x^2])/8 - (5*(3*a*B - 2*A*b*x)*(a + b*x^2)^
(3/2))/(12*x) - ((2*A - B*x)*(a + b*x^2)^(5/2))/(4*x^2) + (15*a^2*Sqrt[b]*B
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/8 - (5*a^(3/2)*A*b*ArcTanh[Sqrt[a +
b*x^2]/Sqrt[a]])/2
```

<u>Rule 65</u>

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 214</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] && !Rational
Q[m])) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx &= -\frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2}-\frac{5}{16}\int \frac{(-4aB-8Abx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx \\
&= -\frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x}-\frac{(2A-Bx)\left(a+bx^2\right)^{5/2}}{4x^2}+\frac{5}{32}\int \frac{(16aAb+}{} \\
&= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2}-\frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x}-\frac{(2A-Bx)(a-}{4x^2} \\
&= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2}-\frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x}-\frac{(2A-Bx)(a-}{4x^2} \\
&= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2}-\frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x}-\frac{(2A-Bx)(a-}{4x^2} \\
&= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2}-\frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x}-\frac{(2A-Bx)(a-}{4x^2} \\
&= \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2}-\frac{5(3aB-2Abx)\left(a+bx^2\right)^{3/2}}{12x}-\frac{(2A-Bx)(a-}{4x^2}
\end{aligned}
$$

Mathematica [A]
time = 0.50, size = 132, normalized size = 0.94

$$5a^{3/2}Ab\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{24}\left(\frac{\sqrt{a+bx^2}\left(-12a^2(A+2Bx)+2b^2x^4(4A+3Bx)+abx^2(56A+27Bx)\right)}{x^2}-45a^2\sqrt{b}\,B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x^3,x]
```

```
[Out] 5*a^(3/2)*A*b*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + ((Sqrt[a + b
*x^2]*(-12*a^2*(A + 2*B*x) + 2*b^2*x^4*(4*A + 3*B*x) + a*b*x^2*(56*A + 27*B
*x)))/x^2 - 45*a^2*Sqrt[b]*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/24
```

Maple [A]
time = 0.13, size = 187, normalized size = 1.33

method	result
risch	$-\frac{a^2\sqrt{b\,x^2+a}\,(2Bx+A)}{2x^2}+\frac{b^2B\,x^3\sqrt{b\,x^2+a}}{4}+\frac{9bBax\sqrt{b\,x^2+a}}{8}+\frac{15\sqrt{b}\,a^2B\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{8}+$

default	$A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{2a\,x^{2}}+\frac{5b\left(\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)\right)}{2a}\right)+B$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)*(b*x^2+a)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/2/a/x^2*(b*x^2+a)^(7/2)+5/2*b/a*(1/5*(b*x^2+a)^(5/2)+a*(1/3*(b*x^2+a)
^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))
+B*(-1/a/x*(b*x^2+a)^(7/2)+6*b/a*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2
+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)
^(1/2))))))
```

Maxima [A]

time = 0.29, size = 143, normalized size = 1.01

$$\frac{5}{4}\left(bx^{2}+a\right)^{\frac{3}{2}}Bbx+\frac{15}{8}\sqrt{bx^{2}+a}\,Babx+\frac{15}{8}Ba^{2}\sqrt{b}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)-\frac{5}{2}Aa^{\frac{3}{2}}b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)+\frac{5}{6}\left(bx^{2}+a\right)^{\frac{3}{2}}Ab+\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}Ab}{2a}+\frac{5}{2}\sqrt{bx^{2}+a}\,Aab-\frac{\left(bx^{2}+a\right)^{\frac{5}{2}}B}{x}-\frac{\left(bx^{2}+a\right)^{\frac{7}{2}}A}{2ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] 5/4*(b*x^2 + a)^(3/2)*B*b*x + 15/8*sqrt(b*x^2 + a)*B*a*b*x + 15/8*B*a^2*sqr
t(b)*arcsinh(b*x/sqrt(a*b)) - 5/2*A*a^(3/2)*b*arcsinh(a/(sqrt(a*b)*abs(x)))
 + 5/6*(b*x^2 + a)^(3/2)*A*b + 1/2*(b*x^2 + a)^(5/2)*A*b/a + 5/2*sqrt(b*x^2
 + a)*A*a*b - (b*x^2 + a)^(5/2)*B/x - 1/2*(b*x^2 + a)^(7/2)*A/(a*x^2)
```

Fricas [A]

time = 4.25, size = 535, normalized size = 3.79

$$\left[\frac{45\,Ba^{2}\sqrt{b}\,x^{2}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+60\,Aa^{\frac{3}{2}}bx^{2}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,\left(6\,Bb^{2}x^{5}+8\,Ab^{2}x^{4}+27\,Babx^{3}+56\,Aabx^{2}-24\,Ba^{2}x-12\,Aa^{2}\right)\sqrt{bx^{2}+a}}{48\,x^{2}},-\frac{45\,Ba^{2}\sqrt{-b}\,x^{2}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-30\,Aa^{\frac{3}{2}}bx^{2}\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)-\left(6\,Bb^{2}x^{5}+8\,Ab^{2}x^{4}+27\,Babx^{3}+56\,Aabx^{2}-24\,Ba^{2}x-12\,Aa^{2}\right)\sqrt{bx^{2}+a}}{24\,x^{2}},\frac{120\,A\sqrt{-a}\,abx^{2}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+45\,Ba^{2}\sqrt{b}\,x^{2}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,\left(6\,Bb^{2}x^{5}+8\,Ab^{2}x^{4}+27\,Babx^{3}+56\,Aabx^{2}-24\,Ba^{2}x-12\,Aa^{2}\right)\sqrt{bx^{2}+a}}{48\,x^{2}},-\frac{45\,Ba^{2}\sqrt{-b}\,x^{2}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-60\,A\sqrt{-a}\,abx^{2}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)-\left(6\,Bb^{2}x^{5}+8\,Ab^{2}x^{4}+27\,Babx^{3}+56\,Aabx^{2}-24\,Ba^{2}x-12\,Aa^{2}\right)\sqrt{bx^{2}+a}}{24\,x^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/48*(45*B*a^2*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 60*A*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) +
 2*(6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*x^3 + 56*A*a*b*x^2 - 24*B*a^2*x -
12*A*a^2)*sqrt(b*x^2 + a))/x^2, -1/24*(45*B*a^2*sqrt(-b)*x^2*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) - 30*A*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - (6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*x^3 + 56*A*a*b*x
^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2, 1/48*(120*A*sqrt(-a)*a*b*
x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 45*B*a^2*sqrt(b)*x^2*log(-2*b*x^2 -
2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*
x^3 + 56*A*a*b*x^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2, -1/24*(45
*B*a^2*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 60*A*sqrt(-a)*a*b*
x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*
b*x^3 + 56*A*a*b*x^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2]
```

Sympy [A]
time = 6.21, size = 279, normalized size = 1.98

$$- \frac{5 A a^{\frac{3}{2}} b \operatorname{asinh}{\left(\frac{\sqrt{a}}{\sqrt{b} x} \right)}}{2} - \frac{A a^{2} \sqrt{b} \sqrt{\frac{a}{b x^{2}} + 1}}{2 x} + \frac{2 A a^{2} \sqrt{b}}{x \sqrt{\frac{a}{b x^{2}} + 1}} + \frac{2 A a b^{\frac{3}{2}} x}{\sqrt{\frac{a}{b x^{2}} + 1}} + A b^{2} \left(\begin{cases} \frac{\sqrt{a} x^{2}}{2} & \text{for}\: b = 0 \\\frac{\left(a + b x^{2}\right)^{\frac{3}{2}}}{3 b} & \text{otherwise} \end{cases}\right) - \frac{B a^{\frac{5}{2}}}{x \sqrt{1 + \frac{b x^{2}}{a}}} + B a^{\frac{3}{2}} b x \sqrt{1 + \frac{b x^{2}}{a}} - \frac{7 B a^{\frac{3}{2}} b x}{8 \sqrt{1 + \frac{b x^{2}}{a}}} + \frac{3 B \sqrt{a} b^{2} x^{3}}{8 \sqrt{1 + \frac{b x^{2}}{a}}} + \frac{15 B a^{2} \sqrt{b} \operatorname{asinh}{\left(\frac{\sqrt{b} x}{\sqrt{a}} \right)}}{8} + \frac{B b^{3} x^{5}}{4 \sqrt{a} \sqrt{1 + \frac{b x^{2}}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x**2+a)**(5/2)/x**3,x)
```

```
[Out] -5*A*a**(3/2)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a**2*sqrt(b)*sqrt(a/(b*x**
2) + 1)/(2*x) + 2*A*a**2*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + 2*A*a*b**(3/2)*
x/sqrt(a/(b*x**2) + 1) + A*b**2*Piecewise((sqrt(a)*x**2/2, Eq(b, 0)), ((a +
 b*x**2)**(3/2)/(3*b), True)) - B*a**(5/2)/(x*sqrt(1 + b*x**2/a)) + B*a**(3
/2)*b*x*sqrt(1 + b*x**2/a) - 7*B*a**(3/2)*b*x/(8*sqrt(1 + b*x**2/a)) + 3*B*
sqrt(a)*b**2*x**3/(8*sqrt(1 + b*x**2/a)) + 15*B*a**2*sqrt(b)*asinh(sqrt(b)*
x/sqrt(a))/8 + B*b**3*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.76, size = 219, normalized size = 1.55

$$\frac{5 \, A a^{2} b \arctan\left(-\frac{\sqrt{b} x - \sqrt{b x^{2} + a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{15}{8} \, B a^{2} \sqrt{b} \log\left({\left| -\sqrt{b} x + \sqrt{b x^{2} + a} \right|}\right) + \frac{1}{24} \, {\left(56 \, A a b + {\left(27 \, B a b + 2 \, {\left(3 \, B b^{2} x + 4 \, A b^{2}\right)} x\right)} x\right)} \sqrt{b x^{2} + a} + \frac{{\left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)}^{3} A a^{2} b + 2 \, {\left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)}^{2} B a^{3} \sqrt{b} + {\left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)} A a^{3} b - 2 \, B a^{4} \sqrt{b}}{{\left({\left(\sqrt{b} x - \sqrt{b x^{2} + a}\right)}^{2} - a\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] 5*A*a^2*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 15/8*B
*a^2*sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/24*(56*A*a*b + (27*
B*a*b + 2*(3*B*b^2*x + 4*A*b^2)*x)*x)*sqrt(b*x^2 + a) + ((sqrt(b)*x - sqrt(
```

```
b*x^2 + a))^3*A*a^2*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*sqrt(b) + (
sqrt(b)*x - sqrt(b*x^2 + a))*A*a^3*b - 2*B*a^4*sqrt(b))/((sqrt(b)*x - sqrt(
b*x^2 + a))^2 - a)^2
```

Mupad [B]
time = 2.59, size = 111, normalized size = 0.79

$$\frac{A\,b\,{\left(b\,x^2+a\right)}^{3/2}}{3}+2\,A\,a\,b\,\sqrt{b\,x^2+a}-\frac{A\,a^2\,\sqrt{b\,x^2+a}}{2\,x^2}-\frac{B\,{\left(b\,x^2+a\right)}^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{5/2}}+\frac{A\,a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^(5/2)*(A + B*x))/x^3,x)
```

```
[Out] (A*b*(a + b*x^2)^(3/2))/3 + 2*A*a*b*(a + b*x^2)^(1/2) - (A*a^2*(a + b*x^2)^
(1/2))/(2*x^2) + (A*a^(3/2)*b*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/2 -
(B*(a + b*x^2)^(5/2)*hypergeom([-5/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/
a + 1)^(5/2))
```

3.22 $\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=104

$$\frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2}+\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

```
[Out] 3/8*a^2*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+1/3*A*x^2*(b*x^2+a)^(1
/2)/b+1/4*B*x^3*(b*x^2+a)^(1/2)/b-1/24*a*(9*B*x+16*A)*(b*x^2+a)^(1/2)/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {847, 794, 223, 212}

$$\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}-\frac{a\sqrt{a+bx^2}\,(16A+9Bx)}{24b^2}+\frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (A*x^2*Sqrt[a + b*x^2])/(3*b) + (B*x^3*Sqrt[a + b*x^2])/(4*b) - (a*(16*A +
9*B*x)*Sqrt[a + b*x^2])/(24*b^2) + (3*a^2*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*
x^2]])/(8*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx &= \frac{Bx^3\sqrt{a+bx^2}}{4b}+\frac{\int \frac{x^2(-3aB+4Abx)}{\sqrt{a+bx^2}}\,dx}{4b}\\
&= \frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}+\frac{\int \frac{x(-8aAb-9abBx)}{\sqrt{a+bx^2}}\,dx}{12b^2}\\
&= \frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2}+\frac{(3a^2B)\int \frac{1}{\sqrt{a+bx^2}}\,d}{8b^2}\\
&= \frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2}+\frac{(3a^2B)\,\mathrm{Subst}\left(\int \frac{1}{1-bx^2}\right.}{8b^2}\\
&= \frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2}+\frac{3a^2B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+b}}\right.}{8b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 77, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\,(-16aA-9aBx+8Abx^2+6bBx^3)}{24b^2}-\frac{3a^2B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-16*a*A - 9*a*B*x + 8*A*b*x^2 + 6*b*B*x^3))/(24*b^2) - (3
*a^2*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Maple [A]
time = 0.11, size = 101, normalized size = 0.97

method	result
risch	$-\frac{\left(-6bB\,x^{3}-8Ab\,x^{2}+9Bax+16Aa\right)\sqrt{b\,x^{2}+a}}{24b^{2}}+\frac{3a^{2}B\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{5}{2}}}$
default	$B\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+A\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+}}{3b^{2}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)
*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))+A*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*
x^2+a)^(1/2))
```

Maxima [A]
time = 0.28, size = 88, normalized size = 0.85

$$\frac{\sqrt{bx^{2}+a}\,Bx^{3}}{4\,b}+\frac{\sqrt{bx^{2}+a}\,Ax^{2}}{3\,b}-\frac{3\,\sqrt{bx^{2}+a}\,Bax}{8\,b^{2}}+\frac{3\,Ba^{2}\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{2\,\sqrt{bx^{2}+a}\,Aa}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*B*x^3/b + 1/3*sqrt(b*x^2 + a)*A*x^2/b - 3/8*sqrt(b*x^2
+ a)*B*a*x/b^2 + 3/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 2/3*sqrt(b*x^2
+ a)*A*a/b^2
```

Fricas [A]
time = 7.74, size = 158, normalized size = 1.52

$$\left[\frac{9\,Ba^{2}\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,(6\,Bb^{2}x^{3}+8\,Ab^{2}x^{2}-9\,Babx-16\,Aab)\sqrt{bx^{2}+a}}{48\,b^{3}},-\frac{9\,Ba^{2}\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-(6\,Bb^{2}x^{3}+8\,Ab^{2}x^{2}-9\,Babx-16\,Aab)\sqrt{bx^{2}+a}}{24\,b^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*
(6*B*b^2*x^3 + 8*A*b^2*x^2 - 9*B*a*b*x - 16*A*a*b)*sqrt(b*x^2 + a))/b^3, -1
/24*(9*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (6*B*b^2*x^3 + 8
*A*b^2*x^2 - 9*B*a*b*x - 16*A*a*b)*sqrt(b*x^2 + a))/b^3]
```

Sympy [A]
time = 3.32, size = 150, normalized size = 1.44

$$A\left(\begin{cases}-\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for } b\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise}\end{cases}\right)-\frac{3Ba^{\frac{3}{2}}x}{8b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{B\sqrt{a}\,x^3}{8b\sqrt{1+\frac{bx^2}{a}}}+\frac{3Ba^2\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}+\frac{Bx^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b),
Ne(b, 0)), (x**4/(4*sqrt(a)), True)) - 3*B*a**(3/2)*x/(8*b**2*sqrt(1 + b*x*
*2/a)) - B*sqrt(a)*x**3/(8*b*sqrt(1 + b*x**2/a)) + 3*B*a**2*asinh(sqrt(b)*x
/sqrt(a))/(8*b**(5/2)) + B*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.85, size = 74, normalized size = 0.71

$$\frac{1}{24}\sqrt{bx^2+a}\left(\left(2\left(\frac{3\,Bx}{b}+\frac{4\,A}{b}\right)x-\frac{9\,Ba}{b^2}\right)x-\frac{16\,Aa}{b^2}\right)-\frac{3\,Ba^2\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(b*x^2 + a)*((2*(3*B*x/b + 4*A/b)*x - 9*B*a/b^2)*x - 16*A*a/b^2) -
 3/8*B*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,(A+B\,x)}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x))/(a + b*x^2)^(1/2),x)
```

```
[Out] int((x^3*(A + B*x))/(a + b*x^2)^(1/2), x)
```

3.23 $\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=81

$$\frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2}-\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] -1/2*a*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/3*B*x^2*(b*x^2+a)^(1/
2)/b-1/6*(-3*A*b*x+4*B*a)*(b*x^2+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {847, 794, 223, 212}

$$-\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}-\frac{\sqrt{a+bx^2}\,(4aB-3Abx)}{6b^2}+\frac{Bx^2\sqrt{a+bx^2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (B*x^2*Sqrt[a + b*x^2])/(3*b) - ((4*a*B - 3*A*b*x)*Sqrt[a + b*x^2])/(6*b^2)
 - (a*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx &= \frac{Bx^2\sqrt{a+bx^2}}{3b}+\frac{\int \frac{x(-2aB+3Abx)}{\sqrt{a+bx^2}}\,dx}{3b}\\
&= \frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2}-\frac{(aA)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}\\
&= \frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2}-\frac{(aA)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b}\\
&= \frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2}-\frac{aA\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 68, normalized size = 0.84

$$\frac{\sqrt{a+bx^2}\,(-4aB+3Abx+2bBx^2)}{6b^2}+\frac{aA\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-4*a*B + 3*A*b*x + 2*b*B*x^2))/(6*b^2) + (a*A*Log[-(Sqrt[
b]*x) + Sqrt[a + b*x^2]])/(2*b^(3/2))
```

Maple [A]
time = 0.12, size = 77, normalized size = 0.95

method	result	size
risch	$\frac{(2bB\,x^2+3Abx-4Ba)\sqrt{b\,x^2+a}}{6b^2}-\frac{aA\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}$	56

default	$B\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)+A\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))+A*(1/2*x*(b*x^2+a)^
(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))
```

Maxima [**A**]
time = 0.28, size = 67, normalized size = 0.83

$$\frac{\sqrt{bx^{2}+a}\,Bx^{2}}{3\,b}+\frac{\sqrt{bx^{2}+a}\,Ax}{2\,b}-\frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}-\frac{2\,\sqrt{bx^{2}+a}\,Ba}{3\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(b*x^2 + a)*B*x^2/b + 1/2*sqrt(b*x^2 + a)*A*x/b - 1/2*A*a*arcsinh(b
*x/sqrt(a*b))/b^(3/2) - 2/3*sqrt(b*x^2 + a)*B*a/b^2
```

Fricas [**A**]
time = 10.87, size = 127, normalized size = 1.57

$$\left[\frac{3\,Aa\sqrt{b}\,\log\left(-2\,bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,(2\,Bbx^{2}+3\,Abx-4\,Ba)\sqrt{bx^{2}+a}}{12\,b^{2}},\frac{3\,Aa\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+(2\,Bbx^{2}+3\,Abx-4\,Ba)\sqrt{bx^{2}+a}}{6\,b^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*A*a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(2
*B*b*x^2 + 3*A*b*x - 4*B*a)*sqrt(b*x^2 + a))/b^2, 1/6*(3*A*a*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*B*b*x^2 + 3*A*b*x - 4*B*a)*sqrt(b*x^2 +
a))/b^2]
```

Sympy [**A**]
time = 1.92, size = 94, normalized size = 1.16

$$\frac{A\sqrt{a}\,x\sqrt{1+\frac{bx^{2}}{a}}}{2b}-\frac{Aa\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+B\left(\begin{cases}-\frac{2a\sqrt{a+bx^{2}}}{3b^{2}}+\frac{x^{2}\sqrt{a+bx^{2}}}{3b} & \text{for}\ b\neq 0\\ \frac{x^{4}}{4\sqrt{a}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*sqrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - A*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(
3/2)) + B*Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)
/(3*b), Ne(b, 0)), (x**4/(4*sqrt(a)), True))
```

Giac [**A**]
time = 0.96, size = 61, normalized size = 0.75

$$\frac{1}{6}\sqrt{bx^2+a}\left(\left(\frac{2\,Bx}{b}+\frac{3\,A}{b}\right)x-\frac{4\,Ba}{b^2}\right)+\frac{Aa\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(b*x^2 + a)*((2*B*x/b + 3*A/b)*x - 4*B*a/b^2) + 1/2*A*a*log(abs(-sq
rt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

Mupad [**B**]
time = 1.47, size = 93, normalized size = 1.15

$$\begin{cases} \frac{3\,B\,x^4+4\,A\,x^3}{12\,\sqrt{a}} & \text{if}\ \ b=0\\ \frac{A\,x\,\sqrt{b\,x^2+a}}{2\,b}-\frac{A\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}-\frac{B\,\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2} & \text{if}\ \ b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x))/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, (4*A*x^3 + 3*B*x^4)/(12*a^(1/2)), b ~= 0, - (A*a*log(2*b^
(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)) + (A*x*(a + b*x^2)^(1/2))/(2*b)
 - (B*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2))
```

3.24 $\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=56

$$\frac{(2A+Bx)\sqrt{a+bx^2}}{2b}-\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] -1/2*a*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/2*(B*x+2*A)*(b*x^2+a)
^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {794, 223, 212}

$$\frac{\sqrt{a+bx^2}\,(2A+Bx)}{2b}-\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/(2*b) - (a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(2*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx &= \frac{(2A+Bx)\sqrt{a+bx^2}}{2b}-\frac{(aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b}\\ &= \frac{(2A+Bx)\sqrt{a+bx^2}}{2b}-\frac{(aB)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b}\\ &= \frac{(2A+Bx)\sqrt{a+bx^2}}{2b}-\frac{aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 58, normalized size = 1.04

$$\frac{(2A+Bx)\sqrt{a+bx^2}}{2b}+\frac{aB\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x))/Sqrt[a + b*x^2],x]
```

```
[Out] ((2*A + B*x)*Sqrt[a + b*x^2])/(2*b) + (a*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^
2]])/(2*b^(3/2))
```

Maple [A]
time = 0.11, size = 56, normalized size = 1.00

method	result	size
risch	$\frac{(Bx+2A)\sqrt{b\,x^2+a}}{2b}-\frac{aB\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}$	46
default	$B\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)+\frac{A\sqrt{b\,x^2+a}}{b}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))+A*(
b*x^2+a)^(1/2)/b
```

Maxima [A]
time = 0.27, size = 47, normalized size = 0.84

$$\frac{\sqrt{bx^2+a}\,Bx}{2\,b}-\frac{Ba\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{\sqrt{bx^2+a}\,A}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*B*x/b - 1/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(3/2) + sqrt(b
*x^2 + a)*A/b
```

Fricas [**A**]
time = 9.06, size = 109, normalized size = 1.95

$$\left[\frac{Ba\sqrt{b}\,\log\left(-2\,bx^2 + 2\,\sqrt{bx^2 + a}\,\sqrt{b}\,x - a\right) + 2\,(Bbx + 2\,Ab)\sqrt{bx^2 + a}}{4\,b^2}, \frac{Ba\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2 + a}}\right) + (Bbx + 2\,Ab)\sqrt{bx^2 + a}}{2\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(B*a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(B*b*
x + 2*A*b)*sqrt(b*x^2 + a))/b^2, 1/2*(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b
*x^2 + a)) + (B*b*x + 2*A*b)*sqrt(b*x^2 + a))/b^2]
```

Sympy [**A**]
time = 1.89, size = 70, normalized size = 1.25

$$A\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for}\ b = 0\\ \frac{\sqrt{a + bx^2}}{b} & \text{otherwise}\end{cases}\right) + \frac{B\sqrt{a}\,x\sqrt{1 + \frac{bx^2}{a}}}{2b} - \frac{Ba\,\text{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((x**2/(2*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**2)/b, True)) + B*s
qrt(a)*x*sqrt(1 + b*x**2/a)/(2*b) - B*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2
))
```

Giac [**A**]
time = 0.87, size = 50, normalized size = 0.89

$$\frac{1}{2}\,\sqrt{bx^2 + a}\,\left(\frac{Bx}{b} + \frac{2\,A}{b}\right) + \frac{Ba\log\left(\left|-\sqrt{b}\,x + \sqrt{bx^2 + a}\,\right|\right)}{2\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*(B*x/b + 2*A/b) + 1/2*B*a*log(abs(-sqrt(b)*x + sqrt(b*x
^2 + a)))/b^(3/2)
```

Mupad [B]

time = 1.24, size = 82, normalized size = 1.46

$$\begin{cases} \frac{2\,B\,x^3+3\,A\,x^2}{6\,\sqrt{a}} & \text{if}\ \ b=0\\ \frac{A\,\sqrt{b\,x^2+a}}{b}-\frac{B\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}+\frac{B\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if}\ \ b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x))/(a + b*x^2)^(1/2),x)
```

```
[Out] piecewise(b == 0, (3*A*x^2 + 2*B*x^3)/(6*a^(1/2)), b ~= 0, (A*(a + b*x^2)^(
1/2))/b - (B*a*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)) + (B*x*(
a + b*x^2)^(1/2))/(2*b))
```

3.25 $\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=43

$$\frac{B\sqrt{a+bx^2}}{b}+\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+B*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {655, 223, 212}

$$\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b + (A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx &= \frac{B\sqrt{a+bx^2}}{b}+A\int \frac{1}{\sqrt{a+bx^2}}\,dx\\ &= \frac{B\sqrt{a+bx^2}}{b}+A\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)\\ &= \frac{B\sqrt{a+bx^2}}{b}+\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 46, normalized size = 1.07

$$\frac{B\sqrt{a+bx^2}}{b}-\frac{A\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/Sqrt[a + b*x^2],x]
```

```
[Out] (B*Sqrt[a + b*x^2])/b - (A*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b]
```

Maple [A]
time = 0.11, size = 37, normalized size = 0.86

method	result	size
default	$\frac{B\sqrt{b\,x^2+a}}{b}+\frac{A\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$	37
risch	$\frac{B\sqrt{b\,x^2+a}}{b}+\frac{A\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(b*x^2+a)^(1/2)/b+A*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.28, size = 29, normalized size = 0.67

$$\frac{A\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}+\frac{\sqrt{bx^2+a}\,B}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] A*arcsinh(b*x/sqrt(a*b))/sqrt(b) + sqrt(b*x^2 + a)*B/b
```

Fricas [A]
time = 6.04, size = 92, normalized size = 2.14

$$\left[\frac{A\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,\sqrt{bx^{2}+a}\,B}{2\,b},-\frac{A\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-\sqrt{bx^{2}+a}\,B}{b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(A*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqrt(b*
x^2 + a)*B)/b, -(A*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt(b*x^2
 + a)*B)/b]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(37) = 74$.
time = 0.57, size = 102, normalized size = 2.37

$$A\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\,\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\,\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\,\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\ b>0\wedge a<0\end{cases}\right)+B\left(\begin{cases}\frac{x^{2}}{2\sqrt{a}} & \text{for}\ b=0\\ \frac{\sqrt{a+bx^{2}}}{b} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x**2+a)**(1/2),x)
```

```
[Out] A*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sq
rt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x
*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) + B*Piecewise((x**2/(2*sqrt(a)),
 Eq(b, 0)), (sqrt(a + b*x**2)/b, True))
```

Giac [A]
time = 1.21, size = 39, normalized size = 0.91

$$-\frac{A\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{\sqrt{b}}+\frac{\sqrt{bx^{2}+a}\,B}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -A*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + sqrt(b*x^2 + a)*B/b
```

Mupad [B]
time = 1.14, size = 36, normalized size = 0.84

$$\frac{B\,\sqrt{b\,x^2+a}}{b}+\frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(a + b*x^2)^(1/2),x)
```

```
[Out] (B*(a + b*x^2)^(1/2))/b + (A*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2)
```

3.26 $\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=53

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)+B*arctanh(x*b^(1/2)/(b*x^2+a)^(
1/2))/b^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {858, 223, 212, 272, 65, 214}

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (A*ArcTanh[Sqrt[a + b*x^
2]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx &= A\int \frac{1}{x\sqrt{a+bx^2}}\,dx + B\int \frac{1}{\sqrt{a+bx^2}}\,dx \\
&= \frac{1}{2}A\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, x^2\right) + B\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right) \\
&= \frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} + \frac{A\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx^2}\right)}{b} \\
&= \frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}} - \frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 66, normalized size = 1.25

$$\frac{2A\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (2*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a] - (B*Log[-(Sqr
t[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b]
```

Maple [A]
time = 0.11, size = 52, normalized size = 0.98

method	result	size
default	$\frac{B\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{A\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)-A/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+
a)^(1/2))/x)
```

Maxima [A]
time = 0.27, size = 33, normalized size = 0.62

$$\frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] B*arcsinh(b*x/sqrt(a*b))/sqrt(b) - A*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a)
```

Fricas [A]
time = 3.86, size = 273, normalized size = 5.15

$$\left[\frac{Ba\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+A\sqrt{a}\,b\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)}{2\,ab},-\frac{2\,Ba\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-A\sqrt{a}\,b\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)}{2\,ab},\frac{2\,A\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+Ba\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)}{2\,ab},-\frac{Ba\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-A\sqrt{-a}\,b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + A*sqrt(
a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*b), -1/2*(2*B*
a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - A*sqrt(a)*b*log(-(b*x^2 - 2
*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*b), 1/2*(2*A*sqrt(-a)*b*arctan(sqr
t(-a)/sqrt(b*x^2 + a)) + B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a))/(a*b), -(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - A*sqr
t(-a)*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/(a*b)]
```

Sympy [A]
time = 1.39, size = 99, normalized size = 1.87

$$-\frac{A\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}}+B\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for } a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for } b>0\wedge a<0\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + B*Piecewise((sqrt(-a/b)*asin(x*sqrt
(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a),
 (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a
 < 0)))
```

Giac [A]
time = 0.95, size = 58, normalized size = 1.09

$$\frac{2\,A\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{B\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - B*log(abs(-s
qrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b)
```

Mupad [B]
time = 1.30, size = 42, normalized size = 0.79

$$\frac{B\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{A\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x*(a + b*x^2)^(1/2)),x)
```

```
[Out] (B*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (A*atanh((a + b*x^2)^(1/2)
/a^(1/2)))/a^(1/2)
```

3.27 $\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=47

$$-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -B*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)-A*(b*x^2+a)^(1/2)/a/x
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {821, 272, 65, 214}

$$-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
```

```
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{ax}+B\int \frac{1}{x\sqrt{a+bx^2}}\,dx\\ &= -\frac{A\sqrt{a+bx^2}}{ax}+\frac{1}{2}B\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= -\frac{A\sqrt{a+bx^2}}{ax}+\frac{B\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{b}\\ &= -\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [**A**]
time = 0.13, size = 57, normalized size = 1.21

$$-\frac{A\sqrt{a+bx^2}}{ax}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2])/(a*x)) + (2*B*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/S
qrt[a]])/Sqrt[a]
```

Maple [**A**]
time = 0.12, size = 49, normalized size = 1.04

method	result	size
default	$-\frac{B\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}-\frac{A\sqrt{b\,x^2+a}}{ax}$	49
risch	$-\frac{B\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}-\frac{A\sqrt{b\,x^2+a}}{ax}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -B/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-A*(b*x^2+a)^(1/2)/a/x
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.79

$$-\frac{B \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}} - \frac{\sqrt{bx^2+a}\,A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -B*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) - sqrt(b*x^2 + a)*A/(a*x)
```

Fricas [A]
time = 3.90, size = 101, normalized size = 2.15

$$\left[\frac{B\sqrt{a}\,x\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\,\sqrt{a}+2a}{x^2}\right)-2\sqrt{bx^2+a}\,A}{2\,ax},\frac{B\sqrt{-a}\,x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}\,A}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(B*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*s
qrt(b*x^2 + a)*A)/(a*x), (B*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - s
qrt(b*x^2 + a)*A)/(a*x)]
```

Sympy [A]
time = 1.17, size = 41, normalized size = 0.87

$$-\frac{A\sqrt{b}\,\sqrt{\frac{a}{bx^2}+1}}{a} - \frac{B\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/a - B*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

Giac [A]
time = 1.03, size = 65, normalized size = 1.38

$$\frac{2\,B\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{2\,A\sqrt{b}}{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) + 2*A*sqrt(b)/
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

Mupad [B]
time = 1.20, size = 39, normalized size = 0.83

$$-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{A\,\sqrt{b\,x^2+a}}{a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^2*(a + b*x^2)^(1/2)),x)
```

```
[Out] - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (A*(a + b*x^2)^(1/2))/(a*x
)
```

3.28 $\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=72

$$-\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}+\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

```
[Out] 1/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)-1/2*A*(b*x^2+a)^(1/2)/a/x^
2-B*(b*x^2+a)^(1/2)/a/x
```

Rubi [A]
time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {849, 821, 272, 65, 214}

$$\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/2*(A*Sqrt[a + b*x^2])/(a*x^2) - (B*Sqrt[a + b*x^2])/(a*x) + (A*b*ArcTanh
[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx &= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{\int \frac{-2aB+Abx}{x^2\sqrt{a+bx^2}}\,dx}{2a}\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}-\frac{(Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{2a}\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}-\frac{(Ab)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{4a}\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}-\frac{A\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{2a}\\
&= -\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}+\frac{Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.22, size = 65, normalized size = 0.90

$$
-\frac{(A+2Bx)\sqrt{a+bx^2}}{2ax^2}-\frac{Ab\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/2*((A + 2*B*x)*Sqrt[a + b*x^2])/(a*x^2) - (A*b*ArcTanh[(Sqrt[b]*x - Sqrt
[a + b*x^2])/Sqrt[a]])/a^(3/2)
```

Maple [A]
time = 0.12, size = 69, normalized size = 0.96

method	result	size
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(2 B x +A)}{2 a\,x^{2}}+\frac{A b \ln\left(\frac{2 a +2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2 a^{\frac{3}{2}}}$	55
default	$A \left(-\frac{\sqrt{b\,x^{2}+a}}{2 a\,x^{2}}+\frac{b \ln\left(\frac{2 a +2 \sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2 a^{\frac{3}{2}}}\right)-\frac{B \sqrt{b\,x^{2}+a}}{a x}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/
2))/x))-B*(b*x^2+a)^(1/2)/a/x
```

Maxima [A]
time = 0.27, size = 56, normalized size = 0.78

$$\frac{Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{\sqrt{bx^2+a}\,B}{ax}-\frac{\sqrt{bx^2+a}\,A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - sqrt(b*x^2 + a)*B/(a*x) - 1
/2*sqrt(b*x^2 + a)*A/(a*x^2)
```

Fricas [A]
time = 4.60, size = 123, normalized size = 1.71

$$\left[\frac{A\sqrt{a}\,bx^2\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,(2\,Bax+Aa)\sqrt{bx^2+a}}{4\,a^2x^2},-\frac{A\sqrt{-a}\,bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(2\,Bax+Aa)\sqrt{bx^2+a}}{2\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(A*sqrt(a)*b*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) -
 2*(2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*x^2), -1/2*(A*sqrt(-a)*b*x^2*arcta
n(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*x^2)]
```

Sympy [A]
time = 1.76, size = 66, normalized size = 0.92

$$-\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax}+\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{3}{2}}}-\frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] -A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2
*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(58) = 116.
time = 0.97, size = 146, normalized size = 2.03

$$-\frac{Ab\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a) + ((sqrt(
b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(
b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/(((sqrt(b)*x -
sqrt(b*x^2 + a))^2 - a)^2*a)
```

Mupad [B]
time = 1.35, size = 58, normalized size = 0.81

$$\frac{A\,b\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{B\,\sqrt{b\,x^2+a}}{a\,x}-\frac{A\,\sqrt{b\,x^2+a}}{2\,a\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^3*(a + b*x^2)^(1/2)),x)
```

```
[Out] (A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) - (B*(a + b*x^2)^(1/2))/
(a*x) - (A*(a + b*x^2)^(1/2))/(2*a*x^2)
```

3.29 $\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=81

$$-\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2}-\frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

```
[Out] -3/2*a*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-x^2*(B*x+A)/b/(b*x^2+a)
^(1/2)+1/2*(3*B*x+4*A)*(b*x^2+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {833, 794, 223, 212}

$$\frac{\sqrt{a+bx^2}\,(4A+3Bx)}{2b^2}-\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}-\frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((x^2*(A + B*x))/(b*Sqrt[a + b*x^2])) + ((4*A + 3*B*x)*Sqrt[a + b*x^2])/(2
*b^2) - (3*a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(A+Bx)}{(a+bx^2)^{3/2}}\,dx &= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{\int \frac{x(2aA+3aBx)}{\sqrt{a+bx^2}}\,dx}{ab}\\ &= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2}-\frac{(3aB)\int \frac{1}{\sqrt{a+bx^2}}\,dx}{2b^2}\\ &= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2}-\frac{(3aB)\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{2b^2}\\ &= -\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2}-\frac{3aB\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.26, size = 74, normalized size = 0.91

$$\frac{4aA+3aBx+2Abx^2+bBx^3}{2b^2\sqrt{a+bx^2}}+\frac{3aB\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (4*a*A + 3*a*B*x + 2*A*b*x^2 + b*B*x^3)/(2*b^2*Sqrt[a + b*x^2]) + (3*a*B*Lo
g[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(5/2))
```

Maple [A]
time = 0.12, size = 98, normalized size = 1.21

method	result

risch	$\frac{(Bx+2A)\sqrt{b\,x^{2}+a}}{2b^{2}}+\frac{aBx}{b^{2}\sqrt{b\,x^{2}+a}}-\frac{3aB\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{5}{2}}}+\frac{aA}{b^{2}\sqrt{b\,x^{2}+a}}$
default	$B\left(\frac{x^{3}}{2b\sqrt{b\,x^{2}+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)+A\left(\frac{x^{2}}{b\sqrt{b\,x^{2}+a}}+\frac{2a}{b^{2}\sqrt{b\,x^{2}+a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/2*x^3/b/(b*x^2+a)^(1/2)-3/2*a/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b
^(1/2)+(b*x^2+a)^(1/2))))+A*(x^2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.29, size = 85, normalized size = 1.05

$$\frac{Bx^{3}}{2\sqrt{bx^{2}+a}\,b}+\frac{Ax^{2}}{\sqrt{bx^{2}+a}\,b}+\frac{3\,Bax}{2\sqrt{bx^{2}+a}\,b^{2}}-\frac{3\,Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}}+\frac{2\,Aa}{\sqrt{bx^{2}+a}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*B*x^3/(sqrt(b*x^2 + a)*b) + A*x^2/(sqrt(b*x^2 + a)*b) + 3/2*B*a*x/(sqrt
(b*x^2 + a)*b^2) - 3/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 2*A*a/(sqrt(b*x
^2 + a)*b^2)
```

Fricas [A]
time = 7.08, size = 197, normalized size = 2.43

$$\left[\frac{3\,(Babx^{2}+Ba^{2})\sqrt{b}\,\log\left(-2\,bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,(Bb^{2}x^{3}+2\,Ab^{2}x^{2}+3\,Babx+4\,Aab)\sqrt{bx^{2}+a}}{4\,(b^{4}x^{2}+ab^{3})},\ \frac{3\,(Babx^{2}+Ba^{2})\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+(Bb^{2}x^{3}+2\,Ab^{2}x^{2}+3\,Babx+4\,Aab)\sqrt{bx^{2}+a}}{2\,(b^{4}x^{2}+ab^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(B*a*b*x^2 + B*a^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b
)*x - a) + 2*(B*b^2*x^3 + 2*A*b^2*x^2 + 3*B*a*b*x + 4*A*a*b)*sqrt(b*x^2 + a
))/(b^4*x^2 + a*b^3), 1/2*(3*(B*a*b*x^2 + B*a^2)*sqrt(-b)*arctan(sqrt(-b)*x
/sqrt(b*x^2 + a)) + (B*b^2*x^3 + 2*A*b^2*x^2 + 3*B*a*b*x + 4*A*a*b)*sqrt(b*
x^2 + a))/(b^4*x^2 + a*b^3)]
```

Sympy [**A**]
time = 4.51, size = 117, normalized size = 1.44

$$A\left(\begin{cases}\frac{2a}{b^2\sqrt{a+bx^2}}+\frac{x^2}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+B\left(\frac{3\sqrt{a}\,x}{2b^2\sqrt{1+\frac{bx^2}{a}}}-\frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}}+\frac{x^3}{2\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b,
 0)), (x**4/(4*a**(3/2)), True)) + B*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2/a
)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(1 +
 b*x**2/a)))
```

Giac [**A**]
time = 1.39, size = 70, normalized size = 0.86

$$\frac{\left(\left(\frac{Bx}{b}+\frac{2A}{b}\right)x+\frac{3Ba}{b^2}\right)x+\frac{4Aa}{b^2}}{2\sqrt{bx^2+a}}+\frac{3Ba\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{2b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(((B*x/b + 2*A/b)*x + 3*B*a/b^2)*x + 4*A*a/b^2)/sqrt(b*x^2 + a) + 3/2*B
*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(A+B\,x\right)}{{\left(b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x))/(a + b*x^2)^(3/2),x)
```

```
[Out] int((x^3*(A + B*x))/(a + b*x^2)^(3/2), x)
```

3.30 $\int \frac{x^2(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}+\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

```
[Out] A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-x*(B*x+A)/b/(b*x^2+a)^(1/2)+2*
B*(b*x^2+a)^(1/2)/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {833, 655, 223, 212}

$$\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((x*(A + B*x))/(b*Sqrt[a + b*x^2])) + (2*B*Sqrt[a + b*x^2])/b^2 + (A*ArcTa
nh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(A+Bx)}{(a+bx^2)^{3/2}}\,dx &= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{\int \frac{aA+2aBx}{\sqrt{a+bx^2}}\,dx}{ab}\\
&= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}+\frac{A\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}\\
&= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}+\frac{A\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b}\\
&= -\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}+\frac{A\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.27, size = 61, normalized size = 0.92

$$\frac{2aB-Abx+bBx^2}{b^2\sqrt{a+bx^2}}-\frac{A\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (2*a*B - A*b*x + b*B*x^2)/(b^2*Sqrt[a + b*x^2]) - (A*Log[-(Sqrt[b]*x) + Sqr
t[a + b*x^2]])/b^(3/2)
```

Maple [A]
time = 0.13, size = 74, normalized size = 1.12

method	result	size

risch	$\frac{B\sqrt{b\,x^{2}+a}}{b^{2}}-\frac{Ax}{b\sqrt{b\,x^{2}+a}}+\frac{A\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}+\frac{aB}{b^{2}\sqrt{b\,x^{2}+a}}$	68
default	$B\left(\frac{x^{2}}{b\sqrt{b\,x^{2}+a}}+\frac{2a}{b^{2}\sqrt{b\,x^{2}+a}}\right)+A\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(x^2/b/(b*x^2+a)^(1/2)+2*a/b^2/(b*x^2+a)^(1/2))+A*(-x/b/(b*x^2+a)^(1/2)+1
/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))
```

Maxima [**A**]
time = 0.27, size = 64, normalized size = 0.97

$$\frac{Bx^{2}}{\sqrt{bx^{2}+a}\,b}-\frac{Ax}{\sqrt{bx^{2}+a}\,b}+\frac{A\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}+\frac{2\,Ba}{\sqrt{bx^{2}+a}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] B*x^2/(sqrt(b*x^2 + a)*b) - A*x/(sqrt(b*x^2 + a)*b) + A*arcsinh(b*x/sqrt(a*
b))/b^(3/2) + 2*B*a/(sqrt(b*x^2 + a)*b^2)
```

Fricas [**A**]
time = 2.74, size = 164, normalized size = 2.48

$$\left[\frac{(Abx^{2}+Aa)\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)+2\,(Bbx^{2}-Abx+2\,Ba)\sqrt{bx^{2}+a}}{2\,(b^{3}x^{2}+ab^{2})},-\frac{(Abx^{2}+Aa)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)-(Bbx^{2}-Abx+2\,Ba)\sqrt{bx^{2}+a}}{b^{3}x^{2}+ab^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((A*b*x^2 + A*a)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) + 2*(B*b*x^2 - A*b*x + 2*B*a)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2), -((A*b
*x^2 + A*a)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (B*b*x^2 - A*b*x
+ 2*B*a)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2)]
```

Sympy [**A**]
time = 3.57, size = 83, normalized size = 1.26

$$A\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,b\sqrt{1+\frac{bx^{2}}{a}}}\right)+B\left(\begin{cases}\frac{2a}{b^{2}\sqrt{a+bx^{2}}}+\frac{x^{2}}{b\sqrt{a+bx^{2}}} & \text{for}\ b\neq 0\\ \frac{x^{4}}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) +
B*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b,
 0)), (x**4/(4*a**(3/2)), True))
```

Giac [**A**]
time = 1.18, size = 58, normalized size = 0.88

$$\frac{\left(\frac{Bx}{b}-\frac{A}{b}\right)x+\frac{2Ba}{b^2}}{\sqrt{bx^2+a}}-\frac{A\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] ((B*x/b - A/b)*x + 2*B*a/b^2)/sqrt(b*x^2 + a) - A*log(abs(-sqrt(b)*x + sqrt
(b*x^2 + a)))/b^(3/2)
```

Mupad [**B**]
time = 1.34, size = 61, normalized size = 0.92

$$\frac{A\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}}-\frac{A\,x}{b\,\sqrt{b\,x^2+a}}+\frac{B\,\left(b\,x^2+2\,a\right)}{b^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x))/(a + b*x^2)^(3/2),x)
```

```
[Out] (A*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) - (A*x)/(b*(a + b*x^2)^(1/2)
) + (B*(2*a + b*x^2))/(b^2*(a + b*x^2)^(1/2))
```

3.31 $\int \frac{x(A+Bx)}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{A+Bx}{b\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

```
[Out] B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+(-B*x-A)/b/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {792, 223, 212}

$$\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{A+Bx}{b\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] -((A + B*x)/(b*Sqrt[a + b*x^2])) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/b^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx &= -\frac{A+Bx}{b\sqrt{a+bx^2}}+\frac{B\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b}\\ &= -\frac{A+Bx}{b\sqrt{a+bx^2}}+\frac{B\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b}\\ &= -\frac{A+Bx}{b\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}\end{aligned}$$

Mathematica [**A**]
time = 0.25, size = 53, normalized size = 1.10

$$\frac{-A-Bx}{b\sqrt{a+bx^2}}-\frac{B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x))/(a + b*x^2)^(3/2),x]
```

```
[Out] (-A - B*x)/(b*Sqrt[a + b*x^2]) - (B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^
(3/2)
```

Maple [**A**]
time = 0.11, size = 55, normalized size = 1.15

method	result	size
default	$B\left(-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}\right)-\frac{A}{b\sqrt{b\,x^2+a}}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))-A/b/(b*x^2
+a)^(1/2)
```

Maxima [**A**]
time = 0.28, size = 46, normalized size = 0.96

$$-\frac{Bx}{\sqrt{bx^2+a}\,b}+\frac{B\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}-\frac{A}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -B*x/(sqrt(b*x^2 + a)*b) + B*arcsinh(b*x/sqrt(a*b))/b^(3/2) - A/(sqrt(b*x^2
 + a)*b)
```

Fricas [A]
time = 1.02, size = 147, normalized size = 3.06

$$\left[\frac{(Bbx^2+Ba)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(Bbx+Ab)\sqrt{bx^2+a}}{2\,(b^3x^2+ab^2)},-\frac{(Bbx^2+Ba)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(Bbx+Ab)\sqrt{bx^2+a}}{b^3x^2+ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b*x^2 + B*a)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) - 2*(B*b*x + A*b)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2), -((B*b*x^2 + B*a)*
sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (B*b*x + A*b)*sqrt(b*x^2 + a)
)/(b^3*x^2 + a*b^2)]
```

Sympy [A]
time = 3.07, size = 66, normalized size = 1.38

$$A\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)+B\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}}-\frac{x}{\sqrt{a}\,b\sqrt{1+\frac{bx^2}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True))
 + B*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a)))
```

Giac [A]
time = 0.68, size = 48, normalized size = 1.00

$$-\frac{\frac{Bx}{b}+\frac{A}{b}}{\sqrt{bx^2+a}}-\frac{B\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*x/b + A/b)/sqrt(b*x^2 + a) - B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b
^(3/2)
```

Mupad [B]
time = 1.06, size = 53, normalized size = 1.10

$$\frac{B\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}}-\frac{A}{b\,\sqrt{b\,x^2+a}}-\frac{B\,x}{b\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x))/(a + b*x^2)^(3/2),x)
[Out] (B*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) - A/(b*(a + b*x^2)^(1/2)) -
(B*x)/(b*(a + b*x^2)^(1/2))
```

3.32 $\int \frac{A+Bx}{\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$-\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

```
[Out] (A*b*x-B*a)/a/b/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {651}

$$-\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(a + b*x^2)^(3/2),x]
```

```
[Out] -((a*B - A*b*x)/(a*b*Sqrt[a + b*x^2]))
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rubi steps

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{3/2}}\,dx=-\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

Mathematica [A]
time = 0.23, size = 27, normalized size = 0.96

$$\frac{-aB+Abx}{ab\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(a + b*x^2)^(3/2),x]
```

```
[Out] (-(a*B) + A*b*x)/(a*b*Sqrt[a + b*x^2])
```

Maple [A]
time = 0.11, size = 32, normalized size = 1.14

method	result	size
gosper	$\frac{Abx-Ba}{ab\sqrt{b\,x^{2}+a}}$	26
trager	$\frac{Abx-Ba}{ab\sqrt{b\,x^{2}+a}}$	26
default	$-\frac{B}{b\sqrt{b\,x^{2}+a}}+\frac{Ax}{a\sqrt{b\,x^{2}+a}}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -B/b/(b*x^2+a)^(1/2)+A*x/a/(b*x^2+a)^(1/2)
```

Maxima [A]
time = 0.27, size = 31, normalized size = 1.11

$$\frac{Ax}{\sqrt{bx^2+a}\,a}-\frac{B}{\sqrt{bx^2+a}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] A*x/(sqrt(b*x^2 + a)*a) - B/(sqrt(b*x^2 + a)*b)
```

Fricas [A]
time = 2.08, size = 35, normalized size = 1.25

$$\frac{(Abx-Ba)\sqrt{bx^2+a}}{ab^2x^2+a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] (A*b*x - B*a)*sqrt(b*x^2 + a)/(a*b^2*x^2 + a^2*b)
```

Sympy [A]
time = 2.61, size = 46, normalized size = 1.64

$$\frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}+B\left(\begin{cases}-\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x**2+a)**(3/2),x)
```

```
[Out] A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*Piecewise((-1/(b*sqrt(a + b*x**2)), N
e(b, 0)), (x**2/(2*a**(3/2)), True))
```

Giac [A]
time = 0.60, size = 23, normalized size = 0.82

$$\frac{\frac{Ax}{a} - \frac{B}{b}}{\sqrt{bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (A*x/a - B/b)/sqrt(b*x^2 + a)
```

Mupad [B]
time = 0.91, size = 24, normalized size = 0.86

$$-\frac{\frac{B}{b} - \frac{A\,x}{a}}{\sqrt{b\,x^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(a + b*x^2)^(3/2),x)
```

```
[Out] -(B/b - (A*x)/a)/(a + b*x^2)^(1/2)
```

3.33 $\int \frac{A+Bx}{x\left(a+bx^2\right)^{3/2}} \, dx$

Optimal. Leaf size=47

$$\frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)+(B*x+A)/a/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {837, 12, 272, 65, 214}

$$\frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx}{x\left(a+bx^2\right)^{3/2}}\,dx &= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{\int \frac{aAb}{x\sqrt{a+bx^2}}\,dx}{a^2b}\\
&= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{A\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a}\\
&= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{A\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}\\
&= \frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{A\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{ab}\\
&= \frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.22, size = 57, normalized size = 1.21

$$\frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*Sqrt[a + b*x^2]) + (2*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/
Sqrt[a]])/a^(3/2)
```

Maple [A]
time = 0.12, size = 61, normalized size = 1.30

method	result	size
default	$\frac{Bx}{a\sqrt{b\,x^{2}+a}}+A\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*x/a/(b*x^2+a)^(1/2)+A*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b
*x^2+a)^(1/2))/x))
```

Maxima [A]
time = 0.27, size = 48, normalized size = 1.02

$$\frac{Bx}{\sqrt{bx^{2}+a}\,a}-\frac{A\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{3}{2}}}+\frac{A}{\sqrt{bx^{2}+a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] B*x/(sqrt(b*x^2 + a)*a) - A*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + A/(sqrt
(b*x^2 + a)*a)
```

Fricas [A]
time = 1.17, size = 146, normalized size = 3.11

$$\left[\frac{\left(Abx^{2}+Aa\right)\sqrt{a}\,\log\left(-\frac{bx^{2}-2\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\left(Bax+Aa\right)\sqrt{bx^{2}+a}}{2\left(a^{2}bx^{2}+a^{3}\right)},\ \frac{\left(Abx^{2}+Aa\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(Bax+Aa\right)\sqrt{bx^{2}+a}}{a^{2}bx^{2}+a^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((A*b*x^2 + A*a)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a
)/x^2) + 2*(B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^2 + a^3), ((A*b*x^2 + A*
a)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (B*a*x + A*a)*sqrt(b*x^2 + a
))/(a^2*b*x^2 + a^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(39) = 78$.
time = 3.58, size = 206, normalized size = 4.38

$$A\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)+\frac{Bx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b*
x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a) +
1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9/2)
 + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a**(9/
2) + 2*a**(7/2)*b*x**2)) + B*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 0.68, size = 59, normalized size = 1.26

$$\frac{\frac{Bx}{a}+\frac{A}{a}}{\sqrt{bx^2+a}}+\frac{2\,A\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] (B*x/a + A/a)/sqrt(b*x^2 + a) + 2*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/s
qrt(-a))/(sqrt(-a)*a)
```

Mupad [B]
time = 1.29, size = 50, normalized size = 1.06

$$\frac{A}{a\sqrt{b\,x^2+a}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{B\,x}{a\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x*(a + b*x^2)^(3/2)),x)
```

```
[Out] A/(a*(a + b*x^2)^(1/2)) - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(3/2) + (B
*x)/(a*(a + b*x^2)^(1/2))
```

3.34 $\int \frac{A+Bx}{x^2(a+bx^2)^{3/2}}\,dx$

Optimal. Leaf size=70

$$\frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

```
[Out] -B*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)+(B*x+A)/a/x/(b*x^2+a)^(1/2)-2*A
*(b*x^2+a)^(1/2)/a^2/x
```

Rubi [A]

time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {837, 821, 272, 65, 214}

$$-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}+\frac{A+Bx}{ax\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*x*Sqrt[a + b*x^2]) - (2*A*Sqrt[a + b*x^2])/(a^2*x) - (B*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{3/2}}\,dx &= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{\int \frac{-2aAb-abBx}{x^2\sqrt{a+bx^2}}\,dx}{a^2b}\\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}+\frac{B\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a}\\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}+\frac{B\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}\\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}+\frac{B\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{ab}\\
&= \frac{A+Bx}{ax\sqrt{a+bx^2}}-\frac{2A\sqrt{a+bx^2}}{a^2x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.23, size = 71, normalized size = 1.01

$$\frac{-aA+aBx-2Abx^2}{a^2x\sqrt{a+bx^2}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^2*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a*A) + a*B*x - 2*A*b*x^2)/(a^2*x*Sqrt[a + b*x^2]) + (2*B*ArcTanh[(Sqrt[b
]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2)
```

Maple [A]
time = 0.13, size = 82, normalized size = 1.17

method	result	size
risch	$-\frac{A\sqrt{b\,x^{2}+a}}{a^{2}x}-\frac{Abx}{a^{2}\sqrt{b\,x^{2}+a}}+\frac{B}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)B}{a^{\frac{3}{2}}}$	80
default	$B\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)+A\left(-\frac{1}{ax\sqrt{b\,x^{2}+a}}-\frac{2bx}{a^{2}\sqrt{b\,x^{2}+a}}\right)$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))+A*(
-1/a/x/(b*x^2+a)^(1/2)-2*b/a^2*x/(b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.29, size = 68, normalized size = 0.97

$$-\frac{2\,Abx}{\sqrt{bx^{2}+a}\,a^{2}}-\frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{3}{2}}}+\frac{B}{\sqrt{bx^{2}+a}\,a}-\frac{A}{\sqrt{bx^{2}+a}\,ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*A*b*x/(sqrt(b*x^2 + a)*a^2) - B*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) +
B/(sqrt(b*x^2 + a)*a) - A/(sqrt(b*x^2 + a)*a*x)
```

Fricas [A]
time = 2.52, size = 169, normalized size = 2.41

$$\left[\frac{(Bbx^{3}+Bax)\sqrt{a}\,\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)-2\,(2\,Abx^{2}-Bax+Aa)\sqrt{bx^{2}+a}}{2\,(a^{2}bx^{3}+a^{3}x)},\ \frac{(Bbx^{3}+Bax)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)-(2\,Abx^{2}-Bax+Aa)\sqrt{bx^{2}+a}}{a^{2}bx^{3}+a^{3}x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((B*b*x^3 + B*a*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) - 2*(2*A*b*x^2 - B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^3 + a^3*x)
, ((B*b*x^3 + B*a*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (2*A*b*x^2
 - B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^3 + a^3*x)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(60) = 120$.
time = 5.22, size = 235, normalized size = 3.36

$$A\left(-\frac{1}{a\sqrt{b}\,x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)+B\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**2/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**
2) + 1))) + B*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) +
 a**3*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 +
b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)
/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) +
1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2))
```

Giac [**A**]
time = 0.71, size = 96, normalized size = 1.37

$$-\frac{\frac{Abx}{a^2}-\frac{B}{a}}{\sqrt{bx^2+a}}+\frac{2\,B\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(A*b*x/a^2 - B/a)/sqrt(b*x^2 + a) + 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 +
a))/sqrt(-a))/(sqrt(-a)*a) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2
- a)*a)
```

Mupad [**B**]
time = 1.45, size = 70, normalized size = 1.00

$$\frac{B}{a\,\sqrt{b\,x^2+a}}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{A}{a\,x\,\sqrt{b\,x^2+a}}-\frac{2\,A\,b\,x}{a^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^2*(a + b*x^2)^(3/2)),x)
```

```
[Out] B/(a*(a + b*x^2)^(1/2)) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(3/2) - A/
(a*x*(a + b*x^2)^(1/2)) - (2*A*b*x)/(a^2*(a + b*x^2)^(1/2))
```

3.35 $\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{A+Bx}{ax^2\sqrt{a+bx^2}}-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}-\frac{2B\sqrt{a+bx^2}}{a^2x}+\frac{3Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

```
[Out] 3/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+(B*x+A)/a/x^2/(b*x^2+a)^(1
/2)-3/2*A*(b*x^2+a)^(1/2)/a^2/x^2-2*B*(b*x^2+a)^(1/2)/a^2/x
```

Rubi [A]
time = 0.06, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {837, 849, 821, 272, 65, 214}

$$\frac{3Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}-\frac{2B\sqrt{a+bx^2}}{a^2x}+\frac{A+Bx}{ax^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (A + B*x)/(a*x^2*Sqrt[a + b*x^2]) - (3*A*Sqrt[a + b*x^2])/(2*a^2*x^2) - (2*
B*Sqrt[a + b*x^2])/(a^2*x) + (3*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^
(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{\int \frac{-3aAb-2abBx}{x^3\sqrt{a+bx^2}}\,dx}{a^2b}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} + \frac{\int \frac{4a^2bB-3aAb^2x}{x^2\sqrt{a+bx^2}}\,dx}{2a^3b}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} - \frac{2B\sqrt{a+bx^2}}{a^2x} - \frac{(3Ab)\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{2a^2}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} - \frac{2B\sqrt{a+bx^2}}{a^2x} - \frac{(3Ab)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x,\right.}{4a^2}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} - \frac{2B\sqrt{a+bx^2}}{a^2x} - \frac{(3A)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a-}\right.}{2a^2}$$

$$= \frac{A+Bx}{ax^2\sqrt{a+bx^2}} - \frac{3A\sqrt{a+bx^2}}{2a^2x^2} - \frac{2B\sqrt{a+bx^2}}{a^2x} + \frac{3Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

<u>Mathematica</u> [A]
time = 0.34, size = 83, normalized size = 0.87

$$\frac{-a(A+2Bx)-bx^2(3A+4Bx)}{2a^2x^2\sqrt{a+bx^2}} - \frac{3Ab\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^3*(a + b*x^2)^(3/2)),x]
```

```
[Out] (-(a*(A + 2*B*x)) - b*x^2*(3*A + 4*B*x))/(2*a^2*x^2*Sqrt[a + b*x^2]) - (3*A
*b*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(5/2)
```

<u>Maple</u> [A]
time = 0.13, size = 106, normalized size = 1.12

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(2Bx+A)}{2a^2x^2} - \frac{bA}{a^2\sqrt{b\,x^2+a}} - \frac{bBx}{a^2\sqrt{b\,x^2+a}} + \frac{3\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)Ab}{2a^{\frac{5}{2}}}$

default	$A\left(-\frac{1}{2a\,x^2\sqrt{b\,x^2+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)+B\Big(-\frac{1}{ax\sqrt{b\,x^2+a}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/2/a/x^2/(b*x^2+a)^(1/2)-3/2*b/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*
a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+B*(-1/a/x/(b*x^2+a)^(1/2)-2*b/a^2*x/(b*x^
2+a)^(1/2))
```

Maxima [A]
time = 0.32, size = 89, normalized size = 0.94

$$-\frac{2\,Bbx}{\sqrt{bx^2+a}\,a^2}+\frac{3\,Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{5}{2}}}-\frac{3\,Ab}{2\,\sqrt{bx^2+a}\,a^2}-\frac{B}{\sqrt{bx^2+a}\,ax}-\frac{A}{2\,\sqrt{bx^2+a}\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] -2*B*b*x/(sqrt(b*x^2 + a)*a^2) + 3/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5
/2) - 3/2*A*b/(sqrt(b*x^2 + a)*a^2) - B/(sqrt(b*x^2 + a)*a*x) - 1/2*A/(sqrt
(b*x^2 + a)*a*x^2)
```

Fricas [A]
time = 2.40, size = 211, normalized size = 2.22

$$\left[\frac{3\,(Ab^2x^4+Aabx^2)\sqrt{a}\,\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,(4\,Babx^3+3\,Aabx^2+2\,Ba^2x+Aa^2)\sqrt{bx^2+a}}{4\,(a^3bx^4+a^4x^2)},-\frac{3\,(Ab^2x^4+Aabx^2)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(4\,Babx^3+3\,Aabx^2+2\,Ba^2x+Aa^2)\sqrt{bx^2+a}}{2\,(a^3bx^4+a^4x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(3*(A*b^2*x^4 + A*a*b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqr
t(a) + 2*a)/x^2) - 2*(4*B*a*b*x^3 + 3*A*a*b*x^2 + 2*B*a^2*x + A*a^2)*sqrt(b
*x^2 + a))/(a^3*b*x^4 + a^4*x^2), -1/2*(3*(A*b^2*x^4 + A*a*b*x^2)*sqrt(-a)*
arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (4*B*a*b*x^3 + 3*A*a*b*x^2 + 2*B*a^2*x +
 A*a^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2)]
```

Sympy [A]
time = 4.25, size = 124, normalized size = 1.31

$$A\left(-\frac{1}{2a\sqrt{b}\,x^{3}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{3\sqrt{b}}{2a^{2}x\sqrt{\frac{a}{bx^{2}}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{5}{2}}}\right)+B\left(-\frac{1}{a\sqrt{b}\,x^{2}\sqrt{\frac{a}{bx^{2}}+1}}-\frac{2\sqrt{b}}{a^{2}\sqrt{\frac{a}{bx^{2}}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**3/(b*x**2+a)**(3/2),x)
```

```
[Out] A*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a/
(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))) + B*(-1/(a*sq
rt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**2) + 1)))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(79) = 158.
time = 0.78, size = 171, normalized size = 1.80

$$-\frac{\frac{Bbx}{a^{2}}+\frac{Ab}{a^{2}}}{\sqrt{bx^{2}+a}}-\frac{3\,Ab\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}}+\frac{\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{3}Ab+2\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}Ba\sqrt{b}+\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)Aab-2\,Ba^{2}\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{2}a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -(B*b*x/a^2 + A*b/a^2)/sqrt(b*x^2 + a) - 3*A*b*arctan(-(sqrt(b)*x - sqrt(b*
x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b +
 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 +
a))*A*a*b - 2*B*a^2*sqrt(b))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^2)
```

Mupad [B]
time = 1.59, size = 94, normalized size = 0.99

$$\frac{3\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)}{2\,a^{5/2}}-\frac{3\,A\,b}{2\,a^{2}\,\sqrt{b\,x^{2}+a}}-\frac{A}{2\,a\,x^{2}\,\sqrt{b\,x^{2}+a}}-\frac{\sqrt{b\,x^{2}+a}\,\left(\frac{B}{a}+\frac{2\,B\,b\,x^{2}}{a^{2}}\right)}{b\,x^{3}+a\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^3*(a + b*x^2)^(3/2)),x)
```

```
[Out] (3*A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(5/2)) - (3*A*b)/(2*a^2*(a +
b*x^2)^(1/2)) - A/(2*a*x^2*(a + b*x^2)^(1/2)) - ((a + b*x^2)^(1/2)*(B/a + (
2*B*b*x^2)/a^2))/(a*x + b*x^3)
```

3.36 $\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=79

$$-\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

```
[Out] -1/3*x^2*(B*x+A)/b/(b*x^2+a)^(3/2)+B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(
5/2)+1/3*(-3*B*x-2*A)/b^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {833, 792, 223, 212}

$$-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}-\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*(x^2*(A + B*x))/(b*(a + b*x^2)^(3/2)) - (2*A + 3*B*x)/(3*b^2*Sqrt[a +
b*x^2]) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}+\frac{\int \frac{x(2aA+3aBx)}{\left(a+bx^2\right)^{3/2}}\,dx}{3ab}\\
&= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}+\frac{B\int \frac{1}{\sqrt{a+bx^2}}\,dx}{b^2}\\
&= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}+\frac{B\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{b^2}\\
&= -\frac{x^2(A+Bx)}{3b\left(a+bx^2\right)^{3/2}}-\frac{2A+3Bx}{3b^2\sqrt{a+bx^2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.37, size = 72, normalized size = 0.91

$$
\frac{-2aA-3aBx-3Abx^2-4bBx^3}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{B\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{b^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-2*a*A - 3*a*B*x - 3*A*b*x^2 - 4*b*B*x^3)/(3*b^2*(a + b*x^2)^(3/2)) - (B*L
og[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(5/2)
```

Maple [A]
time = 0.12, size = 97, normalized size = 1.23

method	result	size

default	$B\left(-\frac{x^{3}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}\right)+A\left(-\frac{x^{2}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{2a}{3b^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}\right)$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1
/2)+(b*x^2+a)^(1/2))))+A*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2))
```

Maxima [A]
time = 0.30, size = 102, normalized size = 1.29

$$-\frac{1}{3}\,Bx\left(\frac{3\,x^{2}}{\left(bx^{2}+a\right)^{\frac{3}{2}}b}+\frac{2\,a}{\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}}\right)-\frac{Ax^{2}}{\left(bx^{2}+a\right)^{\frac{3}{2}}b}-\frac{Bx}{3\,\sqrt{bx^{2}+a}\,b^{2}}+\frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}-\frac{2\,Aa}{3\,\left(bx^{2}+a\right)^{\frac{3}{2}}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*B*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) - A*x^
2/((b*x^2 + a)^(3/2)*b) - 1/3*B*x/(sqrt(b*x^2 + a)*b^2) + B*arcsinh(b*x/sqr
t(a*b))/b^(5/2) - 2/3*A*a/((b*x^2 + a)^(3/2)*b^2)
```

Fricas [A]
time = 2.57, size = 239, normalized size = 3.03

$$\left[\frac{3\,\left(Bb^{2}x^{4}+2\,Babx^{2}+Ba^{2}\right)\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\,\left(4\,Bb^{2}x^{3}+3\,Ab^{2}x^{2}+3\,Babx+2\,Aab\right)\sqrt{bx^{2}+a}}{6\,\left(b^{5}x^{4}+2\,ab^{4}x^{2}+a^{2}b^{3}\right)},-\frac{3\,\left(Bb^{2}x^{4}+2\,Babx^{2}+Ba^{2}\right)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+\left(4\,Bb^{2}x^{3}+3\,Ab^{2}x^{2}+3\,Babx+2\,Aab\right)\sqrt{bx^{2}+a}}{3\,\left(b^{5}x^{4}+2\,ab^{4}x^{2}+a^{2}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(B*b^2*x^4 + 2*B*a*b*x^2 + B*a^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x
^2 + a)*sqrt(b)*x - a) - 2*(4*B*b^2*x^3 + 3*A*b^2*x^2 + 3*B*a*b*x + 2*A*a*b
)*sqrt(b*x^2 + a))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3), -1/3*(3*(B*b^2*x^4 +
2*B*a*b*x^2 + B*a^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (4*B*b^2
*x^3 + 3*A*b^2*x^2 + 3*B*a*b*x + 2*A*a*b)*sqrt(b*x^2 + a))/(b^5*x^4 + 2*a*b
^4*x^2 + a^2*b^3)]
```

Sympy [A]
time = 6.30, size = 400, normalized size = 5.06

$$A\left(\begin{cases}-\frac{2a}{3ab^{2}\sqrt{a+bx^{2}}+3b^{3}x^{2}\sqrt{a+bx^{2}}}-\frac{3bx^{2}}{3ab^{2}\sqrt{a+bx^{2}}+3b^{3}x^{2}\sqrt{a+bx^{2}}} & \text{for}\ b\neq 0\\ \frac{x^{4}}{4a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)+B\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^{2}}{a}}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{3a^{\frac{37}{2}}b^{12}x^{2}\sqrt{1+\frac{bx^{2}}{a}}\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}-\frac{4a^{18}b^{\frac{25}{2}}x^{3}}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^{2}\sqrt{1+\frac{bx^{2}}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)
) - 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)), Ne
(b, 0)), (x**4/(4*a**(5/2)), True)) + B*(3*a**(39/2)*b**11*sqrt(1 + b*x**2/
a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a
**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**(37/2)*b**12*x**2*sqrt(1
 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**
2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 3*a**19*b**(23/2)*x
/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqr
t(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x**3/(3*a**(39/2)*b**(27/2)*sqrt(1 + b
*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)))
```

Giac [**A**]
time = 1.01, size = 70, normalized size = 0.89

$$-\frac{\left(\left(\frac{4\,Bx}{b}+\frac{3\,A}{b}\right)x+\frac{3\,Ba}{b^2}\right)x+\frac{2\,Aa}{b^2}}{3\,(bx^2+a)^{\frac{3}{2}}}-\frac{B\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((4*B*x/b + 3*A/b)*x + 3*B*a/b^2)*x + 2*A*a/b^2)/(b*x^2 + a)^(3/2) -
B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,(A+B\,x)}{{(b\,x^2+a)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x))/(a + b*x^2)^(5/2),x)
```

```
[Out] int((x^3*(A + B*x))/(a + b*x^2)^(5/2), x)
```

3.37 $\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=53

$$-\frac{x^2(aB-Abx)}{3ab\,(a+bx^2)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^2}}$$

```
[Out] -1/3*x^2*(-A*b*x+B*a)/a/b/(b*x^2+a)^(3/2)-2/3*B/b^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {819, 267}

$$-\frac{x^2(aB-Abx)}{3ab\,(a+bx^2)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*(x^2*(a*B - A*b*x))/(a*b*(a + b*x^2)^(3/2)) - (2*B)/(3*b^2*Sqrt[a + b*
x^2])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[m*((c*d*f + a*e*g)/(2*a*c*(p + 1))), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx &= -\frac{x^2(aB-Abx)}{3ab\,(a+bx^2)^{3/2}}+\frac{(2B)\int \frac{x}{(a+bx^2)^{3/2}}\,dx}{3b}\\ &= -\frac{x^2(aB-Abx)}{3ab\,(a+bx^2)^{3/2}}-\frac{2B}{3b^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [**A**]
time = 0.39, size = 44, normalized size = 0.83

$$\frac{-2a^2B-3abBx^2+Ab^2x^3}{3ab^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-2*a^2*B - 3*a*b*B*x^2 + A*b^2*x^3)/(3*a*b^2*(a + b*x^2)^(3/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(45) = 90.
time = 0.12, size = 92, normalized size = 1.74

method	result	size
gosper	$\frac{A\,b^{2} x^{3}-3 B a b\,x^{2}-2 a^{2} B}{3 \left(b\,x^{2}+a \right)^{\frac{3}{2}} a\,b^{2}}$	41
trager	$\frac{A\,b^{2} x^{3}-3 B a b\,x^{2}-2 a^{2} B}{3 \left(b\,x^{2}+a \right)^{\frac{3}{2}} a\,b^{2}}$	41
default	$B \left(-\frac{x^{2}}{b \left(b\,x^{2}+a \right)^{\frac{3}{2}}}-\frac{2 a}{3 b^{2} \left(b\,x^{2}+a \right)^{\frac{3}{2}}}\right)+A \left(-\frac{x}{2 b \left(b\,x^{2}+a \right)^{\frac{3}{2}}}+\frac{a \left(\frac{x}{3 a \left(b\,x^{2}+a \right)^{\frac{3}{2}}}+\frac{2 x}{3 a^{2} \sqrt{b\,x^{2}+a}}\right)}{2 b}\right)$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2))+A*(-1/2*x/b/(b*x^2+a)^
(3/2)+1/2*a/b*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))
```

Maxima [**A**]
time = 0.29, size = 70, normalized size = 1.32

$$-\frac{Bx^2}{\left(bx^2+a\right)^{\frac{3}{2}}b}-\frac{Ax}{3\left(bx^2+a\right)^{\frac{3}{2}}b}+\frac{Ax}{3\sqrt{bx^2+a}\,ab}-\frac{2\,Ba}{3\left(bx^2+a\right)^{\frac{3}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -B*x^2/((b*x^2 + a)^(3/2)*b) - 1/3*A*x/((b*x^2 + a)^(3/2)*b) + 1/3*A*x/(sqr
t(b*x^2 + a)*a*b) - 2/3*B*a/((b*x^2 + a)^(3/2)*b^2)
```

Fricas [**A**]
time = 3.13, size = 63, normalized size = 1.19

$$\frac{\left(Ab^2x^3-3\,Babx^2-2\,Ba^2\right)\sqrt{bx^2+a}}{3\left(ab^4x^4+2\,a^2b^3x^2+a^3b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(A*b^2*x^3 - 3*B*a*b*x^2 - 2*B*a^2)*sqrt(b*x^2 + a)/(a*b^4*x^4 + 2*a^2*
b^3*x^2 + a^3*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(46) = 92$.
time = 5.19, size = 141, normalized size = 2.66

$$\frac{Ax^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+B\left(\begin{cases}-\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}}-\frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a
)) + B*Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*
x**2)) - 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)
), Ne(b, 0)), (x**4/(4*a**(5/2)), True))
```

Giac [A]
time = 1.03, size = 36, normalized size = 0.68

$$\frac{\left(\frac{Ax}{a}-\frac{3\,B}{b}\right)x^2-\frac{2\,Ba}{b^2}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((A*x/a - 3*B/b)*x^2 - 2*B*a/b^2)/(b*x^2 + a)^(3/2)
```

Mupad [B]
time = 0.97, size = 51, normalized size = 0.96

$$\frac{B\,a^2-3\,B\,a\,(b\,x^2+a)+A\,b\,x\,(b\,x^2+a)-A\,a\,b\,x}{3\,a\,b^2\,(b\,x^2+a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x))/(a + b*x^2)^(5/2),x)
```

```
[Out] (B*a^2 - 3*B*a*(a + b*x^2) + A*b*x*(a + b*x^2) - A*a*b*x)/(3*a*b^2*(a + b*x
^2)^(3/2))
```

3.38 $\int \frac{x(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=50

$$\frac{-A-Bx}{3b\left(a+bx^2\right)^{3/2}}+\frac{Bx}{3ab\sqrt{a+bx^2}}$$

```
[Out] 1/3*(-B*x-A)/b/(b*x^2+a)^(3/2)+1/3*B*x/a/b/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 47, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {792, 197}

$$\frac{Bx}{3ab\sqrt{a+bx^2}}-\frac{A+Bx}{3b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*(A + B*x)/(b*(a + b*x^2)^(3/2)) + (B*x)/(3*a*b*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{A+Bx}{3b\left(a+bx^2\right)^{3/2}}+\frac{B\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3b}\\ &= -\frac{A+Bx}{3b\left(a+bx^2\right)^{3/2}}+\frac{Bx}{3ab\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 32, normalized size = 0.64

$$\frac{-aA+bBx^3}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x))/(a + b*x^2)^(5/2),x]
```

```
[Out] (-(a*A) + b*B*x^3)/(3*a*b*(a + b*x^2)^(3/2))
```

Maple [A]

time = 0.11, size = 72, normalized size = 1.44

method	result	size
gosper	$-\frac{-bB\,x^{3}+Aa}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}} a b}$	29
trager	$-\frac{-bB\,x^{3}+Aa}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}} a b}$	29
default	$B\left(-\frac{x}{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)}{2b}\right)-\frac{A}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/2*x/b/(b*x^2+a)^(3/2)+1/2*a/b*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x
^2+a)^(1/2)))-1/3*A/b/(b*x^2+a)^(3/2)
```

Maxima [A]

time = 0.28, size = 51, normalized size = 1.02

$$-\frac{Bx}{3\left(bx^{2}+a\right)^{\frac{3}{2}}b}+\frac{Bx}{3\sqrt{bx^{2}+a}\,ab}-\frac{A}{3\left(bx^{2}+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*B*x/((b*x^2 + a)^(3/2)*b) + 1/3*B*x/(sqrt(b*x^2 + a)*a*b) - 1/3*A/((b*
x^2 + a)^(3/2)*b)
```

Fricas [A]

time = 3.13, size = 49, normalized size = 0.98

$$\frac{\left(Bbx^{3}-Aa\right)\sqrt{bx^{2}+a}}{3\left(ab^{3}x^{4}+2\,a^{2}b^{2}x^{2}+a^{3}b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(B*b*x^3 - A*a)*sqrt(b*x^2 + a)/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

Sympy [A]
time = 4.69, size = 95, normalized size = 1.90

$$A\left(\begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right)+\frac{Bx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne
(b, 0)), (x**2/(2*a**(5/2)), True)) + B*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a)
 + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 1.47, size = 26, normalized size = 0.52

$$\frac{\frac{Bx^3}{a}-\frac{A}{b}}{3\left(bx^2+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(B*x^3/a - A/b)/(b*x^2 + a)^(3/2)
```

Mupad [B]
time = 0.92, size = 34, normalized size = 0.68

$$\frac{B\,x^3}{3\,a\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{A}{3\,b\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x))/(a + b*x^2)^(5/2),x)
```

```
[Out] (B*x^3)/(3*a*(a + b*x^2)^(3/2)) - A/(3*b*(a + b*x^2)^(3/2))
```

3.39 $\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx$

Optimal. Leaf size=51

$$\frac{-aB+Abx}{3ab\left(a+bx^2\right)^{3/2}}+\frac{2Ax}{3a^2\sqrt{a+bx^2}}$$

```
[Out] 1/3*(A*b*x-B*a)/a/b/(b*x^2+a)^(3/2)+2/3*A*x/a^2/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {653, 197}

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}-\frac{aB-Abx}{3ab\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(a + b*x^2)^(5/2),x]
```

```
[Out] -1/3*(a*B - A*b*x)/(a*b*(a + b*x^2)^(3/2)) + (2*A*x)/(3*a^2*Sqrt[a + b*x^2]
)
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx}{\left(a+bx^2\right)^{5/2}}\,dx &= -\frac{aB-Abx}{3ab\left(a+bx^2\right)^{3/2}}+\frac{(2A)\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx}{3a}\\ &= -\frac{aB-Abx}{3ab\left(a+bx^2\right)^{3/2}}+\frac{2Ax}{3a^2\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 43, normalized size = 0.84

$$\frac{-a^2B+3aAbx+2Ab^2x^3}{3a^2b\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(a + b*x^2)^(5/2),x]
```

```
[Out] (-(a^2*B) + 3*a*A*b*x + 2*A*b^2*x^3)/(3*a^2*b*(a + b*x^2)^(3/2))
```

Maple [A]
time = 0.11, size = 50, normalized size = 0.98

method	result	size
gosper	$\frac{2A\,b^{2}x^{3}+3abAx-a^{2}B}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b}$	40
trager	$\frac{2A\,b^{2}x^{3}+3abAx-a^{2}B}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}}a^{2}b}$	40
default	$-\frac{B}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+A\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*B/b/(b*x^2+a)^(3/2)+A*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/
2))
```

Maxima [A]
time = 0.28, size = 48, normalized size = 0.94

$$\frac{2\,Ax}{3\,\sqrt{bx^2+a}\,a^2}+\frac{Ax}{3\,\left(bx^2+a\right)^{\frac{3}{2}}a}-\frac{B}{3\,\left(bx^2+a\right)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*x^2 + a)^(3/2)*a) - 1/3*B/((b*x
^2 + a)^(3/2)*b)
```

Fricas [A]
time = 3.53, size = 62, normalized size = 1.22

$$\frac{\left(2\,Ab^2x^3+3\,Aabx-Ba^2\right)\sqrt{bx^2+a}}{3\,\left(a^2b^3x^4+2\,a^3b^2x^2+a^4b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*A*b^2*x^3 + 3*A*a*b*x - B*a^2)*sqrt(b*x^2 + a)/(a^2*b^3*x^4 + 2*a^3*
b^2*x^2 + a^4*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(44) = 88$.
time = 4.40, size = 146, normalized size = 2.86

$$A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right)+B\left(\begin{cases}-\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b \neq 0\\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2
/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
 b*x**2/a))) + B*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a
 + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True))
```

Giac [A]
time = 0.98, size = 37, normalized size = 0.73

$$\frac{\left(\frac{2\,Abx^2}{a^2}+\frac{3\,A}{a}\right)x-\frac{B}{b}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((2*A*b*x^2/a^2 + 3*A/a)*x - B/b)/(b*x^2 + a)^(3/2)
```

Mupad [B]
time = 0.93, size = 41, normalized size = 0.80

$$\frac{2\,A\,b\,x\,\left(b\,x^2+a\right)-B\,a^2+A\,a\,b\,x}{3\,a^2\,b\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(a + b*x^2)^(5/2),x)
```

```
[Out] (2*A*b*x*(a + b*x^2) - B*a^2 + A*a*b*x)/(3*a^2*b*(a + b*x^2)^(3/2))
```

3.40 $\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=76

$$\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] 1/3*(B*x+A)/a/(b*x^2+a)^(3/2)-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+1/
3*(2*B*x+3*A)/a^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {837, 12, 272, 65, 214}

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A + B*x)/(3*a*(a + b*x^2)^(3/2)) + (3*A + 2*B*x)/(3*a^2*Sqrt[a + b*x^2]) -
 (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(5/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx &= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{-3aAb-2abBx}{x\left(a+bx^2\right)^{3/2}}\,dx}{3a^2b}\\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{\int \frac{3a^2Ab^2}{x\sqrt{a+bx^2}}\,dx}{3a^4b^2}\\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{A\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a^2}\\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{A\mathrm{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a^2}\\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}+\frac{A\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}\right)}{a^2b}\\
&= \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.36, size = 79, normalized size = 1.04

$$
\frac{bx^2(3A+2Bx)+a(4A+3Bx)}{3a^2\left(a+bx^2\right)^{3/2}}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x*(a + b*x^2)^(5/2)),x]
```

```
[Out] (b*x^2*(3*A + 2*B*x) + a*(4*A + 3*B*x))/(3*a^2*(a + b*x^2)^(3/2)) + (2*A*Ar
cTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(5/2)
```

Maple [A]
time = 0.12, size = 98, normalized size = 1.29

method	result	siz
default	$B\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)+A\left(\frac{1}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}\right)$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))+A*(1/3/a/(b*x^2+a)^(3
/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x
)))
```

Maxima [A]
time = 0.28, size = 80, normalized size = 1.05

$$\frac{2\,Bx}{3\,\sqrt{bx^{2}+a}\,a^{2}}+\frac{Bx}{3\,\left(bx^{2}+a\right)^{\frac{3}{2}}a}-\frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{5}{2}}}+\frac{A}{\sqrt{bx^{2}+a}\,a^{2}}+\frac{A}{3\,\left(bx^{2}+a\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*B*x/(sqrt(b*x^2 + a)*a^2) + 1/3*B*x/((b*x^2 + a)^(3/2)*a) - A*arcsinh(a
/(sqrt(a*b)*abs(x)))/a^(5/2) + A/(sqrt(b*x^2 + a)*a^2) + 1/3*A/((b*x^2 + a)
^(3/2)*a)
```

Fricas [A]
time = 4.71, size = 239, normalized size = 3.14

$$\left[\frac{3\,\left(Ab^{2}x^{4}+2\,Aabx^{2}+Aa^{2}\right)\sqrt{a}\,\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,\left(2\,Babx^{3}+3\,Aabx^{2}+3\,Ba^{2}x+4\,Aa^{2}\right)\sqrt{bx^{2}+a}}{6\,\left(a^{3}b^{2}x^{4}+2\,a^{4}bx^{2}+a^{5}\right)},\ \frac{3\,\left(Ab^{2}x^{4}+2\,Aabx^{2}+Aa^{2}\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+\left(2\,Babx^{3}+3\,Aabx^{2}+3\,Ba^{2}x+4\,Aa^{2}\right)\sqrt{bx^{2}+a}}{3\,\left(a^{3}b^{2}x^{4}+2\,a^{4}bx^{2}+a^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(A*b^2*x^4 + 2*A*a*b*x^2 + A*a^2)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^
2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*B*a*b*x^3 + 3*A*a*b*x^2 + 3*B*a^2*x + 4*A
*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5), 1/3*(3*(A*b^2*x^4
 + 2*A*a*b*x^2 + A*a^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*
b*x^3 + 3*A*a*b*x^2 + 3*B*a^2*x + 4*A*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4 +
2*a^4*b*x^2 + a^5)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(65) = 130$.
time = 9.76, size = 840, normalized size = 11.05

$$A\left(\frac{8a^{7}\sqrt{1+\frac{bx^{2}}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{3a^{7}\log\left(\frac{bx^{2}}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}-\frac{6a^{7}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{14a^{6}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{9a^{6}bx^{2}\log\left(\frac{bx^{2}}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}-\frac{18a^{6}bx^{2}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{6a^{5}b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{9a^{5}b^{2}x^{4}\log\left(\frac{bx^{2}}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}-\frac{18a^{5}b^{2}x^{4}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{3a^{4}b^{3}x^{6}\log\left(\frac{bx^{2}}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}-\frac{6a^{4}b^{3}x^{6}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}\right)+B\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{5}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{2bx^{3}}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^{2}}{a}}+3a^{\frac{5}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15
/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2)
+ 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6
*a**7*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a
**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**
2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13
/2)*b**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*
x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*x**2*log
(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*
b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6
*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**
3*x**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2
 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(
sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b
**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**(1
9/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6
) - 6*a**4*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2
)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6)) + B*(3*a*x/(3*a
**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)) + 2*b*x*
*3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**2/a)))
```

Giac [**A**]
time = 1.49, size = 82, normalized size = 1.08

$$\frac{\left(\left(\frac{2\,Bbx}{a^{2}}+\frac{3\,Ab}{a^{2}}\right)x+\frac{3\,B}{a}\right)x+\frac{4\,A}{a}}{3\,(bx^{2}+a)^{\frac{3}{2}}}+\frac{2\,A\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(((2*B*b*x/a^2 + 3*A*b/a^2)*x + 3*B/a)*x + 4*A/a)/(b*x^2 + a)^(3/2) + 2
*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2)
```

Mupad [B]
time = 1.38, size = 80, normalized size = 1.05

$$\frac{\frac{A}{3\,a}+\frac{A\,\left(b\,x^2+a\right)}{a^2}}{{\left(b\,x^2+a\right)}^{3/2}}+\frac{2\,B\,x\,\left(b\,x^2+a\right)+B\,a\,x}{3\,a^2\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x*(a + b*x^2)^(5/2)),x)
```

```
[Out] (A/(3*a) + (A*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) + (2*B*x*(a + b*x^2) + B*
a*x)/(3*a^2*(a + b*x^2)^(3/2)) - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(5/
2)
```

3.41 $\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=104

$$\frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

```
[Out] 1/3*(B*x+A)/a/x/(b*x^2+a)^(3/2)-B*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+
1/3*(3*B*x+4*A)/a^2/x/(b*x^2+a)^(1/2)-8/3*A*(b*x^2+a)^(1/2)/a^3/x
```

Rubi [A]

time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {837, 821, 272, 65, 214}

$$-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}+\frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A + B*x)/(3*a*x*(a + b*x^2)^(3/2)) + (4*A + 3*B*x)/(3*a^2*x*Sqrt[a + b*x^2
]) - (8*A*Sqrt[a + b*x^2])/(3*a^3*x) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/a^(5/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx &= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{-4aAb-3abBx}{x^2\left(a+bx^2\right)^{3/2}}\,dx}{3a^2b}\\
&= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}+\frac{\int \frac{8a^2Ab^2+3a^2b^2Bx}{x^2\sqrt{a+bx^2}}\,dx}{3a^4b^2}\\
&= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}+\frac{B\int \frac{1}{x\sqrt{a+bx^2}}\,dx}{a^2}\\
&= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}+\frac{B\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\right.}{2a^2}\\
&= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}+\frac{B\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a}\right.}{a^2b}\\
&= \frac{A+Bx}{3ax\left(a+bx^2\right)^{3/2}}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}
$$

Mathematica [A]

time = 0.40, size = 94, normalized size = 0.90

$$\frac{-8Ab^2x^4+3abx^2(-4A+Bx)+a^2(-3A+4Bx)}{3a^3x\left(a+bx^2\right)^{3/2}}+\frac{2B\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^2*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-8*A*b^2*x^4 + 3*a*b*x^2*(-4*A + B*x) + a^2*(-3*A + 4*B*x))/(3*a^3*x*(a +
b*x^2)^(3/2)) + (2*B*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(5/2
)
```

Maple [A]
time = 0.15, size = 122, normalized size = 1.17

method	result
default	$B\left(\frac{1}{3a(bx^2+a)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{bx^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{bx^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}\right)+A\left(-\frac{1}{ax(bx^2+a)^{\frac{3}{2}}}-\frac{4b\left(\frac{x}{3a\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{}{3a^2\sqrt{b}}\right.}{a}\right.$
risch	$-\frac{A\sqrt{bx^2+a}}{a^3x}+\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2b-2\sqrt{-ab}\,\left(x+\frac{\sqrt{-ab}}{b}\right)}\,A}{12a^2\sqrt{-ab}\,\left(x+\frac{\sqrt{-ab}}{b}\right)^2}-\frac{\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2b-2\sqrt{}}}{12a^2b\left(x+\sqrt{}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/
2)*(b*x^2+a)^(1/2))/x)))+A*(-1/a/x/(b*x^2+a)^(3/2)-4*b/a*(1/3*x/a/(b*x^2+a)
^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.28, size = 100, normalized size = 0.96

$$-\frac{8\,Abx}{3\,\sqrt{bx^2+a}\,a^3}-\frac{4\,Abx}{3\,(bx^2+a)^{\frac{3}{2}}a^2}-\frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{5}{2}}}+\frac{B}{\sqrt{bx^2+a}\,a^2}+\frac{B}{3\,(bx^2+a)^{\frac{3}{2}}a}-\frac{A}{(bx^2+a)^{\frac{3}{2}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -8/3*A*b*x/(sqrt(b*x^2 + a)*a^3) - 4/3*A*b*x/((b*x^2 + a)^(3/2)*a^2) - B*ar
csinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + B/(sqrt(b*x^2 + a)*a^2) + 1/3*B/((b*x
^2 + a)^(3/2)*a) - A/((b*x^2 + a)^(3/2)*a*x)
```

Fricas [**A**]
time = 6.19, size = 264, normalized size = 2.54

$$\left[\frac{3\,{\left(B b^{2} x^{5} + 2\,B a b x^{3} + B a^{2} x\right)}\sqrt{a} \log\left(-\frac{b x^{2} - 2\,\sqrt{b x^{2} + a} \sqrt{a} + 2\,a}{x^{2}}\right) - 2\,{\left(8\,A b^{2} x^{4} - 3\,B a b x^{3} + 12\,A a b x^{2} - 4\,B a^{2} x + 3\,A a^{2}\right)}\sqrt{b x^{2} + a}}{6\,{\left(a^{3} b^{2} x^{5} + 2\,a^{4} b x^{3} + a^{5} x\right)}}, \frac{3\,{\left(B b^{2} x^{5} + 2\,B a b x^{3} + B a^{2} x\right)}\sqrt{-a} \arctan\left(\frac{\sqrt{-a}}{\sqrt{b x^{2} + a}}\right) - {\left(8\,A b^{2} x^{4} - 3\,B a b x^{3} + 12\,A a b x^{2} - 4\,B a^{2} x + 3\,A a^{2}\right)}\sqrt{b x^{2} + a}}{3\,{\left(a^{3} b^{2} x^{5} + 2\,a^{4} b x^{3} + a^{5} x\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(B*b^2*x^5 + 2*B*a*b*x^3 + B*a^2*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(8*A*b^2*x^4 - 3*B*a*b*x^3 + 12*A*a*b*x^2
- 4*B*a^2*x + 3*A*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)
, 1/3*(3*(B*b^2*x^5 + 2*B*a*b*x^3 + B*a^2*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(
b*x^2 + a)) - (8*A*b^2*x^4 - 3*B*a*b*x^3 + 12*A*a*b*x^2 - 4*B*a^2*x + 3*A*a
^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(88) = 176$.
time = 8.49, size = 910, normalized size = 8.75

$$A\left(-\frac{3a^{2}b^{\frac{9}{2}}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}+6a^{4}b^{5}x^{2}+3a^{3}b^{6}x^{4}}-\frac{12ab^{\frac{11}{2}}x^{2}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}+6a^{4}b^{5}x^{2}+3a^{3}b^{6}x^{4}}-\frac{8b^{\frac{13}{2}}x^{4}\sqrt{\frac{a}{bx^{2}}+1}}{3a^{5}b^{4}+6a^{4}b^{5}x^{2}+3a^{3}b^{6}x^{4}}\right)+B\left(\frac{8a^{7}\sqrt{1+\frac{bx^{2}}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{3a^{7}\log\left(\frac{bx^{2}}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}-\frac{6a^{7}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{14a^{6}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{9a^{6}bx^{2}\log\left(\frac{bx^{2}}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}-\frac{18a^{6}bx^{2}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{6a^{5}b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{9a^{5}b^{2}x^{4}\log\left(\frac{bx^{2}}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}-\frac{18a^{5}b^{2}x^{4}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}+\frac{3a^{4}b^{3}x^{6}\log\left(\frac{bx^{2}}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}-\frac{6a^{4}b^{3}x^{6}\log\left(\sqrt{1+\frac{bx^{2}}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^{2}+18a^{\frac{15}{2}}b^{2}x^{4}+6a^{\frac{13}{2}}b^{3}x^{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**2/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 +
3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 +
 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2) +
1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4)) + B*(8*a**7*sqrt(1
+ b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6
*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*
x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**7*log(sqrt(1
+ b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**
4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**2/a)/(6*a**(19/2)
 + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) +
9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/
2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*x**2*log(sqrt(1 + b*x**2/
a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**
(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a
**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 9*a**5*
b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b
**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(sqrt(1 + b*x**2/a
```

```
) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(
13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/
2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**4*b**3*x
**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**
(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6))
```

Giac [A]
time = 1.16, size = 119, normalized size = 1.14

$$-\frac{\left(\left(\frac{5\,Ab^2x}{a^3}-\frac{3\,Bb}{a^2}\right)x+\frac{6\,Ab}{a^2}\right)x-\frac{4\,B}{a}}{3\,(bx^2+a)^{\frac{3}{2}}}+\frac{2\,B\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((5*A*b^2*x/a^3 - 3*B*b/a^2)*x + 6*A*b/a^2)*x - 4*B/a)/(b*x^2 + a)^(3
/2) + 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) +
2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^2)
```

Mupad [B]
time = 1.58, size = 96, normalized size = 0.92

$$\frac{\frac{B}{3\,a}+\frac{B\,(b\,x^2+a)}{a^2}}{(b\,x^2+a)^{3/2}}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{5/2}}+\frac{A\,a^2-8\,A\,(b\,x^2+a)^2+4\,A\,a\,(b\,x^2+a)}{3\,a^3\,x\,(b\,x^2+a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^2*(a + b*x^2)^(5/2)),x)
```

```
[Out] (B/(3*a) + (B*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) - (B*atanh((a + b*x^2)^(1
/2)/a^(1/2)))/a^(5/2) + (A*a^2 - 8*A*(a + b*x^2)^2 + 4*A*a*(a + b*x^2))/(3*
a^3*x*(a + b*x^2)^(3/2))
```

3.42 $\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=129

$$\frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}}+\frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}}-\frac{5A\sqrt{a+bx^2}}{2a^3x^2}-\frac{8B\sqrt{a+bx^2}}{3a^3x}+\frac{5Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}$$

```
[Out] 1/3*(B*x+A)/a/x^2/(b*x^2+a)^(3/2)+5/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/
a^(7/2)+1/3*(4*B*x+5*A)/a^2/x^2/(b*x^2+a)^(1/2)-5/2*A*(b*x^2+a)^(1/2)/a^3/x
^2-8/3*B*(b*x^2+a)^(1/2)/a^3/x
```

Rubi [A]
time = 0.08, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {837, 849, 821, 272, 65, 214}

$$\frac{5Ab\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}-\frac{5A\sqrt{a+bx^2}}{2a^3x^2}-\frac{8B\sqrt{a+bx^2}}{3a^3x}+\frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}}+\frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] (A + B*x)/(3*a*x^2*(a + b*x^2)^(3/2)) + (5*A + 4*B*x)/(3*a^2*x^2*Sqrt[a + b
*x^2]) - (5*A*Sqrt[a + b*x^2])/(2*a^3*x^2) - (8*B*Sqrt[a + b*x^2])/(3*a^3*x
) + (5*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(7/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx &= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-5aAb-4abBx}{x^3(a+bx^2)^{3/2}}\,dx}{3a^2b} \\
&= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} + \frac{\int \frac{15a^2Ab^2+8a^2b^2Bx}{x^3\sqrt{a+bx^2}}\,dx}{3a^4b^2} \\
&= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{\int \frac{-16a^3b^2B+15a^2Ab^3x}{x^2\sqrt{a+bx^2}}\,dx}{6a^5b^2} \\
&= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} - \frac{(5Ab)\int \frac{-}{x}}{} \\
&= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} - \frac{(5Ab)\mathrm{Sub}}{} \\
&= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} - \frac{(5A)\mathrm{Subs}}{} \\
&= \frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}} + \frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}} - \frac{5A\sqrt{a+bx^2}}{2a^3x^2} - \frac{8B\sqrt{a+bx^2}}{3a^3x} + \frac{5Ab\tanh}{}
\end{aligned}$$

Mathematica [A]
time = 0.47, size = 102, normalized size = 0.79

$$\frac{-3a^2(A+2Bx)-4abx^2(5A+6Bx)-b^2x^4(15A+16Bx)}{6a^3x^2\left(a+bx^2\right)^{3/2}} - \frac{5Ab\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x)/(x^3*(a + b*x^2)^(5/2)),x]
```

```
[Out] (-3*a^2*(A + 2*B*x) - 4*a*b*x^2*(5*A + 6*B*x) - b^2*x^4*(15*A + 16*B*x))/(6
*a^3*x^2*(a + b*x^2)^(3/2)) - (5*A*b*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/
Sqrt[a]])/a^(7/2)
```

Maple [A]
time = 0.15, size = 146, normalized size = 1.13

method	result

default	$A\left(-\frac{1}{2a\,x^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{5b\left(\frac{1}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}\right)}{2a}\right)+B\left(-\frac{1}{ax\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\right.$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(2Bx+A\right)}{2a^{3}x^{2}}+\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}b+2\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}\,A}{12a^{3}\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}}-\frac{\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}b}}{12a^{2}\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x+A)/x^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/2/a/x^2/(b*x^2+a)^(3/2)-5/2*b/a*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^
2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))+B*(-1/a/x/(b*
x^2+a)^(3/2)-4*b/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))
```

Maxima [A]
time = 0.28, size = 122, normalized size = 0.95

$$-\frac{8\,Bbx}{3\,\sqrt{bx^{2}+a}\,a^{3}}-\frac{4\,Bbx}{3\,(bx^{2}+a)^{\frac{3}{2}}a^{2}}+\frac{5\,Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{7}{2}}}-\frac{5\,Ab}{2\,\sqrt{bx^{2}+a}\,a^{3}}-\frac{5\,Ab}{6\,(bx^{2}+a)^{\frac{3}{2}}a^{2}}-\frac{B}{(bx^{2}+a)^{\frac{3}{2}}ax}-\frac{A}{2\,(bx^{2}+a)^{\frac{3}{2}}ax^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

```
[Out] -8/3*B*b*x/(sqrt(b*x^2 + a)*a^3) - 4/3*B*b*x/((b*x^2 + a)^(3/2)*a^2) + 5/2*
A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 5/2*A*b/(sqrt(b*x^2 + a)*a^3) -
 5/6*A*b/((b*x^2 + a)^(3/2)*a^2) - B/((b*x^2 + a)^(3/2)*a*x) - 1/2*A/((b*x^
2 + a)^(3/2)*a*x^2)
```

Fricas [A]
time = 9.97, size = 307, normalized size = 2.38

$$\left[\frac{15\,(Ab^{3}x^{6}+2\,Aab^{2}x^{4}+Aa^{2}bx^{2})\sqrt{a}\,\log\left(-\frac{bx^{2}+2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)-2\,(16\,Bab^{2}x^{5}+15\,Aab^{2}x^{4}+24\,Ba^{2}bx^{3}+20\,Aa^{2}bx^{2}+6\,Ba^{3}x+3\,Aa^{3})\sqrt{bx^{2}+a}}{12\,(a^{4}b^{2}x^{6}+2\,a^{5}bx^{4}+a^{6}x^{2})},-\frac{15\,(Ab^{3}x^{6}+2\,Aab^{2}x^{4}+Aa^{2}bx^{2})\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(16\,Bab^{2}x^{5}+15\,Aab^{2}x^{4}+24\,Ba^{2}bx^{3}+20\,Aa^{2}bx^{2}+6\,Ba^{3}x+3\,Aa^{3})\sqrt{bx^{2}+a}}{6\,(a^{4}b^{2}x^{6}+2\,a^{5}bx^{4}+a^{6}x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(15*(A*b^3*x^6 + 2*A*a*b^2*x^4 + A*a^2*b*x^2)*sqrt(a)*log(-(b*x^2 + 2
*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(16*B*a*b^2*x^5 + 15*A*a*b^2*x^4 +
 24*B*a^2*b*x^3 + 20*A*a^2*b*x^2 + 6*B*a^3*x + 3*A*a^3)*sqrt(b*x^2 + a))/(a
^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2), -1/6*(15*(A*b^3*x^6 + 2*A*a*b^2*x^4 +
A*a^2*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (16*B*a*b^2*x^5 +
15*A*a*b^2*x^4 + 24*B*a^2*b*x^3 + 20*A*a^2*b*x^2 + 6*B*a^3*x + 3*A*a^3)*sqr
t(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. $2(122) = 244$.
time = 8.43, size = 1034, normalized size = 8.02

$$A\left(-\frac{6a^{17}\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{46a^{16}bx^2\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{15a^{16}bx^2\log\left(\frac{bx^2}{a}\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}+\frac{30a^{16}bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{70a^{15}b^2x^4\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{45a^{15}b^2x^4\log\left(\frac{bx^2}{a}\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}+\frac{90a^{15}b^2x^4\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{30a^{14}b^3x^6\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{45a^{14}b^3x^6\log\left(\frac{bx^2}{a}\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}+\frac{90a^{14}b^3x^6\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}-\frac{15a^{13}b^4x^8\log\left(\frac{bx^2}{a}\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}+\frac{30a^{13}b^4x^8\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}\right)+B\left(-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x**3/(b*x**2+a)**(5/2),x)
```

```
[Out] A*(-6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 3
6*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1 +
b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6
 + 12*a**(33/2)*b**3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2)*x*
*2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8)
 + 30*a**16*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(
37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70*a**15*
b**2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*
a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4*log(b*x*
*2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 1
2*a**(33/2)*b**3*x**8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(12
*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/
2)*b**3*x**8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 +
36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45
*a**14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 3
6*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*b**3*x**6*log(sq
rt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/
2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**13*b**4*x**8*log(b*x**2/a)/(
12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(3
3/2)*b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39
/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3
*x**8)) + B*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b*
*5*x**2 + 3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a
**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(
b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4))
```

Giac [**A**]

time = 0.93, size = 197, normalized size = 1.53

$$-\frac{\left(\left(\frac{5\,Bb^2x}{a^3}+\frac{6\,Ab^2}{a^3}\right)x+\frac{6\,Bb}{a^2}\right)x+\frac{7\,Ab}{a^2}}{3\,(bx^2+a)^{\frac{3}{2}}}-\frac{5\,Ab\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((5*B*b^2*x/a^3 + 6*A*b^2/a^3)*x + 6*B*b/a^2)*x + 7*A*b/a^2)/(b*x^2 +
 a)^(3/2) - 5*A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)
*a^3) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/
(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^3)
```

Mupad [B]

time = 1.62, size = 123, normalized size = 0.95

$$\frac{B\,a^2-8\,B\,{\left(b\,x^2+a\right)}^2+4\,B\,a\,\left(b\,x^2+a\right)}{3\,a^3\,x\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{10\,A\,b}{3\,a^2\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{A}{2\,a\,x^2\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{5\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{7/2}}-\frac{5\,A\,b^2\,x^2}{2\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x)/(x^3*(a + b*x^2)^(5/2)),x)
```

```
[Out] (B*a^2 - 8*B*(a + b*x^2)^2 + 4*B*a*(a + b*x^2))/(3*a^3*x*(a + b*x^2)^(3/2))
 - (10*A*b)/(3*a^2*(a + b*x^2)^(3/2)) - A/(2*a*x^2*(a + b*x^2)^(3/2)) + (5*
A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(7/2)) - (5*A*b^2*x^2)/(2*a^3*(a
 + b*x^2)^(3/2))
```

3.43 $\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)$$

```
[Out] -1/2*arcsin(x)-1/2*(2-x)*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {794, 222}

$$-\frac{\text{ArcSin}(x)}{2}-\frac{1}{2}\sqrt{1-x^2}\,(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x)*x)/Sqrt[1 - x^2],x]
```

```
[Out] -1/2*((2 - x)*Sqrt[1 - x^2]) - ArcSin[x]/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.08, size = 39, normalized size = 1.44

$$-\frac{1}{2}(2-x)\sqrt{1-x^2}+\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x)*x)/Sqrt[1 - x^2],x]
```

```
[Out] -1/2*((2 - x)*Sqrt[1 - x^2]) + ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A]
time = 0.15, size = 29, normalized size = 1.07

method	result	size
risch	$-\frac{(x-2)\left(x^{2}-1\right)}{2\sqrt{-x^{2}+1}}-\frac{\arcsin(x)}{2}$	25
default	$\frac{\sqrt{-x^{2}+1}\,x}{2}-\frac{\arcsin(x)}{2}-\sqrt{-x^{2}+1}$	29
trager	$\left(\frac{x}{2}-1\right)\sqrt{-x^{2}+1}+\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^{2}+1\right)\sqrt{-x^{2}+1}+x\right)}{2}$	45
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^{2}+1}}{2\sqrt{\pi}}-\frac{i\left(i\sqrt{\pi}\,x\sqrt{-x^{2}+1}-i\sqrt{\pi}\,\arcsin(x)\right)}{2\sqrt{\pi}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)*x/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*x-1/2*arcsin(x)-(-x^2+1)^(1/2)
```

Maxima [A]
time = 0.49, size = 28, normalized size = 1.04

$$\frac{1}{2}\sqrt{-x^{2}+1}\,x-\sqrt{-x^{2}+1}-\frac{1}{2}\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

Fricas [A]
time = 9.00, size = 31, normalized size = 1.15

$$\frac{1}{2}\sqrt{-x^{2}+1}\,(x-2)+\arctan\left(\frac{\sqrt{-x^{2}+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A]
time = 0.07, size = 24, normalized size = 0.89

$$\frac{x\sqrt{1-x^2}}{2}-\sqrt{1-x^2}-\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2
```

Giac [A]
time = 0.88, size = 19, normalized size = 0.70

$$\frac{1}{2}\sqrt{-x^2+1}\,(x-2)-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

Mupad [B]
time = 0.04, size = 20, normalized size = 0.74

$$\left(\frac{x}{2}-1\right)\sqrt{1-x^2}-\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(x - 1))/(1 - x^2)^(1/2),x)
```

```
[Out] (x/2 - 1)*(1 - x^2)^(1/2) - asin(x)/2
```

3.44 $\int \frac{x-x^2}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=27

$$-\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)$$

```
[Out] -1/2*arcsin(x)-1/2*(2-x)*(-x^2+1)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1607, 794, 222}

$$-\frac{\text{ArcSin}(x)}{2}-\frac{1}{2}\sqrt{1-x^2}\,(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x - x^2)/Sqrt[1 - x^2],x]
```

```
[Out] -1/2*((2 - x)*Sqrt[1 - x^2]) - ArcSin[x]/2
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx &= \int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx \\ &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 39, normalized size = 1.44

$$-\frac{1}{2}(2-x)\sqrt{1-x^2}+\tan^{-1}\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - x^2)/Sqrt[1 - x^2],x]
```

```
[Out] -1/2*((2 - x)*Sqrt[1 - x^2]) + ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

Maple [A]
time = 0.12, size = 29, normalized size = 1.07

method	result	size
risch	$-\frac{(x-2)(x^2-1)}{2\sqrt{-x^2+1}}-\frac{\arcsin(x)}{2}$	25
default	$\frac{\sqrt{-x^2+1}\,x}{2}-\frac{\arcsin(x)}{2}-\sqrt{-x^2+1}$	29
trager	$\left(\frac{x}{2}-1\right)\sqrt{-x^2+1}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	45
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^2+1}}{2\sqrt{\pi}}-\frac{i\left(i\sqrt{\pi}\,x\sqrt{-x^2+1}-i\sqrt{\pi}\,\arcsin(x)\right)}{2\sqrt{\pi}}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-x^2+1)^(1/2)*x-1/2*arcsin(x)-(-x^2+1)^(1/2)
```

Maxima [A]
time = 0.51, size = 28, normalized size = 1.04

$$\frac{1}{2}\sqrt{-x^2+1}\,x-\sqrt{-x^2+1}-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

Fricas [A]
time = 4.15, size = 31, normalized size = 1.15

$$\frac{1}{2}\sqrt{-x^2+1}\,(x-2)+\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A]
time = 0.07, size = 24, normalized size = 0.89

$$\frac{x\sqrt{1-x^2}}{2}-\sqrt{1-x^2}-\frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+x)/(-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2
```

Giac [A]
time = 0.71, size = 19, normalized size = 0.70

$$\frac{1}{2}\sqrt{-x^2+1}\,(x-2)-\frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

Mupad [B]
time = 0.03, size = 20, normalized size = 0.74

$$\left(\frac{x}{2}-1\right)\sqrt{1-x^2}-\frac{\mathrm{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - x^2)/(1 - x^2)^(1/2),x)
```

```
[Out] (x/2 - 1)*(1 - x^2)^(1/2) - asin(x)/2
```

3.45 $\int \frac{3+x^2}{-3+x^2}\,dx$

Optimal. Leaf size=17

$$x-2\sqrt{3}\,\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

```
[Out] x-2*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {396, 213}

$$x-2\sqrt{3}\,\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x^2)/(-3 + x^2),x]
```

```
[Out] x - 2*Sqrt[3]*ArcTanh[x/Sqrt[3]]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{3+x^2}{-3+x^2}\,dx &= x+6\int \frac{1}{-3+x^2}\,dx\\ &= x-2\sqrt{3}\,\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 33, normalized size = 1.94

$$x+\sqrt{3}\,\log\left(\sqrt{3}-x\right)-\sqrt{3}\,\log\left(\sqrt{3}+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x^2)/(-3 + x^2),x]
```

```
[Out] x + Sqrt[3]*Log[Sqrt[3] - x] - Sqrt[3]*Log[Sqrt[3] + x]
```

Maple [A]
time = 0.14, size = 15, normalized size = 0.88

method	result	size
default	$x-2\operatorname{arctanh}\left(\frac{\sqrt{3}\,x}{3}\right)\sqrt{3}$	15
risch	$x+\sqrt{3}\,\ln\left(x-\sqrt{3}\right)-\sqrt{3}\,\ln\left(x+\sqrt{3}\right)$	26
meijerg	$-\operatorname{arctanh}\left(\frac{\sqrt{3}\,x}{3}\right)\sqrt{3}-\frac{i\sqrt{3}\,\left(\frac{2i\sqrt{3}\,x}{3}-2i\operatorname{arctanh}\left(\frac{\sqrt{3}\,x}{3}\right)\right)}{2}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3)/(x^2-3),x,method=_RETURNVERBOSE)
```

```
[Out] x-2*arctanh(1/3*3^(1/2)*x)*3^(1/2)
```

Maxima [A]
time = 0.51, size = 22, normalized size = 1.29

$$\sqrt{3}\,\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2-3),x, algorithm="maxima")
```

```
[Out] sqrt(3)*log((x - sqrt(3))/(x + sqrt(3))) + x
```

Fricas [A]
time = 3.29, size = 26, normalized size = 1.53

$$\sqrt{3}\,\log\left(\frac{x^2-2\,\sqrt{3}\,x+3}{x^2-3}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2-3),x, algorithm="fricas")
```

```
[Out] sqrt(3)*log((x^2 - 2*sqrt(3)*x + 3)/(x^2 - 3)) + x
```

Sympy [**A**]
time = 0.02, size = 27, normalized size = 1.59

$$x+\sqrt{3}\log\left(x-\sqrt{3}\right)-\sqrt{3}\log\left(x+\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3)/(x**2-3),x)
```

```
[Out] x + sqrt(3)*log(x - sqrt(3)) - sqrt(3)*log(x + sqrt(3))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(14) = 28$.
time = 0.82, size = 30, normalized size = 1.76

$$\sqrt{3}\log\left(\frac{\left|2\,x-2\,\sqrt{3}\right|}{\left|2\,x+2\,\sqrt{3}\right|}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)/(x^2-3),x, algorithm="giac")
```

```
[Out] sqrt(3)*log(abs(2*x - 2*sqrt(3))/abs(2*x + 2*sqrt(3))) + x
```

Mupad [**B**]
time = 0.92, size = 14, normalized size = 0.82

$$x-2\,\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3)/(x^2 - 3),x)
```

```
[Out] x - 2*3^(1/2)*atanh((3^(1/2)*x)/3)
```

3.46 $\int \frac{-1+x^2}{1+x^2}\,dx$

Optimal. Leaf size=6

$$x - 2\tan^{-1}(x)$$

```
[Out] x-2*arctan(x)
```

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {396, 209}

$$x - 2\text{ArcTan}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(1 + x^2),x]
```

```
[Out] x - 2*ArcTan[x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{1+x^2}\,dx &= x-2\int \frac{1}{1+x^2}\,dx\\ &= x-2\tan^{-1}(x)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$x - 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(1 + x^2),x]
```

```
[Out] x - 2*ArcTan[x]
```

Maple [A]
time = 0.12, size = 7, normalized size = 1.17

method	result	size
default	$x - 2\arctan\left(x\right)$	7
meijerg	$x - 2\arctan\left(x\right)$	7
risch	$x - 2\arctan\left(x\right)$	7

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] x-2*arctan(x)
```

Maxima [A]
time = 0.49, size = 6, normalized size = 1.00

$$x - 2\ \arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1),x, algorithm="maxima")
```

```
[Out] x - 2*arctan(x)
```

Fricas [A]
time = 4.42, size = 6, normalized size = 1.00

$$x - 2\ \arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1),x, algorithm="fricas")
```

```
[Out] x - 2*arctan(x)
```

Sympy [A]
time = 0.02, size = 5, normalized size = 0.83

$$x - 2\,\mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1),x)
```

```
[Out] x - 2*atan(x)
```

Giac [**A**]
time = 0.98, size = 6, normalized size = 1.00

$$x - 2 \arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/(x^2+1),x, algorithm="giac")
```

```
[Out] x - 2*arctan(x)
```

Mupad [**B**]
time = 0.04, size = 6, normalized size = 1.00

$$x - 2\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 - 1)/(x^2 + 1),x)
```

```
[Out] x - 2*atan(x)
```

3.47 $\int \frac{x^7\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=213

$$-\frac{x^7(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x(35aB-8(Ab-8aC)x}{35ab^4\sqrt{a+bx^2}}$$

```
[Out] -1/7*x^7*(a*B-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x^5*(7*a*B-(A*b-8*C*a)*
x)/a/b^2/(b*x^2+a)^(5/2)-1/105*x^3*(35*a*B-6*(A*b-8*C*a)*x)/a/b^3/(b*x^2+a)
^(3/2)+B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(9/2)-1/35*x*(35*a*B-8*(A*b-8
*C*a)*x)/a/b^4/(b*x^2+a)^(1/2)-16/35*(A*b-8*C*a)*(b*x^2+a)^(1/2)/a/b^5
```

Rubi [A]

time = 0.23, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{1818, 833, 655, 223, 212\}$

$$-\frac{16\sqrt{a+bx^2}\,(Ab-8aC)}{35ab^5}-\frac{x(35aB-8x(Ab-8aC))}{35ab^4\sqrt{a+bx^2}}-\frac{x^3(35aB-6x(Ab-8aC))}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x^5(7aB-x(Ab-8aC))}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}+\frac{B\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*(x^7*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) - (x^5*(7*a*B - (A
*b - 8*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (x^3*(35*a*B - 6*(A*b - 8*a*
C)*x))/(105*a*b^3*(a + b*x^2)^(3/2)) - (x*(35*a*B - 8*(A*b - 8*a*C)*x))/(35
*a*b^4*Sqrt[a + b*x^2]) - (16*(A*b - 8*a*C)*Sqrt[a + b*x^2])/(35*a*b^5) + (
B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
```

```
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx &= -\frac{x^7(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{\int \frac{x^6(-7aB+(Ab-8aC)x)}{(a+bx^2)^{7/2}}\,dx}{7ab} \\
&= -\frac{x^7(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\,(a+bx^2)^{5/2}} - \frac{\int \frac{x^4(-35a^2B+6a(Ab-8aC)x}{(a+bx^2)^{5/2}}}{35a^2b^2} \\
&= -\frac{x^7(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\,(a+bx^2)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC}{105ab^3\,(a+bx^2)^{3/2}} \\
&= -\frac{x^7(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\,(a+bx^2)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC}{105ab^3\,(a+bx^2)^{3/2}} \\
&= -\frac{x^7(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\,(a+bx^2)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC}{105ab^3\,(a+bx^2)^{3/2}} \\
&= -\frac{x^7(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\,(a+bx^2)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC}{105ab^3\,(a+bx^2)^{3/2}} \\
&= -\frac{x^7(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^5(7aB-(Ab-8aC)x)}{35ab^2\,(a+bx^2)^{5/2}} - \frac{x^3(35aB-6(Ab-8aC}{105ab^3\,(a+bx^2)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 1.15, size = 156, normalized size = 0.73

$$\frac{384a^4C-3a^3b(16A+7x(5B-64Cx))+14a^2b^2x^2(-12A+5x(-5B+24Cx))+14ab^3x^4(-15A+x(-29B+60Cx))+b^4x^6(-105A+x(-176B+105Cx))-105\sqrt{b}\,B(a+bx^2)^{7/2}\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{105b^5\,(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (384*a^4*C - 3*a^3*b*(16*A + 7*x*(5*B - 64*C*x)) + 14*a^2*b^2*x^2*(-12*A +
5*x*(-5*B + 24*C*x)) + 14*a*b^3*x^4*(-15*A + x*(-29*B + 60*C*x)) + b^4*x^6*
(-105*A + x*(-176*B + 105*C*x)) - 105*Sqrt[b]*B*(a + b*x^2)^(7/2)*Log[-(Sqr
t[b]*x) + Sqrt[a + b*x^2]])/(105*b^5*(a + b*x^2)^(7/2))
```

Maple [A]
time = 0.17, size = 295, normalized size = 1.38

method	result

default	$C\left(\frac{x^{8}}{b(b\,x^{2}+a)^{\frac{7}{2}}}-\frac{8a\left(-\frac{x^{6}}{b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{6a\left(-\frac{x^{4}}{3b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{4a\left(-\frac{x^{2}}{5b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}\right)}{3b}\right)}{b}\right)}{b}\right)+B\left(-\frac{x^{7}}{7b(b\,x^{2}+a)^{\frac{7}{2}}}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(x^8/b/(b*x^2+a)^(7/2)-8*a/b*(-x^6/b/(b*x^2+a)^(7/2)+6*a/b*(-1/3*x^4/b/(b
*x^2+a)^(7/2)+4/3*a/b*(-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2
)))))+B*(-1/7*x^7/b/(b*x^2+a)^(7/2)+1/b*(-1/5*x^5/b/(b*x^2+a)^(5/2)+1/b*(-1
/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(
b*x^2+a)^(1/2))))))+A*(-x^6/b/(b*x^2+a)^(7/2)+6*a/b*(-1/3*x^4/b/(b*x^2+a)^(
7/2)+4/3*a/b*(-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(189) = 378$.
time $= 0.30$, size $= 435$, normalized size $= 2.04$

$$\frac{Cx^{8}}{(bx^{2}+a)^{\frac{7}{2}}b}-\frac{1}{35}\left(\frac{35\,x^{6}}{(bx^{2}+a)^{\frac{7}{2}}b}+\frac{70\,ax^{4}}{(bx^{2}+a)^{\frac{7}{2}}b^{2}}+\frac{56\,a^{2}x^{2}}{(bx^{2}+a)^{\frac{7}{2}}b^{3}}+\frac{16\,a^{3}}{(bx^{2}+a)^{\frac{7}{2}}b^{4}}\right)Bx+\frac{8\,Cax^{6}}{(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{Ax^{6}}{(bx^{2}+a)^{\frac{7}{2}}b}-\frac{Bx\left(\frac{15\,x^{4}}{(bx^{2}+a)^{\frac{5}{2}}b}+\frac{20\,ax^{2}}{(bx^{2}+a)^{\frac{5}{2}}b^{2}}+\frac{8\,a^{2}}{(bx^{2}+a)^{\frac{5}{2}}b^{3}}\right)}{15\,b}-\frac{Bx\left(\frac{3\,x^{2}}{(bx^{2}+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^{2}+a)^{\frac{3}{2}}b^{2}}\right)}{3\,b^{2}}+\frac{16\,Ca^{2}x^{4}}{(bx^{2}+a)^{\frac{7}{2}}b^{3}}-\frac{2\,Aax^{4}}{(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{Bax^{3}}{(bx^{2}+a)^{\frac{5}{2}}b^{3}}+\frac{64\,Ca^{3}x^{2}}{5\,(bx^{2}+a)^{\frac{7}{2}}b^{4}}-\frac{8\,Aa^{2}x^{2}}{5\,(bx^{2}+a)^{\frac{7}{2}}b^{3}}+\frac{139\,Bx}{105\,\sqrt{bx^{2}+a}\,b^{4}}+\frac{17\,Bax}{105\,(bx^{2}+a)^{\frac{3}{2}}b^{4}}-\frac{29\,Ba^{2}x}{35\,(bx^{2}+a)^{\frac{5}{2}}b^{4}}+\frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}+\frac{128\,Ca^{4}}{35\,(bx^{2}+a)^{\frac{7}{2}}b^{5}}-\frac{16\,Aa^{3}}{35\,(bx^{2}+a)^{\frac{7}{2}}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] C*x^8/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4
/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*
x^2 + a)^(7/2)*b^4))*B*x + 8*C*a*x^6/((b*x^2 + a)^(7/2)*b^2) - A*x^6/((b*x^
2 + a)^(7/2)*b) - 1/15*B*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2
 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/3*B*x*(3*x^2/((b*x^
2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 16*C*a^2*x^4/((b*x^2 +
```

```
 a)^(7/2)*b^3) - 2*A*a*x^4/((b*x^2 + a)^(7/2)*b^2) - B*a*x^3/((b*x^2 + a)^(
5/2)*b^3) + 64/5*C*a^3*x^2/((b*x^2 + a)^(7/2)*b^4) - 8/5*A*a^2*x^2/((b*x^2
+ a)^(7/2)*b^3) + 139/105*B*x/(sqrt(b*x^2 + a)*b^4) + 17/105*B*a*x/((b*x^2
+ a)^(3/2)*b^4) - 29/35*B*a^2*x/((b*x^2 + a)^(5/2)*b^4) + B*arcsinh(b*x/sqr
t(a*b))/b^(9/2) + 128/35*C*a^4/((b*x^2 + a)^(7/2)*b^5) - 16/35*A*a^3/((b*x^
2 + a)^(7/2)*b^4)
```

Fricas [A]
time = 3.53, size = 522, normalized size = 2.45

$$\left[\frac{105\,{\left(B b^{4} x^{8} + 4\,B a b^{3} x^{6} + 6\,B a^{2} b^{2} x^{4} + 4\,B a^{3} b x^{2} + B a^{4}\right)} \sqrt{b} \log\left(-2\,b x^{2} - 2\,\sqrt{b x^{2} + a} \sqrt{b} x - a\right) + 2\,{\left(105\,C b^{4} x^{8} - 176\,B b^{4} x^{7} - 406\,B a b^{3} x^{5} - 350\,B a^{2} b^{2} x^{3} + 105\,{\left(8\,C a b^{3} - A b^{4}\right)} x^{6} - 105\,B a^{3} b x + 384\,C a^{4} - 48\,A a^{3} b + 210\,{\left(8\,C a^{2} b^{2} - A a b^{3}\right)} x^{4} + 168\,{\left(8\,C a^{3} b - A a^{2} b^{2}\right)} x^{2}\right)} \sqrt{b x^{2} + a}}{210\,{\left(b^{9} x^{8} + 4\,a b^{8} x^{6} + 6\,a^{2} b^{7} x^{4} + 4\,a^{3} b^{6} x^{2} + a^{4} b^{5}\right)}}, -\frac{105\,{\left(B b^{4} x^{8} + 4\,B a b^{3} x^{6} + 6\,B a^{2} b^{2} x^{4} + 4\,B a^{3} b x^{2} + B a^{4}\right)} \sqrt{-b} \arctan\left(\frac{\sqrt{-b} x}{\sqrt{b x^{2} + a}}\right) - {\left(105\,C b^{4} x^{8} - 176\,B b^{4} x^{7} - 406\,B a b^{3} x^{5} - 350\,B a^{2} b^{2} x^{3} + 105\,{\left(8\,C a b^{3} - A b^{4}\right)} x^{6} - 105\,B a^{3} b x + 384\,C a^{4} - 48\,A a^{3} b + 210\,{\left(8\,C a^{2} b^{2} - A a b^{3}\right)} x^{4} + 168\,{\left(8\,C a^{3} b - A a^{2} b^{2}\right)} x^{2}\right)} \sqrt{b x^{2} + a}}{105\,{\left(b^{9} x^{8} + 4\,a b^{8} x^{6} + 6\,a^{2} b^{7} x^{4} + 4\,a^{3} b^{6} x^{2} + a^{4} b^{5}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(B*b^4*x^8 + 4*B*a*b^3*x^6 + 6*B*a^2*b^2*x^4 + 4*B*a^3*b*x^2 +
B*a^4)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(105*C*b
^4*x^8 - 176*B*b^4*x^7 - 406*B*a*b^3*x^5 - 350*B*a^2*b^2*x^3 + 105*(8*C*a*b
^3 - A*b^4)*x^6 - 105*B*a^3*b*x + 384*C*a^4 - 48*A*a^3*b + 210*(8*C*a^2*b^2
 - A*a*b^3)*x^4 + 168*(8*C*a^3*b - A*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(b^9*x^
8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*(105*(B*
b^4*x^8 + 4*B*a*b^3*x^6 + 6*B*a^2*b^2*x^4 + 4*B*a^3*b*x^2 + B*a^4)*sqrt(-b)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (105*C*b^4*x^8 - 176*B*b^4*x^7 - 406*
B*a*b^3*x^5 - 350*B*a^2*b^2*x^3 + 105*(8*C*a*b^3 - A*b^4)*x^6 - 105*B*a^3*b
*x + 384*C*a^4 - 48*A*a^3*b + 210*(8*C*a^2*b^2 - A*a*b^3)*x^4 + 168*(8*C*a^
3*b - A*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(b^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x
^4 + 4*a^3*b^6*x^2 + a^4*b^5)]
```

Sympy [A]
time = 103.21, size = 3806, normalized size = 17.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2*s
qrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a +
b*x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x*
*2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(
a + b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**
5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*s
qrt(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b
**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6
*sqrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)), True)) + B*(105*a**(205/
2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99
```

```
/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a)
+ 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(
105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b
*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193
/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(203/2)*b**46*x**2*sqrt(1
 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*
x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/
2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqr
t(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*
a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*
x**12*sqrt(1 + b*x**2/a)) + 1575*a**(201/2)*b**47*x**4*sqrt(1 + b*x**2/a)*a
sinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*
a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*
x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a
) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**
(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 +
 b*x**2/a)) + 2100*a**(199/2)*b**48*x**6*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x
/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**
(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 +
b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(1
97/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*
sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) +
1575*a**(197/2)*b**49*x**8*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105
*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*s
qrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 21
00*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/
2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**
2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(195/2)
*b**50*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b*
*(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2
/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*
b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1
 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**
(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 105*a**(193/2)*b**51*x**12*s
qrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1
 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**
(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**
6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) +
 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(11
1/2)*x**12*sqrt(1 + b*x**2/a)) - 105*a**102*b**(91/2)*x/(105*a**(205/2)*b**
(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/
a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b
**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1
+ b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(
193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 665*a**101*b**(93/2)*x**3/(10
```

```
5*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*
sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2
100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107
/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x*
*2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 1771*a**100*b
**(95/2)*x**5/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)
*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(
1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a
**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x*
*10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)
) - 2549*a**99*b**(97/2)*x**7/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a)
+ 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**...
```

Giac [A]

time = 0.73, size = 204, normalized size = 0.96

$$\frac{\left(\left(\left(\left(\left(\left(\left(\frac{105\,Cx}{b}-\frac{176\,B}{b}\right)x+\frac{105\,(8\,Ca^4b^7-Aa^3b^8)}{a^3b^9}\right)x-\frac{406\,Ba}{b^2}\right)x+\frac{210\,(8\,Ca^5b^6-Aa^4b^7)}{a^3b^9}\right)x-\frac{350\,Ba^2}{b^3}\right)x+\frac{168\,(8\,Ca^6b^5-Aa^5b^6)}{a^3b^9}\right)x-\frac{105\,Ba^3}{b^4}\right)x+\frac{48\,(8\,Ca^7b^4-Aa^6b^5)}{a^3b^9}}{105\,(bx^2+a)^{\frac{7}{2}}}-\frac{B\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((((((((105*C*x/b - 176*B/b)*x + 105*(8*C*a^4*b^7 - A*a^3*b^8)/(a^3*b
^9))*x - 406*B*a/b^2)*x + 210*(8*C*a^5*b^6 - A*a^4*b^7)/(a^3*b^9))*x - 350*
B*a^2/b^3)*x + 168*(8*C*a^6*b^5 - A*a^5*b^6)/(a^3*b^9))*x - 105*B*a^3/b^4)*
x + 48*(8*C*a^7*b^4 - A*a^6*b^5)/(a^3*b^9))/(b*x^2 + a)^(7/2) - B*log(abs(-
sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,\left(C\,x^2+B\,x+A\right)}{{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2), x)
```

3.48 $\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=150

$$-\frac{x^6(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}}-\frac{x^4(6B+7Cx)}{35b^2\,(a+bx^2)^{5/2}}-\frac{x^2(24B+35Cx)}{105b^3\,(a+bx^2)^{3/2}}-\frac{16B+35Cx}{35b^4\sqrt{a+bx^2}}+\frac{C\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

```
[Out] -1/7*x^6*(a*B-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x^4*(7*C*x+6*B)/b^2/(b*
x^2+a)^(5/2)-1/105*x^2*(35*C*x+24*B)/b^3/(b*x^2+a)^(3/2)+C*arctanh(x*b^(1/2
)/(b*x^2+a)^(1/2))/b^(9/2)+1/35*(-35*C*x-16*B)/b^4/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1818, 833, 792, 223, 212}

$$-\frac{x^6(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}+\frac{C\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}-\frac{16B+35Cx}{35b^4\sqrt{a+bx^2}}-\frac{x^2(24B+35Cx)}{105b^3\,(a+bx^2)^{3/2}}-\frac{x^4(6B+7Cx)}{35b^2\,(a+bx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*(x^6*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) - (x^4*(6*B + 7*C*
x))/(35*b^2*(a + b*x^2)^(5/2)) - (x^2*(24*B + 35*C*x))/(105*b^3*(a + b*x^2)
^(3/2)) - (16*B + 35*C*x)/(35*b^4*Sqrt[a + b*x^2]) + (C*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/b^(9/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
```

```
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx &= -\frac{x^6(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{\int \frac{x^5(-6aB-7aCx)}{(a+bx^2)^{7/2}}\,dx}{7ab} \\
&= -\frac{x^6(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\,(a+bx^2)^{5/2}} - \frac{\int \frac{x^3(-24a^2B-35a^2Cx)}{(a+bx^2)^{5/2}}\,dx}{35a^2b^2} \\
&= -\frac{x^6(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\,(a+bx^2)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3\,(a+bx^2)^{3/2}} - \frac{\int \frac{x(-48a^3B}{(a+b}}{10} \\
&= -\frac{x^6(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\,(a+bx^2)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3\,(a+bx^2)^{3/2}} - \frac{16B+3}{35b^4\sqrt{a+}} \\
&= -\frac{x^6(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\,(a+bx^2)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3\,(a+bx^2)^{3/2}} - \frac{16B+3}{35b^4\sqrt{a+}} \\
&= -\frac{x^6(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\,(a+bx^2)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3\,(a+bx^2)^{3/2}} - \frac{16B+3}{35b^4\sqrt{a+}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.93, size = 127, normalized size = 0.85

$$\frac{15Ab^4x^7-14a^3bx^2(12B+25Cx)-14a^2b^2x^4(15B+29Cx)-3a^4(16B+35Cx)-ab^3x^6(105B+176Cx)}{105ab^4\left(a+bx^2\right)^{7/2}}-\frac{C\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (15*A*b^4*x^7 - 14*a^3*b*x^2*(12*B + 25*C*x) - 14*a^2*b^2*x^4*(15*B + 29*C*
x) - 3*a^4*(16*B + 35*C*x) - a*b^3*x^6*(105*B + 176*C*x))/(105*a*b^4*(a + b
*x^2)^(7/2)) - (C*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(9/2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(128) = 256.
time = 0.12, size = 334, normalized size = 2.23

method	result

default	$C\left(-\frac{x^{7}}{7b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{-\frac{x^{5}}{5b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^{3}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)+B\left(-\frac{x^{6}}{b(b\,x^{2}+a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-1/7*x^7/b/(b*x^2+a)^(7/2)+1/b*(-1/5*x^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^3
/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+
a)^(1/2))))))+B*(-x^6/b/(b*x^2+a)^(7/2)+6*a/b*(-1/3*x^4/b/(b*x^2+a)^(7/2)+4
/3*a/b*(-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2))))+A*(-1/2*x^
5/b/(b*x^2+a)^(7/2)+5/2*a/b*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/6*x/b/(
b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5
/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))))
```

<u>Maxima</u> [**B**] Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(127) = 254$.
time = 0.29, size = 447, normalized size = 2.98

$$-\frac{1}{35}\left(\frac{35\,x^{6}}{(bx^{2}+a)^{\frac{7}{2}}b}+\frac{70\,ax^{4}}{(bx^{2}+a)^{\frac{7}{2}}b^{2}}+\frac{56\,a^{2}x^{2}}{(bx^{2}+a)^{\frac{7}{2}}b^{3}}+\frac{16\,a^{3}}{(bx^{2}+a)^{\frac{7}{2}}b^{4}}\right)Cx-\frac{Bx^{6}}{(bx^{2}+a)^{\frac{7}{2}}b}-\frac{Cx\left(\frac{15\,x^{4}}{(bx^{2}+a)^{\frac{5}{2}}b}+\frac{20\,ax^{2}}{(bx^{2}+a)^{\frac{5}{2}}b^{2}}+\frac{8\,a^{2}}{(bx^{2}+a)^{\frac{5}{2}}b^{3}}\right)}{15\,b}-\frac{Ax^{5}}{2\,(bx^{2}+a)^{\frac{7}{2}}b}-\frac{Cx\left(\frac{3\,x^{2}}{(bx^{2}+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^{2}+a)^{\frac{3}{2}}b^{2}}\right)}{3\,b^{2}}-\frac{2\,Bax^{4}}{(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{Cax^{3}}{(bx^{2}+a)^{\frac{5}{2}}b^{3}}-\frac{5\,Aax^{3}}{8\,(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{8\,Ba^{2}x^{2}}{5\,(bx^{2}+a)^{\frac{7}{2}}b^{3}}+\frac{139\,Cx}{105\,\sqrt{bx^{2}+a}\,b^{4}}+\frac{17\,Cax}{105\,(bx^{2}+a)^{\frac{3}{2}}b^{4}}-\frac{29\,Ca^{2}x}{35\,(bx^{2}+a)^{\frac{5}{2}}b^{4}}+\frac{Ax}{14\,(bx^{2}+a)^{\frac{3}{2}}b^{3}}+\frac{Ax}{7\,\sqrt{bx^{2}+a}\,ab^{3}}+\frac{3\,Aax}{56\,(bx^{2}+a)^{\frac{5}{2}}b^{3}}-\frac{15\,Aa^{2}x}{56\,(bx^{2}+a)^{\frac{7}{2}}b^{3}}+\frac{C\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}-\frac{16\,Ba^{3}}{35\,(bx^{2}+a)^{\frac{7}{2}}b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56
*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*C*x - B*
x^6/((b*x^2 + a)^(7/2)*b) - 1/15*C*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x
^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*A*x^5/(
(b*x^2 + a)^(7/2)*b) - 1/3*C*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 +
 a)^(3/2)*b^2))/b^2 - 2*B*a*x^4/((b*x^2 + a)^(7/2)*b^2) - C*a*x^3/((b*x^2 +
 a)^(5/2)*b^3) - 5/8*A*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 8/5*B*a^2*x^2/((b*x^
2 + a)^(7/2)*b^3) + 139/105*C*x/(sqrt(b*x^2 + a)*b^4) + 17/105*C*a*x/((b*x^
2 + a)^(3/2)*b^4) - 29/35*C*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*A*x/((b*x^
2 + a)^(3/2)*b^3) + 1/7*A*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*A*a*x/((b*x^2 +
a)^(5/2)*b^3) - 15/56*A*a^2*x/((b*x^2 + a)^(7/2)*b^3) + C*arcsinh(b*x/sqrt(
a*b))/b^(9/2) - 16/35*B*a^3/((b*x^2 + a)^(7/2)*b^4)
```

<u>Fricas</u> [A]
time = 4.62, size = 467, normalized size = 3.11

$$\left[\frac{105\,(Cab^{4}x^{8}+4\,Ca^{2}b^{3}x^{6}+6\,Ca^{3}b^{2}x^{4}+4\,Ca^{4}bx^{2}+Ca^{5})\sqrt{b}\,\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{b}\,x-a\right)-2\,(105\,Bab^{4}x^{6}+406\,Ca^{2}b^{3}x^{5}+210\,Ba^{2}b^{3}x^{4}+350\,Ca^{3}b^{2}x^{3}+168\,Ba^{3}b^{2}x^{2}+(176\,Cab^{4}-15\,Ab^{5})x^{7}+105\,Ca^{4}bx+48\,Ba^{4}b)\sqrt{bx^{2}+a}}{210\,(ab^{9}x^{8}+4\,a^{2}b^{8}x^{6}+6\,a^{3}b^{7}x^{4}+4\,a^{4}b^{6}x^{2}+a^{5}b^{5})},-\frac{105\,(Cab^{4}x^{8}+4\,Ca^{2}b^{3}x^{6}+6\,Ca^{3}b^{2}x^{4}+4\,Ca^{4}bx^{2}+Ca^{5})\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^{2}+a}}\right)+(105\,Bab^{4}x^{6}+406\,Ca^{2}b^{3}x^{5}+210\,Ba^{2}b^{3}x^{4}+350\,Ca^{3}b^{2}x^{3}+168\,Ba^{3}b^{2}x^{2}+(176\,Cab^{4}-15\,Ab^{5})x^{7}+105\,Ca^{4}bx+48\,Ba^{4}b)\sqrt{bx^{2}+a}}{105\,(ab^{9}x^{8}+4\,a^{2}b^{8}x^{6}+6\,a^{3}b^{7}x^{4}+4\,a^{4}b^{6}x^{2}+a^{5}b^{5})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(C*a*b^4*x^8 + 4*C*a^2*b^3*x^6 + 6*C*a^3*b^2*x^4 + 4*C*a^4*b*x^
2 + C*a^5)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(105
```

```
*B*a*b^4*x^6 + 406*C*a^2*b^3*x^5 + 210*B*a^2*b^3*x^4 + 350*C*a^3*b^2*x^3 +
168*B*a^3*b^2*x^2 + (176*C*a*b^4 - 15*A*b^5)*x^7 + 105*C*a^4*b*x + 48*B*a^4
*b)*sqrt(b*x^2 + a))/(a*b^9*x^8 + 4*a^2*b^8*x^6 + 6*a^3*b^7*x^4 + 4*a^4*b^6
*x^2 + a^5*b^5), -1/105*(105*(C*a*b^4*x^8 + 4*C*a^2*b^3*x^6 + 6*C*a^3*b^2*x
^4 + 4*C*a^4*b*x^2 + C*a^5)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (
105*B*a*b^4*x^6 + 406*C*a^2*b^3*x^5 + 210*B*a^2*b^3*x^4 + 350*C*a^3*b^2*x^3
 + 168*B*a^3*b^2*x^2 + (176*C*a*b^4 - 15*A*b^5)*x^7 + 105*C*a^4*b*x + 48*B*
a^4*b)*sqrt(b*x^2 + a))/(a*b^9*x^8 + 4*a^2*b^8*x^6 + 6*a^3*b^7*x^4 + 4*a^4*
b^6*x^2 + a^5*b^5)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(133) = 266$.
time = 77.58, size = 3448, normalized size = 22.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/
a) + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1
 + b*x**2/a)) + B*Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 105*
a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**
7*x**6*sqrt(a + b*x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**2) +
105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35
*b**7*x**6*sqrt(a + b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a + b*x**2
) + 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2)
+ 35*b**7*x**6*sqrt(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt(a + b*x*
*2) + 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2
) + 35*b**7*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)), True)) +
 C*(105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a
**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqr
t(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100
*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)
*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/
a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(203/2)*b
**46*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(9
9/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a)
 + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**
(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 +
b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(19
3/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(201/2)*b**47*x**4*sqrt
(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 +
b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(20
1/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*s
```

```
qrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 63
0*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2
)*x**12*sqrt(1 + b*x**2/a)) + 2100*a**(199/2)*b**48*x**6*sqrt(1 + b*x**2/a)
*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 63
0*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2
)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2
/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b
**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1
 + b*x**2/a)) + 1575*a**(197/2)*b**49*x**8*sqrt(1 + b*x**2/a)*asinh(sqrt(b)
*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b
**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1
+ b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**
(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**1
0*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a))
+ 630*a**(195/2)*b**50*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(1
05*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2
*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) +
2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(10
7/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x
**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 105*a**(193/
2)*b**51*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*
b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x*
*2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2
)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt
(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a
**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 105*a**102*b**(91/2)*x/(10
5*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*
sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2
100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107
/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x*
*2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 665*a**101*b*
*(93/2)*x**3/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*
b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1
 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a*
*(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**
10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a))
 - 1771*a**100*b**(95/2)*x**5/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a)
+ 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(1
03/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*
x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/
2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a*...
```

Giac [**A**]

time = 1.59, size = 138, normalized size = 0.92

$$-\frac{\left(\left(\left(\left(\left(x\left(\frac{105\,B}{b}+\frac{(176\,Ca^3b^7-15\,Aa^2b^8)x}{a^3b^8}\right)+\frac{406\,Ca}{b^2}\right)x+\frac{210\,Ba}{b^2}\right)x+\frac{350\,Ca^2}{b^3}\right)x+\frac{168\,Ba^2}{b^3}\right)x+\frac{105\,Ca^3}{b^4}\right)x+\frac{48\,Ba^3}{b^4}}{105\,(bx^2+a)^{\frac{7}{2}}}-\frac{C\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\,\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((((((x*(105*B/b + (176*C*a^3*b^7 - 15*A*a^2*b^8)*x/(a^3*b^8)) + 406
*C*a/b^2)*x + 210*B*a/b^2)*x + 350*C*a^2/b^3)*x + 168*B*a^2/b^3)*x + 105*C*
a^3/b^4)*x + 48*B*a^3/b^4)/(b*x^2 + a)^(7/2) - C*log(abs(-sqrt(b)*x + sqrt(
b*x^2 + a)))/b^(9/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^6\,\left(C\,x^2+B\,x+A\right)}{{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2), x)
```

3.49 $\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=132

$$-\frac{x^5(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^4(Ab+6aC-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{4(Ab+6aC)}{105b^4\left(a+bx^2\right)^{3/2}}-\frac{4(Ab+6aC)}{35ab^4\sqrt{a+bx^2}}$$

```
[Out] -1/7*x^5*(a*B-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x^4*(-5*B*b*x+A*b+6*C*a
)/a/b^2/(b*x^2+a)^(5/2)+4/105*(A*b+6*C*a)/b^4/(b*x^2+a)^(3/2)-4/35*(A*b+6*C
*a)/a/b^4/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1818, 819, 272, 45}

$$-\frac{4(6aC+Ab)}{35ab^4\sqrt{a+bx^2}}+\frac{4(6aC+Ab)}{105b^4\left(a+bx^2\right)^{3/2}}-\frac{x^4(6aC+Ab-5bBx)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^5(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*(x^5*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) - (x^4*(A*b + 6*a*
C - 5*b*B*x))/(35*a*b^2*(a + b*x^2)^(5/2)) + (4*(A*b + 6*a*C))/(105*b^4*(a
+ b*x^2)^(3/2)) - (4*(A*b + 6*a*C))/(35*a*b^4*Sqrt[a + b*x^2])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[m*((c*d*f + a*e*g)/(2*a*c*(p + 1))), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx &= -\frac{x^5(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{\int \frac{x^4(-5aB-(Ab+6aC)x)}{(a+bx^2)^{7/2}}\,dx}{7ab} \\
&= -\frac{x^5(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^4(Ab+6aC-5bBx)}{35ab^2\,(a+bx^2)^{5/2}} + \frac{(4(Ab+6aC))\int \frac{x^3}{(a+bx^2)^{5/}}}{35ab^2} \\
&= -\frac{x^5(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^4(Ab+6aC-5bBx)}{35ab^2\,(a+bx^2)^{5/2}} + \frac{(2(Ab+6aC))\mathrm{Subst}\Big(\int \frac{}{(}}{35ab^2} \\
&= -\frac{x^5(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^4(Ab+6aC-5bBx)}{35ab^2\,(a+bx^2)^{5/2}} + \frac{(2(Ab+6aC))\mathrm{Subst}\Big(\int \Big(}{} \\
&= -\frac{x^5(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^4(Ab+6aC-5bBx)}{35ab^2\,(a+bx^2)^{5/2}} + \frac{4(Ab+6aC)}{105b^4\,(a+bx^2)^{3/2}} - \frac{4(}{35a}
\end{aligned}
$$

Mathematica [A]
time = 0.96, size = 89, normalized size = 0.67

$$
\frac{-48a^4C+15b^4Bx^7-35ab^3x^4(A+3Cx^2)-14a^2b^2x^2(2A+15Cx^2)-8a^3b(A+21Cx^2)}{105ab^4\,(a+bx^2)^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-48*a^4*C + 15*b^4*B*x^7 - 35*a*b^3*x^4*(A + 3*C*x^2) - 14*a^2*b^2*x^2*(2*
A + 15*C*x^2) - 8*a^3*b*(A + 21*C*x^2))/(105*a*b^4*(a + b*x^2)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(116) = 232$.
time = 0.12, size = 289, normalized size = 2.19

method	result
gosper	$-\frac{-15B\,x^{7} b^{4}+105C\,x^{6} a\,b^{3}+35Aa\,b^{3} x^{4}+210C\,a^{2} b^{2} x^{4}+28A\,a^{2} b^{2} x^{2}+168C\,a^{3} b\,x^{2}+8A\,a^{3} b+48C\,a^{4}}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}} a\,b^{4}}$
trager	$-\frac{-15B\,x^{7} b^{4}+105C\,x^{6} a\,b^{3}+35Aa\,b^{3} x^{4}+210C\,a^{2} b^{2} x^{4}+28A\,a^{2} b^{2} x^{2}+168C\,a^{3} b\,x^{2}+8A\,a^{3} b+48C\,a^{4}}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}} a\,b^{4}}$

default	$C\left(-\frac{x^{6}}{b(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{6a\left(-\frac{x^{4}}{3b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{4a\left(-\frac{x^{2}}{5b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}\right)}{3b}\right)}{b}\right)+B\left(-\frac{x^{5}}{2b(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^{3}}{4b\left(b\,x^{2}\right.}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-x^6/b/(b*x^2+a)^(7/2)+6*a/b*(-1/3*x^4/b/(b*x^2+a)^(7/2)+4/3*a/b*(-1/5*x
^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2))))+B*(-1/2*x^5/b/(b*x^2+a)^
(7/2)+5/2*a/b*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/6*x/b/(b*x^2+a)^(7/2)
+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3
*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))))+A*(-1/3*x^4/b/(b*x^2+a
)^(7/2)+4/3*a/b*(-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(116) = 232$.
time = 0.28, size = 240, normalized size = 1.82

$$-\frac{Cx^{6}}{(bx^{2}+a)^{\frac{7}{2}}b}-\frac{Bx^{5}}{2\,(bx^{2}+a)^{\frac{7}{2}}b}-\frac{2\,Cax^{4}}{(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{Ax^{4}}{3\,(bx^{2}+a)^{\frac{7}{2}}b}-\frac{5\,Bax^{3}}{8\,(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{8\,Ca^{2}x^{2}}{5\,(bx^{2}+a)^{\frac{7}{2}}b^{3}}-\frac{4\,Aax^{2}}{15\,(bx^{2}+a)^{\frac{7}{2}}b^{2}}+\frac{Bx}{14\,(bx^{2}+a)^{\frac{3}{2}}b^{3}}+\frac{Bx}{7\,\sqrt{bx^{2}+a}\,ab^{3}}+\frac{3\,Bax}{56\,(bx^{2}+a)^{\frac{5}{2}}b^{3}}-\frac{15\,Ba^{2}x}{56\,(bx^{2}+a)^{\frac{7}{2}}b^{3}}-\frac{16\,Ca^{3}}{35\,(bx^{2}+a)^{\frac{7}{2}}b^{4}}-\frac{8\,Aa^{2}}{105\,(bx^{2}+a)^{\frac{7}{2}}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -C*x^6/((b*x^2 + a)^(7/2)*b) - 1/2*B*x^5/((b*x^2 + a)^(7/2)*b) - 2*C*a*x^4/
((b*x^2 + a)^(7/2)*b^2) - 1/3*A*x^4/((b*x^2 + a)^(7/2)*b) - 5/8*B*a*x^3/((b
*x^2 + a)^(7/2)*b^2) - 8/5*C*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) - 4/15*A*a*x^2
/((b*x^2 + a)^(7/2)*b^2) + 1/14*B*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*B*x/(sqrt
(b*x^2 + a)*a*b^3) + 3/56*B*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*B*a^2*x/((b
*x^2 + a)^(7/2)*b^3) - 16/35*C*a^3/((b*x^2 + a)^(7/2)*b^4) - 8/105*A*a^2/((
b*x^2 + a)^(7/2)*b^3)
```

Fricas [A]
time = 2.93, size = 137, normalized size = 1.04

$$\frac{\left(15\,Bb^{4}x^{7}-105\,Cab^{3}x^{6}-48\,Ca^{4}-8\,Aa^{3}b-35\,(6\,Ca^{2}b^{2}+Aab^{3})x^{4}-28\,(6\,Ca^{3}b+Aa^{2}b^{2})x^{2}\right)\sqrt{bx^{2}+a}}{105\,(ab^{8}x^{8}+4\,a^{2}b^{7}x^{6}+6\,a^{3}b^{6}x^{4}+4\,a^{4}b^{5}x^{2}+a^{5}b^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*B*b^4*x^7 - 105*C*a*b^3*x^6 - 48*C*a^4 - 8*A*a^3*b - 35*(6*C*a^2*
b^2 + A*a*b^3)*x^4 - 28*(6*C*a^3*b + A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)/(a*b^8
*x^8 + 4*a^2*b^7*x^6 + 6*a^3*b^6*x^4 + 4*a^4*b^5*x^2 + a^5*b^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(121) = 242$.
time = 82.04, size = 740, normalized size = 5.61

$$A\left(\begin{cases}-\frac{8a^{2}}{105a^{3}b^{3}\sqrt{a+bx^{2}}+315a^{2}b^{4}x^{2}\sqrt{a+bx^{2}}+315ab^{5}x^{4}\sqrt{a+bx^{2}}+105b^{6}x^{6}\sqrt{a+bx^{2}}}-\frac{28abx^{2}}{105a^{3}b^{3}\sqrt{a+bx^{2}}+315a^{2}b^{4}x^{2}\sqrt{a+bx^{2}}+315ab^{5}x^{4}\sqrt{a+bx^{2}}+105b^{6}x^{6}\sqrt{a+bx^{2}}}-\frac{35b^{2}x^{4}}{105a^{3}b^{3}\sqrt{a+bx^{2}}+315a^{2}b^{4}x^{2}\sqrt{a+bx^{2}}+315ab^{5}x^{4}\sqrt{a+bx^{2}}+105b^{6}x^{6}\sqrt{a+bx^{2}}} & \text{for}\: b \neq 0 \\ \frac{x^{6}}{6a^{\frac{9}{2}}} & \text{otherwise}\end{cases}\right)+\frac{Bx^{7}}{7a^{\frac{9}{2}}\sqrt{1+\frac{bx^{2}}{a}}+21a^{\frac{7}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}+21a^{\frac{5}{2}}b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}+7a^{\frac{3}{2}}b^{3}x^{6}\sqrt{1+\frac{bx^{2}}{a}}}+C\left(\begin{cases}-\frac{16a^{3}}{35a^{3}b^{4}\sqrt{a+bx^{2}}+105a^{2}b^{5}x^{2}\sqrt{a+bx^{2}}+105ab^{6}x^{4}\sqrt{a+bx^{2}}+35b^{7}x^{6}\sqrt{a+bx^{2}}}-\frac{56a^{2}bx^{2}}{35a^{3}b^{4}\sqrt{a+bx^{2}}+105a^{2}b^{5}x^{2}\sqrt{a+bx^{2}}+105ab^{6}x^{4}\sqrt{a+bx^{2}}+35b^{7}x^{6}\sqrt{a+bx^{2}}}-\frac{70ab^{2}x^{4}}{35a^{3}b^{4}\sqrt{a+bx^{2}}+105a^{2}b^{5}x^{2}\sqrt{a+bx^{2}}+105ab^{6}x^{4}\sqrt{a+bx^{2}}+35b^{7}x^{6}\sqrt{a+bx^{2}}}-\frac{35b^{3}x^{6}}{35a^{3}b^{4}\sqrt{a+bx^{2}}+105a^{2}b^{5}x^{2}\sqrt{a+bx^{2}}+105ab^{6}x^{4}\sqrt{a+bx^{2}}+35b^{7}x^{6}\sqrt{a+bx^{2}}} & \text{for}\: b \neq 0 \\ \frac{x^{8}}{8a^{\frac{9}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*Piecewise((-8*a**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*s
qrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a +
 b*x**2)) - 28*a*b*x**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**
2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(
a + b*x**2)) - 35*b**2*x**4/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4
*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*s
qrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(9/2)), True)) + B*x**7/(7*a**(9/2
)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 21*a**(5/2)*
b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + C
*Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2*sq
rt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a + b
*x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**
2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a
 + b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5
*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sq
rt(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b*
*5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*
sqrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)), True))
```

Giac [A]
time = 1.05, size = 112, normalized size = 0.85

$$\frac{\left(5\left(3\left(\frac{Bx}{a}-\frac{7C}{b}\right)x^{2}-\frac{7\left(6Ca^{4}b^{2}+Aa^{3}b^{3}\right)}{a^{3}b^{4}}\right)x^{2}-\frac{28\left(6Ca^{5}b+Aa^{4}b^{2}\right)}{a^{3}b^{4}}\right)x^{2}-\frac{8\left(6Ca^{6}+Aa^{5}b\right)}{a^{3}b^{4}}}{105\left(bx^{2}+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((5*(3*(B*x/a - 7*C/b)*x^2 - 7*(6*C*a^4*b^2 + A*a^3*b^3)/(a^3*b^4))*x
^2 - 28*(6*C*a^5*b + A*a^4*b^2)/(a^3*b^4))*x^2 - 8*(6*C*a^6 + A*a^5*b)/(a^3
*b^4))/(b*x^2 + a)^(7/2)
```

Mupad [B]
time = 1.27, size = 196, normalized size = 1.48

$$\frac{\frac{a\,\left(\frac{C}{3\,b^3}-\frac{7\,A\,b-14\,C\,a}{21\,a\,b^3}\right)}{b}-\frac{3\,B\,x}{7\,b^3}}{{\left(b\,x^2+a\right)}^{3/2}}-\frac{\frac{a^2\,\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)}{b^2}+\frac{B\,a^2\,x}{7\,b^3}}{{\left(b\,x^2+a\right)}^{7/2}}-\frac{\frac{C}{b^4}-\frac{B\,x}{7\,a\,b^3}}{\sqrt{b\,x^2+a}}-\frac{\frac{a\,\left(\frac{7\,C\,a^2-7\,A\,a\,b}{35\,a\,b^3}+\frac{a\,\left(\frac{C}{5\,b^2}-\frac{7\,A\,b^2-7\,C\,a\,b}{35\,a\,b^3}\right)}{b}\right)}{b}-\frac{3\,B\,a\,x}{7\,b^3}}{{\left(b\,x^2+a\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] ((a*(C/(3*b^3) - (7*A*b - 14*C*a)/(21*a*b^3)))/b - (3*B*x)/(7*b^3))/(a + b*
x^2)^(3/2) - ((a^2*(A/(7*b) - (C*a)/(7*b^2)))/b^2 + (B*a^2*x)/(7*b^3))/(a +
 b*x^2)^(7/2) - (C/b^4 - (B*x)/(7*a*b^3))/(a + b*x^2)^(1/2) - ((a*((7*C*a^2
 - 7*A*a*b)/(35*a*b^3) + (a*(C/(5*b^2) - (7*A*b^2 - 7*C*a*b)/(35*a*b^3)))/b
))/b - (3*B*a*x)/(7*b^3))/(a + b*x^2)^(5/2)
```

3.50 $\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{x^4(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{8aB+3(2Ab+5aC)x}{105ab^3\left(a+bx^2\right)^{3/2}}+\frac{(2Ab+5aC)x}{35a^2b^3\sqrt{a+bx^2}}$$

```
[Out] -1/7*x^4*(a*B-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x^2*(4*a*B+(2*A*b+5*C*a
)*x)/a/b^2/(b*x^2+a)^(5/2)+1/105*(-8*a*B-3*(2*A*b+5*C*a)*x)/a/b^3/(b*x^2+a)
^(3/2)+1/35*(2*A*b+5*C*a)*x/a^2/b^3/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1818, 833, 792, 197}

$$\frac{x(5aC+2Ab)}{35a^2b^3\sqrt{a+bx^2}}-\frac{3x(5aC+2Ab)+8aB}{105ab^3\left(a+bx^2\right)^{3/2}}-\frac{x^2(x(5aC+2Ab)+4aB)}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^4(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*(x^4*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) - (x^2*(4*a*B + (2
*A*b + 5*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (8*a*B + 3*(2*A*b + 5*a*C)
*x)/(105*a*b^3*(a + b*x^2)^(3/2)) + ((2*A*b + 5*a*C)*x)/(35*a^2*b^3*Sqrt[a
+ b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
```

```
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx &= -\frac{x^4(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}}-\frac{\int \frac{x^3(-4aB-(2Ab+5aC)x)}{(a+bx^2)^{7/2}}\,dx}{7ab}\\
&= -\frac{x^4(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}}-\frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\,(a+bx^2)^{5/2}}-\frac{\int \frac{x(-8a^2B-3a(2Ab+5aC)x)}{(a+bx^2)^{5/2}}}{35a^2b^2}\\
&= -\frac{x^4(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}}-\frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\,(a+bx^2)^{5/2}}-\frac{8aB+3(2Ab+5aC)x}{105ab^3\,(a+bx^2)^{3/2}}\\
&= -\frac{x^4(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}}-\frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\,(a+bx^2)^{5/2}}-\frac{8aB+3(2Ab+5aC)x}{105ab^3\,(a+bx^2)^{3/2}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.75, size = 79, normalized size = 0.53

$$
\frac{-8a^4B-28a^3bBx^2-35a^2b^2Bx^4+21aAb^3x^5+6Ab^4x^7+15ab^3Cx^7}{105a^2b^3\,(a+bx^2)^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-8*a^4*B - 28*a^3*b*B*x^2 - 35*a^2*b^2*B*x^4 + 21*a*A*b^3*x^5 + 6*A*b^4*x^
7 + 15*a*b^3*C*x^7)/(105*a^2*b^3*(a + b*x^2)^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(133) = 266.
time = 0.12, size = 327, normalized size = 2.19

method	result
gosper	$\frac{6A\,b^4x^7+15Ca\,x^7b^3+21Aa\,b^3x^5-35B\,x^4a^2b^2-28B\,a^3b\,x^2-8B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}\,a^2b^3}$
trager	$\frac{6A\,b^4x^7+15Ca\,x^7b^3+21Aa\,b^3x^5-35B\,x^4a^2b^2-28B\,a^3b\,x^2-8B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}\,a^2b^3}$

default	$C\left(-\frac{x^5}{2b(b x^2+a)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^3}{4b\left(b x^2+a\right)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b x^2+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b x^2}}\right.}{7a}}{a}\right.}{6b}\right.}{4b}\right.}{2b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-1/2*x^5/b/(b*x^2+a)^(7/2)+5/2*a/b*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(
-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b
*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))))
)+B*(-1/3*x^4/b/(b*x^2+a)^(7/2)+4/3*a/b*(-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/
b^2/(b*x^2+a)^(7/2)))+A*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/6*x/b/(b*x^
2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+
4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))
```

Maxima [A]
time = 0.28, size = 253, normalized size = 1.70

$$-\frac{Cx^5}{2\,(bx^2+a)^{\frac{7}{2}}b}-\frac{Bx^4}{3\,(bx^2+a)^{\frac{7}{2}}b}-\frac{5\,Cax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{Ax^3}{4\,(bx^2+a)^{\frac{7}{2}}b}-\frac{4\,Bax^2}{15\,(bx^2+a)^{\frac{7}{2}}b^2}+\frac{Cx}{14\,(bx^2+a)^{\frac{3}{2}}b^3}+\frac{Cx}{7\,\sqrt{bx^2+a}\,ab^3}+\frac{3\,Cax}{56\,(bx^2+a)^{\frac{5}{2}}b^3}-\frac{15\,Ca^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3}+\frac{3\,Ax}{140\,(bx^2+a)^{\frac{5}{2}}b^2}+\frac{2\,Ax}{35\,\sqrt{bx^2+a}\,a^2b^2}+\frac{Ax}{35\,(bx^2+a)^{\frac{3}{2}}ab^2}-\frac{3\,Aax}{28\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{8\,Ba^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*C*x^5/((b*x^2 + a)^(7/2)*b) - 1/3*B*x^4/((b*x^2 + a)^(7/2)*b) - 5/8*C*
a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*A*x^3/((b*x^2 + a)^(7/2)*b) - 4/15*B*a*
x^2/((b*x^2 + a)^(7/2)*b^2) + 1/14*C*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*C*x/(s
qrt(b*x^2 + a)*a*b^3) + 3/56*C*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*C*a^2*x/
((b*x^2 + a)^(7/2)*b^3) + 3/140*A*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*A*x/(sqr
t(b*x^2 + a)*a^2*b^2) + 1/35*A*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*A*a*x/((b
*x^2 + a)^(7/2)*b^2) - 8/105*B*a^2/((b*x^2 + a)^(7/2)*b^3)
```

Fricas [A]
time = 4.45, size = 122, normalized size = 0.82

$$\frac{(21\,Aab^3x^5-35\,Ba^2b^2x^4+3\,(5\,Cab^3+2\,Ab^4)x^7-28\,Ba^3bx^2-8\,Ba^4)\sqrt{bx^2+a}}{105\,(a^2b^7x^8+4\,a^3b^6x^6+6\,a^4b^5x^4+4\,a^5b^4x^2+a^6b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(21*A*a*b^3*x^5 - 35*B*a^2*b^2*x^4 + 3*(5*C*a*b^3 + 2*A*b^4)*x^7 - 28
*B*a^3*b*x^2 - 8*B*a^4)*sqrt(b*x^2 + a)/(a^2*b^7*x^8 + 4*a^3*b^6*x^6 + 6*a^
4*b^5*x^4 + 4*a^5*b^4*x^2 + a^6*b^3)
```

Sympy [A]
time = 61.13, size = 575, normalized size = 3.86

$$A\left(\frac{7ax^5}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^7}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right)+B\left(\begin{cases}-\frac{8a^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}}-\frac{28abx^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}}-\frac{35b^2x^4}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} & \text{for}\ b\neq 0\\ \frac{x^6}{6a^{\frac{9}{2}}} & \text{otherwise}\end{cases}\right)+\frac{Cx^7}{7a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}+21a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+21a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+7a^{\frac{3}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 +
 b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x
**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a
**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/
a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + B*Piecewise((-8*a**2/(105
*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*a*b
**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b*x**2)) - 28*a*b*x**2/(
105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*
a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b*x**2)) - 35*b**2*x*
*4/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2) +
315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b*x**2)), Ne(b, 0
)), (x**6/(6*a**(9/2)), True)) + C*x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21
*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2
/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 1.24, size = 81, normalized size = 0.54

$$\frac{\left(\left(3\,x\left(\frac{7\,A}{a}+\frac{\left(5\,Ca^2b^3+2\,Aab^4\right)x^2}{a^3b^3}\right)-\frac{35\,B}{b}\right)x^2-\frac{28\,Ba}{b^2}\right)x^2-\frac{8\,Ba^2}{b^3}}{105\,\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((3*x*(7*A/a + (5*C*a^2*b^3 + 2*A*a*b^4)*x^2/(a^3*b^3)) - 35*B/b)*x^
2 - 28*B*a/b^2)*x^2 - 8*B*a^2/b^3)/(b*x^2 + a)^(7/2)
```

Mupad [B]
time = 1.19, size = 186, normalized size = 1.25

$$\frac{x\left(\frac{C\,a^2-A\,a\,b}{35\,a\,b^3}+\frac{a\,\left(\frac{C}{5\,b^2}-\frac{7\,A\,b^2-7\,C\,a\,b}{35\,a\,b^3}\right)}{b}\right)+\frac{2\,B\,a}{5\,b^3}}{{\left(b\,x^2+a\right)}^{5/2}}-\frac{\frac{B}{3\,b^3}+x\,\left(\frac{C}{3\,b^3}-\frac{3\,A\,b-10\,C\,a}{105\,a\,b^3}\right)}{{\left(b\,x^2+a\right)}^{3/2}}-\frac{\frac{B\,a^2}{7\,b^3}-\frac{a\,x\,\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)}{b}}{{\left(b\,x^2+a\right)}^{7/2}}+\frac{x\,\left(2\,A\,b+5\,C\,a\right)}{35\,a^2\,b^3\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] (x*((C*a^2 - A*a*b)/(35*a*b^3) + (a*(C/(5*b^2) - (7*A*b^2 - 7*C*a*b)/(35*a*
b^3)))/b) + (2*B*a)/(5*b^3))/(a + b*x^2)^(5/2) - (B/(3*b^3) + x*(C/(3*b^3)
- (3*A*b - 10*C*a)/(105*a*b^3)))/(a + b*x^2)^(3/2) - ((B*a^2)/(7*b^3) - (a*
x*(A/(7*b) - (C*a)/(7*b^2)))/b)/(a + b*x^2)^(7/2) + (x*(2*A*b + 5*C*a))/(35
*a^2*b^3*(a + b*x^2)^(1/2))
```

3.51 $\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{x^3(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}}-\frac{x(3aB+(3Ab+4aC)x)}{35ab^2\,(a+bx^2)^{5/2}}-\frac{2(3Ab+4aC)-3bBx}{105ab^3\,(a+bx^2)^{3/2}}+\frac{2Bx}{35a^2b^2\sqrt{a+bx^2}}$$

```
[Out] -1/7*x^3*(a*B-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x*(3*a*B+(3*A*b+4*C*a)*
x)/a/b^2/(b*x^2+a)^(5/2)+1/105*(3*B*b*x-6*A*b-8*C*a)/a/b^3/(b*x^2+a)^(3/2)+
2/35*B*x/a^2/b^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1818, 833, 653, 197}

$$\frac{2Bx}{35a^2b^2\sqrt{a+bx^2}}-\frac{2(4aC+3Ab)-3bBx}{105ab^3\,(a+bx^2)^{3/2}}-\frac{x(x(4aC+3Ab)+3aB)}{35ab^2\,(a+bx^2)^{5/2}}-\frac{x^3(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*(x^3*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) - (x*(3*a*B + (3*A
*b + 4*a*C)*x))/(35*a*b^2*(a + b*x^2)^(5/2)) - (2*(3*A*b + 4*a*C) - 3*b*B*x
)/(105*a*b^3*(a + b*x^2)^(3/2)) + (2*B*x)/(35*a^2*b^2*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
```

```
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx &= -\frac{x^3(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}}-\frac{\int \frac{x^2(-3aB-(3Ab+4aC)x)}{(a+bx^2)^{7/2}}\,dx}{7ab}\\ &= -\frac{x^3(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}}-\frac{x(3aB+(3Ab+4aC)x)}{35ab^2\,(a+bx^2)^{5/2}}-\frac{\int \frac{-3a^2B-2a(3Ab+4aC)x}{(a+bx^2)^{5/2}}\,dx}{35a^2b^2}\\ &= -\frac{x^3(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}}-\frac{x(3aB+(3Ab+4aC)x)}{35ab^2\,(a+bx^2)^{5/2}}-\frac{2(3Ab+4aC)-3bBx}{105ab^3\,(a+bx^2)^{3/2}}+\\ &= -\frac{x^3(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}}-\frac{x(3aB+(3Ab+4aC)x)}{35ab^2\,(a+bx^2)^{5/2}}-\frac{2(3Ab+4aC)-3bBx}{105ab^3\,(a+bx^2)^{3/2}}+\end{aligned}$$

Mathematica [A]
time = 0.90, size = 84, normalized size = 0.60

$$\frac{-8a^4C+21ab^3Bx^5+6b^4Bx^7-7a^2b^2x^2(3A+5Cx^2)-2a^3b(3A+14Cx^2)}{105a^2b^3\,(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-8*a^4*C + 21*a*b^3*B*x^5 + 6*b^4*B*x^7 - 7*a^2*b^2*x^2*(3*A + 5*C*x^2) -
2*a^3*b*(3*A + 14*C*x^2))/(105*a^2*b^3*(a + b*x^2)^(7/2))
```

Maple [A]
time = 0.12, size = 217, normalized size = 1.56

method	result
gosper	$-\frac{-6B\,x^7b^4-21B\,x^5a\,b^3+35C\,a^2b^2x^4+21A\,a^2b^2x^2+28C\,a^3b\,x^2+6A\,a^3b+8C\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^2b^3}$
trager	$-\frac{-6B\,x^7b^4-21B\,x^5a\,b^3+35C\,a^2b^2x^4+21A\,a^2b^2x^2+28C\,a^3b\,x^2+6A\,a^3b+8C\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^2b^3}$
default	$C\left(-\frac{x^4}{3b(b\,x^2+a)^{\frac{7}{2}}}+\frac{4a\left(-\frac{x^2}{5b\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^2\left(b\,x^2+a\right)^{\frac{7}{2}}}\right)}{3b}\right)+B\left(-\frac{x^3}{4b(b\,x^2+a)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^2+a\right)}\right.}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-1/3*x^4/b/(b*x^2+a)^(7/2)+4/3*a/b*(-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^
2/(b*x^2+a)^(7/2)))+B*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/6*x/b/(b*x^2+
a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/
5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))))+A*(-1/5*x^2/b/(
b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2))
```

Maxima [A]

time = 0.30, size = 179, normalized size = 1.29

$$-\frac{Cx^4}{3\,(bx^2+a)^{\frac{7}{2}}b}-\frac{Bx^3}{4\,(bx^2+a)^{\frac{7}{2}}b}-\frac{4\,Cax^2}{15\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{Ax^2}{5\,(bx^2+a)^{\frac{7}{2}}b}+\frac{3\,Bx}{140\,(bx^2+a)^{\frac{5}{2}}b^2}+\frac{2\,Bx}{35\,\sqrt{bx^2+a}\,a^2b^2}+\frac{Bx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2}-\frac{3\,Bax}{28\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{8\,Ca^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3}-\frac{2\,Aa}{35\,(bx^2+a)^{\frac{7}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/3*C*x^4/((b*x^2 + a)^(7/2)*b) - 1/4*B*x^3/((b*x^2 + a)^(7/2)*b) - 4/15*C
*a*x^2/((b*x^2 + a)^(7/2)*b^2) - 1/5*A*x^2/((b*x^2 + a)^(7/2)*b) + 3/140*B*
x/((b*x^2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*B*x/(
(b*x^2 + a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)*b^2) - 8/105*C*a^2
/((b*x^2 + a)^(7/2)*b^3) - 2/35*A*a/((b*x^2 + a)^(7/2)*b^2)
```

Fricas [A]
time = 3.05, size = 131, normalized size = 0.94

$$\frac{\left(6\,Bb^{4}x^{7}+21\,Bab^{3}x^{5}-35\,Ca^{2}b^{2}x^{4}-8\,Ca^{4}-6\,Aa^{3}b-7\,(4\,Ca^{3}b+3\,Aa^{2}b^{2})x^{2}\right)\sqrt{bx^{2}+a}}{105\,(a^{2}b^{7}x^{8}+4\,a^{3}b^{6}x^{6}+6\,a^{4}b^{5}x^{4}+4\,a^{5}b^{4}x^{2}+a^{6}b^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(6*B*b^4*x^7 + 21*B*a*b^3*x^5 - 35*C*a^2*b^2*x^4 - 8*C*a^4 - 6*A*a^3*
b - 7*(4*C*a^3*b + 3*A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)/(a^2*b^7*x^8 + 4*a^3*b
^6*x^6 + 6*a^4*b^5*x^4 + 4*a^5*b^4*x^2 + a^6*b^3)
```

Sympy [A]
time = 50.28, size = 660, normalized size = 4.75

$$A\left(\begin{cases}-\frac{2a}{35a^{3}b^{2}\sqrt{a+bx^{2}}+105a^{2}b^{3}x^{2}\sqrt{a+bx^{2}}+105ab^{4}x^{4}\sqrt{a+bx^{2}}+35b^{5}x^{6}\sqrt{a+bx^{2}}}-\frac{7bx^{2}}{35a^{3}b^{2}\sqrt{a+bx^{2}}+105a^{2}b^{3}x^{2}\sqrt{a+bx^{2}}+105ab^{4}x^{4}\sqrt{a+bx^{2}}+35b^{5}x^{6}\sqrt{a+bx^{2}}} & \text{for}\: b\neq 0\\\frac{x^{4}}{4a^{\frac{9}{2}}} & \text{otherwise}\end{cases}\right)+B\left(\frac{7ax^{5}}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^{2}}{a}}+105a^{\frac{9}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}+105a^{\frac{7}{2}}b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}+35a^{\frac{5}{2}}b^{3}x^{6}\sqrt{1+\frac{bx^{2}}{a}}}+\frac{2bx^{7}}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^{2}}{a}}+105a^{\frac{9}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}+105a^{\frac{7}{2}}b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}+35a^{\frac{5}{2}}b^{3}x^{6}\sqrt{1+\frac{bx^{2}}{a}}}\right)+C\left(\begin{cases}-\frac{8a^{2}}{105a^{3}b^{3}\sqrt{a+bx^{2}}+315a^{2}b^{4}x^{2}\sqrt{a+bx^{2}}+315ab^{5}x^{4}\sqrt{a+bx^{2}}+105b^{6}x^{6}\sqrt{a+bx^{2}}}-\frac{28abx^{2}}{105a^{3}b^{3}\sqrt{a+bx^{2}}+315a^{2}b^{4}x^{2}\sqrt{a+bx^{2}}+315ab^{5}x^{4}\sqrt{a+bx^{2}}+105b^{6}x^{6}\sqrt{a+bx^{2}}}-\frac{35b^{2}x^{4}}{105a^{3}b^{3}\sqrt{a+bx^{2}}+315a^{2}b^{4}x^{2}\sqrt{a+bx^{2}}+315ab^{5}x^{4}\sqrt{a+bx^{2}}+105b^{6}x^{6}\sqrt{a+bx^{2}}} & \text{for}\: b\neq 0\\\frac{x^{6}}{6a^{\frac{9}{2}}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*Piecewise((-2*a/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(
a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x*
*2)) - 7*b*x**2/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a
+ b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2
)), Ne(b, 0)), (x**4/(4*a**(9/2)), True)) + B*(7*a*x**5/(35*a**(11/2)*sqrt(
1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*
x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x
**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/
a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt
(1 + b*x**2/a))) + C*Piecewise((-8*a**2/(105*a**3*b**3*sqrt(a + b*x**2) + 3
15*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105
*b**6*x**6*sqrt(a + b*x**2)) - 28*a*b*x**2/(105*a**3*b**3*sqrt(a + b*x**2)
+ 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) +
105*b**6*x**6*sqrt(a + b*x**2)) - 35*b**2*x**4/(105*a**3*b**3*sqrt(a + b*x*
*2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2
) + 105*b**6*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(9/2)), True))
```

Giac [A]
time = 1.10, size = 95, normalized size = 0.68

$$\frac{\left(\left(3\left(\frac{2Bbx^2}{a^2}+\frac{7B}{a}\right)x-\frac{35C}{b}\right)x^2-\frac{7\left(4Ca^4b+3Aa^3b^2\right)}{a^3b^3}\right)x^2-\frac{2\left(4Ca^5+3Aa^4b\right)}{a^3b^3}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((3*(2*B*b*x^2/a^2 + 7*B/a)*x - 35*C/b)*x^2 - 7*(4*C*a^4*b + 3*A*a^3
*b^2)/(a^3*b^3))*x^2 - 2*(4*C*a^5 + 3*A*a^4*b)/(a^3*b^3))/(b*x^2 + a)^(7/2)
```

Mupad [B]
time = 1.14, size = 133, normalized size = 0.96

$$\frac{\frac{a\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)}{b}+\frac{B\,a\,x}{7\,b^2}}{{\left(b\,x^2+a\right)}^{7/2}}-\frac{\frac{C}{3\,b^3}-\frac{B\,x}{35\,a\,b^2}}{{\left(b\,x^2+a\right)}^{3/2}}+\frac{\frac{a\left(\frac{C}{5\,b^2}-\frac{7\,A\,b-7\,C\,a}{35\,a\,b^2}\right)}{b}-\frac{8\,B\,x}{35\,b^2}}{{\left(b\,x^2+a\right)}^{5/2}}+\frac{2\,B\,x}{35\,a^2\,b^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] ((a*(A/(7*b) - (C*a)/(7*b^2)))/b + (B*a*x)/(7*b^2))/(a + b*x^2)^(7/2) - (C/
(3*b^3) - (B*x)/(35*a*b^2))/(a + b*x^2)^(3/2) + ((a*(C/(5*b^2) - (7*A*b - 7
*C*a)/(35*a*b^2)))/b - (8*B*x)/(35*b^2))/(a + b*x^2)^(5/2) + (2*B*x)/(35*a^
2*b^2*(a + b*x^2)^(1/2))
```

3.52 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{x^2(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{2aB+(4Ab+3aC)x}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{(4Ab+3aC)x}{105a^2b^2\left(a+bx^2\right)^{3/2}}+\frac{2(4Ab+3aC)x}{105a^3b^2\sqrt{a+bx^2}}$$

```
[Out] -1/7*x^2*(a*B-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)+1/35*(-2*a*B-(4*A*b+3*C*a)*x
)/a/b^2/(b*x^2+a)^(5/2)+1/105*(4*A*b+3*C*a)*x/a^2/b^2/(b*x^2+a)^(3/2)+2/105
*(4*A*b+3*C*a)*x/a^3/b^2/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1818, 792, 198, 197}

$$\frac{2x(3aC+4Ab)}{105a^3b^2\sqrt{a+bx^2}}+\frac{x(3aC+4Ab)}{105a^2b^2\left(a+bx^2\right)^{3/2}}-\frac{x(3aC+4Ab)+2aB}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x^2(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*(x^2*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) - (2*a*B + (4*A*b
+ 3*a*C)*x)/(35*a*b^2*(a + b*x^2)^(5/2)) + ((4*A*b + 3*a*C)*x)/(105*a^2*b^2
*(a + b*x^2)^(3/2)) + (2*(4*A*b + 3*a*C)*x)/(105*a^3*b^2*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx &= -\frac{x^2(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{\int \frac{x(-2aB-(4Ab+3aC)x)}{(a+bx^2)^{7/2}}\,dx}{7ab} \\
&= -\frac{x^2(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{2aB+(4Ab+3aC)x}{35ab^2\,(a+bx^2)^{5/2}} + \frac{(4Ab+3aC)\int \frac{1}{(a+bx^2)^{5/2}}\,d}{35ab^2} \\
&= -\frac{x^2(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{2aB+(4Ab+3aC)x}{35ab^2\,(a+bx^2)^{5/2}} + \frac{(4Ab+3aC)x}{105a^2b^2\,(a+bx^2)^{3/2}} + \frac{(2(}{} \\
&= -\frac{x^2(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{2aB+(4Ab+3aC)x}{35ab^2\,(a+bx^2)^{5/2}} + \frac{(4Ab+3aC)x}{105a^2b^2\,(a+bx^2)^{3/2}} + \frac{2}{105}
\end{aligned}
$$

Mathematica [**A**]
time = 0.71, size = 87, normalized size = 0.63

$$\frac{-6a^4B-21a^3bBx^2+8Ab^4x^7+7a^2b^2x^3(5A+3Cx^2)+2ab^3x^5(14A+3Cx^2)}{105a^3b^2\,(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-6*a^4*B - 21*a^3*b*B*x^2 + 8*A*b^4*x^7 + 7*a^2*b^2*x^3*(5*A + 3*C*x^2) +
2*a*b^3*x^5*(14*A + 3*C*x^2))/(105*a^3*b^2*(a + b*x^2)^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(124) = 248$.
time = 0.12, size = 255, normalized size = 1.83

method	result
gosper	$\frac{8A\,b^4x^7+6Ca\,x^7b^3+28Aa\,b^3x^5+21C\,a^2x^5b^2+35A\,a^2b^2x^3-21B\,a^3b\,x^2-6B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^3b^2}$

trager	$\frac{8Ab^4x^7+6Cax^7b^3+28Aab^3x^5+21Ca^2x^5b^2+35Aa^2b^2x^3-21Ba^3bx^2-6Ba^4}{105(bx^2+a)^{\frac{7}{2}}a^3b^2}$
default	$C\left(-\frac{x^3}{4b(bx^2+a)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b(bx^2+a)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a(bx^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a(bx^2+a)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a(bx^2+a)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{bx^2+a}}\right)}{7a}}{a}\right)}{6b}\right)}{4b}\right)+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/
7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+
a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))))+B*(-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*
a/b^2/(b*x^2+a)^(7/2))+A*(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+
a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*
x/a^2/(b*x^2+a)^(1/2)))))
```

Maxima [A]

time = 0.30, size = 197, normalized size = 1.42

$$-\frac{Cx^3}{4\,(bx^2+a)^{\frac{7}{2}}b}-\frac{Bx^2}{5\,(bx^2+a)^{\frac{7}{2}}b}+\frac{3\,Cx}{140\,(bx^2+a)^{\frac{5}{2}}b^2}+\frac{2\,Cx}{35\,\sqrt{bx^2+a}\,a^2b^2}+\frac{Cx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2}-\frac{3\,Cax}{28\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}b}+\frac{8\,Ax}{105\,\sqrt{bx^2+a}\,a^3b}+\frac{4\,Ax}{105\,(bx^2+a)^{\frac{3}{2}}a^2b}+\frac{Ax}{35\,(bx^2+a)^{\frac{5}{2}}ab}-\frac{2\,Ba}{35\,(bx^2+a)^{\frac{7}{2}}b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*C*x^3/((b*x^2 + a)^(7/2)*b) - 1/5*B*x^2/((b*x^2 + a)^(7/2)*b) + 3/140*
C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*C*x
/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*A*x/(
(b*x^2 + a)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((b*x^
2 + a)^(3/2)*a^2*b) + 1/35*A*x/((b*x^2 + a)^(5/2)*a*b) - 2/35*B*a/((b*x^2 +
 a)^(7/2)*b^2)
```

Fricas [A]
time = 3.74, size = 134, normalized size = 0.96

$$\frac{\left(35\,Aa^{2}b^{2}x^{3}+2\,(3\,Cab^{3}+4\,Ab^{4})x^{7}-21\,Ba^{3}bx^{2}+7\,(3\,Ca^{2}b^{2}+4\,Aab^{3})x^{5}-6\,Ba^{4}\right)\sqrt{bx^{2}+a}}{105\,(a^{3}b^{6}x^{8}+4\,a^{4}b^{5}x^{6}+6\,a^{5}b^{4}x^{4}+4\,a^{6}b^{3}x^{2}+a^{7}b^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(35*A*a^2*b^2*x^3 + 2*(3*C*a*b^3 + 4*A*b^4)*x^7 - 21*B*a^3*b*x^2 + 7*
(3*C*a^2*b^2 + 4*A*a*b^3)*x^5 - 6*B*a^4)*sqrt(b*x^2 + a)/(a^3*b^6*x^8 + 4*a
^4*b^5*x^6 + 6*a^5*b^4*x^4 + 4*a^6*b^3*x^2 + a^7*b^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(129) = 258$.
time = 33.53, size = 904, normalized size = 6.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sq
rt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/
2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a
)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**
2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**
(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x*
*2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2
)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a*
*(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2
/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*
sqrt(1 + b*x**2/a))) + B*Piecewise((-2*a/(35*a**3*b**2*sqrt(a + b*x**2) + 1
05*a**2*b**3*x**2*sqrt(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*
b**5*x**6*sqrt(a + b*x**2)) - 7*b*x**2/(35*a**3*b**2*sqrt(a + b*x**2) + 105
*a**2*b**3*x**2*sqrt(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b*
*5*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(9/2)), True)) + C*(7*a*x
```

```
**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/
a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt
(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*
b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*
a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)))
```

Giac [A]
time = 1.05, size = 94, normalized size = 0.68

$$\frac{\left(\left(x^2\left(\frac{2\left(3\,Cab^4+4\,Ab^5\right)x^2}{a^3b^3}+\frac{7\left(3\,Ca^2b^3+4\,Aab^4\right)}{a^3b^3}\right)+\frac{35\,A}{a}\right)x-\frac{21\,B}{b}\right)x^2-\frac{6\,Ba}{b^2}}{105\left(bx^2+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((x^2*(2*(3*C*a*b^4 + 4*A*b^5)*x^2/(a^3*b^3) + 7*(3*C*a^2*b^3 + 4*A*
a*b^4)/(a^3*b^3)) + 35*A/a)*x - 21*B/b)*x^2 - 6*B*a/b^2)/(b*x^2 + a)^(7/2)
```

Mupad [B]
time = 1.09, size = 133, normalized size = 0.96

$$\frac{x\,\left(4\,A\,b+3\,C\,a\right)}{105\,a^2\,b^2\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{\frac{B}{5\,b^2}+x\,\left(\frac{C}{5\,b^2}-\frac{A\,b-C\,a}{35\,a\,b^2}\right)}{{\left(b\,x^2+a\right)}^{5/2}}-\frac{x\,\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)-\frac{B\,a}{7\,b^2}}{{\left(b\,x^2+a\right)}^{7/2}}+\frac{x\,\left(8\,A\,b+6\,C\,a\right)}{105\,a^3\,b^2\,\sqrt{b\,x^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] (x*(4*A*b + 3*C*a))/(105*a^2*b^2*(a + b*x^2)^(3/2)) - (B/(5*b^2) + x*(C/(5*
b^2) - (A*b - C*a)/(35*a*b^2)))/(a + b*x^2)^(5/2) - (x*(A/(7*b) - (C*a)/(7*
b^2)) - (B*a)/(7*b^2))/(a + b*x^2)^(7/2) + (x*(8*A*b + 6*C*a))/(105*a^3*b^2
*(a + b*x^2)^(1/2))
```

3.53 $\int \frac{x\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{x(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}}-\frac{5Ab+2aC-bBx}{35ab^2\left(a+bx^2\right)^{5/2}}+\frac{4Bx}{105a^2b\left(a+bx^2\right)^{3/2}}+\frac{8Bx}{105a^3b\sqrt{a+bx^2}}$$

```
[Out] -1/7*x*(a*B-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)+1/35*(B*b*x-5*A*b-2*C*a)/a/b^2
/(b*x^2+a)^(5/2)+4/105*B*x/a^2/b/(b*x^2+a)^(3/2)+8/105*B*x/a^3/b/(b*x^2+a)^
(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1818, 653, 198, 197}

$$\frac{8Bx}{105a^3b\sqrt{a+bx^2}}+\frac{4Bx}{105a^2b\left(a+bx^2\right)^{3/2}}-\frac{2aC+5Ab-bBx}{35ab^2\left(a+bx^2\right)^{5/2}}-\frac{x(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*(x*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) - (5*A*b + 2*a*C - b
*B*x)/(35*a*b^2*(a + b*x^2)^(5/2)) + (4*B*x)/(105*a^2*b*(a + b*x^2)^(3/2))
+ (8*B*x)/(105*a^3*b*Sqrt[a + b*x^2])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx &= -\frac{x(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}}-\frac{\int \frac{-aB-(5Ab+2aC)x}{(a+bx^2)^{7/2}}\,dx}{7ab}\\
&= -\frac{x(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}}-\frac{5Ab+2aC-bBx}{35ab^2\,(a+bx^2)^{5/2}}+\frac{(4B)\int \frac{1}{(a+bx^2)^{5/2}}\,dx}{35ab}\\
&= -\frac{x(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}}-\frac{5Ab+2aC-bBx}{35ab^2\,(a+bx^2)^{5/2}}+\frac{4Bx}{105a^2b\,(a+bx^2)^{3/2}}+\frac{(8B)\int \frac{}{(a+}}{105a}\\
&= -\frac{x(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}}-\frac{5Ab+2aC-bBx}{35ab^2\,(a+bx^2)^{5/2}}+\frac{4Bx}{105a^2b\,(a+bx^2)^{3/2}}+\frac{8Bx}{105a^3b\sqrt{a}}
\end{aligned}$$

Mathematica [A]
time = 0.82, size = 76, normalized size = 0.64

$$\frac{-15a^3Ab-6a^4C-21a^3bCx^2+35a^2b^2Bx^3+28ab^3Bx^5+8b^4Bx^7}{105a^3b^2\,(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-15*a^3*A*b - 6*a^4*C - 21*a^3*b*C*x^2 + 35*a^2*b^2*B*x^3 + 28*a*b^3*B*x^5
 + 8*b^4*B*x^7)/(105*a^3*b^2*(a + b*x^2)^(7/2))
```

Maple [A]
time = 0.12, size = 149, normalized size = 1.25

method	result
gosper	$-\frac{-8B\,x^7b^4-28B\,x^5a\,b^3-35B\,x^3a^2b^2+21C\,a^3b\,x^2+15A\,a^3b+6C\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^3b^2}$

trager	$-\frac{-8B\,x^7 b^4-28B\,x^5 a\,b^3-35B\,x^3 a^2 b^2+21C\,a^3 b\,x^2+15A\,a^3 b+6C\,a^4}{105\left(b\,x^2+a\right)^{\frac{7}{2}} a^3 b^2}$
default	$C\left(-\frac{x^2}{5b\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^2\left(b\,x^2+a\right)^{\frac{7}{2}}}\right)+B\left(-\frac{x}{6b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{}{15a^2}\right)}{7a}}{a}\right.}{6b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2))+B*(-1/6*x/b/(b*x^
2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+
4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))-1/7*A/b/(b*x^2
+a)^(7/2)
```

Maxima [A]
time = 0.28, size = 123, normalized size = 1.03

$$-\frac{Cx^2}{5\,(bx^2+a)^{\frac{7}{2}}b}-\frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}b}+\frac{8\,Bx}{105\,\sqrt{bx^2+a}\,a^3b}+\frac{4\,Bx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b}+\frac{Bx}{35\,(bx^2+a)^{\frac{5}{2}}ab}-\frac{2\,Ca}{35\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{A}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/5*C*x^2/((b*x^2 + a)^(7/2)*b) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8/105*B*
x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*B*x/
((b*x^2 + a)^(5/2)*a*b) - 2/35*C*a/((b*x^2 + a)^(7/2)*b^2) - 1/7*A/((b*x^2
+ a)^(7/2)*b)
```

Fricas [A]
time = 3.22, size = 119, normalized size = 1.00

$$\frac{\left(8\,Bb^4x^7+28\,Bab^3x^5+35\,Ba^2b^2x^3-21\,Ca^3bx^2-6\,Ca^4-15\,Aa^3b\right)\sqrt{bx^2+a}}{105\,\left(a^3b^6x^8+4\,a^4b^5x^6+6\,a^5b^4x^4+4\,a^6b^3x^2+a^7b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*B*b^4*x^7 + 28*B*a*b^3*x^5 + 35*B*a^2*b^2*x^3 - 21*C*a^3*b*x^2 - 6
*C*a^4 - 15*A*a^3*b)*sqrt(b*x^2 + a)/(a^3*b^6*x^8 + 4*a^4*b^5*x^6 + 6*a^5*b
^4*x^4 + 4*a^6*b^3*x^2 + a^7*b^2)
```

Sympy [A]
time = 22.65, size = 796, normalized size = 6.69

$$A\left(\begin{cases} -\frac{1}{7a^{3}b\sqrt{a+bx^{2}}+21a^{2}b^{2}x^{2}\sqrt{a+bx^{2}}+21ab^{3}x^{4}\sqrt{a+bx^{2}}+7b^{4}x^{6}\sqrt{a+bx^{2}}} & \text{for}\ b \neq 0 \\ \frac{x^{2}}{2a^{\frac{9}{2}}} & \text{otherwise} \end{cases}\right) + B\left(\frac{35a^{5}x^{3}}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^{2}}{a}}+420a^{\frac{17}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}+630a^{\frac{15}{2}}b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}+420a^{\frac{13}{2}}b^{3}x^{6}\sqrt{1+\frac{bx^{2}}{a}}+105a^{\frac{11}{2}}b^{4}x^{8}\sqrt{1+\frac{bx^{2}}{a}}} + \frac{63a^{4}bx^{5}}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^{2}}{a}}+420a^{\frac{17}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}+630a^{\frac{15}{2}}b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}+420a^{\frac{13}{2}}b^{3}x^{6}\sqrt{1+\frac{bx^{2}}{a}}+105a^{\frac{11}{2}}b^{4}x^{8}\sqrt{1+\frac{bx^{2}}{a}}} + \frac{36a^{3}b^{2}x^{7}}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^{2}}{a}}+420a^{\frac{17}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}+630a^{\frac{15}{2}}b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}+420a^{\frac{13}{2}}b^{3}x^{6}\sqrt{1+\frac{bx^{2}}{a}}+105a^{\frac{11}{2}}b^{4}x^{8}\sqrt{1+\frac{bx^{2}}{a}}} + \frac{8a^{2}b^{3}x^{9}}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^{2}}{a}}+420a^{\frac{17}{2}}bx^{2}\sqrt{1+\frac{bx^{2}}{a}}+630a^{\frac{15}{2}}b^{2}x^{4}\sqrt{1+\frac{bx^{2}}{a}}+420a^{\frac{13}{2}}b^{3}x^{6}\sqrt{1+\frac{bx^{2}}{a}}+105a^{\frac{11}{2}}b^{4}x^{8}\sqrt{1+\frac{bx^{2}}{a}}}\right) + C\left(\begin{cases} -\frac{2a}{35a^{3}b^{2}\sqrt{a+bx^{2}}+105a^{2}b^{3}x^{2}\sqrt{a+bx^{2}}+105ab^{4}x^{4}\sqrt{a+bx^{2}}+35b^{5}x^{6}\sqrt{a+bx^{2}}} - \frac{7bx^{2}}{35a^{3}b^{2}\sqrt{a+bx^{2}}+105a^{2}b^{3}x^{2}\sqrt{a+bx^{2}}+105ab^{4}x^{4}\sqrt{a+bx^{2}}+35b^{5}x^{6}\sqrt{a+bx^{2}}} & \text{for}\ b \neq 0 \\ \frac{x^{4}}{4a^{\frac{9}{2}}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqrt(a + b*x
**2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x**2)), Ne(
b, 0)), (x**2/(2*a**(9/2)), True)) + B*(35*a**5*x**3/(105*a**(19/2)*sqrt(1
+ b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*
x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*
a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 63*a**4*b*x**5/(105*a**(19/2)*sqr
t(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b
**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 36*a**3*b**2*x**7/(105*a**(19
/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(
15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**
2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*
a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 63
0*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 +
 b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a))) + C*Piecewise((-2
*a/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a + b*x**2) + 1
05*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2)) - 7*b*x**2
/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt(a + b*x**2) + 105
*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2)), Ne(b, 0)),
(x**4/(4*a**(9/2)), True))
```

Giac [A]
time = 0.94, size = 82, normalized size = 0.69

$$\frac{\left(\left(4\left(\frac{2\,Bb^{2}x^{2}}{a^{3}}+\frac{7\,Bb}{a^{2}}\right)x^{2}+\frac{35\,B}{a}\right)x-\frac{21\,C}{b}\right)x^{2}-\frac{3\,\left(2\,Ca^{4}b+5\,Aa^{3}b^{2}\right)}{a^{3}b^{3}}}{105\,\left(bx^{2}+a\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*(2*B*b^2*x^2/a^3 + 7*B*b/a^2)*x^2 + 35*B/a)*x - 21*C/b)*x^2 - 3*
(2*C*a^4*b + 5*A*a^3*b^2)/(a^3*b^3))/(b*x^2 + a)^(7/2)
```

Mupad [B]
time = 1.05, size = 99, normalized size = 0.83

$$\frac{8\,B\,x}{105\,a^3\,b\,\sqrt{b\,x^2+a}}-\frac{\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}+\frac{B\,x}{7\,b}}{{\left(b\,x^2+a\right)}^{7/2}}-\frac{\frac{C}{5\,b^2}-\frac{B\,x}{35\,a\,b}}{{\left(b\,x^2+a\right)}^{5/2}}+\frac{4\,B\,x}{105\,a^2\,b\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

```
[Out] (8*B*x)/(105*a^3*b*(a + b*x^2)^(1/2)) - (A/(7*b) - (C*a)/(7*b^2) + (B*x)/(7
*b))/(a + b*x^2)^(7/2) - (C/(5*b^2) - (B*x)/(35*a*b))/(a + b*x^2)^(5/2) + (
4*B*x)/(105*a^2*b*(a + b*x^2)^(3/2))
```

3.54 $\int \frac{A+Bx+Cx^2}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=127

$$-\frac{aB-(Ab-aC)x}{7ab\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aC)x}{35a^2b\left(a+bx^2\right)^{5/2}}+\frac{4(6Ab+aC)x}{105a^3b\left(a+bx^2\right)^{3/2}}+\frac{8(6Ab+aC)x}{105a^4b\sqrt{a+bx^2}}$$

```
[Out] 1/7*(-a*B+(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)+1/35*(6*A*b+C*a)*x/a^2/b/(b*x^2+
a)^(5/2)+4/105*(6*A*b+C*a)*x/a^3/b/(b*x^2+a)^(3/2)+8/105*(6*A*b+C*a)*x/a^4/
b/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1828, 12, 198, 197}

$$\frac{8x(aC+6Ab)}{105a^4b\sqrt{a+bx^2}}+\frac{4x(aC+6Ab)}{105a^3b\left(a+bx^2\right)^{3/2}}+\frac{x(aC+6Ab)}{35a^2b\left(a+bx^2\right)^{5/2}}-\frac{aB-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```

```
[Out] -1/7*(a*B - (A*b - a*C)*x)/(a*b*(a + b*x^2)^(7/2)) + ((6*A*b + a*C)*x)/(35*
a^2*b*(a + b*x^2)^(5/2)) + (4*(6*A*b + a*C)*x)/(105*a^3*b*(a + b*x^2)^(3/2)
) + (8*(6*A*b + a*C)*x)/(105*a^4*b*Sqrt[a + b*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx &= -\frac{aB-(Ab-aC)x}{7ab\,(a+bx^2)^{7/2}}-\frac{\int \frac{-6A-\frac{aC}{b}}{(a+bx^2)^{7/2}}\,dx}{7a}\\
&= -\frac{aB-(Ab-aC)x}{7ab\,(a+bx^2)^{7/2}}+\frac{(6Ab+aC)\int \frac{1}{(a+bx^2)^{7/2}}\,dx}{7ab}\\
&= -\frac{aB-(Ab-aC)x}{7ab\,(a+bx^2)^{7/2}}+\frac{(6Ab+aC)x}{35a^2b\,(a+bx^2)^{5/2}}+\frac{(4(6Ab+aC))\int \frac{1}{(a+bx^2)^{5/2}}\,dx}{35a^2b}\\
&= -\frac{aB-(Ab-aC)x}{7ab\,(a+bx^2)^{7/2}}+\frac{(6Ab+aC)x}{35a^2b\,(a+bx^2)^{5/2}}+\frac{4(6Ab+aC)x}{105a^3b\,(a+bx^2)^{3/2}}+\frac{(8(6Ab+aC))}{105a}\\
&= -\frac{aB-(Ab-aC)x}{7ab\,(a+bx^2)^{7/2}}+\frac{(6Ab+aC)x}{35a^2b\,(a+bx^2)^{5/2}}+\frac{4(6Ab+aC)x}{105a^3b\,(a+bx^2)^{3/2}}+\frac{8(6Ab+aC)x}{105a^4b\sqrt{a+bx}}
\end{aligned}$$

Mathematica [A]
time = 0.64, size = 92, normalized size = 0.72

$$\frac{-15a^4B+48Ab^4x^7+35a^3bx(3A+Cx^2)+8ab^3x^5(21A+Cx^2)+14a^2b^2x^3(15A+2Cx^2)}{105a^4b\,(a+bx^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```

```
[Out] (-15*a^4*B + 48*A*b^4*x^7 + 35*a^3*b*x*(3*A + C*x^2) + 8*a*b^3*x^5*(21*A +
C*x^2) + 14*a^2*b^2*x^3*(15*A + 2*C*x^2))/(105*a^4*b*(a + b*x^2)^(7/2))
```

Maple [A]
time = 0.12, size = 189, normalized size = 1.49

method	result
gosper	$\frac{48A\,b^4x^7+8Ca\,x^7b^3+168Aa\,b^3x^5+28C\,a^2x^5b^2+210A\,a^2b^2x^3+35C\,a^3x^3b+105A\,a^3bx-15B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}\,a^4b}$

trager	$\frac{48A\,b^4x^7+8C a\,x^7b^3+168Aa\,b^3x^5+28C\,a^2x^5b^2+210A\,a^2b^2x^3+35C\,a^3x^3b+105A\,a^3bx-15B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^4b}$
default	$C\left(-\frac{x}{6b(b\,x^2+a)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)}{6b}\right)-\frac{B}{7b(b\,x^2+a)^{\frac{7}{2}}}+A\left(\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a
/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))
))-1/7*B/b/(b*x^2+a)^(7/2)+A*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2
+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))
```

Maxima [A]
time = 0.28, size = 153, normalized size = 1.20

$$\frac{16\,Ax}{35\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Ax}{35\,(bx^2+a)^{\frac{3}{2}}a^3}+\frac{6\,Ax}{35\,(bx^2+a)^{\frac{5}{2}}a^2}+\frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}a}-\frac{Cx}{7\,(bx^2+a)^{\frac{7}{2}}b}+\frac{8\,Cx}{105\,\sqrt{bx^2+a}\,a^3b}+\frac{4\,Cx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b}+\frac{Cx}{35\,(bx^2+a)^{\frac{5}{2}}ab}-\frac{B}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*A
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^2 + a)^(7/2)*a) - 1/7*C*x/((b*x^
2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*C*x/((b*x^2 + a
)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5/2)*a*b) - 1/7*B/((b*x^2 + a)^(7/2
)*b)
```

Fricas [A]
time = 3.20, size = 137, normalized size = 1.08

$$\frac{(8\,(Cab^3+6\,Ab^4)x^7+105\,Aa^3bx+28\,(Ca^2b^2+6\,Aab^3)x^5-15\,Ba^4+35\,(Ca^3b+6\,Aa^2b^2)x^3)\sqrt{bx^2+a}}{105\,(a^4b^5x^8+4\,a^5b^4x^6+6\,a^6b^3x^4+4\,a^7b^2x^2+a^8b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(C*a*b^3 + 6*A*b^4)*x^7 + 105*A*a^3*b*x + 28*(C*a^2*b^2 + 6*A*a*b^
3)*x^5 - 15*B*a^4 + 35*(C*a^3*b + 6*A*a^2*b^2)*x^3)*sqrt(b*x^2 + a)/(a^4*b^
5*x^8 + 4*a^5*b^4*x^6 + 6*a^6*b^3*x^4 + 4*a^7*b^2*x^2 + a^8*b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. $2(117) = 234$.
time = 26.84, size = 1880, normalized size = 14.80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(
1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*
b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) +
 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt
(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a
**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**
2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8
*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(
25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sq
rt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*
b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt
(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b*
*3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*
x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x
**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a
**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b
*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(
35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a
) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sq
rt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/
2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(
1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 52
5*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1
+ b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x*
*13/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2
/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*
sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(2
7/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**
2/a))) + B*Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqr
```

```
t(a + b*x**2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x*
*2)), Ne(b, 0)), (x**2/(2*a**(9/2)), True)) + C*(35*a**5*x**3/(105*a**(19/2
)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15
/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/
a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 63*a**4*b*x**5/(105*a**(
19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a*
*(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x
**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 36*a**3*b**2*x**7/(1
05*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) +
 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(
1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x
**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**
2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6
*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)))
```

Giac [A]

time = 0.99, size = 112, normalized size = 0.88

$$\frac{\left(\left(4\,x^{2}\left(\frac{2\,(Cab^{5}+6\,Ab^{6})x^{2}}{a^{4}b^{3}}+\frac{7\,(Ca^{2}b^{4}+6\,Aab^{5})}{a^{4}b^{3}}\right)+\frac{35\,(Ca^{3}b^{3}+6\,Aa^{2}b^{4})}{a^{4}b^{3}}\right)x^{2}+\frac{105\,A}{a}\right)x-\frac{15\,B}{b}}{105\,(bx^{2}+a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((4*x^2*(2*(C*a*b^5 + 6*A*b^6)*x^2/(a^4*b^3) + 7*(C*a^2*b^4 + 6*A*a*
b^5)/(a^4*b^3)) + 35*(C*a^3*b^3 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*x
- 15*B/b)/(b*x^2 + a)^(7/2)
```

Mupad [B]

time = 1.03, size = 115, normalized size = 0.91

$$\frac{x\,(6\,A\,b+C\,a)}{35\,a^{2}\,b\,(b\,x^{2}+a)^{5/2}}-\frac{\frac{B}{7\,b}-x\left(\frac{A}{7\,a}-\frac{C}{7\,b}\right)}{(b\,x^{2}+a)^{7/2}}+\frac{x\,(24\,A\,b+4\,C\,a)}{105\,a^{3}\,b\,(b\,x^{2}+a)^{3/2}}+\frac{x\,(48\,A\,b+8\,C\,a)}{105\,a^{4}\,b\,\sqrt{b\,x^{2}+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + b*x^2)^(9/2),x)
```

```
[Out] (x*(6*A*b + C*a))/(35*a^2*b*(a + b*x^2)^(5/2)) - (B/(7*b) - x*(A/(7*a) - C/
(7*b)))/(a + b*x^2)^(7/2) + (x*(24*A*b + 4*C*a))/(105*a^3*b*(a + b*x^2)^(3/
2)) + (x*(48*A*b + 8*C*a))/(105*a^4*b*(a + b*x^2)^(1/2))
```

3.55 $\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=138

$$\frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}}+\frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{35A+16Bx}{35a^4\sqrt{a+bx^2}}-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

```
[Out] 1/7*(B*b*x+A*b-C*a)/a/b/(b*x^2+a)^(7/2)+1/35*(6*B*x+7*A)/a^2/(b*x^2+a)^(5/2
)+1/105*(24*B*x+35*A)/a^3/(b*x^2+a)^(3/2)-A*arctanh((b*x^2+a)^(1/2)/a^(1/2)
)/a^(9/2)+1/35*(16*B*x+35*A)/a^4/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1819, 837, 12, 272, 65, 214}

$$-\frac{A\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{35A+16Bx}{35a^4\sqrt{a+bx^2}}+\frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] (A*b - a*C + b*B*x)/(7*a*b*(a + b*x^2)^(7/2)) + (7*A + 6*B*x)/(35*a^2*(a +
b*x^2)^(5/2)) + (35*A + 24*B*x)/(105*a^3*(a + b*x^2)^(3/2)) + (35*A + 16*B*
x)/(35*a^4*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx &= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-7A-6Bx}{x(a+bx^2)^{7/2}}\,dx}{7a} \\
&= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{35aAb+24abBx}{x(a+bx^2)^{5/2}}\,dx}{35a^3b} \\
&= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} - \frac{\int \frac{-105a^2Ab^2-48a^2b^2Bx}{x(a+bx^2)^{3/2}}\,d}{105a^5b^2} \\
&= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \\
&= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \\
&= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \\
&= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} + \\
&= \frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}} + \frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}} + \frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}} + \frac{35A+16Bx}{35a^4\sqrt{a+bx^2}} -
\end{aligned}$$

Mathematica [A]
time = 0.97, size = 130, normalized size = 0.94

$$\frac{-15a^4C+14ab^3x^4(25A+12Bx)+14a^2b^2x^2(29A+15Bx)+3b^4x^6(35A+16Bx)+a^3b(176A+105Bx)}{105a^4b\left(a+bx^2\right)^{7/2}}+\frac{2A\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-15*a^4*C + 14*a*b^3*x^4*(25*A + 12*B*x) + 14*a^2*b^2*x^2*(29*A + 15*B*x)
+ 3*b^4*x^6*(35*A + 16*B*x) + a^3*b*(176*A + 105*B*x))/(105*a^4*b*(a + b*x^
2)^(7/2)) + (2*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(9/2)
```

Maple [A]
time = 0.12, size = 193, normalized size = 1.40

method	result
default	$-\frac{C}{7b(b\,x^{2}+a)^{\frac{7}{2}}}+B\left(\frac{x}{7a(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}\right)}{7a}}{a}\right)+A\left(\frac{1}{7a(b\,x^{2}+a)^{\frac{7}{2}}}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*C/b/(b*x^2+a)^(7/2)+B*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a
)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))+A*(1/7/
a/(b*x^2+a)^(7/2)+1/a*(1/5/a/(b*x^2+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(3/2)+1/a
*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))
```

Maxima [A]

time = 0.32, size = 157, normalized size = 1.14

$$\frac{16\,Bx}{35\,\sqrt{bx^{2}+a}\,a^{4}}+\frac{8\,Bx}{35\,(bx^{2}+a)^{\frac{3}{2}}a^{3}}+\frac{6\,Bx}{35\,(bx^{2}+a)^{\frac{5}{2}}a^{2}}+\frac{Bx}{7\,(bx^{2}+a)^{\frac{7}{2}}a}-\frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{9}{2}}}+\frac{A}{\sqrt{bx^{2}+a}\,a^{4}}+\frac{A}{3\,(bx^{2}+a)^{\frac{3}{2}}a^{3}}+\frac{A}{5\,(bx^{2}+a)^{\frac{5}{2}}a^{2}}+\frac{A}{7\,(bx^{2}+a)^{\frac{7}{2}}a}-\frac{C}{7\,(bx^{2}+a)^{\frac{7}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*B*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*B
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/((b*x^2 + a)^(7/2)*a) - A*arcsinh(a/(s
qrt(a*b)*abs(x)))/a^(9/2) + A/(sqrt(b*x^2 + a)*a^4) + 1/3*A/((b*x^2 + a)^(3
/2)*a^3) + 1/5*A/((b*x^2 + a)^(5/2)*a^2) + 1/7*A/((b*x^2 + a)^(7/2)*a) - 1/
7*C/((b*x^2 + a)^(7/2)*b)
```

Fricas [A]

time = 2.30, size = 465, normalized size = 3.37

$$\left[\frac{105\,(Ab^{5}x^{8}+4\,Aab^{4}x^{6}+6\,Aa^{2}b^{3}x^{4}+4\,Aa^{3}b^{2}x^{2}+Aa^{4}b)\sqrt{a}\,\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,(48\,Bab^{4}x^{7}+105\,Aab^{4}x^{6}+168\,Ba^{2}b^{3}x^{5}+350\,Aa^{2}b^{3}x^{4}+210\,Ba^{3}b^{2}x^{3}+406\,Aa^{3}b^{2}x^{2}+105\,Ba^{4}bx-15\,Ca^{5}+176\,Aa^{4}b)\sqrt{bx^{2}+a}}{210\,(a^{5}b^{5}x^{8}+4\,a^{6}b^{4}x^{6}+6\,a^{7}b^{3}x^{4}+4\,a^{8}b^{2}x^{2}+a^{9}b)},\ \frac{105\,(Ab^{5}x^{8}+4\,Aab^{4}x^{6}+6\,Aa^{2}b^{3}x^{4}+4\,Aa^{3}b^{2}x^{2}+Aa^{4}b)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(48\,Bab^{4}x^{7}+105\,Aab^{4}x^{6}+168\,Ba^{2}b^{3}x^{5}+350\,Aa^{2}b^{3}x^{4}+210\,Ba^{3}b^{2}x^{3}+406\,Aa^{3}b^{2}x^{2}+105\,Ba^{4}bx-15\,Ca^{5}+176\,Aa^{4}b)\sqrt{bx^{2}+a}}{105\,(a^{5}b^{5}x^{8}+4\,a^{6}b^{4}x^{6}+6\,a^{7}b^{3}x^{4}+4\,a^{8}b^{2}x^{2}+a^{9}b)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(A*b^5*x^8 + 4*A*a*b^4*x^6 + 6*A*a^2*b^3*x^4 + 4*A*a^3*b^2*x^2
+ A*a^4*b)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*
```

```
(48*B*a*b^4*x^7 + 105*A*a*b^4*x^6 + 168*B*a^2*b^3*x^5 + 350*A*a^2*b^3*x^4 +
 210*B*a^3*b^2*x^3 + 406*A*a^3*b^2*x^2 + 105*B*a^4*b*x - 15*C*a^5 + 176*A*a
^4*b)*sqrt(b*x^2 + a))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 + 4*a^8
*b^2*x^2 + a^9*b), 1/105*(105*(A*b^5*x^8 + 4*A*a*b^4*x^6 + 6*A*a^2*b^3*x^4
+ 4*A*a^3*b^2*x^2 + A*a^4*b)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (4
8*B*a*b^4*x^7 + 105*A*a*b^4*x^6 + 168*B*a^2*b^3*x^5 + 350*A*a^2*b^3*x^4 + 2
10*B*a^3*b^2*x^3 + 406*A*a^3*b^2*x^2 + 105*B*a^4*b*x - 15*C*a^5 + 176*A*a^4
*b)*sqrt(b*x^2 + a))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 + 4*a^8*b
^2*x^2 + a^9*b)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 5251 vs. $2(122) = 244$.
time = 36.95, size = 6613, normalized size = 47.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(352*a**32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 94
50*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x
**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(5
9/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 2
10*a**(53/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) + 2100*a
**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44
100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6
*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(
55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqrt(1 + b*x*
*2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**
4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)
*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 945
0*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x*
*20) + 2924*a**31*b*x**2*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)
*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(
65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 +
 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b*
*9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**31*b*x**2*log(b*x**2/a)/(21
0*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(
67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 +
44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b*
*8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 2100*a*
*31*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x*
*2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)
*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2520
0*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x*
```

```
*18 + 210*a**(53/2)*b**10*x**20) + 10852*a**30*b**2*x**4*sqrt(1 + b*x**2/a)
/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*
a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**1
0 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2
)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 472
5*a**30*b**2*x**4*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 94
50*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x
**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(5
9/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 2
10*a**(53/2)*b**10*x**20) - 9450*a**30*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1
)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200
*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**
10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/
2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 23
630*a**29*b**3*x**6*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x*
*2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)
*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2520
0*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x*
*18 + 210*a**(53/2)*b**10*x**20) + 12600*a**29*b**3*x**6*log(b*x**2/a)/(210
*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(6
7/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 4
4100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**
8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 25200*a*
*29*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b
*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65
/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 2
5200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9
*x**18 + 210*a**(53/2)*b**10*x**20) + 33280*a**28*b**4*x**8*sqrt(1 + b*x**2
/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 252
00*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x
**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(5
7/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) +
22050*a**28*b**4*x**8*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2
+ 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b*
*4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a
**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
 + 210*a**(53/2)*b**10*x**20) - 44100*a**28*b**4*x**8*log(sqrt(1 + b*x**2/a
) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 +
25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**
5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a*
*(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 ...
```

Giac [A]

time = 1.68, size = 152, normalized size = 1.10

$$\frac{((((3((\frac{16Bb^3x}{a^4}+\frac{35Ab^3}{a^4})x+\frac{56Bb^2}{a^3})x+\frac{350Ab^2}{a^3})x+\frac{210Bb}{a^2})x+\frac{406Ab}{a^2})x+\frac{105B}{a})x-\frac{15Ca^{14}b^2-176Aa^{13}b^3}{a^{14}b^3}}{105\,(bx^2+a)^{\frac{7}{2}}}+\frac{2A\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((((((3*((16*B*b^3*x/a^4 + 35*A*b^3/a^4)*x + 56*B*b^2/a^3)*x + 350*A*b
^2/a^3)*x + 210*B*b/a^2)*x + 406*A*b/a^2)*x + 105*B/a)*x - (15*C*a^14*b^2 -
 176*A*a^13*b^3)/(a^14*b^3))/(b*x^2 + a)^(7/2) + 2*A*arctan(-(sqrt(b)*x - s
qrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4)
```

Mupad [B]

time = 1.62, size = 159, normalized size = 1.15

$$\frac{\frac{A}{7a}+\frac{A\,(b\,x^2+a)^2}{3\,a^3}+\frac{A\,(b\,x^2+a)^3}{a^4}+\frac{A\,(b\,x^2+a)}{5\,a^2}}{(b\,x^2+a)^{7/2}}-\frac{C}{7\,b\,(b\,x^2+a)^{7/2}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{16\,B\,x}{35\,a^4\,\sqrt{b\,x^2+a}}+\frac{8\,B\,x}{35\,a^3\,(b\,x^2+a)^{3/2}}+\frac{6\,B\,x}{35\,a^2\,(b\,x^2+a)^{5/2}}+\frac{B\,x}{7\,a\,(b\,x^2+a)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x)
```

```
[Out] (A/(7*a) + (A*(a + b*x^2)^2)/(3*a^3) + (A*(a + b*x^2)^3)/a^4 + (A*(a + b*x^
2))/(5*a^2))/(a + b*x^2)^(7/2) - C/(7*b*(a + b*x^2)^(7/2)) - (A*atanh((a +
b*x^2)^(1/2)/a^(1/2)))/a^(9/2) + (16*B*x)/(35*a^4*(a + b*x^2)^(1/2)) + (8*B
*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*B*x)/(35*a^2*(a + b*x^2)^(5/2)) + (B*x)
/(7*a*(a + b*x^2)^(7/2))
```

3.56 $\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=188

$$\frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}}+\frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}}-\frac{A\sqrt{a+bx^2}}{a^5x}-\frac{B\,t}{}$$

```
[Out] 1/7*(B-(A*b/a-C)*x)/a/(b*x^2+a)^(7/2)+1/35*(7*B-(13*A*b/a-6*C)*x)/a^2/(b*x^
2+a)^(5/2)+1/105*(35*B-3*(29*A*b/a-8*C)*x)/a^3/(b*x^2+a)^(3/2)-B*arctanh((b
*x^2+a)^(1/2)/a^(1/2))/a^(9/2)+1/35*(35*B-(93*A*b/a-16*C)*x)/a^4/(b*x^2+a)^
(1/2)-A*(b*x^2+a)^(1/2)/a^5/x
```

Rubi [A]
time = 0.25, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1819, 821, 272, 65, 214}

$$-\frac{B\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}-\frac{A\sqrt{a+bx^2}}{a^5x}+\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{35a^4\sqrt{a+bx^2}}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (B - ((A*b)/a - C)*x)/(7*a*(a + b*x^2)^(7/2)) + (7*B - ((13*A*b)/a - 6*C)*x
)/(35*a^2*(a + b*x^2)^(5/2)) + (35*B - 3*((29*A*b)/a - 8*C)*x)/(105*a^3*(a
+ b*x^2)^(3/2)) + (35*B - ((93*A*b)/a - 16*C)*x)/(35*a^4*Sqrt[a + b*x^2]) -
 (A*Sqrt[a + b*x^2])/(a^5*x) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(9/2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx &= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{-7A-7Bx+6\left(\frac{Ab}{a}-C\right)x^2}{x^2(a+bx^2)^{7/2}}\,dx}{7a}\\
&= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}}+\frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{\int \frac{35A+35Bx-4\left(\frac{13Ab}{a}-6C\right)x^2}{x^2(a+bx^2)^{5/2}}\,dx}{35a^2}\\
&= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}}+\frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{-105A-105Bx+}{x^2(a+b}}{105}\\
&= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}}+\frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{35B-\left(\frac{93Ab}{a}-\right.}{35a^4\sqrt{a+b}}\\
&= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}}+\frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{35B-\left(\frac{93Ab}{a}-\right.}{35a^4\sqrt{a+b}}\\
&= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}}+\frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{35B-\left(\frac{93Ab}{a}-\right.}{35a^4\sqrt{a+b}}\\
&= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}}+\frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{35B-\left(\frac{93Ab}{a}-\right.}{35a^4\sqrt{a+b}}\\
&= \frac{B-\left(\frac{Ab}{a}-C\right)x}{7a\left(a+bx^2\right)^{7/2}}+\frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{35B-\left(\frac{93Ab}{a}-\right.}{35a^4\sqrt{a+b}}
\end{aligned}
$$

Mathematica [A]
time = 0.95, size = 168, normalized size = 0.89

$$\frac{-384Ab^4x^8+14a^2b^2x^4(-120A+x(25B+12Cx))+14a^3bx^2(-60A+x(29B+15Cx))+3ab^3x^6(-448A+x(35B+16Cx))+a^4(-105A+x(176B+105Cx))+210\sqrt{a}\,Bx(a+bx^2)^{7/2}\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-384*A*b^4*x^8 + 14*a^2*b^2*x^4*(-120*A + x*(25*B + 12*C*x)) + 14*a^3*b*x^
2*(-60*A + x*(29*B + 15*C*x)) + 3*a*b^3*x^6*(-448*A + x*(35*B + 16*C*x)) +
a^4*(-105*A + x*(176*B + 105*C*x)) + 210*Sqrt[a]*B*x*(a + b*x^2)^(7/2)*ArcT
anh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(105*a^5*x*(a + b*x^2)^(7/2))
```

Maple [A]
time = 0.16, size = 277, normalized size = 1.47

method	result
default	$C\left(\frac{x}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)+B\left(\frac{1}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{1}{5a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{}{3a\left(b\,x\right.}}{}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b
*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))+B*(1/7/a/(b*x^2+a)^(7/2)+1/a*(1/
5/a/(b*x^2+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a
^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))+A*(-1/a/x/(b*x^2+a)^(7/2)-
8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/
a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))
```

Maxima [A]

time = 0.31, size = 228, normalized size = 1.21

$$\frac{16\,Cx}{35\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Cx}{35\,(bx^2+a)^{\frac{3}{2}}a^3}+\frac{6\,Cx}{35\,(bx^2+a)^{\frac{5}{2}}a^2}+\frac{Cx}{7\,(bx^2+a)^{\frac{7}{2}}a}-\frac{128\,Abx}{35\,\sqrt{bx^2+a}\,a^5}-\frac{64\,Abx}{35\,(bx^2+a)^{\frac{3}{2}}a^4}-\frac{48\,Abx}{35\,(bx^2+a)^{\frac{5}{2}}a^3}-\frac{8\,Abx}{7\,(bx^2+a)^{\frac{7}{2}}a^2}-\frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{a^{\frac{9}{2}}}+\frac{B}{\sqrt{bx^2+a}\,a^4}+\frac{B}{3\,(bx^2+a)^{\frac{3}{2}}a^3}+\frac{B}{5\,(bx^2+a)^{\frac{5}{2}}a^2}+\frac{B}{7\,(bx^2+a)^{\frac{7}{2}}a}-\frac{A}{(bx^2+a)^{\frac{7}{2}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*C*x/(sqrt(b*x^2 + a)*a^4) + 8/35*C*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*C
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*C*x/((b*x^2 + a)^(7/2)*a) - 128/35*A*b*x/(
sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*A*b*x/((
b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 + a)^(7/2)*a^2) - B*arcsinh(a/(sq
rt(a*b)*abs(x)))/a^(9/2) + B/(sqrt(b*x^2 + a)*a^4) + 1/3*B/((b*x^2 + a)^(3/
2)*a^3) + 1/5*B/((b*x^2 + a)^(5/2)*a^2) + 1/7*B/((b*x^2 + a)^(7/2)*a) - A/(
(b*x^2 + a)^(7/2)*a*x)
```

Fricas [A]

time = 1.40, size = 525, normalized size = 2.79

$$\left[\frac{105\,(Bb^4x^9+4\,Bab^3x^7+6\,Ba^2b^2x^5+4\,Ba^3bx^3+Ba^4x)\sqrt{a}\,\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2a}{x^2}\right)+2\,(105\,Bab^3x^7+350\,Ba^2b^2x^5+48\,(Cab^3-8\,Ab^4)x^8+406\,Ba^3bx^3+168\,(Ca^2b^2-8\,Aab^3)x^6+176\,Ba^4x-105\,Aa^4+210\,(Ca^3b-8\,Aa^2b^2)x^4+105\,(Ca^4-8\,Aa^3b)x^2)\sqrt{bx^2+a}}{210\,(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x)},\ \frac{105\,(Bb^4x^9+4\,Bab^3x^7+6\,Ba^2b^2x^5+4\,Ba^3bx^3+Ba^4x)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(105\,Bab^3x^7+350\,Ba^2b^2x^5+48\,(Cab^3-8\,Ab^4)x^8+406\,Ba^3bx^3+168\,(Ca^2b^2-8\,Aab^3)x^6+176\,Ba^4x-105\,Aa^4+210\,(Ca^3b-8\,Aa^2b^2)x^4+105\,(Ca^4-8\,Aa^3b)x^2)\sqrt{bx^2+a}}{105\,(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(B*b^4*x^9 + 4*B*a*b^3*x^7 + 6*B*a^2*b^2*x^5 + 4*B*a^3*b*x^3 +
B*a^4*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(1
05*B*a*b^3*x^7 + 350*B*a^2*b^2*x^5 + 48*(C*a*b^3 - 8*A*b^4)*x^8 + 406*B*a^3
*b*x^3 + 168*(C*a^2*b^2 - 8*A*a*b^3)*x^6 + 176*B*a^4*x - 105*A*a^4 + 210*(C
*a^3*b - 8*A*a^2*b^2)*x^4 + 105*(C*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(
a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x), 1/105*(
105*(B*b^4*x^9 + 4*B*a*b^3*x^7 + 6*B*a^2*b^2*x^5 + 4*B*a^3*b*x^3 + B*a^4*x)
*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (105*B*a*b^3*x^7 + 350*B*a^2*b
^2*x^5 + 48*(C*a*b^3 - 8*A*b^4)*x^8 + 406*B*a^3*b*x^3 + 168*(C*a^2*b^2 - 8*
A*a*b^3)*x^6 + 176*B*a^4*x - 105*A*a^4 + 210*(C*a^3*b - 8*A*a^2*b^2)*x^4 +
105*(C*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7
+ 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6922 vs. $2(155) = 310$.
time = 47.11, size = 6922, normalized size = 36.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(-35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 28
0*a**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 56
0*a**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 44
8*a*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 128*b
**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 2
10*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)) + B*(352*a*
*32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69
/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 529
20*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7
*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53
/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*
b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(6
5/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 +
25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**
9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqrt(1 + b*x**2/a) + 1
)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200
```

```
*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**
10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/
2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 29
24*a**31*b*x**2*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 +
 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**
4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a*
*(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
+ 210*a**(53/2)*b**10*x**20) + 1050*a**31*b*x**2*log(b*x**2/a)/(210*a**(73/
2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**
3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**
(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16
+ 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 2100*a**31*b*x**
2*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450
*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**
8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/
2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210
*a**(53/2)*b**10*x**20) + 10852*a**30*b**2*x**4*sqrt(1 + b*x**2/a)/(210*a**
(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)
*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100
*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x*
*16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 4725*a**30*b
**2*x**4*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69
/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 529
20*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7
*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53
/2)*b**10*x**20) - 9450*a**30*b**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(210*a*
*(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2
)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 4410
0*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x
**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 23630*a**29
*b**3*x**6*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450
*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**
8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/
2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210
*a**(53/2)*b**10*x**20) + 12600*a**29*b**3*x**6*log(b*x**2/a)/(210*a**(73/2
) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3
*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(
61/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 +
 2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 25200*a**29*b**3*
x**6*log(sqrt(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9
450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*
x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(
59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 +
210*a**(53/2)*b**10*x**20) + 33280*a**28*b**4*x**8*sqrt(1 + b*x**2/a)/(210*
a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67
```

```
/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44
100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7...
```

Giac [A]
time = 1.47, size = 239, normalized size = 1.27

$$\frac{\left(\left(\left(\left(3\left(x\left(\frac{35\,Bb^{3}}{a^{4}}+\frac{\left(16\,Ca^{20}b^{6}-93\,Aa^{19}b^{7}\right)x}{a^{24}b^{3}}\right)+\frac{28\,\left(2\,Ca^{21}b^{5}-11\,Aa^{20}b^{6}\right)}{a^{24}b^{3}}\right)x+\frac{350\,Bb^{2}}{a^{3}}\right)x+\frac{210\,\left(Ca^{22}b^{4}-5\,Aa^{21}b^{5}\right)}{a^{24}b^{3}}\right)x+\frac{406\,Bb}{a^{2}}\right)x+\frac{105\,\left(Ca^{23}b^{3}-4\,Aa^{22}b^{4}\right)}{a^{24}b^{3}}\right)x+\frac{176\,B}{a}}{105\,\left(bx^{2}+a\right)^{\frac{7}{2}}}+\frac{2\,B\arctan\left(-\frac{\sqrt{b}\,x-\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{4}}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}-a\right)a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*(((((3*(x*(35*B*b^3/a^4 + (16*C*a^20*b^6 - 93*A*a^19*b^7)*x/(a^24*b^3
)) + 28*(2*C*a^21*b^5 - 11*A*a^20*b^6)/(a^24*b^3))*x + 350*B*b^2/a^3)*x + 2
10*(C*a^22*b^4 - 5*A*a^21*b^5)/(a^24*b^3))*x + 406*B*b/a^2)*x + 105*(C*a^23
*b^3 - 4*A*a^22*b^4)/(a^24*b^3))*x + 176*B/a)/(b*x^2 + a)^(7/2) + 2*B*arcta
n(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4) + 2*A*sqrt(b)/(((
sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

Mupad [B]
time = 2.10, size = 225, normalized size = 1.20

$$\frac{\frac{B}{7\,a}+\frac{B\,{\left(b\,x^2+a\right)}^2}{3\,a^3}+\frac{B\,{\left(b\,x^2+a\right)}^3}{a^4}+\frac{B\,\left(b\,x^2+a\right)}{5\,a^2}}{{\left(b\,x^2+a\right)}^{7/2}}-\frac{\frac{A}{a^4}+\frac{128\,A\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}}-\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{16\,C\,x}{35\,a^4\,\sqrt{b\,x^2+a}}+\frac{8\,C\,x}{35\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{6\,C\,x}{35\,a^2\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{C\,x}{7\,a\,{\left(b\,x^2+a\right)}^{7/2}}-\frac{29\,A\,b\,x}{35\,a^4\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{13\,A\,b\,x}{35\,a^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{A\,b\,x}{7\,a^2\,{\left(b\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x)
```

```
[Out] (B/(7*a) + (B*(a + b*x^2)^2)/(3*a^3) + (B*(a + b*x^2)^3)/a^4 + (B*(a + b*x^
2))/(5*a^2))/(a + b*x^2)^(7/2) - (A/a^4 + (128*A*b*x^2)/(35*a^5))/(x*(a + b
*x^2)^(1/2)) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(9/2) + (16*C*x)/(35*
a^4*(a + b*x^2)^(1/2)) + (8*C*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*C*x)/(35*a
^2*(a + b*x^2)^(5/2)) + (C*x)/(7*a*(a + b*x^2)^(7/2)) - (29*A*b*x)/(35*a^4*
(a + b*x^2)^(3/2)) - (13*A*b*x)/(35*a^3*(a + b*x^2)^(5/2)) - (A*b*x)/(7*a^2
*(a + b*x^2)^(7/2))
```

3.57 $\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=219

$$-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}-\frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}}-\frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}}-\frac{A\sqrt{a+}}{2a^5x}$$

```
[Out] 1/7*(-a*(A*b/a-C)-b*B*x)/a^2/(b*x^2+a)^(7/2)+1/35*(-13*B*b*x-14*A*b+7*C*a)/
a^3/(b*x^2+a)^(5/2)+1/105*(-87*B*b*x-105*A*b+35*C*a)/a^4/(b*x^2+a)^(3/2)+1/
2*(9*A*b-2*C*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(11/2)+1/35*(-93*B*b*x-1
40*A*b+35*C*a)/a^5/(b*x^2+a)^(1/2)-1/2*A*(b*x^2+a)^(1/2)/a^5/x^2-B*(b*x^2+a
)^(1/2)/a^5/x
```

Rubi [A]
time = 0.33, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1819, 1821, 821, 272, 65, 214}

$$\frac{(9Ab-2aC)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}-\frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}}-\frac{A\sqrt{a+bx^2}}{2a^5x^2}-\frac{B\sqrt{a+bx^2}}{a^5x}-\frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}}-\frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] -1/7*(a*((A*b)/a - C) + b*B*x)/(a^2*(a + b*x^2)^(7/2)) - (7*(2*A*b - a*C) +
 13*b*B*x)/(35*a^3*(a + b*x^2)^(5/2)) - (35*(3*A*b - a*C) + 87*b*B*x)/(105*
a^4*(a + b*x^2)^(3/2)) - (35*(4*A*b - a*C) + 93*b*B*x)/(35*a^5*Sqrt[a + b*x
^2]) - (A*Sqrt[a + b*x^2])/(2*a^5*x^2) - (B*Sqrt[a + b*x^2])/(a^5*x) + ((9*
A*b - 2*a*C)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(11/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx &= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{-7A-7Bx+7\left(\frac{Ab}{a}-C\right)x^2+\frac{6bBx^3}{a}}{x^3\left(a+bx^2\right)^{7/2}}\,dx}{7a} \\
&= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} + \frac{\int \frac{35A+35Bx-35\left(\frac{2Ab}{a}-C\right)x^2-\frac{52bBx^3}{a}}{x^3\left(a+bx^2\right)^{5/2}}\,dx}{35a^2} \\
&= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \int \frac{-105}{} \\
&= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4A}{35} \\
&= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4A}{35} \\
&= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4A}{35} \\
&= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4A}{35} \\
&= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4A}{35} \\
&= -\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}} - \frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}} - \frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}} - \frac{35(4A}{35}
\end{aligned}$$

<u>Mathematica</u> [A]
time = 1.23, size = 173, normalized size = 0.79

$$\frac{-3b^4x^8(315A+256Bx)+a^4\left(-105A-210Bx+352Cx^2\right)-4a^3bx^2(396A+7x(60B-29Cx))+42ab^3x^6(-75A+x(-64B+5Cx))+14a^2b^2x^4(-261A+10x(-24B+5Cx))}{210a^5x^2\left(a+bx^2\right)^{7/2}}+\frac{(-9Ab+2aC)\tanh^{-1}\left(\frac{\sqrt{b}\,x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-3*b^4*x^8*(315*A + 256*B*x) + a^4*(-105*A - 210*B*x + 352*C*x^2) - 4*a^3*
b*x^2*(396*A + 7*x*(60*B - 29*C*x)) + 42*a*b^3*x^6*(-75*A + x*(-64*B + 5*C*
x)) + 14*a^2*b^2*x^4*(-261*A + 10*x*(-24*B + 5*C*x)))/(210*a^5*x^2*(a + b*x
```

```
^2)^(7/2)) + ((-9*A*b + 2*a*C)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a
]])/a^(11/2)
```

Maple [A]
time = 0.16, size = 327, normalized size = 1.49

method	result
default	$A \left(-\frac{1}{2 a \,x^{2} \left(b \,x^{2}+a \right)^{\frac{7}{2}}}-\frac{9 b \left(\frac{1}{7 a \left(b \,x^{2}+a \right)^{\frac{7}{2}}}+\frac{\frac{1}{5 a \left(b \,x^{2}+a \right)^{\frac{5}{2}}}+\frac{\frac{1}{3 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{\frac{1}{a \sqrt{b \,x^{2}+a}}-\frac{\ln \left(\frac{2 a +2 \sqrt{a}\, \sqrt{b \,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}}{a}}{a}\right)}{2 a}\right)$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/2/a/x^2/(b*x^2+a)^(7/2)-9/2*b/a*(1/7/a/(b*x^2+a)^(7/2)+1/a*(1/5/a/(b*
x^2+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*
ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))+C*(1/7/a/(b*x^2+a)^(7/2)+1/a*(1/
5/a/(b*x^2+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a
^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))+B*(-1/a/x/(b*x^2+a)^(7/2)-
8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/
a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))
```

Maxima [A]
time = 0.28, size = 265, normalized size = 1.21

$$-\frac{128 \, B b x}{35 \, \sqrt{b x^{2} + a} a^{5}} - \frac{64 \, B b x}{35 \, {\left(b x^{2} + a\right)}^{\frac{3}{2}} a^{4}} - \frac{48 \, B b x}{35 \, {\left(b x^{2} + a\right)}^{\frac{5}{2}} a^{3}} - \frac{8 \, B b x}{7 \, {\left(b x^{2} + a\right)}^{\frac{7}{2}} a^{2}} - \frac{C \operatorname{arsinh}\left(\frac{a}{\sqrt{a b} {\left| x \right|}}\right)}{a^{\frac{9}{2}}} + \frac{9 \, A b \operatorname{arsinh}\left(\frac{a}{\sqrt{a b} {\left| x \right|}}\right)}{2 \, a^{\frac{11}{2}}} + \frac{C}{\sqrt{b x^{2} + a} a^{4}} + \frac{C}{3 \, {\left(b x^{2} + a\right)}^{\frac{3}{2}} a^{3}} + \frac{C}{5 \, {\left(b x^{2} + a\right)}^{\frac{5}{2}} a^{2}} + \frac{C}{7 \, {\left(b x^{2} + a\right)}^{\frac{7}{2}} a} - \frac{9 \, A b}{2 \, \sqrt{b x^{2} + a} a^{5}} - \frac{3 \, A b}{2 \, {\left(b x^{2} + a\right)}^{\frac{3}{2}} a^{4}} - \frac{9 \, A b}{10 \, {\left(b x^{2} + a\right)}^{\frac{5}{2}} a^{3}} - \frac{9 \, A b}{14 \, {\left(b x^{2} + a\right)}^{\frac{7}{2}} a^{2}} - \frac{B}{{\left(b x^{2} + a\right)}^{\frac{7}{2}} a x} - \frac{A}{2 \, {\left(b x^{2} + a\right)}^{\frac{7}{2}} a x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -128/35*B*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*B*b*x/((b*x^2 + a)^(3/2)*a^4) -
 48/35*B*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*B*b*x/((b*x^2 + a)^(7/2)*a^2) -
C*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2) + 9/2*A*b*arcsinh(a/(sqrt(a*b)*abs(
x)))/a^(11/2) + C/(sqrt(b*x^2 + a)*a^4) + 1/3*C/((b*x^2 + a)^(3/2)*a^3) + 1
/5*C/((b*x^2 + a)^(5/2)*a^2) + 1/7*C/((b*x^2 + a)^(7/2)*a) - 9/2*A*b/(sqrt(
b*x^2 + a)*a^5) - 3/2*A*b/((b*x^2 + a)^(3/2)*a^4) - 9/10*A*b/((b*x^2 + a)^(
5/2)*a^3) - 9/14*A*b/((b*x^2 + a)^(7/2)*a^2) - B/((b*x^2 + a)^(7/2)*a*x) -
1/2*A/((b*x^2 + a)^(7/2)*a*x^2)
```

Fricas [**A**]
time = 1.62, size = 688, normalized size = 3.14

$$\left[-\frac{105\left((2\,Cab^4-9\,Ab^5)x^{10}+4\,(2\,Ca^2b^3-9\,Aab^4)x^8+6\,(2\,Ca^3b^2-9\,Aa^2b^3)x^6+4\,(2\,Ca^4b-9\,Aa^3b^2)x^4+(2\,Ca^5-9\,Aa^4b)x^2\right)\sqrt{a}\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\,(768\,Bab^4x^9+2688\,Ba^2b^3x^7+3360\,Ba^3b^2x^5+1680\,Ba^4bx^3-105\,(2\,Ca^2b^3-9\,Aab^4)x^8+210\,Ba^5x-350\,(2\,Ca^3b^2-9\,Aa^2b^3)x^6+105\,Aa^5-406\,(2\,Ca^4b-9\,Aa^3b^2)x^4-176\,(2\,Ca^5-9\,Aa^4b)x^2)\sqrt{bx^2+a}}{420\,(a^6b^4x^{10}+4\,a^7b^3x^8+6\,a^8b^2x^6+4\,a^9bx^4+a^{10}x^2)},\ \frac{105\left((2\,Cab^4-9\,Ab^5)x^{10}+4\,(2\,Ca^2b^3-9\,Aab^4)x^8+6\,(2\,Ca^3b^2-9\,Aa^2b^3)x^6+4\,(2\,Ca^4b-9\,Aa^3b^2)x^4+(2\,Ca^5-9\,Aa^4b)x^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(768\,Bab^4x^9+2688\,Ba^2b^3x^7+3360\,Ba^3b^2x^5+1680\,Ba^4bx^3-105\,(2\,Ca^2b^3-9\,Aab^4)x^8+210\,Ba^5x-350\,(2\,Ca^3b^2-9\,Aa^2b^3)x^6+105\,Aa^5-406\,(2\,Ca^4b-9\,Aa^3b^2)x^4-176\,(2\,Ca^5-9\,Aa^4b)x^2)\sqrt{bx^2+a}}{210\,(a^6b^4x^{10}+4\,a^7b^3x^8+6\,a^8b^2x^6+4\,a^9bx^4+a^{10}x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [-1/420*(105*((2*C*a*b^4 - 9*A*b^5)*x^10 + 4*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8
+ 6*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 4*(2*C*a^4*b - 9*A*a^3*b^2)*x^4 + (2*
C*a^5 - 9*A*a^4*b)*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2
*a)/x^2) + 2*(768*B*a*b^4*x^9 + 2688*B*a^2*b^3*x^7 + 3360*B*a^3*b^2*x^5 + 1
680*B*a^4*b*x^3 - 105*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8 + 210*B*a^5*x - 350*(2*
C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 105*A*a^5 - 406*(2*C*a^4*b - 9*A*a^3*b^2)*x^
4 - 176*(2*C*a^5 - 9*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^6*b^4*x^10 + 4*a^7*b
^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10*x^2), 1/210*(105*((2*C*a*b^4 -
9*A*b^5)*x^10 + 4*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8 + 6*(2*C*a^3*b^2 - 9*A*a^2*
b^3)*x^6 + 4*(2*C*a^4*b - 9*A*a^3*b^2)*x^4 + (2*C*a^5 - 9*A*a^4*b)*x^2)*sqr
t(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (768*B*a*b^4*x^9 + 2688*B*a^2*b^3*
x^7 + 3360*B*a^3*b^2*x^5 + 1680*B*a^4*b*x^3 - 105*(2*C*a^2*b^3 - 9*A*a*b^4)
*x^8 + 210*B*a^5*x - 350*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 105*A*a^5 - 406*
(2*C*a^4*b - 9*A*a^3*b^2)*x^4 - 176*(2*C*a^5 - 9*A*a^4*b)*x^2)*sqrt(b*x^2 +
 a))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10*x^2
)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 11198 vs. 2(196) = 392.
time = 66.60, size = 11198, normalized size = 51.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**3/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(-70*a**49*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x*
*4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/
```

```
2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 1
6800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9
*x**20 + 140*a**(87/2)*b**10*x**22) - 1476*a**48*b*x**2*sqrt(1 + b*x**2/a)/
(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 +
 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*
b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300
*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**
22) - 315*a**48*b*x**2*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)
*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a*
*(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**1
4 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)
*b**9*x**20 + 140*a**(87/2)*b**10*x**22) + 630*a**48*b*x**2*log(sqrt(1 + b*
x**2/a) + 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2
)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 352
80*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7
*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87
/2)*b**10*x**22) - 9822*a**47*b**2*x**4*sqrt(1 + b*x**2/a)/(140*a**(107/2)*
x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2
)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 294
00*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*
x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 3150*a**47
*b**2*x**4*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 63
00*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4
*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a*
*(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20
 + 140*a**(87/2)*b**10*x**22) + 6300*a**47*b**2*x**4*log(sqrt(1 + b*x**2/a)
 + 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x
**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(9
7/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 +
 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**1
0*x**22) - 33956*a**46*b**3*x**6*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 +
1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*
x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(
95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 +
 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 14175*a**46*b**3*
x**6*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**
(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10
 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2
)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*
a**(87/2)*b**10*x**22) + 28350*a**46*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/
(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 +
 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*
b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300
*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**
22) - 71940*a**45*b**4*x**8*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*
```

```
a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8
+ 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)
*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400
*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 37800*a**45*b**4*x**8*
log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/
2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35
280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**
7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(8
7/2)*b**10*x**22) + 75600*a**45*b**4*x**8*log(sqrt(1 + b*x**2/a) + 1)/(140*
a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 1680
0*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*
x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(
91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) -
 100260*a**44*b**5*x**10*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**
(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 2
9400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b*
*6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a*
*(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 66150*a**44*b**5*x**10*lo
g(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)
*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400...
```

Giac [A]
time = 1.38, size = 325, normalized size = 1.48

$$-\frac{\left(\left(\left(\left(3\left(\left(\frac{93\,Bb^{4}x}{a^{5}}-\frac{35\,(Ca^{24}b^{6}-4\,Aa^{23}b^{7})}{a^{28}b^{3}}\right)x+\frac{308\,Bb^{3}}{a^{4}}\right)x-\frac{35\,(10\,Ca^{25}b^{5}-39\,Aa^{24}b^{6})}{a^{28}b^{3}}\right)x+\frac{1050\,Bb^{2}}{a^{3}}\right)x-\frac{14\,(29\,Ca^{26}b^{4}-108\,Aa^{25}b^{5})}{a^{28}b^{3}}\right)x+\frac{420\,Bb}{a^{2}}\right)x-\frac{2\,(88\,Ca^{27}b^{3}-291\,Aa^{26}b^{4})}{a^{28}b^{3}}}{105\,(bx^{2}+a)^{\frac{7}{2}}}+\frac{(2\,Ca-9\,Ab)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{5}}+\frac{\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{3}Ab+2\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)Aab-2\,Ba^{2}\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{2}a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*(((((3*((93*B*b^4*x/a^5 - 35*(C*a^24*b^6 - 4*A*a^23*b^7)/(a^28*b^3))
*x + 308*B*b^3/a^4)*x - 35*(10*C*a^25*b^5 - 39*A*a^24*b^6)/(a^28*b^3))*x +
1050*B*b^2/a^3)*x - 14*(29*C*a^26*b^4 - 108*A*a^25*b^5)/(a^28*b^3))*x + 420
*B*b/a^2)*x - 2*(88*C*a^27*b^3 - 291*A*a^26*b^4)/(a^28*b^3))/(b*x^2 + a)^(7
/2) + (2*C*a - 9*A*b)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt
(-a)*a^5) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(
b))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^5)
```

Mupad [B]
time = 2.52, size = 279, normalized size = 1.27

$$\frac{\frac{C}{7\,a}+\frac{C\,{\left(b\,x^2+a\right)}^2}{3\,a^3}+\frac{C\,{\left(b\,x^2+a\right)}^3}{a^4}+\frac{C\,\left(b\,x^2+a\right)}{5\,a^2}}{{\left(b\,x^2+a\right)}^{7/2}}-\frac{\frac{A\,b}{7\,a}+\frac{9\,A\,b\,\left(b\,x^2+a\right)}{35\,a^2}+\frac{3\,A\,b\,{\left(b\,x^2+a\right)}^2}{5\,a^3}+\frac{3\,A\,b\,{\left(b\,x^2+a\right)}^3}{a^4}-\frac{9\,A\,b\,{\left(b\,x^2+a\right)}^4}{2\,a^5}}{a\,{\left(b\,x^2+a\right)}^{7/2}-{\left(b\,x^2+a\right)}^{9/2}}-\frac{\frac{B}{a^4}+\frac{128\,B\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}}-\frac{C\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{9\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{11/2}}-\frac{29\,B\,b\,x}{35\,a^4\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{13\,B\,b\,x}{35\,a^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{B\,b\,x}{7\,a^2\,{\left(b\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x)
```

```
[Out] (C/(7*a) + (C*(a + b*x^2)^2)/(3*a^3) + (C*(a + b*x^2)^3)/a^4 + (C*(a + b*x^
2))/(5*a^2))/(a + b*x^2)^(7/2) - ((A*b)/(7*a) + (9*A*b*(a + b*x^2))/(35*a^2
) + (3*A*b*(a + b*x^2)^2)/(5*a^3) + (3*A*b*(a + b*x^2)^3)/a^4 - (9*A*b*(a +
 b*x^2)^4)/(2*a^5))/(a*(a + b*x^2)^(7/2) - (a + b*x^2)^(9/2)) - (B/a^4 + (1
28*B*b*x^2)/(35*a^5))/(x*(a + b*x^2)^(1/2)) - (C*atanh((a + b*x^2)^(1/2)/a^
(1/2)))/a^(9/2) + (9*A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(11/2)) - (
29*B*b*x)/(35*a^4*(a + b*x^2)^(3/2)) - (13*B*b*x)/(35*a^3*(a + b*x^2)^(5/2)
) - (B*b*x)/(7*a^2*(a + b*x^2)^(7/2))
```

3.58 $\int \frac{A(cx)^m}{a+bx^2}\,dx$

Optimal. Leaf size=45

$$\frac{A(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ac(1+m)}$$

```
[Out] A*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/c/(1+m)
```

Rubi [A]
time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {12, 371}

$$\frac{A(cx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ac(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(A*(c*x)^m)/(a + b*x^2),x]
```

```
[Out] (A*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/
(a*c*(1 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{A(cx)^m}{a+bx^2}\,dx &= A\int \frac{(cx)^m}{a+bx^2}\,dx\\ &= \frac{A(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ac(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 43, normalized size = 0.96

$$\frac{Ax(cx)^m\,{}_2F_1\left(1,\frac{1+m}{2};1+\frac{1+m}{2};-\frac{bx^2}{a}\right)}{a(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A*(c*x)^m)/(a + b*x^2),x]
```

```
[Out] (A*x*(c*x)^m*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])/
(a*(1 + m))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{A(cx)^m}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(A*(c*x)^m/(b*x^2+a),x)
```

```
[Out] int(A*(c*x)^m/(b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A*(c*x)^m/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] A*integrate((c*x)^m/(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A*(c*x)^m/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((c*x)^m*A/(b*x^2 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.54, size = 97, normalized size = 2.16

$$A\left(\frac{c^m m x x^m \Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{c^m x x^m \Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A*(c*x)**m/(b*x**2+a),x)
```

```
[Out] A*(c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
 + 1/2)/(4*a*gamma(m/2 + 3/2)) + c**m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi
)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A*(c*x)^m/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((c*x)^m*A/(b*x^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{A\,{\left(c\,x\right)}^m}{b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A*(c*x)^m)/(a + b*x^2),x)
```

```
[Out] int((A*(c*x)^m)/(a + b*x^2), x)
```

3.59 $\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx$

Optimal. Leaf size=91

$$\frac{A(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ac(1+m)}+\frac{B(cx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{ac^2(2+m)}$$

```
[Out] A*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/c/(1+m)+B*(c
*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-b*x^2/a)/a/c^2/(2+m)
```

Rubi [A]
time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {822, 371}

$$\frac{A(cx)^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{ac(m+1)}+\frac{B(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac^2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x)^m*(A + B*x))/(a + b*x^2),x]
```

```
[Out] (A*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/
(a*c*(1 + m)) + (B*(c*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2,
 -((b*x^2)/a)])/(a*c^2*(2 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] && !RationalQ[m
] && !IGtQ[p, 0]
```

Rubi steps

$$\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx = A\int \frac{(cx)^m}{a+bx^2}\,dx + \frac{B\int \frac{(cx)^{1+m}}{a+bx^2}\,dx}{c}$$
$$= \frac{A(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{ac(1+m)} + \frac{B(cx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{ac^2(2+m)}$$

Mathematica [**A**]
time = 0.05, size = 82, normalized size = 0.90

$$\frac{x(cx)^m\left(B(1+m)x\,{}_2F_1\left(1,1+\frac{m}{2};2+\frac{m}{2};-\frac{bx^2}{a}\right)+A(2+m)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)\right)}{a(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x)^m*(A + B*x))/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^m*(B*(1 + m)*x*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, -((b*x^2)/a)
] + A*(2 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a
*(1 + m)*(2 + m))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m\,(Bx+A)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(B*x+A)/(b*x^2+a),x)
```

```
[Out] int((c*x)^m*(B*x+A)/(b*x^2+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.83, size = 192, normalized size = 2.11

$$\frac{Ac^m mxx^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Ac^m xx^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Bc^m mx^2x^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{4a\Gamma\left(\frac{m}{2}+2\right)} + \frac{Bc^m x^2x^m \Phi\left(\frac{bx^2e^{i\pi}}{a}, 1, \frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{2a\Gamma\left(\frac{m}{2}+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(B*x+A)/(b*x**2+a),x)
```

```
[Out] A*c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
+ 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x*x**m*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + B*c**m*m*x**2
*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(4*a*ga
mma(m/2 + 2)) + B*c**m*x**2*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2
+ 1)*gamma(m/2 + 1)/(2*a*gamma(m/2 + 2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x\right)}^m\,\left(A+B\,x\right)}{b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x)^m*(A + B*x))/(a + b*x^2),x)
```

```
[Out] int(((c*x)^m*(A + B*x))/(a + b*x^2), x)
```

3.60 $\int \frac{(cx)^m (A+Cx^2)}{a+bx^2} \, dx$

Optimal. Leaf size=76

$$\frac{C(cx)^{1+m}}{bc(1+m)} + \frac{(Ab-aC)(cx)^{1+m} \, {}_2F_1\left(1, \frac{1+m}{2}; \frac{3+m}{2}; -\frac{bx^2}{a}\right)}{abc(1+m)}$$

```
[Out] C*(c*x)^(1+m)/b/c/(1+m)+(A*b-C*a)*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2
+1/2*m],-b*x^2/a)/a/b/c/(1+m)
```

Rubi [A]

time $= 0.03$, antiderivative size $= 76$, normalized size of antiderivative $= 1.00$, number of steps used $= 2$, number of rules used $= 2$, integrand size $= 22$, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used $= \{470, 371\}$

$$\frac{(cx)^{m+1}(Ab-aC) \, {}_2F_1\left(1, \frac{m+1}{2}; \frac{m+3}{2}; -\frac{bx^2}{a}\right)}{abc(m+1)} + \frac{C(cx)^{m+1}}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x)^m*(A + C*x^2))/(a + b*x^2),x]
```

```
[Out] (C*(c*x)^(1 + m))/(b*c*(1 + m)) + ((A*b - a*C)*(c*x)^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*c*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rubi steps

$$\int \frac{(cx)^m (A+Cx^2)}{a+bx^2}\,dx = \frac{C(cx)^{1+m}}{bc(1+m)} - \frac{(-Ab(1+m)+aC(1+m))\int \frac{(cx)^m}{a+bx^2}\,dx}{b(1+m)}$$
$$= \frac{C(cx)^{1+m}}{bc(1+m)} + \frac{(Ab-aC)(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{abc(1+m)}$$

Mathematica [**A**]
time = 0.08, size = 56, normalized size = 0.74

$$\frac{x(cx)^m \left(aC+(Ab-aC)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)\right)}{ab(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x)^m*(A + C*x^2))/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^m*(a*C + (A*b - a*C)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -(
(b*x^2)/a)]))/(a*b*(1 + m))
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m \left(C\,x^2+A\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(C*x^2+A)/(b*x^2+a),x)
```

```
[Out] int((c*x)^m*(C*x^2+A)/(b*x^2+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.95, size = 204, normalized size = 2.68

$$\frac{Ac^m mxx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{Ac^m xx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{Cc^m mx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{3Cc^m x^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(C*x**2+A)/(b*x**2+a),x)
```

```
[Out] A*c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
+ 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x*x**m*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + C*c**m*m*x**3
*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*
a*gamma(m/2 + 5/2)) + 3*C*c**m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a,
 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(C\,x^2+A\right)\,{\left(c\,x\right)}^m}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + C*x^2)*(c*x)^m)/(a + b*x^2),x)
```

```
[Out] int(((A + C*x^2)*(c*x)^m)/(a + b*x^2), x)
```

3.61 $\int \frac{(cx)^m(A+Bx+Cx^2)}{a+bx^2}\,dx$

Optimal. Leaf size=121

$$\frac{C(cx)^{1+m}}{bc(1+m)}+\frac{(Ab-aC)(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{abc(1+m)}+\frac{B(cx)^{2+m}\,{}_2F_1\left(1,\frac{2+m}{2};\frac{4+m}{2};-\frac{bx^2}{a}\right)}{ac^2(2+m)}$$

```
[Out] C*(c*x)^(1+m)/b/c/(1+m)+(A*b-C*a)*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2
+1/2*m],-b*x^2/a)/a/b/c/(1+m)+B*(c*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m
],-b*x^2/a)/a/c^2/(2+m)
```

Rubi [A]
time = 0.08, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1816, 822, 371}

$$\frac{(cx)^{m+1}(Ab-aC)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abc(m+1)}+\frac{B(cx)^{m+2}\,{}_2F_1\left(1,\frac{m+2}{2};\frac{m+4}{2};-\frac{bx^2}{a}\right)}{ac^2(m+2)}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2),x]
```

```
[Out] (C*(c*x)^(1 + m))/(b*c*(1 + m)) + ((A*b - a*C)*(c*x)^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*c*(1 + m)) + (B*(c*x)^(2 +
 m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a*c^2*(2 + m
))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 822

```
Int[((e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[f, Int[(e*x)^m*(a + c*x^2)^p, x], x] + Dist[g/e, Int[(e*x)^(m
+ 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, e, f, g, p}, x] &&  !RationalQ[m
] &&  !IGtQ[p, 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(cx)^m\left(A+Bx+Cx^2\right)}{a+bx^2}\,dx &= \int \left(\frac{C(cx)^m}{b}+\frac{(cx)^m(Ab-aC+bBx)}{b\left(a+bx^2\right)}\right)dx\\ &= \frac{C(cx)^{1+m}}{bc(1+m)}+\frac{\int \frac{(cx)^m(Ab-aC+bBx)}{a+bx^2}\,dx}{b}\\ &= \frac{C(cx)^{1+m}}{bc(1+m)}+\frac{B\int \frac{(cx)^{1+m}}{a+bx^2}\,dx}{c}+\frac{(Ab-aC)\int \frac{(cx)^m}{a+bx^2}\,dx}{b}\\ &= \frac{C(cx)^{1+m}}{bc(1+m)}+\frac{(Ab-aC)(cx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{abc(1+m)}+\frac{B(cx)^{2+m}\,{}_2F_1\left(\right.}{ac^2}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 99, normalized size = 0.82

$$\frac{x(cx)^m\left(aC(2+m)+bB(1+m)x\,{}_2F_1\left(1,1+\frac{m}{2};2+\frac{m}{2};-\frac{bx^2}{a}\right)+(Ab-aC)(2+m)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)\right)}{ab(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2),x]
```

```
[Out] (x*(c*x)^m*(a*C*(2 + m) + b*B*(1 + m)*x*Hypergeometric2F1[1, 1 + m/2, 2 + m
/2, -((b*x^2)/a)] + (A*b - a*C)*(2 + m)*Hypergeometric2F1[1, (1 + m)/2, (3
+ m)/2, -((b*x^2)/a)]))/(a*b*(1 + m)*(2 + m))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(cx)^m\left(C\,x^2+Bx+A\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x)
```

```
[Out] int((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.59, size = 298, normalized size = 2.46

$$\frac{Ac^mmxx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{Ac^mxx^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}+\frac{Bc^mmx^2x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{4a\Gamma\left(\frac{m}{2}+2\right)}+\frac{Bc^mx^2x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{2a\Gamma\left(\frac{m}{2}+2\right)}+\frac{Cc^mmx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}+\frac{3Cc^mx^3x^m\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)**m*(C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] A*c**m*m*x*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2
+ 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x*x**m*lerchphi(b*x**2*exp_polar(I*p
i)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + B*c**m*m*x**2
*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(4*a*ga
mma(m/2 + 2)) + B*c**m*x**2*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2
+ 1)*gamma(m/2 + 1)/(2*a*gamma(m/2 + 2)) + C*c**m*m*x**3*x**m*lerchphi(b*x*
*2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
 + 3*C*c**m*x**3*x**m*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamm
a(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c\,x\right)}^m\,\left(C\,x^2+B\,x+A\right)}{b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2),x)
[Out] int(((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2), x)
```

3.62 $\int x^3(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=65

$$\frac{1}{4}aAx^4+\frac{1}{5}aBx^5+\frac{1}{6}(Ab+aC)x^6+\frac{1}{7}(bB+aD)x^7+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

```
[Out] 1/4*a*A*x^4+1/5*a*B*x^5+1/6*(A*b+C*a)*x^6+1/7*(B*b+D*a)*x^7+1/8*b*C*x^8+1/9
*b*D*x^9
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.038$, Rules used = $\{1816\}$

$$\frac{1}{6}x^6(aC+Ab)+\frac{1}{4}aAx^4+\frac{1}{7}x^7(aD+bB)+\frac{1}{5}aBx^5+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^7)/7 + (b*
C*x^8)/8 + (b*D*x^9)/9
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx&=\int\left(aAx^3+aBx^4+(Ab+aC)x^5+(bB+aD)x^6+bCx^7+b\right.\\&=\frac{1}{4}aAx^4+\frac{1}{5}aBx^5+\frac{1}{6}(Ab+aC)x^6+\frac{1}{7}(bB+aD)x^7+\frac{1}{8}bCx^8\end{aligned}$$

Mathematica [A]
time = 0.01, size = 65, normalized size = 1.00

$$\frac{1}{4}aAx^4+\frac{1}{5}aBx^5+\frac{1}{6}(Ab+aC)x^6+\frac{1}{7}(bB+aD)x^7+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^7)/7 + (b*
C*x^8)/8 + (b*D*x^9)/9
```

Maple [A]
time = 0.05, size = 54, normalized size = 0.83

method	result	size
default	$\frac{Aa\,x^{4}}{4}+\frac{aB\,x^{5}}{5}+\frac{(Ab+aC)x^{6}}{6}+\frac{(Bb+aD)x^{7}}{7}+\frac{bC\,x^{8}}{8}+\frac{bD x^{9}}{9}$	54
norman	$\frac{bD x^{9}}{9}+\frac{bC\,x^{8}}{8}+\left(\frac{Bb}{7}+\frac{aD}{7}\right)x^{7}+\left(\frac{Ab}{6}+\frac{aC}{6}\right)x^{6}+\frac{aB\,x^{5}}{5}+\frac{Aa\,x^{4}}{4}$	56
gosper	$\frac{1}{9}bD x^{9}+\frac{1}{8}bC\,x^{8}+\frac{1}{7}bB\,x^{7}+\frac{1}{7}x^{7}aD+\frac{1}{6}x^{6}Ab+\frac{1}{6}x^{6}aC+\frac{1}{5}aB\,x^{5}+\frac{1}{4}Aa\,x^{4}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*A*a*x^4+1/5*a*B*x^5+1/6*(A*b+C*a)*x^6+1/7*(B*b+D*a)*x^7+1/8*b*C*x^8+1/9
*b*D*x^9
```

Maxima [A]
time = 0.27, size = 53, normalized size = 0.82

$$\frac{1}{9}\,Dbx^{9}+\frac{1}{8}\,Cbx^{8}+\frac{1}{7}\,(Da+Bb)x^{7}+\frac{1}{5}\,Bax^{5}+\frac{1}{6}\,(Ca+Ab)x^{6}+\frac{1}{4}\,Aax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*(D*a + B*b)*x^7 + 1/5*B*a*x^5 + 1/6*(C*a +
A*b)*x^6 + 1/4*A*a*x^4
```

Fricas [A]
time = 3.35, size = 53, normalized size = 0.82

$$\frac{1}{9}\,Dbx^{9}+\frac{1}{8}\,Cbx^{8}+\frac{1}{7}\,(Da+Bb)x^{7}+\frac{1}{5}\,Bax^{5}+\frac{1}{6}\,(Ca+Ab)x^{6}+\frac{1}{4}\,Aax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*(D*a + B*b)*x^7 + 1/5*B*a*x^5 + 1/6*(C*a +
A*b)*x^6 + 1/4*A*a*x^4
```

Sympy [A]
time = 0.01, size = 60, normalized size = 0.92

$$\frac{Aax^{4}}{4}+\frac{Bax^{5}}{5}+\frac{Cbx^{8}}{8}+\frac{Dbx^{9}}{9}+x^{7}\left(\frac{Bb}{7}+\frac{Da}{7}\right)+x^{6}\left(\frac{Ab}{6}+\frac{Ca}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**4/4 + B*a*x**5/5 + C*b*x**8/8 + D*b*x**9/9 + x**7*(B*b/7 + D*a/7) +
x**6*(A*b/6 + C*a/6)
```

Giac [**A**]
time = 1.20, size = 57, normalized size = 0.88

$$\frac{1}{9}\,Dbx^{9} + \frac{1}{8}\,Cbx^{8} + \frac{1}{7}\,Dax^{7} + \frac{1}{7}\,Bbx^{7} + \frac{1}{6}\,Cax^{6} + \frac{1}{6}\,Abx^{6} + \frac{1}{5}\,Bax^{5} + \frac{1}{4}\,Aax^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*D*a*x^7 + 1/7*B*b*x^7 + 1/6*C*a*x^6 + 1/6*A
*b*x^6 + 1/5*B*a*x^5 + 1/4*A*a*x^4
```

Mupad [**B**]
time = 1.20, size = 57, normalized size = 0.88

$$\frac{a\,x^7\,D}{7}+\frac{b\,x^9\,D}{9}+\frac{A\,a\,x^4}{4}+\frac{B\,a\,x^5}{5}+\frac{A\,b\,x^6}{6}+\frac{C\,a\,x^6}{6}+\frac{B\,b\,x^7}{7}+\frac{C\,b\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a*x^7*D)/7 + (b*x^9*D)/9 + (A*a*x^4)/4 + (B*a*x^5)/5 + (A*b*x^6)/6 + (C*a*
x^6)/6 + (B*b*x^7)/7 + (C*b*x^8)/8
```

3.63 $\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=65

$$\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}(bB+aD)x^6+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

```
[Out] 1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*(B*b+D*a)*x^6+1/7*b*C*x^7+1/8
*b*D*x^8
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1816}

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{6}x^6(aD+bB)+\frac{1}{4}aBx^4+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^6)/6 + (b*
C*x^7)/7 + (b*D*x^8)/8
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx&=\int \left(aAx^2+aBx^3+(Ab+aC)x^4+(bB+aD)x^5+bCx^6+b\right.\\&=\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}(bB+aD)x^6+\frac{1}{7}bCx\end{aligned}$$

Mathematica [A]
time = 0.01, size = 65, normalized size = 1.00

$$\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}(bB+aD)x^6+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^6)/6 + (b*
C*x^7)/7 + (b*D*x^8)/8
```

Maple [A]
time = 0.06, size = 54, normalized size = 0.83

method	result	size
default	$\frac{aA\,x^{3}}{3}+\frac{Ba\,x^{4}}{4}+\frac{(Ab+aC)x^{5}}{5}+\frac{(Bb+aD)x^{6}}{6}+\frac{bC\,x^{7}}{7}+\frac{bDx^{8}}{8}$	54
norman	$\frac{bDx^{8}}{8}+\frac{bC\,x^{7}}{7}+\left(\frac{Bb}{6}+\frac{aD}{6}\right)x^{6}+\left(\frac{Ab}{5}+\frac{aC}{5}\right)x^{5}+\frac{Ba\,x^{4}}{4}+\frac{aA\,x^{3}}{3}$	56
gosper	$\frac{1}{8}bDx^{8}+\frac{1}{7}bC\,x^{7}+\frac{1}{6}bB\,x^{6}+\frac{1}{6}x^{6}aD+\frac{1}{5}x^{5}Ab+\frac{1}{5}x^{5}aC+\frac{1}{4}Ba\,x^{4}+\frac{1}{3}aA\,x^{3}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a*A*x^3+1/4*B*a*x^4+1/5*(A*b+C*a)*x^5+1/6*(B*b+D*a)*x^6+1/7*b*C*x^7+1/8
*b*D*x^8
```

Maxima [A]
time = 0.27, size = 53, normalized size = 0.82

$$\frac{1}{8}\,Dbx^{8}+\frac{1}{7}\,Cbx^{7}+\frac{1}{6}\,(Da+Bb)x^{6}+\frac{1}{4}\,Bax^{4}+\frac{1}{5}\,(Ca+Ab)x^{5}+\frac{1}{3}\,Aax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*(D*a + B*b)*x^6 + 1/4*B*a*x^4 + 1/5*(C*a +
A*b)*x^5 + 1/3*A*a*x^3
```

Fricas [A]
time = 3.53, size = 53, normalized size = 0.82

$$\frac{1}{8}\,Dbx^{8}+\frac{1}{7}\,Cbx^{7}+\frac{1}{6}\,(Da+Bb)x^{6}+\frac{1}{4}\,Bax^{4}+\frac{1}{5}\,(Ca+Ab)x^{5}+\frac{1}{3}\,Aax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*(D*a + B*b)*x^6 + 1/4*B*a*x^4 + 1/5*(C*a +
A*b)*x^5 + 1/3*A*a*x^3
```

Sympy [A]
time = 0.01, size = 60, normalized size = 0.92

$$\frac{Aax^{3}}{3}+\frac{Bax^{4}}{4}+\frac{Cbx^{7}}{7}+\frac{Dbx^{8}}{8}+x^{6}\left(\frac{Bb}{6}+\frac{Da}{6}\right)+x^{5}\left(\frac{Ab}{5}+\frac{Ca}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**3/3 + B*a*x**4/4 + C*b*x**7/7 + D*b*x**8/8 + x**6*(B*b/6 + D*a/6) +
x**5*(A*b/5 + C*a/5)
```

Giac [A]
time = 1.12, size = 57, normalized size = 0.88

$$\frac{1}{8}\,Dbx^{8}+\frac{1}{7}\,Cbx^{7}+\frac{1}{6}\,Dax^{6}+\frac{1}{6}\,Bbx^{6}+\frac{1}{5}\,Cax^{5}+\frac{1}{5}\,Abx^{5}+\frac{1}{4}\,Bax^{4}+\frac{1}{3}\,Aax^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*D*a*x^6 + 1/6*B*b*x^6 + 1/5*C*a*x^5 + 1/5*A
*b*x^5 + 1/4*B*a*x^4 + 1/3*A*a*x^3
```

Mupad [B]
time = 1.18, size = 57, normalized size = 0.88

$$\frac{a\,x^6\,D}{6}+\frac{b\,x^8\,D}{8}+\frac{A\,a\,x^3}{3}+\frac{B\,a\,x^4}{4}+\frac{A\,b\,x^5}{5}+\frac{C\,a\,x^5}{5}+\frac{B\,b\,x^6}{6}+\frac{C\,b\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a*x^6*D)/6 + (b*x^8*D)/8 + (A*a*x^3)/3 + (B*a*x^4)/4 + (A*b*x^5)/5 + (C*a*
x^5)/5 + (B*b*x^6)/6 + (C*b*x^7)/7
```

3.64 $\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=65

$$\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}(bB+aD)x^5+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

```
[Out] 1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*(B*b+D*a)*x^5+1/6*b*C*x^6+1/7
*b*D*x^7
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.042$, Rules used = $\{1816\}$

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{5}x^5(aD+bB)+\frac{1}{3}aBx^3+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^5)/5 + (b*
C*x^6)/6 + (b*D*x^7)/7
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \int \left(aAx+aBx^2+(Ab+aC)x^3+(bB+aD)x^4+bCx^5+bDx\right.\\ &= \frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}(bB+aD)x^5+\frac{1}{6}bCx^6+\end{aligned}$$

Mathematica [A]
time = 0.01, size = 65, normalized size = 1.00

$$\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}(bB+aD)x^5+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^5)/5 + (b*
C*x^6)/6 + (b*D*x^7)/7
```

Maple [A]
time = 0.05, size = 54, normalized size = 0.83

method	result	size
default	$\frac{aA\,x^{2}}{2}+\frac{Ba\,x^{3}}{3}+\frac{(Ab+aC)x^{4}}{4}+\frac{(Bb+aD)x^{5}}{5}+\frac{bC\,x^{6}}{6}+\frac{bDx^{7}}{7}$	54
norman	$\frac{bDx^{7}}{7}+\frac{bC\,x^{6}}{6}+\left(\frac{Bb}{5}+\frac{aD}{5}\right)x^{5}+\left(\frac{Ab}{4}+\frac{aC}{4}\right)x^{4}+\frac{Ba\,x^{3}}{3}+\frac{aA\,x^{2}}{2}$	56
gosper	$\frac{1}{7}bDx^{7}+\frac{1}{6}bC\,x^{6}+\frac{1}{5}bB\,x^{5}+\frac{1}{5}x^{5}aD+\frac{1}{4}x^{4}Ab+\frac{1}{4}x^{4}aC+\frac{1}{3}Ba\,x^{3}+\frac{1}{2}aA\,x^{2}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a*A*x^2+1/3*B*a*x^3+1/4*(A*b+C*a)*x^4+1/5*(B*b+D*a)*x^5+1/6*b*C*x^6+1/7
*b*D*x^7
```

Maxima [A]
time = 0.28, size = 53, normalized size = 0.82

$$\frac{1}{7}\,Dbx^{7}+\frac{1}{6}\,Cbx^{6}+\frac{1}{5}\,(Da+Bb)x^{5}+\frac{1}{3}\,Bax^{3}+\frac{1}{4}\,(Ca+Ab)x^{4}+\frac{1}{2}\,Aax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*(D*a + B*b)*x^5 + 1/3*B*a*x^3 + 1/4*(C*a +
A*b)*x^4 + 1/2*A*a*x^2
```

Fricas [A]
time = 5.12, size = 53, normalized size = 0.82

$$\frac{1}{7}\,Dbx^{7}+\frac{1}{6}\,Cbx^{6}+\frac{1}{5}\,(Da+Bb)x^{5}+\frac{1}{3}\,Bax^{3}+\frac{1}{4}\,(Ca+Ab)x^{4}+\frac{1}{2}\,Aax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*(D*a + B*b)*x^5 + 1/3*B*a*x^3 + 1/4*(C*a +
A*b)*x^4 + 1/2*A*a*x^2
```

Sympy [A]
time = 0.01, size = 60, normalized size = 0.92

$$\frac{Aax^{2}}{2}+\frac{Bax^{3}}{3}+\frac{Cbx^{6}}{6}+\frac{Dbx^{7}}{7}+x^{5}\left(\frac{Bb}{5}+\frac{Da}{5}\right)+x^{4}\left(\frac{Ab}{4}+\frac{Ca}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x**2/2 + B*a*x**3/3 + C*b*x**6/6 + D*b*x**7/7 + x**5*(B*b/5 + D*a/5) +
x**4*(A*b/4 + C*a/4)
```

Giac [A]
time = 1.55, size = 57, normalized size = 0.88

$$\frac{1}{7}\,Dbx^{7} + \frac{1}{6}\,Cbx^{6} + \frac{1}{5}\,Dax^{5} + \frac{1}{5}\,Bbx^{5} + \frac{1}{4}\,Cax^{4} + \frac{1}{4}\,Abx^{4} + \frac{1}{3}\,Bax^{3} + \frac{1}{2}\,Aax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*D*a*x^5 + 1/5*B*b*x^5 + 1/4*C*a*x^4 + 1/4*A
*b*x^4 + 1/3*B*a*x^3 + 1/2*A*a*x^2
```

Mupad [B]
time = 1.19, size = 57, normalized size = 0.88

$$\frac{a\,x^5\,D}{5}+\frac{b\,x^7\,D}{7}+\frac{A\,a\,x^2}{2}+\frac{B\,a\,x^3}{3}+\frac{A\,b\,x^4}{4}+\frac{C\,a\,x^4}{4}+\frac{B\,b\,x^5}{5}+\frac{C\,b\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a*x^5*D)/5 + (b*x^7*D)/7 + (A*a*x^2)/2 + (B*a*x^3)/3 + (A*b*x^4)/4 + (C*a*
x^4)/4 + (B*b*x^5)/5 + (C*b*x^6)/6
```

3.65 $\int \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal. Leaf size=60

$$aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}(bB+aD)x^4+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

```
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*b+C*a)*x^3+1/4*(B*b+D*a)*x^4+1/5*b*C*x^5+1/6*b*D*x
^6
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = $\{1824\}$

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB)+\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5)
/5 + (b*D*x^6)/6
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) \, dx &= \int \left(aA+aBx+(Ab+aC)x^2+(bB+aD)x^3+bCx^4+bDx^5\right) \\ &= aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}(bB+aD)x^4+\frac{1}{5}bCx^5+\frac{1}{6}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 60, normalized size = 1.00

$$aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}(bB+aD)x^4+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5)
/5 + (b*D*x^6)/6
```

Maple [A]
time = 0.05, size = 51, normalized size = 0.85

method	result	size
default	$aAx+\frac{Ba\,x^{2}}{2}+\frac{(Ab+aC)x^{3}}{3}+\frac{(Bb+aD)x^{4}}{4}+\frac{bC\,x^{5}}{5}+\frac{bD x^{6}}{6}$	51
norman	$\frac{bDx^{6}}{6}+\frac{bC\,x^{5}}{5}+\left(\frac{Bb}{4}+\frac{aD}{4}\right)x^{4}+\left(\frac{Ab}{3}+\frac{aC}{3}\right)x^{3}+\frac{Ba\,x^{2}}{2}+aAx$	53
gosper	$\frac{1}{6}bDx^{6}+\frac{1}{5}bC\,x^{5}+\frac{1}{4}bB\,x^{4}+\frac{1}{4}x^{4}aD+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}x^{3}aC+\frac{1}{2}Ba\,x^{2}+aAx$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] a*A*x+1/2*B*a*x^2+1/3*(A*b+C*a)*x^3+1/4*(B*b+D*a)*x^4+1/5*b*C*x^5+1/6*b*D*x
^6
```

Maxima [A]
time = 0.27, size = 50, normalized size = 0.83

$$\frac{1}{6}\,Dbx^{6}+\frac{1}{5}\,Cbx^{5}+\frac{1}{4}\,(Da+Bb)x^{4}+\frac{1}{2}\,Bax^{2}+\frac{1}{3}\,(Ca+Ab)x^{3}+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*(D*a + B*b)*x^4 + 1/2*B*a*x^2 + 1/3*(C*a +
A*b)*x^3 + A*a*x
```

Fricas [A]
time = 4.94, size = 50, normalized size = 0.83

$$\frac{1}{6}\,Dbx^{6}+\frac{1}{5}\,Cbx^{5}+\frac{1}{4}\,(Da+Bb)x^{4}+\frac{1}{2}\,Bax^{2}+\frac{1}{3}\,(Ca+Ab)x^{3}+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*(D*a + B*b)*x^4 + 1/2*B*a*x^2 + 1/3*(C*a +
A*b)*x^3 + A*a*x
```

Sympy [A]
time = 0.01, size = 56, normalized size = 0.93

$$Aax+\frac{Bax^{2}}{2}+\frac{Cbx^{5}}{5}+\frac{Dbx^{6}}{6}+x^{4}\left(\frac{Bb}{4}+\frac{Da}{4}\right)+x^{3}\left(\frac{Ab}{3}+\frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a*x + B*a*x**2/2 + C*b*x**5/5 + D*b*x**6/6 + x**4*(B*b/4 + D*a/4) + x**3*
(A*b/3 + C*a/3)
```

Giac [A]
time = 1.20, size = 54, normalized size = 0.90

$$\frac{1}{6}\,Dbx^{6}+\frac{1}{5}\,Cbx^{5}+\frac{1}{4}\,Dax^{4}+\frac{1}{4}\,Bbx^{4}+\frac{1}{3}\,Cax^{3}+\frac{1}{3}\,Abx^{3}+\frac{1}{2}\,Bax^{2}+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*D*a*x^4 + 1/4*B*b*x^4 + 1/3*C*a*x^3 + 1/3*A
*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

Mupad [B]
time = 1.16, size = 54, normalized size = 0.90

$$\frac{a\,x^4\,D}{4}+\frac{b\,x^6\,D}{6}+A\,a\,x+\frac{B\,a\,x^2}{2}+\frac{A\,b\,x^3}{3}+\frac{C\,a\,x^3}{3}+\frac{B\,b\,x^4}{4}+\frac{C\,b\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a*x^4*D)/4 + (b*x^6*D)/6 + A*a*x + (B*a*x^2)/2 + (A*b*x^3)/3 + (C*a*x^3)/3
 + (B*b*x^4)/4 + (C*b*x^5)/5
```

3.66 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx$

Optimal. Leaf size=56

$$aBx+\frac{1}{2}(Ab+aC)x^2+\frac{1}{3}(bB+aD)x^3+\frac{1}{4}bCx^4+\frac{1}{5}bDx^5+aA\log(x)$$

```
[Out] a*B*x+1/2*(A*b+C*a)*x^2+1/3*(B*b+D*a)*x^3+1/4*b*C*x^4+1/5*b*D*x^5+a*A*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = $\{1816\}$

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+\frac{1}{3}x^3(aD+bB)+aBx+\frac{1}{4}bCx^4+\frac{1}{5}bDx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 + (b*D*x^5)
/5 + a*A*Log[x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx &= \int \left(aB+\frac{aA}{x}+(Ab+aC)x+(bB+aD)x^2+bCx^3+bDx^4\right) d\\ &= aBx+\frac{1}{2}(Ab+aC)x^2+\frac{1}{3}(bB+aD)x^3+\frac{1}{4}bCx^4+\frac{1}{5}bDx^5+aA\end{aligned}$$

Mathematica [A]
time = 0.01, size = 56, normalized size = 1.00

$$aBx+\frac{1}{2}(Ab+aC)x^2+\frac{1}{3}(bB+aD)x^3+\frac{1}{4}bCx^4+\frac{1}{5}bDx^5+aA\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 + (b*D*x^5)
/5 + a*A*Log[x]
```

Maple [**A**]
time = 0.01, size = 53, normalized size = 0.95

method	result	size
norman	$\left(\frac{Ab}{2}+\frac{aC}{2}\right)x^{2}+\left(\frac{Bb}{3}+\frac{aD}{3}\right)x^{3}+Bax+\frac{bC\,x^{4}}{4}+\frac{bD x^{5}}{5}+aA\ln\left(x\right)$	51
default	$\frac{bD x^{5}}{5}+\frac{bC\,x^{4}}{4}+\frac{bB\,x^{3}}{3}+\frac{Da\,x^{3}}{3}+\frac{Ab\,x^{2}}{2}+\frac{Ca\,x^{2}}{2}+Bax+aA\ln\left(x\right)$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b*D*x^5+1/4*b*C*x^4+1/3*b*B*x^3+1/3*D*a*x^3+1/2*A*b*x^2+1/2*C*a*x^2+B*a
*x+a*A*ln(x)
```

Maxima [**A**]
time = 0.27, size = 48, normalized size = 0.86

$$\frac{1}{5}\,Dbx^{5}+\frac{1}{4}\,Cbx^{4}+\frac{1}{3}\,(Da+Bb)x^{3}+Bax+\frac{1}{2}\,(Ca+Ab)x^{2}+Aa\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

```
[Out] 1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*(D*a + B*b)*x^3 + B*a*x + 1/2*(C*a + A*b)*x
^2 + A*a*log(x)
```

Fricas [**A**]
time = 3.97, size = 48, normalized size = 0.86

$$\frac{1}{5}\,Dbx^{5}+\frac{1}{4}\,Cbx^{4}+\frac{1}{3}\,(Da+Bb)x^{3}+Bax+\frac{1}{2}\,(Ca+Ab)x^{2}+Aa\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

```
[Out] 1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*(D*a + B*b)*x^3 + B*a*x + 1/2*(C*a + A*b)*x
^2 + A*a*log(x)
```

Sympy [**A**]
time = 0.05, size = 54, normalized size = 0.96

$$Aa\log\left(x\right)+Bax+\frac{Cbx^{4}}{4}+\frac{Dbx^{5}}{5}+x^{3}\left(\frac{Bb}{3}+\frac{Da}{3}\right)+x^{2}\left(\frac{Ab}{2}+\frac{Ca}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a*log(x) + B*a*x + C*b*x**4/4 + D*b*x**5/5 + x**3*(B*b/3 + D*a/3) + x**2*
(A*b/2 + C*a/2)
```

Giac [**A**]
time = 1.51, size = 53, normalized size = 0.95

$$\frac{1}{5}Dbx^5 + \frac{1}{4}Cbx^4 + \frac{1}{3}Dax^3 + \frac{1}{3}Bbx^3 + \frac{1}{2}Cax^2 + \frac{1}{2}Abx^2 + Bax + Aa\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

```
[Out] 1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*D*a*x^3 + 1/3*B*b*x^3 + 1/2*C*a*x^2 + 1/2*A
*b*x^2 + B*a*x + A*a*log(abs(x))
```

Mupad [**B**]
time = 1.17, size = 52, normalized size = 0.93

$$\frac{a\,x^3\,D}{3} + \frac{b\,x^5\,D}{5} + B\,a\,x + \frac{A\,b\,x^2}{2} + \frac{C\,a\,x^2}{2} + \frac{B\,b\,x^3}{3} + \frac{C\,b\,x^4}{4} + A\,a\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x,x)
```

```
[Out] (a*x^3*D)/3 + (b*x^5*D)/5 + B*a*x + (A*b*x^2)/2 + (C*a*x^2)/2 + (B*b*x^3)/3
 + (C*b*x^4)/4 + A*a*log(x)
```

3.67 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$

Optimal. Leaf size=54

$$-\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}(bB+aD)x^2+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4+aB\log(x)$$

```
[Out] -a*A/x+(A*b+C*a)*x+1/2*(B*b+D*a)*x^2+1/3*b*C*x^3+1/4*b*D*x^4+a*B*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = $\{1816\}$

$$x(aC+Ab)-\frac{aA}{x}+\frac{1}{2}x^2(aD+bB)+aB\log(x)+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 + (b*D*x^4)/
4 + a*B*Log[x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx &= \int \left(Ab\left(1+\frac{aC}{Ab}\right)+\frac{aA}{x^2}+\frac{aB}{x}+(bB+aD)x+bCx^2+bDx^3\right) \\ &= -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}(bB+aD)x^2+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4+aB\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 1.00

$$-\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}(bB+aD)x^2+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4+aB\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 + (b*D*x^4)/
4 + a*B*Log[x]
```

Maple [A]
time = 0.02, size = 50, normalized size = 0.93

method	result	size
default	$\frac{bD x^4}{4}+\frac{bC x^3}{3}+\frac{bB x^2}{2}+\frac{Da x^2}{2}+Abx+aCx+B\ln\left(x\right)a-\frac{aA}{x}$	50
norman	$\frac{\left(\frac{Bb}{2}+\frac{aD}{2}\right)x^3+\left(Ab+aC\right)x^2-Aa+\frac{bC x^4}{3}+\frac{bD x^5}{4}}{x}+B\ln\left(x\right)a$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b*D*x^4+1/3*b*C*x^3+1/2*b*B*x^2+1/2*D*a*x^2+A*b*x+a*C*x+B*ln(x)*a-a*A/x
```

Maxima [A]
time = 0.27, size = 48, normalized size = 0.89

$$\frac{1}{4}Dbx^4+\frac{1}{3}Cbx^3+\frac{1}{2}(Da+Bb)x^2+Ba\log\left(x\right)+(Ca+Ab)x-\frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*D*b*x^4 + 1/3*C*b*x^3 + 1/2*(D*a + B*b)*x^2 + B*a*log(x) + (C*a + A*b)*
x - A*a/x
```

Fricas [A]
time = 3.62, size = 55, normalized size = 1.02

$$\frac{3\,Dbx^5+4\,Cbx^4+6\,(Da+Bb)x^3+12\,Bax\log\left(x\right)+12\,(Ca+Ab)x^2-12\,Aa}{12\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*D*b*x^5 + 4*C*b*x^4 + 6*(D*a + B*b)*x^3 + 12*B*a*x*log(x) + 12*(C*a
 + A*b)*x^2 - 12*A*a)/x
```

Sympy [A]
time = 0.06, size = 49, normalized size = 0.91

$$-\frac{Aa}{x}+Ba\log\left(x\right)+\frac{Cbx^3}{3}+\frac{Dbx^4}{4}+x^2\left(\frac{Bb}{2}+\frac{Da}{2}\right)+x(Ab+Ca)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a/x + B*a*log(x) + C*b*x**3/3 + D*b*x**4/4 + x**2*(B*b/2 + D*a/2) + x*(A
*b + C*a)
```

Giac [A]
time = 0.96, size = 50, normalized size = 0.93

$$\frac{1}{4}Dbx^4+\frac{1}{3}Cbx^3+\frac{1}{2}Dax^2+\frac{1}{2}Bbx^2+Cax+Abx+Ba\log(|x|)-\frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*D*b*x^4 + 1/3*C*b*x^3 + 1/2*D*a*x^2 + 1/2*B*b*x^2 + C*a*x + A*b*x + B*a
*log(abs(x)) - A*a/x
```

Mupad [B]
time = 1.14, size = 49, normalized size = 0.91

$$\frac{a\,x^2\,D}{2}+\frac{b\,x^4\,D}{4}+A\,b\,x+C\,a\,x-\frac{A\,a}{x}+\frac{B\,b\,x^2}{2}+\frac{C\,b\,x^3}{3}+B\,a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

```
[Out] (a*x^2*D)/2 + (b*x^4*D)/4 + A*b*x + C*a*x - (A*a)/x + (B*b*x^2)/2 + (C*b*x^
3)/3 + B*a*log(x)
```

3.68 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$

Optimal. Leaf size=54

$$-\frac{aA}{2x^2}-\frac{aB}{x}+(bB+aD)x+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3+(Ab+aC)\log(x)$$

```
[Out] -1/2*a*A/x^2-a*B/x+(B*b+D*a)*x+1/2*b*C*x^2+1/3*b*D*x^3+(A*b+C*a)*ln(x)
```

Rubi [A]
time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$,
Rules used = {1816}

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}+x(aD+bB)-\frac{aB}{x}+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -1/2*(a*A)/x^2 - (a*B)/x + (b*B + a*D)*x + (b*C*x^2)/2 + (b*D*x^3)/3 + (A*b
 + a*C)*Log[x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx &= \int \left(bB\left(1+\frac{aD}{bB}\right)+\frac{aA}{x^3}+\frac{aB}{x^2}+\frac{Ab+aC}{x}+bCx+bDx^2\right)\,dx\\ &= -\frac{aA}{2x^2}-\frac{aB}{x}+(bB+aD)x+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3+(Ab+aC)\log(\end{aligned}$$

Mathematica [A]
time = 0.02, size = 51, normalized size = 0.94

$$\frac{1}{6}bx(6B+3Cx+2Dx^2)-\frac{a(A+2Bx-2Dx^3)}{2x^2}+(Ab+aC)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] (b*x*(6*B + 3*C*x + 2*D*x^2))/6 - (a*(A + 2*B*x - 2*D*x^3))/(2*x^2) + (A*b
+ a*C)*Log[x]
```

Maple [A]
time = 0.01, size = 48, normalized size = 0.89

method	result	size
default	$\frac{bDx^3}{3}+\frac{bC\,x^2}{2}+bBx+aDx-\frac{aA}{2x^2}+(Ab+aC)\ln(x)-\frac{aB}{x}$	48
norman	$\frac{(Bb+aD)x^3-\frac{Aa}{2}-Bax+\frac{bC\,x^4}{2}+\frac{bD\,x^5}{3}}{x^2}+(Ab+aC)\ln(x)$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*b*D*x^3+1/2*b*C*x^2+b*B*x+a*D*x-1/2*a*A/x^2+(A*b+C*a)*ln(x)-a*B/x
```

Maxima [A]
time = 0.27, size = 48, normalized size = 0.89

$$\frac{1}{3}\,Dbx^3+\frac{1}{2}\,Cbx^2+(Da+Bb)x+(Ca+Ab)\log(x)-\frac{2\,Bax+Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/3*D*b*x^3 + 1/2*C*b*x^2 + (D*a + B*b)*x + (C*a + A*b)*log(x) - 1/2*(2*B*a
*x + A*a)/x^2
```

Fricas [A]
time = 8.28, size = 55, normalized size = 1.02

$$\frac{2\,Dbx^5+3\,Cbx^4+6\,(Da+Bb)x^3+6\,(Ca+Ab)x^2\log(x)-6\,Bax-3\,Aa}{6\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*D*b*x^5 + 3*C*b*x^4 + 6*(D*a + B*b)*x^3 + 6*(C*a + A*b)*x^2*log(x) -
 6*B*a*x - 3*A*a)/x^2
```

Sympy [A]
time = 0.13, size = 51, normalized size = 0.94

$$\frac{Cbx^2}{2}+\frac{Dbx^3}{3}+x(Bb+Da)+(Ab+Ca)\log(x)+\frac{-Aa-2Bax}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] C*b*x**2/2 + D*b*x**3/3 + x*(B*b + D*a) + (A*b + C*a)*log(x) + (-A*a - 2*B*
a*x)/(2*x**2)
```

Giac [A]
time = 1.02, size = 48, normalized size = 0.89

$$\frac{1}{3}\,Dbx^{3}+\frac{1}{2}\,Cbx^{2}+Dax+Bbx+(Ca+Ab)\log\left(|x|\right)-\frac{2\,Bax+Aa}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/3*D*b*x^3 + 1/2*C*b*x^2 + D*a*x + B*b*x + (C*a + A*b)*log(abs(x)) - 1/2*(
2*B*a*x + A*a)/x^2
```

Mupad [B]
time = 1.14, size = 47, normalized size = 0.87

$$\frac{b\,x^{3}\,D}{3}+B\,b\,x-\frac{A\,a}{2\,x^{2}}-\frac{B\,a}{x}+\frac{C\,b\,x^{2}}{2}+A\,b\,\ln\left(x\right)+C\,a\,\ln\left(x\right)+a\,x\,D$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

```
[Out] (b*x^3*D)/3 + B*b*x - (A*a)/(2*x^2) - (B*a)/x + (C*b*x^2)/2 + A*b*log(x) +
C*a*log(x) + a*x*D
```

3.69 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$

Optimal. Leaf size=54

$$-\frac{aA}{3x^3}-\frac{aB}{2x^2}-\frac{Ab+aC}{x}+bCx+\frac{1}{2}bDx^2+(bB+aD)\log(x)$$

```
[Out] -1/3*a*A/x^3-1/2*a*B/x^2+(-A*b-C*a)/x+b*C*x+1/2*b*D*x^2+(B*b+D*a)*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1816}

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}+\log(x)(aD+bB)-\frac{aB}{2x^2}+bCx+\frac{1}{2}bDx^2$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -1/3*(a*A)/x^3 - (a*B)/(2*x^2) - (A*b + a*C)/x + b*C*x + (b*D*x^2)/2 + (b*B
 + a*D)*Log[x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx &= \int \left(bC+\frac{aA}{x^4}+\frac{aB}{x^3}+\frac{Ab+aC}{x^2}+\frac{bB+aD}{x}+bDx\right)dx\\ &= -\frac{aA}{3x^3}-\frac{aB}{2x^2}-\frac{Ab+aC}{x}+bCx+\frac{1}{2}bDx^2+(bB+aD)\log(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 55, normalized size = 1.02

$$-\frac{aA}{3x^3}-\frac{aB}{2x^2}+\frac{-Ab-aC}{x}+bCx+\frac{1}{2}bDx^2+(bB+aD)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -1/3*(a*A)/x^3 - (a*B)/(2*x^2) + (-(A*b) - a*C)/x + b*C*x + (b*D*x^2)/2 + (
b*B + a*D)*Log[x]
```

Maple [A]
time = 0.01, size = 49, normalized size = 0.91

method	result	size
default	$\frac{bDx^2}{2}+bCx-\frac{aB}{2x^2}-\frac{aA}{3x^3}+(Bb+aD)\ln(x)-\frac{Ab+aC}{x}$	49
norman	$\frac{(-Ab-aC)x^2+bC\,x^4-\frac{Aa}{3}-\frac{Bax}{2}+\frac{bDx^5}{2}}{x^3}+(Bb+aD)\ln(x)$	52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*b*D*x^2+b*C*x-1/2*a*B/x^2-1/3*a*A/x^3+(B*b+D*a)*ln(x)-(A*b+C*a)/x
```

Maxima [A]
time = 0.27, size = 49, normalized size = 0.91

$$\frac{1}{2}\,Dbx^2+Cbx+(Da+Bb)\log(x)-\frac{3\,Bax+6\,(Ca+Ab)x^2+2\,Aa}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/2*D*b*x^2 + C*b*x + (D*a + B*b)*log(x) - 1/6*(3*B*a*x + 6*(C*a + A*b)*x^2
 + 2*A*a)/x^3
```

Fricas [A]
time = 6.77, size = 55, normalized size = 1.02

$$\frac{3\,Dbx^5+6\,Cbx^4+6\,(Da+Bb)x^3\log(x)-3\,Bax-6\,(Ca+Ab)x^2-2\,Aa}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(3*D*b*x^5 + 6*C*b*x^4 + 6*(D*a + B*b)*x^3*log(x) - 3*B*a*x - 6*(C*a +
A*b)*x^2 - 2*A*a)/x^3
```

Sympy [A]
time = 0.38, size = 54, normalized size = 1.00

$$Cbx+\frac{Dbx^2}{2}+(Bb+Da)\log(x)+\frac{-2Aa-3Bax+x^2(-6Ab-6Ca)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] C*b*x + D*b*x**2/2 + (B*b + D*a)*log(x) + (-2*A*a - 3*B*a*x + x**2*(-6*A*b
- 6*C*a))/(6*x**3)
```

Giac [A]
time = 1.20, size = 50, normalized size = 0.93

$$\frac{1}{2}\,Dbx^2+Cbx+(Da+Bb)\log\left(|x|\right)-\frac{3\,Bax+6\,(Ca+Ab)x^2+2\,Aa}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/2*D*b*x^2 + C*b*x + (D*a + B*b)*log(abs(x)) - 1/6*(3*B*a*x + 6*(C*a + A*b
)*x^2 + 2*A*a)/x^3
```

Mupad [B]
time = 1.15, size = 50, normalized size = 0.93

$$\frac{b\,x^2\,D}{2}+a\,\ln\left(x\right)\,D+C\,b\,x-\frac{A\,a}{3\,x^3}-\frac{A\,b}{x}-\frac{B\,a}{2\,x^2}-\frac{C\,a}{x}+B\,b\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

```
[Out] (b*x^2*D)/2 + a*log(x)*D + C*b*x - (A*a)/(3*x^3) - (A*b)/x - (B*a)/(2*x^2)
- (C*a)/x + B*b*log(x)
```

3.70 $\int x^3(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=109

$$\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{6}a(2Ab+aC)x^6+\frac{1}{7}a(2bB+aD)x^7+\frac{1}{8}b(Ab+2aC)x^8+\frac{1}{9}b(bB+2aD)x^9+\frac{1}{10}b^2Cx^{10}+\frac{1}{11}b^2D$$

```
[Out] 1/4*a^2*A*x^4+1/5*a^2*B*x^5+1/6*a*(2*A*b+C*a)*x^6+1/7*a*(2*B*b+D*a)*x^7+1/8
*b*(A*b+2*C*a)*x^8+1/9*b*(B*b+2*D*a)*x^9+1/10*b^2*C*x^10+1/11*b^2*D*x^11
```

Rubi [A]
time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{1816\}$

$$\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{8}bx^8(2aC+Ab)+\frac{1}{6}ax^6(aC+2Ab)+\frac{1}{9}bx^9(2aD+bB)+\frac{1}{7}ax^7(aD+2bB)+\frac{1}{10}b^2Cx^{10}+\frac{1}{11}b^2Dx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*A*x^4)/4 + (a^2*B*x^5)/5 + (a*(2*A*b + a*C)*x^6)/6 + (a*(2*b*B + a*D)*
x^7)/7 + (b*(A*b + 2*a*C)*x^8)/8 + (b*(b*B + 2*a*D)*x^9)/9 + (b^2*C*x^10)/1
0 + (b^2*D*x^11)/11
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \int\left(a^2Ax^3+a^2Bx^4+a(2Ab+aC)x^5+a(2bB+aD)x^6+b(\right.\\ &= \frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{6}a(2Ab+aC)x^6+\frac{1}{7}a(2bB+aD)x^7+\end{aligned}$$

Mathematica [A]
time = 0.03, size = 98, normalized size = 0.90

$$a^2\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}+\frac{1}{42}x^6(7C+6Dx)\right)+\frac{b^2x^8(495A+4x(110B+99Cx+90Dx^2))}{3960}+\frac{1}{252}abx^6(84A+x(72B+7x(9C+8Dx)))$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a^2*((A*x^4)/4 + (B*x^5)/5 + (x^6*(7*C + 6*D*x))/42) + (b^2*x^8*(495*A + 4*
x*(110*B + 99*C*x + 90*D*x^2)))/3960 + (a*b*x^6*(84*A + x*(72*B + 7*x*(9*C
+ 8*D*x))))/252
```

Maple [A]
time = 0.10, size = 102, normalized size = 0.94

method	result
default	$\frac{b^2Dx^{11}}{11}+\frac{b^2C\,x^{10}}{10}+\frac{(b^2B+2abD)x^9}{9}+\frac{(b^2A+2abC)x^8}{8}+\frac{(2abB+a^2D)x^7}{7}+\frac{(2abA+a^2C)x^6}{6}+\frac{a^2B\,x^5}{5}+\frac{a^2A\,x^4}{4}$
norman	$\frac{b^2Dx^{11}}{11}+\frac{b^2C\,x^{10}}{10}+\left(\frac{1}{9}b^2B+\frac{2}{9}abD\right)x^9+\left(\frac{1}{8}b^2A+\frac{1}{4}abC\right)x^8+\left(\frac{2}{7}abB+\frac{1}{7}a^2D\right)x^7+\left(\frac{1}{3}abA+\frac{1}{6}a^2$
gosper	$\frac{1}{11}b^2Dx^{11}+\frac{1}{10}b^2C\,x^{10}+\frac{1}{9}b^2B\,x^9+\frac{2}{9}x^9abD+\frac{1}{8}x^8b^2A+\frac{1}{4}x^8abC+\frac{2}{7}x^7abB+\frac{1}{7}x^7a^2D+\frac{1}{3}x^6ab$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^2*D*x^11+1/10*b^2*C*x^10+1/9*(B*b^2+2*D*a*b)*x^9+1/8*(A*b^2+2*C*a*b)
*x^8+1/7*(2*B*a*b+D*a^2)*x^7+1/6*(2*A*a*b+C*a^2)*x^6+1/5*a^2*B*x^5+1/4*a^2*
A*x^4
```

Maxima [A]
time = 0.27, size = 101, normalized size = 0.93

$$\frac{1}{11}\,Db^2x^{11}+\frac{1}{10}\,Cb^2x^{10}+\frac{1}{9}\,(2\,Dab+Bb^2)x^9+\frac{1}{8}\,(2\,Cab+Ab^2)x^8+\frac{1}{5}\,Ba^2x^5+\frac{1}{7}\,(Da^2+2\,Bab)x^7+\frac{1}{4}\,Aa^2x^4+\frac{1}{6}\,(Ca^2+2\,Aab)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 1/9*(2*D*a*b + B*b^2)*x^9 + 1/8*(2*C*a*
b + A*b^2)*x^8 + 1/5*B*a^2*x^5 + 1/7*(D*a^2 + 2*B*a*b)*x^7 + 1/4*A*a^2*x^4
+ 1/6*(C*a^2 + 2*A*a*b)*x^6
```

Fricas [A]
time = 3.35, size = 101, normalized size = 0.93

$$\frac{1}{11}\,Db^2x^{11}+\frac{1}{10}\,Cb^2x^{10}+\frac{1}{9}\,(2\,Dab+Bb^2)x^9+\frac{1}{8}\,(2\,Cab+Ab^2)x^8+\frac{1}{5}\,Ba^2x^5+\frac{1}{7}\,(Da^2+2\,Bab)x^7+\frac{1}{4}\,Aa^2x^4+\frac{1}{6}\,(Ca^2+2\,Aab)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 1/9*(2*D*a*b + B*b^2)*x^9 + 1/8*(2*C*a*
b + A*b^2)*x^8 + 1/5*B*a^2*x^5 + 1/7*(D*a^2 + 2*B*a*b)*x^7 + 1/4*A*a^2*x^4
+ 1/6*(C*a^2 + 2*A*a*b)*x^6
```

Sympy [A]
time = 0.02, size = 110, normalized size = 1.01

$$\frac{Aa^2x^4}{4}+\frac{Ba^2x^5}{5}+\frac{Cb^2x^{10}}{10}+\frac{Db^2x^{11}}{11}+x^9\left(\frac{Bb^2}{9}+\frac{2Dab}{9}\right)+x^8\left(\frac{Ab^2}{8}+\frac{Cab}{4}\right)+x^7\cdot\left(\frac{2Bab}{7}+\frac{Da^2}{7}\right)+x^6\left(\frac{Aab}{3}+\frac{Ca^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**4/4 + B*a**2*x**5/5 + C*b**2*x**10/10 + D*b**2*x**11/11 + x**9*(B
*b**2/9 + 2*D*a*b/9) + x**8*(A*b**2/8 + C*a*b/4) + x**7*(2*B*a*b/7 + D*a**2
/7) + x**6*(A*a*b/3 + C*a**2/6)
```

Giac [A]
time = 0.91, size = 105, normalized size = 0.96

$$\frac{1}{11}Db^2x^{11}+\frac{1}{10}Cb^2x^{10}+\frac{2}{9}Dabx^9+\frac{1}{9}Bb^2x^9+\frac{1}{4}Cabx^8+\frac{1}{8}Ab^2x^8+\frac{1}{7}Da^2x^7+\frac{2}{7}Babx^7+\frac{1}{6}Ca^2x^6+\frac{1}{3}Aabx^6+\frac{1}{5}Ba^2x^5+\frac{1}{4}Aa^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 2/9*D*a*b*x^9 + 1/9*B*b^2*x^9 + 1/4*C*a
*b*x^8 + 1/8*A*b^2*x^8 + 1/7*D*a^2*x^7 + 2/7*B*a*b*x^7 + 1/6*C*a^2*x^6 + 1/
3*A*a*b*x^6 + 1/5*B*a^2*x^5 + 1/4*A*a^2*x^4
```

Mupad [B]
time = 1.13, size = 108, normalized size = 0.99

$$\frac{a^2\,x^7\,D}{7}+\frac{b^2\,x^{11}\,D}{11}+\frac{A\,x^4\,\left(6\,a^2+8\,a\,b\,x^2+3\,b^2\,x^4\right)}{24}+\frac{B\,x^5\,\left(63\,a^2+90\,a\,b\,x^2+35\,b^2\,x^4\right)}{315}+\frac{C\,x^6\,\left(10\,a^2+15\,a\,b\,x^2+6\,b^2\,x^4\right)}{60}+\frac{2\,a\,b\,x^9\,D}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a^2*x^7*D)/7 + (b^2*x^11*D)/11 + (A*x^4*(6*a^2 + 3*b^2*x^4 + 8*a*b*x^2))/2
4 + (B*x^5*(63*a^2 + 35*b^2*x^4 + 90*a*b*x^2))/315 + (C*x^6*(10*a^2 + 6*b^2
*x^4 + 15*a*b*x^2))/60 + (2*a*b*x^9*D)/9
```

3.71 $\int x^2(a+bx^2)^2 (A+Bx+Cx^2+Dx^3) \, dx$

Optimal. Leaf size=109

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{6}a(2bB+aD)x^6+\frac{1}{7}b(Ab+2aC)x^7+\frac{1}{8}b(bB+2aD)x^8+\frac{1}{9}b^2Cx^9+\frac{1}{10}b^2D$$

```
[Out] 1/3*a^2*A*x^3+1/4*a^2*B*x^4+1/5*a*(2*A*b+C*a)*x^5+1/6*a*(2*B*b+D*a)*x^6+1/7
*b*(A*b+2*C*a)*x^7+1/8*b*(B*b+2*D*a)*x^8+1/9*b^2*C*x^9+1/10*b^2*D*x^10
```

Rubi [A]
time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{1816\}$

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{7}bx^7(2aC+Ab)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}bx^8(2aD+bB)+\frac{1}{6}ax^6(aD+2bB)+\frac{1}{9}b^2Cx^9+\frac{1}{10}b^2Dx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*(2*b*B + a*D)*
x^6)/6 + (b*(A*b + 2*a*C)*x^7)/7 + (b*(b*B + 2*a*D)*x^8)/8 + (b^2*C*x^9)/9
+ (b^2*D*x^10)/10
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \int \left(a^2Ax^2+a^2Bx^3+a(2Ab+aC)x^4+a(2bB+aD)x^5+b\right.\\ &= \frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{6}a(2bB+aD)x^6+\end{aligned}$$

Mathematica [A]
time = 0.04, size = 92, normalized size = 0.84

$$\frac{42a^2x^3(20A+x(15B+2x(6C+5Dx)))+6abx^5(168A+5x(28B+3x(8C+7Dx)))+b^2x^7(360A+7x(45B+4x(10C+9Dx)))}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (42*a^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))) + 6*a*b*x^5*(168*A + 5*x*
(28*B + 3*x*(8*C + 7*D*x))) + b^2*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*
x))))/2520
```

Maple [A]
time = 0.10, size = 102, normalized size = 0.94

method	result
default	$\frac{b^2 D x^{10}}{10}+\frac{b^2 C\,x^9}{9}+\frac{(b^2 B+2abD)x^8}{8}+\frac{(b^2 A+2abC)x^7}{7}+\frac{(2abB+a^2 D)x^6}{6}+\frac{(2abA+a^2 C)x^5}{5}+\frac{a^2 B\,x^4}{4}+\frac{a^2 A\,x^3}{3}$
norman	$\frac{b^2 D x^{10}}{10}+\frac{b^2 C\,x^9}{9}+\left(\frac{1}{8}b^2 B+\frac{1}{4}abD\right)x^8+\left(\frac{1}{7}b^2 A+\frac{2}{7}abC\right)x^7+\left(\frac{1}{3}abB+\frac{1}{6}a^2 D\right)x^6+\left(\frac{2}{5}abA+\frac{1}{5}a^2 C\right)$
gosper	$\frac{1}{10}b^2 D x^{10}+\frac{1}{9}b^2 C\,x^9+\frac{1}{8}b^2 B\,x^8+\frac{1}{4}x^8 abD+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 abC+\frac{1}{3}x^6 abB+\frac{1}{6}x^6 a^2 D+\frac{2}{5}x^5 abA+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^2*D*x^10+1/9*b^2*C*x^9+1/8*(B*b^2+2*D*a*b)*x^8+1/7*(A*b^2+2*C*a*b)*x
^7+1/6*(2*B*a*b+D*a^2)*x^6+1/5*(2*A*a*b+C*a^2)*x^5+1/4*a^2*B*x^4+1/3*a^2*A*
x^3
```

Maxima [A]
time = 0.28, size = 101, normalized size = 0.93

$$\frac{1}{10}\,Db^2x^{10}+\frac{1}{9}\,Cb^2x^9+\frac{1}{8}\,(2\,Dab+Bb^2)x^8+\frac{1}{7}\,(2\,Cab+Ab^2)x^7+\frac{1}{4}\,Ba^2x^4+\frac{1}{6}\,(Da^2+2\,Bab)x^6+\frac{1}{3}\,Aa^2x^3+\frac{1}{5}\,(Ca^2+2\,Aab)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/8*(2*D*a*b + B*b^2)*x^8 + 1/7*(2*C*a*b
+ A*b^2)*x^7 + 1/4*B*a^2*x^4 + 1/6*(D*a^2 + 2*B*a*b)*x^6 + 1/3*A*a^2*x^3 +
1/5*(C*a^2 + 2*A*a*b)*x^5
```

Fricas [A]
time = 6.00, size = 101, normalized size = 0.93

$$\frac{1}{10}\,Db^2x^{10}+\frac{1}{9}\,Cb^2x^9+\frac{1}{8}\,(2\,Dab+Bb^2)x^8+\frac{1}{7}\,(2\,Cab+Ab^2)x^7+\frac{1}{4}\,Ba^2x^4+\frac{1}{6}\,(Da^2+2\,Bab)x^6+\frac{1}{3}\,Aa^2x^3+\frac{1}{5}\,(Ca^2+2\,Aab)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/8*(2*D*a*b + B*b^2)*x^8 + 1/7*(2*C*a*b
+ A*b^2)*x^7 + 1/4*B*a^2*x^4 + 1/6*(D*a^2 + 2*B*a*b)*x^6 + 1/3*A*a^2*x^3 +
1/5*(C*a^2 + 2*A*a*b)*x^5
```

Sympy [A]
time = 0.02, size = 110, normalized size = 1.01

$$\frac{Aa^2x^3}{3} + \frac{Ba^2x^4}{4} + \frac{Cb^2x^9}{9} + \frac{Db^2x^{10}}{10} + x^8\left(\frac{Bb^2}{8} + \frac{Dab}{4}\right) + x^7\left(\frac{Ab^2}{7} + \frac{2Cab}{7}\right) + x^6\left(\frac{Bab}{3} + \frac{Da^2}{6}\right) + x^5 \cdot \left(\frac{2Aab}{5} + \frac{Ca^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**3/3 + B*a**2*x**4/4 + C*b**2*x**9/9 + D*b**2*x**10/10 + x**8*(B*b
**2/8 + D*a*b/4) + x**7*(A*b**2/7 + 2*C*a*b/7) + x**6*(B*a*b/3 + D*a**2/6)
+ x**5*(2*A*a*b/5 + C*a**2/5)
```

Giac [A]
time = 0.87, size = 105, normalized size = 0.96

$$\frac{1}{10}\,Db^2x^{10} + \frac{1}{9}\,Cb^2x^9 + \frac{1}{4}\,Dabx^8 + \frac{1}{8}\,Bb^2x^8 + \frac{2}{7}\,Cabx^7 + \frac{1}{7}\,Ab^2x^7 + \frac{1}{6}\,Da^2x^6 + \frac{1}{3}\,Babx^6 + \frac{1}{5}\,Ca^2x^5 + \frac{2}{5}\,Aabx^5 + \frac{1}{4}\,Ba^2x^4 + \frac{1}{3}\,Aa^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/4*D*a*b*x^8 + 1/8*B*b^2*x^8 + 2/7*C*a*b
*x^7 + 1/7*A*b^2*x^7 + 1/6*D*a^2*x^6 + 1/3*B*a*b*x^6 + 1/5*C*a^2*x^5 + 2/5*
A*a*b*x^5 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3
```

Mupad [B]
time = 1.11, size = 108, normalized size = 0.99

$$\frac{a^2\,x^6\,D}{6} + \frac{b^2\,x^{10}\,D}{10} + \frac{A\,x^3\,\left(35\,a^2 + 42\,a\,b\,x^2 + 15\,b^2\,x^4\right)}{105} + \frac{B\,x^4\,\left(6\,a^2 + 8\,a\,b\,x^2 + 3\,b^2\,x^4\right)}{24} + \frac{C\,x^5\,\left(63\,a^2 + 90\,a\,b\,x^2 + 35\,b^2\,x^4\right)}{315} + \frac{a\,b\,x^8\,D}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a^2*x^6*D)/6 + (b^2*x^10*D)/10 + (A*x^3*(35*a^2 + 15*b^2*x^4 + 42*a*b*x^2)
)/105 + (B*x^4*(6*a^2 + 3*b^2*x^4 + 8*a*b*x^2))/24 + (C*x^5*(63*a^2 + 35*b^
2*x^4 + 90*a*b*x^2))/315 + (a*b*x^8*D)/4
```

3.72 $\int x(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=104

$$\frac{1}{3}a^2Bx^3+\frac{1}{4}a^2Cx^4+\frac{1}{5}a(2bB+aD)x^5+\frac{1}{3}abCx^6+\frac{1}{7}b(bB+2aD)x^7+\frac{1}{8}b^2Cx^8+\frac{1}{9}b^2Dx^9+\frac{A(a+bx^2)^3}{6b}$$

```
[Out] 1/3*a^2*B*x^3+1/4*a^2*C*x^4+1/5*a*(2*B*b+D*a)*x^5+1/3*a*b*C*x^6+1/7*b*(B*b+
2*D*a)*x^7+1/8*b^2*C*x^8+1/9*b^2*D*x^9+1/6*A*(b*x^2+a)^3/b
```

Rubi [A]
time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1596, 1824}

$$\frac{1}{3}a^2Bx^3+\frac{1}{4}a^2Cx^4+\frac{A(a+bx^2)^3}{6b}+\frac{1}{7}bx^7(2aD+bB)+\frac{1}{5}ax^5(aD+2bB)+\frac{1}{3}abCx^6+\frac{1}{8}b^2Cx^8+\frac{1}{9}b^2Dx^9$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^2*B*x^3)/3 + (a^2*C*x^4)/4 + (a*(2*b*B + a*D)*x^5)/5 + (a*b*C*x^6)/3 + (
b*(b*B + 2*a*D)*x^7)/7 + (b^2*C*x^8)/8 + (b^2*D*x^9)/9 + (A*(a + b*x^2)^3)/
(6*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int x(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\, dx = \frac{A(a+bx^2)^3}{6b} + \int (a+bx^2)^2 (-Ax + x(A+Bx+Cx^2+Dx$$
$$= \frac{A(a+bx^2)^3}{6b} + \int (a^2Bx^2 + a^2Cx^3 + a(2bB+aD)x^4 + 2abC$$
$$= \frac{1}{3}a^2Bx^3 + \frac{1}{4}a^2Cx^4 + \frac{1}{5}a(2bB+aD)x^5 + \frac{1}{3}abCx^6 + \frac{1}{7}b(bB +$$

Mathematica [A]
time = 0.03, size = 92, normalized size = 0.88

$$\frac{42a^2x^2(30A + x(20B + 3x(5C + 4Dx))) + 12abx^4(105A + 2x(42B + 5x(7C + 6Dx))) + 5b^2x^6(84A + x(72B + 7x(9C + 8Dx)))}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (42*a^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))) + 12*a*b*x^4*(105*A + 2*x
*(42*B + 5*x*(7*C + 6*D*x))) + 5*b^2*x^6*(84*A + x*(72*B + 7*x*(9*C + 8*D*x
))))/2520
```

Maple [A]
time = 0.10, size = 102, normalized size = 0.98

method	result
default	$\frac{b^2Dx^9}{9} + \frac{b^2C\,x^8}{8} + \frac{(b^2B+2abD)x^7}{7} + \frac{(b^2A+2abC)x^6}{6} + \frac{(2abB+a^2D)x^5}{5} + \frac{(2abA+a^2C)x^4}{4} + \frac{a^2B\,x^3}{3} + \frac{a^2A\,x^2}{2}$
norman	$\frac{b^2Dx^9}{9} + \frac{b^2C\,x^8}{8} + (\frac{1}{7}b^2B + \frac{2}{7}abD)\,x^7 + (\frac{1}{6}b^2A + \frac{1}{3}abC)\,x^6 + (\frac{2}{5}abB + \frac{1}{5}a^2D)\,x^5 + (\frac{1}{2}abA + \frac{1}{4}a^2C$
gosper	$\frac{1}{9}b^2Dx^9 + \frac{1}{8}b^2C\,x^8 + \frac{1}{7}b^2B\,x^7 + \frac{2}{7}x^7abD + \frac{1}{6}x^6b^2A + \frac{1}{3}abC\,x^6 + \frac{2}{5}x^5abB + \frac{1}{5}x^5a^2D + \frac{1}{2}x^4abA +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^2*D*x^9+1/8*b^2*C*x^8+1/7*(B*b^2+2*D*a*b)*x^7+1/6*(A*b^2+2*C*a*b)*x^6
+1/5*(2*B*a*b+D*a^2)*x^5+1/4*(2*A*a*b+C*a^2)*x^4+1/3*a^2*B*x^3+1/2*a^2*A*x^
2
```

Maxima [A]
time = 0.30, size = 101, normalized size = 0.97

$$\frac{1}{9}\,Db^2x^9 + \frac{1}{8}\,Cb^2x^8 + \frac{1}{7}\,(2\,Dab + Bb^2)x^7 + \frac{1}{6}\,(2\,Cab + Ab^2)x^6 + \frac{1}{3}\,Ba^2x^3 + \frac{1}{5}\,(Da^2 + 2\,Bab)x^5 + \frac{1}{2}\,Aa^2x^2 + \frac{1}{4}\,(Ca^2 + 2\,Aab)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 1/7*(2*D*a*b + B*b^2)*x^7 + 1/6*(2*C*a*b +
A*b^2)*x^6 + 1/3*B*a^2*x^3 + 1/5*(D*a^2 + 2*B*a*b)*x^5 + 1/2*A*a^2*x^2 + 1/
4*(C*a^2 + 2*A*a*b)*x^4
```

Fricas [A]
time = 3.19, size = 101, normalized size = 0.97

$$\frac{1}{9}Db^2x^9+\frac{1}{8}Cb^2x^8+\frac{1}{7}\left(2\,Dab+Bb^2\right)x^7+\frac{1}{6}\left(2\,Cab+Ab^2\right)x^6+\frac{1}{3}Ba^2x^3+\frac{1}{5}\left(Da^2+2\,Bab\right)x^5+\frac{1}{2}Aa^2x^2+\frac{1}{4}\left(Ca^2+2\,Aab\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 1/7*(2*D*a*b + B*b^2)*x^7 + 1/6*(2*C*a*b +
A*b^2)*x^6 + 1/3*B*a^2*x^3 + 1/5*(D*a^2 + 2*B*a*b)*x^5 + 1/2*A*a^2*x^2 + 1/
4*(C*a^2 + 2*A*a*b)*x^4
```

Sympy [A]
time = 0.02, size = 110, normalized size = 1.06

$$\frac{Aa^2x^2}{2}+\frac{Ba^2x^3}{3}+\frac{Cb^2x^8}{8}+\frac{Db^2x^9}{9}+x^7\left(\frac{Bb^2}{7}+\frac{2Dab}{7}\right)+x^6\left(\frac{Ab^2}{6}+\frac{Cab}{3}\right)+x^5\cdot\left(\frac{2Bab}{5}+\frac{Da^2}{5}\right)+x^4\left(\frac{Aab}{2}+\frac{Ca^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x**2/2 + B*a**2*x**3/3 + C*b**2*x**8/8 + D*b**2*x**9/9 + x**7*(B*b**
2/7 + 2*D*a*b/7) + x**6*(A*b**2/6 + C*a*b/3) + x**5*(2*B*a*b/5 + D*a**2/5)
 + x**4*(A*a*b/2 + C*a**2/4)
```

Giac [A]
time = 1.16, size = 105, normalized size = 1.01

$$\frac{1}{9}Db^2x^9+\frac{1}{8}Cb^2x^8+\frac{2}{7}Dabx^7+\frac{1}{7}Bb^2x^7+\frac{1}{3}Cabx^6+\frac{1}{6}Ab^2x^6+\frac{1}{5}Da^2x^5+\frac{2}{5}Babx^5+\frac{1}{4}Ca^2x^4+\frac{1}{2}Aabx^4+\frac{1}{3}Ba^2x^3+\frac{1}{2}Aa^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 2/7*D*a*b*x^7 + 1/7*B*b^2*x^7 + 1/3*C*a*b*x
^6 + 1/6*A*b^2*x^6 + 1/5*D*a^2*x^5 + 2/5*B*a*b*x^5 + 1/4*C*a^2*x^4 + 1/2*A*
a*b*x^4 + 1/3*B*a^2*x^3 + 1/2*A*a^2*x^2
```

Mupad [B]
time = 1.11, size = 107, normalized size = 1.03

$$\frac{a^2\,x^5\,D}{5}+\frac{b^2\,x^9\,D}{9}+\frac{A\,x^2\,\left(3\,a^2+3\,a\,b\,x^2+b^2\,x^4\right)}{6}+\frac{B\,x^3\,\left(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4\right)}{105}+\frac{C\,x^4\,\left(6\,a^2+8\,a\,b\,x^2+3\,b^2\,x^4\right)}{24}+\frac{2\,a\,b\,x^7\,D}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (a^2*x^5*D)/5 + (b^2*x^9*D)/9 + (A*x^2*(3*a^2 + b^2*x^4 + 3*a*b*x^2))/6 + (
B*x^3*(35*a^2 + 15*b^2*x^4 + 42*a*b*x^2))/105 + (C*x^4*(6*a^2 + 3*b^2*x^4 +
 8*a*b*x^2))/24 + (2*a*b*x^7*D)/7
```

3.73 $\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx$

Optimal. Leaf size=99

$$a^2Ax+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{4}a^2Dx^4+\frac{1}{5}b(Ab+2aC)x^5+\frac{1}{3}abDx^6+\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8+\frac{B\left(a+bx^2\right)^3}{6b}$$

```
[Out] a^2*A*x+1/3*a*(2*A*b+C*a)*x^3+1/4*a^2*D*x^4+1/5*b*(A*b+2*C*a)*x^5+1/3*a*b*D
*x^6+1/7*b^2*C*x^7+1/8*b^2*D*x^8+1/6*B*(b*x^2+a)^3/b
```

Rubi [A]
time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = $\{1596, 1824\}$

$$a^2Ax+\frac{1}{4}a^2Dx^4+\frac{1}{5}bx^5(2aC+Ab)+\frac{1}{3}ax^3(aC+2Ab)+\frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{3}abDx^6+\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a^2*A*x + (a*(2*A*b + a*C)*x^3)/3 + (a^2*D*x^4)/4 + (b*(A*b + 2*a*C)*x^5)/5
 + (a*b*D*x^6)/3 + (b^2*C*x^7)/7 + (b^2*D*x^8)/8 + (B*(a + b*x^2)^3)/(6*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned} \int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx &= \frac{B\left(a+bx^2\right)^3}{6b}+\int \left(a+bx^2\right)^2 \left(A+Cx^2+Dx^3\right) \, dx \\ &= \frac{B\left(a+bx^2\right)^3}{6b}+\int \left(a^2A+a(2Ab+aC)x^2+a^2Dx^3+b(Ab+2a \\ &= a^2Ax+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{4}a^2Dx^4+\frac{1}{5}b(Ab+2aC)x^5+\frac{1}{3}abD \end{aligned}$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 0.89

$$\frac{1}{840}(70a^2x(12A+x(6B+x(4C+3Dx)))+28abx^3(20A+x(15B+2x(6C+5Dx)))+b^2x^5(168A+5x(28B+3x(8C+7Dx))))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (70*a^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 28*a*b*x^3*(20*A + x*(15*B +
 2*x*(6*C + 5*D*x))) + b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))))/84
0
```

Maple [A]
time = 0.10, size = 99, normalized size = 1.00

method	result
default	$\frac{b^2Dx^8}{8}+\frac{b^2C\,x^7}{7}+\frac{(b^2B+2abD)x^6}{6}+\frac{(b^2A+2abC)x^5}{5}+\frac{(2abB+a^2D)x^4}{4}+\frac{(2abA+a^2C)x^3}{3}+\frac{B\,a^2x^2}{2}+a^2Ax$
norman	$\frac{b^2Dx^8}{8}+\frac{b^2C\,x^7}{7}+\big(\frac{1}{6}b^2B+\frac{1}{3}abD\big)\,x^6+\big(\frac{1}{5}b^2A+\frac{2}{5}abC\big)\,x^5+\big(\frac{1}{2}abB+\frac{1}{4}a^2D\big)\,x^4+\big(\frac{2}{3}abA+\frac{1}{3}a^2C$
gosper	$\frac{1}{8}b^2Dx^8+\frac{1}{7}b^2C\,x^7+\frac{1}{6}b^2B\,x^6+\frac{1}{3}abDx^6+\frac{1}{5}x^5b^2A+\frac{2}{5}x^5abC+\frac{1}{2}Bab\,x^4+\frac{1}{4}a^2Dx^4+\frac{2}{3}x^3abA+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b^2*D*x^8+1/7*b^2*C*x^7+1/6*(B*b^2+2*D*a*b)*x^6+1/5*(A*b^2+2*C*a*b)*x^5
+1/4*(2*B*a*b+D*a^2)*x^4+1/3*(2*A*a*b+C*a^2)*x^3+1/2*B*a^2*x^2+a^2*A*x
```

Maxima [A]
time = 0.28, size = 98, normalized size = 0.99

$$\frac{1}{8}Db^2x^8+\frac{1}{7}Cb^2x^7+\frac{1}{6}(2Dab+Bb^2)x^6+\frac{1}{5}(2Cab+Ab^2)x^5+\frac{1}{2}Ba^2x^2+\frac{1}{4}(Da^2+2Bab)x^4+Aa^2x+\frac{1}{3}(Ca^2+2Aab)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/6*(2*D*a*b + B*b^2)*x^6 + 1/5*(2*C*a*b +
A*b^2)*x^5 + 1/2*B*a^2*x^2 + 1/4*(D*a^2 + 2*B*a*b)*x^4 + A*a^2*x + 1/3*(C*a
^2 + 2*A*a*b)*x^3
```

Fricas [A]
time = 5.33, size = 98, normalized size = 0.99

$$\frac{1}{8}Db^2x^8+\frac{1}{7}Cb^2x^7+\frac{1}{6}(2Dab+Bb^2)x^6+\frac{1}{5}(2Cab+Ab^2)x^5+\frac{1}{2}Ba^2x^2+\frac{1}{4}(Da^2+2Bab)x^4+Aa^2x+\frac{1}{3}(Ca^2+2Aab)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/6*(2*D*a*b + B*b^2)*x^6 + 1/5*(2*C*a*b +
A*b^2)*x^5 + 1/2*B*a^2*x^2 + 1/4*(D*a^2 + 2*B*a*b)*x^4 + A*a^2*x + 1/3*(C*a
^2 + 2*A*a*b)*x^3
```

Sympy [A]
time = 0.02, size = 107, normalized size = 1.08

$$Aa^2x + \frac{Ba^2x^2}{2} + \frac{Cb^2x^7}{7} + \frac{Db^2x^8}{8} + x^6\left(\frac{Bb^2}{6} + \frac{Dab}{3}\right) + x^5\left(\frac{Ab^2}{5} + \frac{2Cab}{5}\right) + x^4\left(\frac{Bab}{2} + \frac{Da^2}{4}\right) + x^3 \cdot \left(\frac{2Aab}{3} + \frac{Ca^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**2*x + B*a**2*x**2/2 + C*b**2*x**7/7 + D*b**2*x**8/8 + x**6*(B*b**2/6 +
 D*a*b/3) + x**5*(A*b**2/5 + 2*C*a*b/5) + x**4*(B*a*b/2 + D*a**2/4) + x**3*
(2*A*a*b/3 + C*a**2/3)
```

Giac [A]
time = 0.94, size = 102, normalized size = 1.03

$$\frac{1}{8}Db^2x^8 + \frac{1}{7}Cb^2x^7 + \frac{1}{3}Dabx^6 + \frac{1}{6}Bb^2x^6 + \frac{2}{5}Cabx^5 + \frac{1}{5}Ab^2x^5 + \frac{1}{4}Da^2x^4 + \frac{1}{2}Babx^4 + \frac{1}{3}Ca^2x^3 + \frac{2}{3}Aabx^3 + \frac{1}{2}Ba^2x^2 + Aa^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/3*D*a*b*x^6 + 1/6*B*b^2*x^6 + 2/5*C*a*b*x
^5 + 1/5*A*b^2*x^5 + 1/4*D*a^2*x^4 + 1/2*B*a*b*x^4 + 1/3*C*a^2*x^3 + 2/3*A*
a*b*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

Mupad [B]
time = 1.11, size = 105, normalized size = 1.06

$$\frac{A\,x\,\left(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4\right)}{15}+\frac{a^2\,x^4\,D}{4}+\frac{b^2\,x^8\,D}{8}+\frac{B\,x^2\,\left(3\,a^2+3\,a\,b\,x^2+b^2\,x^4\right)}{6}+\frac{C\,x^3\,\left(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4\right)}{105}+\frac{a\,b\,x^6\,D}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (A*x*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 + (a^2*x^4*D)/4 + (b^2*x^8*D)/8
+ (B*x^2*(3*a^2 + b^2*x^4 + 3*a*b*x^2))/6 + (C*x^3*(35*a^2 + 15*b^2*x^4 + 4
2*a*b*x^2))/105 + (a*b*x^6*D)/3
```

3.74 $\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)}{x}\,dx$

Optimal. Leaf size=92

$$a^2Bx+aAbx^2+\frac{1}{3}a(2bB+aD)x^3+\frac{1}{4}Ab^2x^4+\frac{1}{5}b(bB+2aD)x^5+\frac{1}{7}b^2Dx^7+\frac{C\left(a+bx^2\right)^3}{6b}+a^2A\log(x)$$

```
[Out] a^2*B*x+a*A*b*x^2+1/3*a*(2*B*b+D*a)*x^3+1/4*A*b^2*x^4+1/5*b*(B*b+2*D*a)*x^5
+1/7*b^2*D*x^7+1/6*C*(b*x^2+a)^3/b+a^2*A*ln(x)
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1597, 1816}

$$a^2A\log(x)+a^2Bx+aAbx^2+\frac{1}{5}bx^5(2aD+bB)+\frac{1}{3}ax^3(aD+2bB)+\frac{C\left(a+bx^2\right)^3}{6b}+\frac{1}{4}Ab^2x^4+\frac{1}{7}b^2Dx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a^2*B*x + a*A*b*x^2 + (a*(2*b*B + a*D)*x^3)/3 + (A*b^2*x^4)/4 + (b*(b*B + 2
*a*D)*x^5)/5 + (b^2*D*x^7)/7 + (C*(a + b*x^2)^3)/(6*b) + a^2*A*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)}{x}\,dx &= \frac{C\left(a+bx^2\right)^3}{6b}+\int \frac{\left(a+bx^2\right)^2\left(A+Bx+Dx^3\right)}{x}\,dx\\ &= \frac{C\left(a+bx^2\right)^3}{6b}+\int \left(a^2B+\frac{a^2A}{x}+2aAbx+a(2bB+aD)x^2+\right.\\ &= a^2Bx+aAbx^2+\frac{1}{3}a(2bB+aD)x^3+\frac{1}{4}Ab^2x^4+\frac{1}{5}b(bB+2aD)\end{aligned}$$

Mathematica [A]
time = 0.03, size = 88, normalized size = 0.96

$$\frac{1}{420}x\left(70a^2(6B+x(3C+2Dx))+14abx(30A+x(20B+3x(5C+4Dx)))+b^2x^3(105A+2x(42B+5x(7C+6Dx)))\right)+a^2A\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] (x*(70*a^2*(6*B + x*(3*C + 2*D*x)) + 14*a*b*x*(30*A + x*(20*B + 3*x*(5*C +
4*D*x))) + b^2*x^3*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x)))))/420 + a^2*A*L
og[x]
```

Maple [A]
time = 0.10, size = 100, normalized size = 1.09

method	result
norman	$\left(\frac{1}{4}b^2A+\frac{1}{2}abC\right)x^4+\left(\frac{1}{5}b^2B+\frac{2}{5}abD\right)x^5+\left(abA+\frac{1}{2}a^2C\right)x^2+\left(\frac{2}{3}abB+\frac{1}{3}a^2D\right)x^3+a^2Bx+\frac{b^2Cx}{6}$
default	$\frac{b^2Dx^7}{7}+\frac{b^2Cx^6}{6}+\frac{b^2Bx^5}{5}+\frac{2Dab\,x^5}{5}+\frac{A\,b^2x^4}{4}+\frac{Cab\,x^4}{2}+\frac{2Bab\,x^3}{3}+\frac{Da^2x^3}{3}+aAb\,x^2+\frac{C\,a^2x^2}{2}+a^2Bx+a^2$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^2*D*x^7+1/6*b^2*C*x^6+1/5*b^2*B*x^5+2/5*D*a*b*x^5+1/4*A*b^2*x^4+1/2*C
*a*b*x^4+2/3*B*a*b*x^3+1/3*D*a^2*x^3+a*A*b*x^2+1/2*C*a^2*x^2+a^2*B*x+a^2*A*
ln(x)
```

Maxima [A]
time = 0.27, size = 96, normalized size = 1.04

$$\frac{1}{7}Db^2x^7+\frac{1}{6}Cb^2x^6+\frac{1}{5}\left(2Dab+Bb^2\right)x^5+\frac{1}{4}\left(2Cab+Ab^2\right)x^4+Ba^2x+\frac{1}{3}\left(Da^2+2Bab\right)x^3+Aa^2\log\left(x\right)+\frac{1}{2}\left(Ca^2+2Aab\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

```
[Out] 1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 1/5*(2*D*a*b + B*b^2)*x^5 + 1/4*(2*C*a*b +
A*b^2)*x^4 + B*a^2*x + 1/3*(D*a^2 + 2*B*a*b)*x^3 + A*a^2*log(x) + 1/2*(C*a^
2 + 2*A*a*b)*x^2
```

Fricas [A]
time = 6.11, size = 96, normalized size = 1.04

$$\frac{1}{7}Db^2x^7+\frac{1}{6}Cb^2x^6+\frac{1}{5}\left(2Dab+Bb^2\right)x^5+\frac{1}{4}\left(2Cab+Ab^2\right)x^4+Ba^2x+\frac{1}{3}\left(Da^2+2Bab\right)x^3+Aa^2\log\left(x\right)+\frac{1}{2}\left(Ca^2+2Aab\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

```
[Out] 1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 1/5*(2*D*a*b + B*b^2)*x^5 + 1/4*(2*C*a*b +
A*b^2)*x^4 + B*a^2*x + 1/3*(D*a^2 + 2*B*a*b)*x^3 + A*a^2*log(x) + 1/2*(C*a^
2 + 2*A*a*b)*x^2
```

Sympy [A]
time = 0.08, size = 104, normalized size = 1.13

$$Aa^2\log(x) + Ba^2x + \frac{Cb^2x^6}{6} + \frac{Db^2x^7}{7} + x^5\left(\frac{Bb^2}{5} + \frac{2Dab}{5}\right) + x^4\left(\frac{Ab^2}{4} + \frac{Cab}{2}\right) + x^3\cdot\left(\frac{2Bab}{3} + \frac{Da^2}{3}\right) + x^2\left(Aab + \frac{Ca^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a**2*log(x) + B*a**2*x + C*b**2*x**6/6 + D*b**2*x**7/7 + x**5*(B*b**2/5 +
 2*D*a*b/5) + x**4*(A*b**2/4 + C*a*b/2) + x**3*(2*B*a*b/3 + D*a**2/3) + x**
2*(A*a*b + C*a**2/2)
```

Giac [A]
time = 1.01, size = 100, normalized size = 1.09

$$\frac{1}{7}Db^2x^7 + \frac{1}{6}Cb^2x^6 + \frac{2}{5}Dabx^5 + \frac{1}{5}Bb^2x^5 + \frac{1}{2}Cabx^4 + \frac{1}{4}Ab^2x^4 + \frac{1}{3}Da^2x^3 + \frac{2}{3}Babx^3 + \frac{1}{2}Ca^2x^2 + Aabx^2 + Ba^2x + Aa^2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

```
[Out] 1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 2/5*D*a*b*x^5 + 1/5*B*b^2*x^5 + 1/2*C*a*b*x
^4 + 1/4*A*b^2*x^4 + 1/3*D*a^2*x^3 + 2/3*B*a*b*x^3 + 1/2*C*a^2*x^2 + A*a*b*
x^2 + B*a^2*x + A*a^2*log(abs(x))
```

Mupad [B]
time = 1.11, size = 103, normalized size = 1.12

$$\frac{A\,(4\,a^2\,\ln(x) + b^2\,x^4 + 4\,a\,b\,x^2)}{4} + \frac{B\,x\,(15\,a^2 + 10\,a\,b\,x^2 + 3\,b^2\,x^4)}{15} + \frac{a^2\,x^3\,D}{3} + \frac{b^2\,x^7\,D}{7} + \frac{C\,x^2\,(3\,a^2 + 3\,a\,b\,x^2 + b^2\,x^4)}{6} + \frac{2\,a\,b\,x^5\,D}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x,x)
```

```
[Out] (A*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + (B*x*(15*a^2 + 3*b^2*x^4 + 10*
a*b*x^2))/15 + (a^2*x^3*D)/3 + (b^2*x^7*D)/7 + (C*x^2*(3*a^2 + b^2*x^4 + 3*
a*b*x^2))/6 + (2*a*b*x^5*D)/5
```

3.75 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$

Optimal. Leaf size=90

$$-\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2+\frac{1}{3}b(Ab+2aC)x^3+\frac{1}{4}b^2Bx^4+\frac{1}{5}b^2Cx^5+\frac{D(a+bx^2)^3}{6b}+a^2B\log(x)$$

```
[Out] -a^2*A/x+a*(2*A*b+C*a)*x+a*b*B*x^2+1/3*b*(A*b+2*C*a)*x^3+1/4*b^2*B*x^4+1/5*
b^2*C*x^5+1/6*D*(b*x^2+a)^3/b+a^2*B*ln(x)
```

Rubi [A]
time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1597, 1642}

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{3}bx^3(2aC+Ab)+ax(aC+2Ab)+abBx^2+\frac{D(a+bx^2)^3}{6b}+\frac{1}{4}b^2Bx^4+\frac{1}{5}b^2Cx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + (b*(A*b + 2*a*C)*x^3)/3 + (b
^2*B*x^4)/4 + (b^2*C*x^5)/5 + (D*(a + b*x^2)^3)/(6*b) + a^2*B*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2\,(A+Bx+Cx^2+Dx^3)}{x^2}\,dx &= \frac{D(a+bx^2)^3}{6b}+\int \frac{(a+bx^2)^2\,(A+Bx+Cx^2)}{x^2}\,dx\\ &= \frac{D(a+bx^2)^3}{6b}+\int \left(a(2Ab+aC)+\frac{a^2A}{x^2}+\frac{a^2B}{x}+2abBx+b(A\right.\\ &= -\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2+\frac{1}{3}b(Ab+2aC)x^3+\frac{1}{4}b^2Bx^4\end{aligned}$$

Mathematica [A]
time = 0.04, size = 88, normalized size = 0.98

$$a^2\left(-\frac{A}{x}+Cx+\frac{Dx^2}{2}\right)+\frac{1}{6}abx(12A+x(6B+x(4C+3Dx)))+\frac{1}{60}b^2x^3(20A+x(15B+2x(6C+5Dx)))+a^2B\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] a^2*(-(A/x) + C*x + (D*x^2)/2) + (a*b*x*(12*A + x*(6*B + x*(4*C + 3*D*x))))
/6 + (b^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))))/60 + a^2*B*Log[x]
```

Maple [A]
time = 0.11, size = 98, normalized size = 1.09

method	result
default	$\frac{b^2Dx^6}{6}+\frac{b^2Cx^5}{5}+\frac{b^2Bx^4}{4}+\frac{Dabx^4}{2}+\frac{Ab^2x^3}{3}+\frac{2Cabx^3}{3}+Babx^2+\frac{Da^2x^2}{2}+2abAx+a^2Cx+a^2B\ln(x$
norman	$\frac{(\frac{1}{3}b^2A+\frac{2}{3}abC)x^4+(\frac{1}{4}b^2B+\frac{1}{2}abD)x^5+(abB+\frac{1}{2}a^2D)x^3+(2abA+a^2C)x^2-a^2A+\frac{b^2Cx^6}{5}+\frac{b^2Dx^7}{6}}{x}+a^2B\ln(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^2*D*x^6+1/5*b^2*C*x^5+1/4*b^2*B*x^4+1/2*D*a*b*x^4+1/3*A*b^2*x^3+2/3*C
*a*b*x^3+B*a*b*x^2+1/2*D*a^2*x^2+2*a*b*A*x+a^2*C*x+a^2*B*ln(x)-a^2*A/x
```

Maxima [A]
time = 0.27, size = 96, normalized size = 1.07

$$\frac{1}{6}Db^2x^6+\frac{1}{5}Cb^2x^5+\frac{1}{4}\left(2Dab+Bb^2\right)x^4+\frac{1}{3}\left(2Cab+Ab^2\right)x^3+Ba^2\log(x)+\frac{1}{2}\left(Da^2+2Bab\right)x^2-\frac{Aa^2}{x}+\left(Ca^2+2Aab\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/6*D*b^2*x^6 + 1/5*C*b^2*x^5 + 1/4*(2*D*a*b + B*b^2)*x^4 + 1/3*(2*C*a*b +
A*b^2)*x^3 + B*a^2*log(x) + 1/2*(D*a^2 + 2*B*a*b)*x^2 - A*a^2/x + (C*a^2 +
2*A*a*b)*x
```

Fricas [A]
time = 4.42, size = 103, normalized size = 1.14

$$\frac{10Db^2x^7+12Cb^2x^6+15\left(2Dab+Bb^2\right)x^5+20\left(2Cab+Ab^2\right)x^4+60Ba^2x\log(x)+30\left(Da^2+2Bab\right)x^3-60Aa^2+60\left(Ca^2+2Aab\right)x^2}{60x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(10*D*b^2*x^7 + 12*C*b^2*x^6 + 15*(2*D*a*b + B*b^2)*x^5 + 20*(2*C*a*b
+ A*b^2)*x^4 + 60*B*a^2*x*log(x) + 30*(D*a^2 + 2*B*a*b)*x^3 - 60*A*a^2 + 60
*(C*a^2 + 2*A*a*b)*x^2)/x
```

Sympy [A]
time = 0.09, size = 99, normalized size = 1.10

$$-\frac{Aa^2}{x} + Ba^2\log{(x)} + \frac{Cb^2x^5}{5} + \frac{Db^2x^6}{6} + x^4\left(\frac{Bb^2}{4} + \frac{Dab}{2}\right) + x^3\left(\frac{Ab^2}{3} + \frac{2Cab}{3}\right) + x^2\left(Bab + \frac{Da^2}{2}\right) + x\left(2Aab + Ca^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a**2/x + B*a**2*log(x) + C*b**2*x**5/5 + D*b**2*x**6/6 + x**4*(B*b**2/4
+ D*a*b/2) + x**3*(A*b**2/3 + 2*C*a*b/3) + x**2*(B*a*b + D*a**2/2) + x*(2*A
*a*b + C*a**2)
```

Giac [A]
time = 1.12, size = 98, normalized size = 1.09

$$\frac{1}{6}\,Db^2x^6 + \frac{1}{5}\,Cb^2x^5 + \frac{1}{2}\,Dabx^4 + \frac{1}{4}\,Bb^2x^4 + \frac{2}{3}\,Cabx^3 + \frac{1}{3}\,Ab^2x^3 + \frac{1}{2}\,Da^2x^2 + Babx^2 + Ca^2x + 2\,Aabx + Ba^2\log{(|x|)} - \frac{Aa^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/6*D*b^2*x^6 + 1/5*C*b^2*x^5 + 1/2*D*a*b*x^4 + 1/4*B*b^2*x^4 + 2/3*C*a*b*x
^3 + 1/3*A*b^2*x^3 + 1/2*D*a^2*x^2 + B*a*b*x^2 + C*a^2*x + 2*A*a*b*x + B*a^
2*log(abs(x)) - A*a^2/x
```

Mupad [B]
time = 1.11, size = 92, normalized size = 1.02

$$\frac{B\,\left(4\,a^2\,\ln\left(x\right)+b^2\,x^4+4\,a\,b\,x^2\right)}{4}+\frac{{\left(b\,x^2+a\right)}^3\,D}{6\,b}+\frac{C\,x\,\left(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4\right)}{15}+\frac{A\,\left(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4\right)}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

```
[Out] (B*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + ((a + b*x^2)^3*D)/(6*b) + (C*x
*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 + (A*(b^2*x^4 - 3*a^2 + 6*a*b*x^2))/
(3*x)
```

3.76 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$

Optimal. Leaf size=98

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+a(2bB+aD)x+\frac{1}{2}b(Ab+2aC)x^2+\frac{1}{3}b(bB+2aD)x^3+\frac{1}{4}b^2Cx^4+\frac{1}{5}b^2Dx^5+a(2Ab+aC)\log(x)$$

```
[Out] -1/2*a^2*A/x^2-a^2*B/x+a*(2*B*b+D*a)*x+1/2*b*(A*b+2*C*a)*x^2+1/3*b*(B*b+2*D
*a)*x^3+1/4*b^2*C*x^4+1/5*b^2*D*x^5+a*(2*A*b+C*a)*ln(x)
```

Rubi [A]
time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{1816\}$

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{2}bx^2(2aC+Ab)+a\log(x)(aC+2Ab)+\frac{1}{3}bx^3(2aD+bB)+ax(aD+2bB)+\frac{1}{4}b^2Cx^4+\frac{1}{5}b^2Dx^5$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -1/2*(a^2*A)/x^2 - (a^2*B)/x + a*(2*b*B + a*D)*x + (b*(A*b + 2*a*C)*x^2)/2
+ (b*(b*B + 2*a*D)*x^3)/3 + (b^2*C*x^4)/4 + (b^2*D*x^5)/5 + a*(2*A*b + a*C)
*Log[x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3}\,dx &= \int \left(a(2bB+aD)+\frac{a^2A}{x^3}+\frac{a^2B}{x^2}+\frac{a(2Ab+aC)}{x}+b(Ab+2aC\right.\\ &= -\frac{a^2A}{2x^2}-\frac{a^2B}{x}+a(2bB+aD)x+\frac{1}{2}b(Ab+2aC)x^2+\frac{1}{3}b(bB+\end{aligned}$$

Mathematica [A]
time = 0.03, size = 87, normalized size = 0.89

$$-\frac{a^2(A+2Bx-2Dx^3)}{2x^2}+\frac{1}{3}abx(6B+x(3C+2Dx))+\frac{1}{60}b^2x^2(30A+x(20B+3x(5C+4Dx)))+a(2Ab+aC)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -1/2*(a^2*(A + 2*B*x - 2*D*x^3))/x^2 + (a*b*x*(6*B + x*(3*C + 2*D*x)))/3 +
(b^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))/60 + a*(2*A*b + a*C)*Log[x]
```

Maple [A]
time = 0.11, size = 95, normalized size = 0.97

method	result
default	$\frac{b^2Dx^5}{5}+\frac{b^2C\,x^4}{4}+\frac{b^2B\,x^3}{3}+\frac{2Dab\,x^3}{3}+\frac{A\,b^2x^2}{2}+Cab\,x^2+2abBx+a^2Dx-\frac{a^2A}{2x^2}+a(2Ab+aC)\ln(x)-$
norman	$\frac{\left(\frac{1}{2}b^2A+abC\right)x^4+\left(\frac{1}{3}b^2B+\frac{2}{3}abD\right)x^5+\left(2abB+a^2D\right)x^3-\frac{a^2A}{2}-a^2Bx+\frac{b^2C\,x^6}{4}+\frac{b^2Dx^7}{5}}{x^2}+\left(2abA+a^2C\right)\ln(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*b^2*D*x^5+1/4*b^2*C*x^4+1/3*b^2*B*x^3+2/3*D*a*b*x^3+1/2*A*b^2*x^2+C*a*b
*x^2+2*a*b*B*x+a^2*D*x-1/2*a^2*A/x^2+a*(2*A*b+C*a)*ln(x)-a^2*B/x
```

Maxima [A]
time = 0.27, size = 96, normalized size = 0.98

$$\frac{1}{5}\,Db^2x^5+\frac{1}{4}\,Cb^2x^4+\frac{1}{3}\,(2\,Dab+Bb^2)x^3+\frac{1}{2}\,(2\,Cab+Ab^2)x^2+\left(Da^2+2\,Bab\right)x+\left(Ca^2+2\,Aab\right)\log(x)-\frac{2\,Ba^2x+Aa^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/5*D*b^2*x^5 + 1/4*C*b^2*x^4 + 1/3*(2*D*a*b + B*b^2)*x^3 + 1/2*(2*C*a*b +
A*b^2)*x^2 + (D*a^2 + 2*B*a*b)*x + (C*a^2 + 2*A*a*b)*log(x) - 1/2*(2*B*a^2*
x + A*a^2)/x^2
```

Fricas [A]
time = 2.30, size = 103, normalized size = 1.05

$$\frac{12\,Db^2x^7+15\,Cb^2x^6+20\,(2\,Dab+Bb^2)x^5+30\,(2\,Cab+Ab^2)x^4-60\,Ba^2x+60\,(Da^2+2\,Bab)x^3+60\,(Ca^2+2\,Aab)x^2\log(x)-30\,Aa^2}{60\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/60*(12*D*b^2*x^7 + 15*C*b^2*x^6 + 20*(2*D*a*b + B*b^2)*x^5 + 30*(2*C*a*b
+ A*b^2)*x^4 - 60*B*a^2*x + 60*(D*a^2 + 2*B*a*b)*x^3 + 60*(C*a^2 + 2*A*a*b)
*x^2*log(x) - 30*A*a^2)/x^2
```

Sympy [A]
time = 0.17, size = 100, normalized size = 1.02

$$\frac{Cb^2x^4}{4}+\frac{Db^2x^5}{5}+a(2Ab+Ca)\log(x)+x^3\left(\frac{Bb^2}{3}+\frac{2Dab}{3}\right)+x^2\left(\frac{Ab^2}{2}+Cab\right)+x\left(2Bab+Da^2\right)+\frac{-Aa^2-2Ba^2x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] C*b**2*x**4/4 + D*b**2*x**5/5 + a*(2*A*b + C*a)*log(x) + x**3*(B*b**2/3 + 2
*D*a*b/3) + x**2*(A*b**2/2 + C*a*b) + x*(2*B*a*b + D*a**2) + (-A*a**2 - 2*B
*a**2*x)/(2*x**2)
```

Giac [A]
time = 1.54, size = 97, normalized size = 0.99

$$\frac{1}{5}\,Db^{2}x^{5}+\frac{1}{4}\,Cb^{2}x^{4}+\frac{2}{3}\,Dabx^{3}+\frac{1}{3}\,Bb^{2}x^{3}+Cabx^{2}+\frac{1}{2}\,Ab^{2}x^{2}+Da^{2}x+2\,Babx+\left(Ca^{2}+2\,Aab\right)\log\left(|x|\right)-\frac{2\,Ba^{2}x+Aa^{2}}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/5*D*b^2*x^5 + 1/4*C*b^2*x^4 + 2/3*D*a*b*x^3 + 1/3*B*b^2*x^3 + C*a*b*x^2 +
 1/2*A*b^2*x^2 + D*a^2*x + 2*B*a*b*x + (C*a^2 + 2*A*a*b)*log(abs(x)) - 1/2*
(2*B*a^2*x + A*a^2)/x^2
```

Mupad [B]
time = 1.11, size = 103, normalized size = 1.05

$$\frac{C\,\left(4\,a^2\,\ln\left(x\right)+b^2\,x^4+4\,a\,b\,x^2\right)}{4}+a^2\,x\,D+\frac{b^2\,x^5\,D}{5}+\frac{A\,\left(b^2\,x^4-a^2+4\,a\,b\,x^2\,\ln\left(x\right)\right)}{2\,x^2}+\frac{B\,\left(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4\right)}{3\,x}+\frac{2\,a\,b\,x^3\,D}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

```
[Out] (C*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + a^2*x*D + (b^2*x^5*D)/5 + (A*(
b^2*x^4 - a^2 + 4*a*b*x^2*log(x)))/(2*x^2) + (B*(b^2*x^4 - 3*a^2 + 6*a*b*x^
2))/(3*x) + (2*a*b*x^3*D)/3
```

3.77 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$

Optimal. Leaf size=98

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}-\frac{a(2Ab+aC)}{x}+b(Ab+2aC)x+\frac{1}{2}b(bB+2aD)x^2+\frac{1}{3}b^2Cx^3+\frac{1}{4}b^2Dx^4+a(2bB+aD)\log(x)$$

```
[Out] -1/3*a^2*A/x^3-1/2*a^2*B/x^2-a*(2*A*b+C*a)/x+b*(A*b+2*C*a)*x+1/2*b*(B*b+2*D
*a)*x^2+1/3*b^2*C*x^3+1/4*b^2*D*x^4+a*(2*B*b+D*a)*ln(x)
```

Rubi [A]
time = 0.06, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{1816\}$

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+bx(2aC+Ab)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}bx^2(2aD+bB)+a\log(x)(aD+2bB)+\frac{1}{3}b^2Cx^3+\frac{1}{4}b^2Dx^4$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -1/3*(a^2*A)/x^3 - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + b*(A*b + 2*a*C)*
x + (b*(b*B + 2*a*D)*x^2)/2 + (b^2*C*x^3)/3 + (b^2*D*x^4)/4 + a*(2*b*B + a*
D)*Log[x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx &= \int \Bigg(b(Ab+2aC)+\frac{a^2A}{x^4}+\frac{a^2B}{x^3}+\frac{a(2Ab+aC)}{x^2}+\frac{a(2bB+aD}{x}\\ &= -\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}-\frac{a(2Ab+aC)}{x}+b(Ab+2aC)x+\frac{1}{2}b(bB+2aD\end{aligned}$$

Mathematica [A]
time = 0.04, size = 83, normalized size = 0.85

$$-\frac{2aAb}{x}+abx(2C+Dx)-\frac{a^2(2A+3x(B+2Cx))}{6x^3}+\frac{1}{12}b^2x\left(12A+x\left(6B+4Cx+3Dx^2\right)\right)+a(2bB+aD)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] (-2*a*A*b)/x + a*b*x*(2*C + D*x) - (a^2*(2*A + 3*x*(B + 2*C*x)))/(6*x^3) +
(b^2*x*(12*A + x*(6*B + 4*C*x + 3*D*x^2)))/12 + a*(2*b*B + a*D)*Log[x]
```

Maple [A]
time = 0.11, size = 92, normalized size = 0.94

method	result
default	$\frac{b^2 D x^4}{4}+\frac{b^2 C x^3}{3}+\frac{b^2 B x^2}{2}+D a b x^2+b^2 A x+2 a b C x-\frac{a^2 B}{2 x^2}-\frac{a^2 A}{3 x^3}+a(2 B b+a D) \ln (x)-\frac{a(2 A b+a C}{x}$
norman	$\frac{\left(\frac{1}{2} b^2 B+a b D\right) x^5+\left(b^2 A+2 a b C\right) x^4+\left(-2 a b A-a^2 C\right) x^2-\frac{a^2 A}{3}-\frac{a^2 B x}{2}+\frac{b^2 C x^6}{3}+\frac{b^2 D x^7}{4}}{x^3}+\left(2 a b B+a^2 D\right) \ln (x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^2*D*x^4+1/3*b^2*C*x^3+1/2*b^2*B*x^2+D*a*b*x^2+b^2*A*x+2*a*b*C*x-1/2*a
^2*B/x^2-1/3*a^2*A/x^3+a*(2*B*b+D*a)*ln(x)-a*(2*A*b+C*a)/x
```

Maxima [A]
time = 0.28, size = 97, normalized size = 0.99

$$\frac{1}{4} D b^2 x^4+\frac{1}{3} C b^2 x^3+\frac{1}{2}\left(2 D a b+B b^2\right) x^2+\left(2 C a b+A b^2\right) x+\left(D a^2+2 B a b\right) \log (x)-\frac{3 B a^2 x+2 A a^2+6\left(C a^2+2 A a b\right) x^2}{6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/4*D*b^2*x^4 + 1/3*C*b^2*x^3 + 1/2*(2*D*a*b + B*b^2)*x^2 + (2*C*a*b + A*b^
2)*x + (D*a^2 + 2*B*a*b)*log(x) - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2 + 2*A
*a*b)*x^2)/x^3
```

Fricas [A]
time = 1.04, size = 103, normalized size = 1.05

$$\frac{3 D b^2 x^7+4 C b^2 x^6+6\left(2 D a b+B b^2\right) x^5+12\left(2 C a b+A b^2\right) x^4+12\left(D a^2+2 B a b\right) x^3 \log (x)-6 B a^2 x-4 A a^2-12\left(C a^2+2 A a b\right) x^2}{12 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*D*b^2*x^7 + 4*C*b^2*x^6 + 6*(2*D*a*b + B*b^2)*x^5 + 12*(2*C*a*b + A
*b^2)*x^4 + 12*(D*a^2 + 2*B*a*b)*x^3*log(x) - 6*B*a^2*x - 4*A*a^2 - 12*(C*a
^2 + 2*A*a*b)*x^2)/x^3
```

Sympy [A]
time = 0.46, size = 100, normalized size = 1.02

$$\frac{C b^2 x^3}{3}+\frac{D b^2 x^4}{4}+a(2 B b+D a) \log (x)+x^2\left(\frac{B b^2}{2}+D a b\right)+x\left(A b^2+2 C a b\right)+\frac{-2 A a^2-3 B a^2 x+x^2\left(-12 A a b-6 C a^2\right)}{6 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] C*b**2*x**3/3 + D*b**2*x**4/4 + a*(2*B*b + D*a)*log(x) + x**2*(B*b**2/2 + D
*a*b) + x*(A*b**2 + 2*C*a*b) + (-2*A*a**2 - 3*B*a**2*x + x**2*(-12*A*a*b -
6*C*a**2))/(6*x**3)
```

Giac [A]
time = 1.21, size = 97, normalized size = 0.99

$$\frac{1}{4}Db^2x^4+\frac{1}{3}Cb^2x^3+Dabx^2+\frac{1}{2}Bb^2x^2+2Cabx+Ab^2x+\left(Da^2+2Bab\right)\log\left(|x|\right)-\frac{3Ba^2x+2Aa^2+6(Ca^2+2Aab)x^2}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/4*D*b^2*x^4 + 1/3*C*b^2*x^3 + D*a*b*x^2 + 1/2*B*b^2*x^2 + 2*C*a*b*x + A*b
^2*x + (D*a^2 + 2*B*a*b)*log(abs(x)) - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2
+ 2*A*a*b)*x^2)/x^3
```

Mupad [B]
time = 1.28, size = 106, normalized size = 1.08

$$\frac{b^2\,x^4\,D}{4}+\frac{a^2\,\ln\left(x^2\right)\,D}{2}-\frac{A\,\left(a^2+6\,a\,b\,x^2-3\,b^2\,x^4\right)}{3\,x^3}+\frac{B\,\left(b^2\,x^4-a^2+4\,a\,b\,x^2\,\ln\left(x\right)\right)}{2\,x^2}+\frac{C\,\left(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4\right)}{3\,x}+a\,b\,x^2\,D$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

```
[Out] (b^2*x^4*D)/4 + (a^2*log(x^2)*D)/2 - (A*(a^2 - 3*b^2*x^4 + 6*a*b*x^2))/(3*x
^3) + (B*(b^2*x^4 - a^2 + 4*a*b*x^2*log(x)))/(2*x^2) + (C*(b^2*x^4 - 3*a^2
+ 6*a*b*x^2))/(3*x) + a*b*x^2*D
```

3.78 $\int x^3(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$

Optimal. Leaf size=149

$$\frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2(3Ab+aC)x^6+\frac{1}{7}a^2(3bB+aD)x^7+\frac{3}{8}ab(Ab+aC)x^8+\frac{1}{3}ab(bB+aD)x^9+\frac{1}{10}b^2(Ab+3aC$$

```
[Out] 1/4*a^3*A*x^4+1/5*a^3*B*x^5+1/6*a^2*(3*A*b+C*a)*x^6+1/7*a^2*(3*B*b+D*a)*x^7
+3/8*a*b*(A*b+C*a)*x^8+1/3*a*b*(B*b+D*a)*x^9+1/10*b^2*(A*b+3*C*a)*x^10+1/11
*b^2*(B*b+3*D*a)*x^11+1/12*b^3*C*x^12+1/13*b^3*D*x^13
```

Rubi [A]
time = 0.13, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1816}

$$\frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2x^6(aC+3Ab)+\frac{1}{7}a^2x^7(aD+3bB)+\frac{1}{10}b^2x^{10}(3aC+Ab)+\frac{3}{8}abx^8(aC+Ab)+\frac{1}{11}b^2x^{11}(3aD+bB)+\frac{1}{3}abx^9(aD+bB)+\frac{1}{12}b^3Cx^{12}+\frac{1}{13}b^3Dx^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*(3*b*B + a
*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*D)*x^9)/3 + (b^2*(A*
b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11)/11 + (b^3*C*x^12)/12 + (b^3*
D*x^13)/13
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^3\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \int \left(a^3Ax^3+a^3Bx^4+a^2(3Ab+aC)x^5+a^2(3bB+aD)x^6+\right.\\ &= \frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2(3Ab+aC)x^6+\frac{1}{7}a^2(3bB+aD)x^7\end{aligned}$$

Mathematica [A]
time = 0.02, size = 149, normalized size = 1.00

$$\frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2(3Ab+aC)x^6+\frac{1}{7}a^2(3bB+aD)x^7+\frac{3}{8}ab(Ab+aC)x^8+\frac{1}{3}ab(bB+aD)x^9+\frac{1}{10}b^2(Ab+3aC)x^{10}+\frac{1}{11}b^2(bB+3aD)x^{11}+\frac{1}{12}b^3Cx^{12}+\frac{1}{13}b^3Dx^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*(3*b*B + a
*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*D)*x^9)/3 + (b^2*(A*
b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11)/11 + (b^3*C*x^12)/12 + (b^3*
D*x^13)/13
```

Maple [A]
time = 0.10, size = 150, normalized size = 1.01

method	result
norman	$\frac{b^3Dx^{13}}{13}+\frac{b^3C\,x^{12}}{12}+\left(\frac{1}{11}b^3B+\frac{3}{11}a\,b^2D\right)x^{11}+\left(\frac{1}{10}A\,b^3+\frac{3}{10}a\,b^2C\right)x^{10}+\left(\frac{1}{3}a\,b^2B+\frac{1}{3}a^2bD\right)x^9+ (\frac{3}{8}A$
default	$\frac{b^3Dx^{13}}{13}+\frac{b^3C\,x^{12}}{12}+\frac{(b^3B+3a\,b^2D)x^{11}}{11}+\frac{(A\,b^3+3a\,b^2C)x^{10}}{10}+\frac{(3a\,b^2B+3a^2bD)x^9}{9}+\frac{(3Aa\,b^2+3a^2bC)x^8}{8}+\frac{(3B\,a^2b+a^3}{7}$
gosper	$\frac{1}{13}b^3Dx^{13}+\frac{1}{12}b^3C\,x^{12}+\frac{1}{11}x^{11}b^3B+\frac{3}{11}x^{11}a\,b^2D+\frac{1}{10}x^{10}A\,b^3+\frac{3}{10}x^{10}a\,b^2C+\frac{1}{3}x^9a\,b^2B+\frac{1}{3}x^9a^2bD$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b^3*D*x^13+1/12*b^3*C*x^12+1/11*(B*b^3+3*D*a*b^2)*x^11+1/10*(A*b^3+3*C
*a*b^2)*x^10+1/9*(3*B*a*b^2+3*D*a^2*b)*x^9+1/8*(3*A*a*b^2+3*C*a^2*b)*x^8+1/
7*(3*B*a^2*b+D*a^3)*x^7+1/6*(3*A*a^2*b+C*a^3)*x^6+1/5*a^3*B*x^5+1/4*a^3*A*x
^4
```

Maxima [A]
time = 0.27, size = 145, normalized size = 0.97

$$\frac{1}{13}\,Db^3x^{13}+\frac{1}{12}\,Cb^3x^{12}+\frac{1}{11}\,(3\,Dab^2+Bb^3)x^{11}+\frac{1}{10}\,(3\,Cab^2+Ab^3)x^{10}+\frac{1}{3}\,(Da^2b+Bab^2)x^9+\frac{1}{5}\,Ba^3x^5+\frac{3}{8}\,(Ca^2b+Aab^2)x^8+\frac{1}{4}\,Aa^3x^4+\frac{1}{7}\,(Da^3+3\,Ba^2b)x^7+\frac{1}{6}\,(Ca^3+3\,Aa^2b)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 1/11*(3*D*a*b^2 + B*b^3)*x^11 + 1/10*(3
*C*a*b^2 + A*b^3)*x^10 + 1/3*(D*a^2*b + B*a*b^2)*x^9 + 1/5*B*a^3*x^5 + 3/8*
(C*a^2*b + A*a*b^2)*x^8 + 1/4*A*a^3*x^4 + 1/7*(D*a^3 + 3*B*a^2*b)*x^7 + 1/6
*(C*a^3 + 3*A*a^2*b)*x^6
```

Fricas [A]
time = 1.42, size = 145, normalized size = 0.97

$$\frac{1}{13}\,Db^3x^{13}+\frac{1}{12}\,Cb^3x^{12}+\frac{1}{11}\,(3\,Dab^2+Bb^3)x^{11}+\frac{1}{10}\,(3\,Cab^2+Ab^3)x^{10}+\frac{1}{3}\,(Da^2b+Bab^2)x^9+\frac{1}{5}\,Ba^3x^5+\frac{3}{8}\,(Ca^2b+Aab^2)x^8+\frac{1}{4}\,Aa^3x^4+\frac{1}{7}\,(Da^3+3\,Ba^2b)x^7+\frac{1}{6}\,(Ca^3+3\,Aa^2b)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 1/11*(3*D*a*b^2 + B*b^3)*x^11 + 1/10*(3
*C*a*b^2 + A*b^3)*x^10 + 1/3*(D*a^2*b + B*a*b^2)*x^9 + 1/5*B*a^3*x^5 + 3/8*
(C*a^2*b + A*a*b^2)*x^8 + 1/4*A*a^3*x^4 + 1/7*(D*a^3 + 3*B*a^2*b)*x^7 + 1/6
*(C*a^3 + 3*A*a^2*b)*x^6
```

Sympy [A]
time = 0.02, size = 163, normalized size = 1.09

$$\frac{Aa^3x^4}{4}+\frac{Ba^3x^5}{5}+\frac{Cb^3x^{12}}{12}+\frac{Db^3x^{13}}{13}+x^{11}\left(\frac{Bb^3}{11}+\frac{3Dab^2}{11}\right)+x^{10}\left(\frac{Ab^3}{10}+\frac{3Cab^2}{10}\right)+x^9\left(\frac{Bab^2}{3}+\frac{Da^2b}{3}\right)+x^8\cdot\left(\frac{3Aab^2}{8}+\frac{3Ca^2b}{8}\right)+x^7\cdot\left(\frac{3Ba^2b}{7}+\frac{Da^3}{7}\right)+x^6\left(\frac{Aa^2b}{2}+\frac{Ca^3}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**4/4 + B*a**3*x**5/5 + C*b**3*x**12/12 + D*b**3*x**13/13 + x**11*(
B*b**3/11 + 3*D*a*b**2/11) + x**10*(A*b**3/10 + 3*C*a*b**2/10) + x**9*(B*a*
b**2/3 + D*a**2*b/3) + x**8*(3*A*a*b**2/8 + 3*C*a**2*b/8) + x**7*(3*B*a**2*
b/7 + D*a**3/7) + x**6*(A*a**2*b/2 + C*a**3/6)
```

Giac [A]
time = 1.63, size = 153, normalized size = 1.03

$$\frac{1}{13}Db^3x^{13}+\frac{1}{12}Cb^3x^{12}+\frac{3}{11}Dab^2x^{11}+\frac{1}{11}Bb^3x^{11}+\frac{3}{10}Cab^2x^{10}+\frac{1}{10}Ab^3x^{10}+\frac{1}{3}Da^2bx^9+\frac{1}{3}Bab^2x^9+\frac{3}{8}Ca^2bx^8+\frac{3}{8}Aab^2x^8+\frac{1}{7}Da^3x^7+\frac{3}{7}Ba^2bx^7+\frac{1}{6}Ca^3x^6+\frac{1}{2}Aa^2bx^6+\frac{1}{5}Ba^3x^5+\frac{1}{4}Aa^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 3/11*D*a*b^2*x^11 + 1/11*B*b^3*x^11 + 3
/10*C*a*b^2*x^10 + 1/10*A*b^3*x^10 + 1/3*D*a^2*b*x^9 + 1/3*B*a*b^2*x^9 + 3/
8*C*a^2*b*x^8 + 3/8*A*a*b^2*x^8 + 1/7*D*a^3*x^7 + 3/7*B*a^2*b*x^7 + 1/6*C*a
^3*x^6 + 1/2*A*a^2*b*x^6 + 1/5*B*a^3*x^5 + 1/4*A*a^3*x^4
```

Mupad [B]
time = 1.30, size = 153, normalized size = 1.03

$$\frac{A\,a^3\,x^4}{4}+\frac{B\,a^3\,x^5}{5}+\frac{A\,b^3\,x^{10}}{10}+\frac{C\,a^3\,x^6}{6}+\frac{B\,b^3\,x^{11}}{11}+\frac{C\,b^3\,x^{12}}{12}+\frac{a^3\,x^7\,D}{7}+\frac{b^3\,x^{13}\,D}{13}+\frac{a^2\,b\,x^9\,D}{3}+\frac{3\,a\,b^2\,x^{11}\,D}{11}+\frac{A\,a^2\,b\,x^6}{2}+\frac{3\,A\,a\,b^2\,x^8}{8}+\frac{3\,B\,a^2\,b\,x^7}{7}+\frac{B\,a\,b^2\,x^9}{3}+\frac{3\,C\,a^2\,b\,x^8}{8}+\frac{3\,C\,a\,b^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (A*a^3*x^4)/4 + (B*a^3*x^5)/5 + (A*b^3*x^10)/10 + (C*a^3*x^6)/6 + (B*b^3*x^
11)/11 + (C*b^3*x^12)/12 + (a^3*x^7*D)/7 + (b^3*x^13*D)/13 + (a^2*b*x^9*D)/
3 + (3*a*b^2*x^11*D)/11 + (A*a^2*b*x^6)/2 + (3*A*a*b^2*x^8)/8 + (3*B*a^2*b*
x^7)/7 + (B*a*b^2*x^9)/3 + (3*C*a^2*b*x^8)/8 + (3*C*a*b^2*x^10)/10
```

3.79 $\int x^2(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)dx$

Optimal. Leaf size=149

$$\frac{1}{3}a^3Ax^3+\frac{1}{4}a^3Bx^4+\frac{1}{5}a^2(3Ab+aC)x^5+\frac{1}{6}a^2(3bB+aD)x^6+\frac{3}{7}ab(Ab+aC)x^7+\frac{3}{8}ab(bB+aD)x^8+\frac{1}{9}b^2(Ab+3aC)x$$

```
[Out] 1/3*a^3*A*x^3+1/4*a^3*B*x^4+1/5*a^2*(3*A*b+C*a)*x^5+1/6*a^2*(3*B*b+D*a)*x^6
+3/7*a*b*(A*b+C*a)*x^7+3/8*a*b*(B*b+D*a)*x^8+1/9*b^2*(A*b+3*C*a)*x^9+1/10*b
^2*(B*b+3*D*a)*x^10+1/11*b^3*C*x^11+1/12*b^3*D*x^12
```

Rubi [A]
time = 0.09, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = $\{1816\}$

$$\frac{1}{3}a^3Ax^3+\frac{1}{4}a^3Bx^4+\frac{1}{5}a^2x^5(aC+3Ab)+\frac{1}{6}a^2x^6(aD+3bB)+\frac{1}{9}b^2x^9(3aC+Ab)+\frac{3}{7}abx^7(aC+Ab)+\frac{1}{10}b^2x^{10}(3aD+bB)+\frac{3}{8}abx^8(aD+bB)+\frac{1}{11}b^3Cx^{11}+\frac{1}{12}b^3Dx^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*A*x^3)/3 + (a^3*B*x^4)/4 + (a^2*(3*A*b + a*C)*x^5)/5 + (a^2*(3*b*B + a
*D)*x^6)/6 + (3*a*b*(A*b + a*C)*x^7)/7 + (3*a*b*(b*B + a*D)*x^8)/8 + (b^2*(
A*b + 3*a*C)*x^9)/9 + (b^2*(b*B + 3*a*D)*x^10)/10 + (b^3*C*x^11)/11 + (b^3*
D*x^12)/12
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \int\left(a^3Ax^2+a^3Bx^3+a^2(3Ab+aC)x^4+a^2(3bB+aD)x^5+\right.\\ &= \frac{1}{3}a^3Ax^3+\frac{1}{4}a^3Bx^4+\frac{1}{5}a^2(3Ab+aC)x^5+\frac{1}{6}a^2(3bB+aD)x^6\end{aligned}$$

Mathematica [A]
time = 0.04, size = 125, normalized size = 0.84

$$\frac{14b^3x^9(220A+3x(66B+60Cx+55Dx^2))+462a^3x^3(20A+x(15B+2x(6C+5Dx)))+99a^2bx^5(168A+5x(28B+3x(8C+7Dx)))+33ab^2x^7(360A+7x(45B+4x(10C+9Dx)))}{27720}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (14*b^3*x^9*(220*A + 3*x*(66*B + 60*C*x + 55*D*x^2)) + 462*a^3*x^3*(20*A +
x*(15*B + 2*x*(6*C + 5*D*x))) + 99*a^2*b*x^5*(168*A + 5*x*(28*B + 3*x*(8*C
+ 7*D*x))) + 33*a*b^2*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*x))))/27720
```

Maple [A]
time = 0.12, size = 150, normalized size = 1.01

method	result
norman	$\frac{b^3 D x^{12}}{12}+\frac{b^3 C\,x^{11}}{11}+\left(\frac{1}{10}b^3 B+\frac{3}{10}a\,b^2 D\right)x^{10}+\left(\frac{1}{9}A\,b^3+\frac{1}{3}a\,b^2 C\right)x^9+\left(\frac{3}{8}a\,b^2 B+\frac{3}{8}a^2 b D\right)x^8+\big(\frac{3}{7}Aa$
default	$\frac{b^3 D x^{12}}{12}+\frac{b^3 C\,x^{11}}{11}+\frac{\left(b^3 B+3a\,b^2 D\right)x^{10}}{10}+\frac{\left(A\,b^3+3a\,b^2 C\right)x^9}{9}+\frac{\left(3a\,b^2 B+3a^2 b D\right)x^8}{8}+\frac{\left(3Aa\,b^2+3a^2 b C\right)x^7}{7}+\frac{(3B\,a^2 b+a^3}{6}$
gosper	$\frac{1}{12}b^3 D x^{12}+\frac{1}{11}b^3 C\,x^{11}+\frac{1}{10}x^{10}b^3 B+\frac{3}{10}x^{10}a\,b^2 D+\frac{1}{9}x^9 A\,b^3+\frac{1}{3}x^9 a\,b^2 C+\frac{3}{8}x^8 a\,b^2 B+\frac{3}{8}x^8 a^2 b D+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*b^3*D*x^12+1/11*b^3*C*x^11+1/10*(B*b^3+3*D*a*b^2)*x^10+1/9*(A*b^3+3*C*
a*b^2)*x^9+1/8*(3*B*a*b^2+3*D*a^2*b)*x^8+1/7*(3*A*a*b^2+3*C*a^2*b)*x^7+1/6*
(3*B*a^2*b+D*a^3)*x^6+1/5*(3*A*a^2*b+C*a^3)*x^5+1/4*a^3*B*x^4+1/3*a^3*A*x^3
```

Maxima [A]
time = 0.28, size = 145, normalized size = 0.97

$$\frac{1}{12}Db^3x^{12}+\frac{1}{11}Cb^3x^{11}+\frac{1}{10}\left(3Dab^2+Bb^3\right)x^{10}+\frac{1}{9}\left(3Cab^2+Ab^3\right)x^9+\frac{3}{8}\left(Da^2b+Bab^2\right)x^8+\frac{1}{4}Ba^3x^4+\frac{3}{7}\left(Ca^2b+Aab^2\right)x^7+\frac{1}{3}Aa^3x^3+\frac{1}{6}\left(Da^3+3Ba^2b\right)x^6+\frac{1}{5}\left(Ca^3+3Aa^2b\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 1/10*(3*D*a*b^2 + B*b^3)*x^10 + 1/9*(3*
C*a*b^2 + A*b^3)*x^9 + 3/8*(D*a^2*b + B*a*b^2)*x^8 + 1/4*B*a^3*x^4 + 3/7*(C
*a^2*b + A*a*b^2)*x^7 + 1/3*A*a^3*x^3 + 1/6*(D*a^3 + 3*B*a^2*b)*x^6 + 1/5*(
C*a^3 + 3*A*a^2*b)*x^5
```

Fricas [A]
time = 3.23, size = 145, normalized size = 0.97

$$\frac{1}{12}Db^3x^{12}+\frac{1}{11}Cb^3x^{11}+\frac{1}{10}\left(3Dab^2+Bb^3\right)x^{10}+\frac{1}{9}\left(3Cab^2+Ab^3\right)x^9+\frac{3}{8}\left(Da^2b+Bab^2\right)x^8+\frac{1}{4}Ba^3x^4+\frac{3}{7}\left(Ca^2b+Aab^2\right)x^7+\frac{1}{3}Aa^3x^3+\frac{1}{6}\left(Da^3+3Ba^2b\right)x^6+\frac{1}{5}\left(Ca^3+3Aa^2b\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 1/10*(3*D*a*b^2 + B*b^3)*x^10 + 1/9*(3*
C*a*b^2 + A*b^3)*x^9 + 3/8*(D*a^2*b + B*a*b^2)*x^8 + 1/4*B*a^3*x^4 + 3/7*(C
```

```
*a^2*b + A*a*b^2)*x^7 + 1/3*A*a^3*x^3 + 1/6*(D*a^3 + 3*B*a^2*b)*x^6 + 1/5*(
C*a^3 + 3*A*a^2*b)*x^5
```

Sympy [A]
time = 0.02, size = 165, normalized size = 1.11

$$\frac{Aa^3x^3}{3} + \frac{Ba^3x^4}{4} + \frac{Cb^3x^{11}}{11} + \frac{Db^3x^{12}}{12} + x^{10}\left(\frac{Bb^3}{10} + \frac{3Dab^2}{10}\right) + x^9\left(\frac{Ab^3}{9} + \frac{Cab^2}{3}\right) + x^8\cdot\left(\frac{3Bab^2}{8} + \frac{3Da^2b}{8}\right) + x^7\cdot\left(\frac{3Aab^2}{7} + \frac{3Ca^2b}{7}\right) + x^6\left(\frac{Ba^2b}{2} + \frac{Da^3}{6}\right) + x^5\cdot\left(\frac{3Aa^2b}{5} + \frac{Ca^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**3/3 + B*a**3*x**4/4 + C*b**3*x**11/11 + D*b**3*x**12/12 + x**10*(
B*b**3/10 + 3*D*a*b**2/10) + x**9*(A*b**3/9 + C*a*b**2/3) + x**8*(3*B*a*b**
2/8 + 3*D*a**2*b/8) + x**7*(3*A*a*b**2/7 + 3*C*a**2*b/7) + x**6*(B*a**2*b/2
 + D*a**3/6) + x**5*(3*A*a**2*b/5 + C*a**3/5)
```

Giac [A]
time = 1.38, size = 153, normalized size = 1.03

$$\frac{1}{12}Db^3x^{12} + \frac{1}{11}Cb^3x^{11} + \frac{3}{10}Dab^2x^{10} + \frac{1}{10}Bb^3x^{10} + \frac{1}{3}Cab^2x^9 + \frac{1}{9}Ab^3x^9 + \frac{3}{8}Da^2bx^8 + \frac{3}{8}Bab^2x^8 + \frac{3}{7}Ca^2bx^7 + \frac{3}{7}Aab^2x^7 + \frac{1}{6}Da^3x^6 + \frac{1}{2}Ba^2bx^6 + \frac{1}{5}Ca^3x^5 + \frac{3}{5}Aa^2bx^5 + \frac{1}{4}Ba^3x^4 + \frac{1}{3}Aa^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 3/10*D*a*b^2*x^10 + 1/10*B*b^3*x^10 + 1
/3*C*a*b^2*x^9 + 1/9*A*b^3*x^9 + 3/8*D*a^2*b*x^8 + 3/8*B*a*b^2*x^8 + 3/7*C*
a^2*b*x^7 + 3/7*A*a*b^2*x^7 + 1/6*D*a^3*x^6 + 1/2*B*a^2*b*x^6 + 1/5*C*a^3*x
^5 + 3/5*A*a^2*b*x^5 + 1/4*B*a^3*x^4 + 1/3*A*a^3*x^3
```

Mupad [B]
time = 1.28, size = 153, normalized size = 1.03

$$\frac{A\,a^3\,x^3}{3} + \frac{B\,a^3\,x^4}{4} + \frac{A\,b^3\,x^9}{9} + \frac{C\,a^3\,x^5}{5} + \frac{B\,b^3\,x^{10}}{10} + \frac{C\,b^3\,x^{11}}{11} + \frac{a^3\,x^6\,D}{6} + \frac{b^3\,x^{12}\,D}{12} + \frac{3\,a^2\,b\,x^8\,D}{8} + \frac{3\,a\,b^2\,x^{10}\,D}{10} + \frac{3\,A\,a^2\,b\,x^5}{5} + \frac{3\,A\,a\,b^2\,x^7}{7} + \frac{B\,a^2\,b\,x^6}{2} + \frac{3\,B\,a\,b^2\,x^8}{8} + \frac{3\,C\,a^2\,b\,x^7}{7} + \frac{C\,a\,b^2\,x^9}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (A*a^3*x^3)/3 + (B*a^3*x^4)/4 + (A*b^3*x^9)/9 + (C*a^3*x^5)/5 + (B*b^3*x^10
)/10 + (C*b^3*x^11)/11 + (a^3*x^6*D)/6 + (b^3*x^12*D)/12 + (3*a^2*b*x^8*D)/
8 + (3*a*b^2*x^10*D)/10 + (3*A*a^2*b*x^5)/5 + (3*A*a*b^2*x^7)/7 + (B*a^2*b*
x^6)/2 + (3*B*a*b^2*x^8)/8 + (3*C*a^2*b*x^7)/7 + (C*a*b^2*x^9)/3
```

3.80 $\int x(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)dx$

Optimal. Leaf size=138

$$\frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{1}{5}a^2(3bB+aD)x^5+\frac{1}{2}a^2bCx^6+\frac{3}{7}ab(bB+aD)x^7+\frac{3}{8}ab^2Cx^8+\frac{1}{9}b^2(bB+3aD)x^9+\frac{1}{10}b^3Cx^{10}$$

```
[Out] 1/3*a^3*B*x^3+1/4*a^3*C*x^4+1/5*a^2*(3*B*b+D*a)*x^5+1/2*a^2*b*C*x^6+3/7*a*b
*(B*b+D*a)*x^7+3/8*a*b^2*C*x^8+1/9*b^2*(B*b+3*D*a)*x^9+1/10*b^3*C*x^10+1/11
*b^3*D*x^11+1/8*A*(b*x^2+a)^4/b
```

Rubi [A]
time = 0.07, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1596, 1824}

$$\frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{1}{5}a^2x^5(aD+3bB)+\frac{1}{2}a^2bCx^6+\frac{A(a+bx^2)^4}{8b}+\frac{1}{9}b^2x^9(3aD+bB)+\frac{3}{8}ab^2Cx^8+\frac{3}{7}abx^7(aD+bB)+\frac{1}{10}b^3Cx^{10}+\frac{1}{11}b^3Dx^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (a^3*B*x^3)/3 + (a^3*C*x^4)/4 + (a^2*(3*b*B + a*D)*x^5)/5 + (a^2*b*C*x^6)/2
 + (3*a*b*(b*B + a*D)*x^7)/7 + (3*a*b^2*C*x^8)/8 + (b^2*(b*B + 3*a*D)*x^9)/
9 + (b^3*C*x^10)/10 + (b^3*D*x^11)/11 + (A*(a + b*x^2)^4)/(8*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int x(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx = \frac{A(a+bx^2)^4}{8b} + \int (a+bx^2)^3 (-Ax+x(A+Bx+Cx^2+Dx^3$$

$$= \frac{A(a+bx^2)^4}{8b} + \int (a^3Bx^2+a^3Cx^3+a^2(3bB+aD)x^4+3a^2b$$

$$= \frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{1}{5}a^2(3bB+aD)x^5+\frac{1}{2}a^2bCx^6+\frac{3}{7}ab(bB$$

Mathematica [A]
time = 0.04, size = 124, normalized size = 0.90

$$\frac{7b^3x^8(495A+4x(110B+99Cx+90Dx^2))+462a^3x^2(30A+x(20B+3x(5C+4Dx)))+198a^2bx^4(105A+2x(42B+5x(7C+6Dx)))+165ab^2x^6(84A+x(72B+7x(9C+8Dx)))}{27720}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (7*b^3*x^8*(495*A + 4*x*(110*B + 99*C*x + 90*D*x^2)) + 462*a^3*x^2*(30*A +
x*(20*B + 3*x*(5*C + 4*D*x))) + 198*a^2*b*x^4*(105*A + 2*x*(42*B + 5*x*(7*C
 + 6*D*x))) + 165*a*b^2*x^6*(84*A + x*(72*B + 7*x*(9*C + 8*D*x))))/27720
```

Maple [A]
time = 0.10, size = 150, normalized size = 1.09

method	result
norman	$\frac{b^3Dx^{11}}{11}+\frac{b^3Cx^{10}}{10}+\left(\frac{1}{9}b^3B+\frac{1}{3}ab^2D\right)x^9+\left(\frac{1}{8}Ab^3+\frac{3}{8}ab^2C\right)x^8+\left(\frac{3}{7}ab^2B+\frac{3}{7}a^2bD\right)x^7+(\frac{1}{2}Aab^2$
default	$\frac{b^3Dx^{11}}{11}+\frac{b^3Cx^{10}}{10}+\frac{(b^3B+3ab^2D)x^9}{9}+\frac{(Ab^3+3ab^2C)x^8}{8}+\frac{(3ab^2B+3a^2bD)x^7}{7}+\frac{(3Aab^2+3a^2bC)x^6}{6}+\frac{(3Ba^2b+a^3D)}{5}$
gosper	$\frac{1}{11}b^3Dx^{11}+\frac{1}{10}b^3Cx^{10}+\frac{1}{9}x^9b^3B+\frac{1}{3}x^9ab^2D+\frac{1}{8}x^8Ab^3+\frac{3}{8}ab^2Cx^8+\frac{3}{7}x^7ab^2B+\frac{3}{7}x^7a^2bD+\frac{1}{2}x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b^3*D*x^11+1/10*b^3*C*x^10+1/9*(B*b^3+3*D*a*b^2)*x^9+1/8*(A*b^3+3*C*a*
b^2)*x^8+1/7*(3*B*a*b^2+3*D*a^2*b)*x^7+1/6*(3*A*a*b^2+3*C*a^2*b)*x^6+1/5*(3
*B*a^2*b+D*a^3)*x^5+1/4*(3*A*a^2*b+C*a^3)*x^4+1/3*a^3*B*x^3+1/2*a^3*A*x^2
```

Maxima [A]
time = 0.32, size = 145, normalized size = 1.05

$$\frac{1}{11}Db^3x^{11}+\frac{1}{10}Cb^3x^{10}+\frac{1}{9}(3\,Dab^2+Bb^3)x^9+\frac{1}{8}(3\,Cab^2+Ab^3)x^8+\frac{3}{7}(Da^2b+Bab^2)x^7+\frac{1}{3}Ba^3x^3+\frac{1}{2}(Ca^2b+Aab^2)x^6+\frac{1}{2}Aa^3x^2+\frac{1}{5}(Da^3+3\,Ba^2b)x^5+\frac{1}{4}(Ca^3+3\,Aa^2b)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/9*(3*D*a*b^2 + B*b^3)*x^9 + 1/8*(3*C*
a*b^2 + A*b^3)*x^8 + 3/7*(D*a^2*b + B*a*b^2)*x^7 + 1/3*B*a^3*x^3 + 1/2*(C*a
^2*b + A*a*b^2)*x^6 + 1/2*A*a^3*x^2 + 1/5*(D*a^3 + 3*B*a^2*b)*x^5 + 1/4*(C*
a^3 + 3*A*a^2*b)*x^4
```

Fricas [A]
time = 2.80, size = 145, normalized size = 1.05

$$\frac{1}{11}\,Db^{3}x^{11}+\frac{1}{10}\,Cb^{3}x^{10}+\frac{1}{9}\,(3\,Dab^{2}+Bb^{3})x^{9}+\frac{1}{8}\,(3\,Cab^{2}+Ab^{3})x^{8}+\frac{3}{7}\,(Da^{2}b+Bab^{2})x^{7}+\frac{1}{3}\,Ba^{3}x^{3}+\frac{1}{2}\,(Ca^{2}b+Aab^{2})x^{6}+\frac{1}{2}\,Aa^{3}x^{2}+\frac{1}{5}\,(Da^{3}+3\,Ba^{2}b)x^{5}+\frac{1}{4}\,(Ca^{3}+3\,Aa^{2}b)x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/9*(3*D*a*b^2 + B*b^3)*x^9 + 1/8*(3*C*
a*b^2 + A*b^3)*x^8 + 3/7*(D*a^2*b + B*a*b^2)*x^7 + 1/3*B*a^3*x^3 + 1/2*(C*a
^2*b + A*a*b^2)*x^6 + 1/2*A*a^3*x^2 + 1/5*(D*a^3 + 3*B*a^2*b)*x^5 + 1/4*(C*
a^3 + 3*A*a^2*b)*x^4
```

Sympy [A]
time = 0.02, size = 163, normalized size = 1.18

$$\frac{Aa^{3}x^{2}}{2}+\frac{Ba^{3}x^{3}}{3}+\frac{Cb^{3}x^{10}}{10}+\frac{Db^{3}x^{11}}{11}+x^{9}\left(\frac{Bb^{3}}{9}+\frac{Dab^{2}}{3}\right)+x^{8}\left(\frac{Ab^{3}}{8}+\frac{3Cab^{2}}{8}\right)+x^{7}\cdot\left(\frac{3Bab^{2}}{7}+\frac{3Da^{2}b}{7}\right)+x^{6}\left(\frac{Aab^{2}}{2}+\frac{Ca^{2}b}{2}\right)+x^{5}\cdot\left(\frac{3Ba^{2}b}{5}+\frac{Da^{3}}{5}\right)+x^{4}\cdot\left(\frac{3Aa^{2}b}{4}+\frac{Ca^{3}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x**2/2 + B*a**3*x**3/3 + C*b**3*x**10/10 + D*b**3*x**11/11 + x**9*(B
*b**3/9 + D*a*b**2/3) + x**8*(A*b**3/8 + 3*C*a*b**2/8) + x**7*(3*B*a*b**2/7
 + 3*D*a**2*b/7) + x**6*(A*a*b**2/2 + C*a**2*b/2) + x**5*(3*B*a**2*b/5 + D*
a**3/5) + x**4*(3*A*a**2*b/4 + C*a**3/4)
```

Giac [A]
time = 0.95, size = 153, normalized size = 1.11

$$\frac{1}{11}\,Db^{3}x^{11}+\frac{1}{10}\,Cb^{3}x^{10}+\frac{1}{3}\,Dab^{2}x^{9}+\frac{1}{9}\,Bb^{3}x^{9}+\frac{3}{8}\,Cab^{2}x^{8}+\frac{1}{8}\,Ab^{3}x^{8}+\frac{3}{7}\,Da^{2}bx^{7}+\frac{3}{7}\,Bab^{2}x^{7}+\frac{1}{2}\,Ca^{2}bx^{6}+\frac{1}{2}\,Aab^{2}x^{6}+\frac{1}{5}\,Da^{3}x^{5}+\frac{3}{5}\,Ba^{2}bx^{5}+\frac{1}{4}\,Ca^{3}x^{4}+\frac{3}{4}\,Aa^{2}bx^{4}+\frac{1}{3}\,Ba^{3}x^{3}+\frac{1}{2}\,Aa^{3}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/3*D*a*b^2*x^9 + 1/9*B*b^3*x^9 + 3/8*C
*a*b^2*x^8 + 1/8*A*b^3*x^8 + 3/7*D*a^2*b*x^7 + 3/7*B*a*b^2*x^7 + 1/2*C*a^2*
b*x^6 + 1/2*A*a*b^2*x^6 + 1/5*D*a^3*x^5 + 3/5*B*a^2*b*x^5 + 1/4*C*a^3*x^4 +
 3/4*A*a^2*b*x^4 + 1/3*B*a^3*x^3 + 1/2*A*a^3*x^2
```

Mupad [B]
time = 1.28, size = 153, normalized size = 1.11

$$\frac{A\,a^3\,x^2}{2}+\frac{B\,a^3\,x^3}{3}+\frac{A\,b^3\,x^8}{8}+\frac{C\,a^3\,x^4}{4}+\frac{B\,b^3\,x^9}{9}+\frac{C\,b^3\,x^{10}}{10}+\frac{a^3\,x^5\,D}{5}+\frac{b^3\,x^{11}\,D}{11}+\frac{3\,a^2\,b\,x^7\,D}{7}+\frac{a\,b^2\,x^9\,D}{3}+\frac{3\,A\,a^2\,b\,x^4}{4}+\frac{A\,a\,b^2\,x^6}{2}+\frac{3\,B\,a^2\,b\,x^5}{5}+\frac{3\,B\,a\,b^2\,x^7}{7}+\frac{C\,a^2\,b\,x^6}{2}+\frac{3\,C\,a\,b^2\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (A*a^3*x^2)/2 + (B*a^3*x^3)/3 + (A*b^3*x^8)/8 + (C*a^3*x^4)/4 + (B*b^3*x^9)
/9 + (C*b^3*x^10)/10 + (a^3*x^5*D)/5 + (b^3*x^11*D)/11 + (3*a^2*b*x^7*D)/7
+ (a*b^2*x^9*D)/3 + (3*A*a^2*b*x^4)/4 + (A*a*b^2*x^6)/2 + (3*B*a^2*b*x^5)/5
 + (3*B*a*b^2*x^7)/7 + (C*a^2*b*x^6)/2 + (3*C*a*b^2*x^8)/8
```

3.81 $\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) dx$

Optimal. Leaf size=133

$$a^3Ax+\frac{1}{3}a^2(3Ab+aC)x^3+\frac{1}{4}a^3Dx^4+\frac{3}{5}ab(Ab+aC)x^5+\frac{1}{2}a^2bDx^6+\frac{1}{7}b^2(Ab+3aC)x^7+\frac{3}{8}ab^2Dx^8+\frac{1}{9}b^3Cx^9+\frac{1}{10}b$$

```
[Out] a^3*A*x+1/3*a^2*(3*A*b+C*a)*x^3+1/4*a^3*D*x^4+3/5*a*b*(A*b+C*a)*x^5+1/2*a^2
*b*D*x^6+1/7*b^2*(A*b+3*C*a)*x^7+3/8*a*b^2*D*x^8+1/9*b^3*C*x^9+1/10*b^3*D*x
^10+1/8*B*(b*x^2+a)^4/b
```

Rubi [A]

time = 0.07, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1596, 1824}

$$a^3Ax+\frac{1}{4}a^3Dx^4+\frac{1}{3}a^2x^3(aC+3Ab)+\frac{1}{2}a^2bDx^6+\frac{1}{7}b^2x^7(3aC+Ab)+\frac{3}{5}abx^5(aC+Ab)+\frac{3}{8}ab^2Dx^8+\frac{B\left(a+bx^2\right)^4}{8b}+\frac{1}{9}b^3Cx^9+\frac{1}{10}b^3Dx^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*C)*x^3)/3 + (a^3*D*x^4)/4 + (3*a*b*(A*b + a*C)*x^
5)/5 + (a^2*b*D*x^6)/2 + (b^2*(A*b + 3*a*C)*x^7)/7 + (3*a*b^2*D*x^8)/8 + (b
^3*C*x^9)/9 + (b^3*D*x^10)/10 + (B*(a + b*x^2)^4)/(8*b)
```

Rule 1596

```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{B(a+bx^2)^4}{8b} + \int \left(a+bx^2\right)^3 \left(A+Cx^2+Dx^3\right)\,dx \\
&= \frac{B(a+bx^2)^4}{8b} + \int \left(a^3A + a^2(3Ab+aC)x^2 + a^3Dx^3 + 3ab(Ab+ \right. \\
&= a^3Ax + \frac{1}{3}a^2(3Ab+aC)x^3 + \frac{1}{4}a^3Dx^4 + \frac{3}{5}ab(Ab+aC)x^5 + \frac{1}{2}a^2
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 121, normalized size = 0.91

$$\frac{210a^3x(12A+x(6B+x(4C+3Dx)))+126a^2bx^3(20A+x(15B+2x(6C+5Dx)))+9ab^2x^5(168A+5x(28B+3x(8C+7Dx)))+b^3x^7(360A+7x(45B+4x(10C+9Dx)))}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

```
[Out] (210*a^3*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 126*a^2*b*x^3*(20*A + x*(15
*B + 2*x*(6*C + 5*D*x))) + 9*a*b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*
x))) + b^3*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*x))))/2520
```

Maple [A]
time = 0.10, size = 147, normalized size = 1.11

method	result
norman	$\frac{b^3Dx^{10}}{10}+\frac{b^3Cx^9}{9}+(\frac{1}{8}b^3B+\frac{3}{8}ab^2D)x^8+(\frac{1}{7}Ab^3+\frac{3}{7}ab^2C)x^7+(\frac{1}{2}ab^2B+\frac{1}{2}a^2bD)x^6+(\frac{3}{5}Aab^2+$
default	$\frac{b^3Dx^{10}}{10}+\frac{b^3Cx^9}{9}+\frac{(b^3B+3ab^2D)x^8}{8}+\frac{(Ab^3+3ab^2C)x^7}{7}+\frac{(3ab^2B+3a^2bD)x^6}{6}+\frac{(3Aab^2+3a^2bC)x^5}{5}+\frac{(3Ba^2b+a^3D)x}{4}$
gosper	$\frac{1}{10}b^3Dx^{10}+\frac{1}{9}b^3Cx^9+\frac{1}{8}Bb^3x^8+\frac{3}{8}ab^2Dx^8+\frac{1}{7}x^7Ab^3+\frac{3}{7}x^7ab^2C+\frac{1}{2}x^6ab^2B+\frac{1}{2}a^2bDx^6+\frac{3}{5}x^5$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*b^3*D*x^10+1/9*b^3*C*x^9+1/8*(B*b^3+3*D*a*b^2)*x^8+1/7*(A*b^3+3*C*a*b^
2)*x^7+1/6*(3*B*a*b^2+3*D*a^2*b)*x^6+1/5*(3*A*a*b^2+3*C*a^2*b)*x^5+1/4*(3*B
*a^2*b+D*a^3)*x^4+1/3*(3*A*a^2*b+C*a^3)*x^3+1/2*B*a^3*x^2+a^3*A*x
```

Maxima [A]
time = 0.28, size = 142, normalized size = 1.07

$$\frac{1}{10}\,Db^3x^{10}+\frac{1}{9}\,Cb^3x^9+\frac{1}{8}\,(3\,Dab^2+Bb^3)x^8+\frac{1}{7}\,(3\,Cab^2+Ab^3)x^7+\frac{1}{2}\,(Da^2b+Bab^2)x^6+\frac{1}{2}\,Ba^3x^2+\frac{3}{5}\,(Ca^2b+Aab^2)x^5+Aa^3x+\frac{1}{4}\,(Da^3+3\,Ba^2b)x^4+\frac{1}{3}\,(Ca^3+3\,Aa^2b)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

```
[Out] 1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 1/8*(3*D*a*b^2 + B*b^3)*x^8 + 1/7*(3*C*a*
b^2 + A*b^3)*x^7 + 1/2*(D*a^2*b + B*a*b^2)*x^6 + 1/2*B*a^3*x^2 + 3/5*(C*a^2
*b + A*a*b^2)*x^5 + A*a^3*x + 1/4*(D*a^3 + 3*B*a^2*b)*x^4 + 1/3*(C*a^3 + 3*
A*a^2*b)*x^3
```

Fricas [A]
time = 3.08, size = 142, normalized size = 1.07

$$\frac{1}{10}\,Db^3x^{10}+\frac{1}{9}\,Cb^3x^9+\frac{1}{8}\,(3\,Dab^2+Bb^3)x^8+\frac{1}{7}\,(3\,Cab^2+Ab^3)x^7+\frac{1}{2}\,(Da^2b+Bab^2)x^6+\frac{1}{2}\,Ba^3x^2+\frac{3}{5}\,(Ca^2b+Aab^2)x^5+Aa^3x+\frac{1}{4}\,(Da^3+3\,Ba^2b)x^4+\frac{1}{3}\,(Ca^3+3\,Aa^2b)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

```
[Out] 1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 1/8*(3*D*a*b^2 + B*b^3)*x^8 + 1/7*(3*C*a*
b^2 + A*b^3)*x^7 + 1/2*(D*a^2*b + B*a*b^2)*x^6 + 1/2*B*a^3*x^2 + 3/5*(C*a^2
*b + A*a*b^2)*x^5 + A*a^3*x + 1/4*(D*a^3 + 3*B*a^2*b)*x^4 + 1/3*(C*a^3 + 3*
A*a^2*b)*x^3
```

Sympy [A]
time = 0.02, size = 158, normalized size = 1.19

$$Aa^3x+\frac{Ba^3x^2}{2}+\frac{Cb^3x^9}{9}+\frac{Db^3x^{10}}{10}+x^8\left(\frac{Bb^3}{8}+\frac{3Dab^2}{8}\right)+x^7\left(\frac{Ab^3}{7}+\frac{3Cab^2}{7}\right)+x^6\left(\frac{Bab^2}{2}+\frac{Da^2b}{2}\right)+x^5\cdot\left(\frac{3Aab^2}{5}+\frac{3Ca^2b}{5}\right)+x^4\cdot\left(\frac{3Ba^2b}{4}+\frac{Da^3}{4}\right)+x^3\left(Aa^2b+\frac{Ca^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

```
[Out] A*a**3*x + B*a**3*x**2/2 + C*b**3*x**9/9 + D*b**3*x**10/10 + x**8*(B*b**3/8
 + 3*D*a*b**2/8) + x**7*(A*b**3/7 + 3*C*a*b**2/7) + x**6*(B*a*b**2/2 + D*a*
*2*b/2) + x**5*(3*A*a*b**2/5 + 3*C*a**2*b/5) + x**4*(3*B*a**2*b/4 + D*a**3/
4) + x**3*(A*a**2*b + C*a**3/3)
```

Giac [A]
time = 1.00, size = 149, normalized size = 1.12

$$\frac{1}{10}\,Db^3x^{10}+\frac{1}{9}\,Cb^3x^9+\frac{3}{8}\,Dab^2x^8+\frac{1}{8}\,Bb^3x^8+\frac{3}{7}\,Cab^2x^7+\frac{1}{7}\,Ab^3x^7+\frac{1}{2}\,Da^2bx^6+\frac{1}{2}\,Bab^2x^6+\frac{3}{5}\,Ca^2bx^5+\frac{3}{5}\,Aab^2x^5+\frac{1}{4}\,Da^3x^4+\frac{3}{4}\,Ba^2bx^4+\frac{1}{3}\,Ca^3x^3+Aa^2bx^3+\frac{1}{2}\,Ba^3x^2+Aa^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

```
[Out] 1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 3/8*D*a*b^2*x^8 + 1/8*B*b^3*x^8 + 3/7*C*a
*b^2*x^7 + 1/7*A*b^3*x^7 + 1/2*D*a^2*b*x^6 + 1/2*B*a*b^2*x^6 + 3/5*C*a^2*b*
x^5 + 3/5*A*a*b^2*x^5 + 1/4*D*a^3*x^4 + 3/4*B*a^2*b*x^4 + 1/3*C*a^3*x^3 + A
*a^2*b*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

Mupad [B]
time = 1.26, size = 149, normalized size = 1.12

$$\frac{B\,a^3\,x^2}{2}+\frac{A\,b^3\,x^7}{7}+\frac{C\,a^3\,x^3}{3}+\frac{B\,b^3\,x^8}{8}+\frac{C\,b^3\,x^9}{9}+\frac{a^3\,x^4\,D}{4}+\frac{b^3\,x^{10}\,D}{10}+A\,a^3\,x+\frac{a^2\,b\,x^6\,D}{2}+\frac{3\,a\,b^2\,x^8\,D}{8}+A\,a^2\,b\,x^3+\frac{3\,A\,a\,b^2\,x^5}{5}+\frac{3\,B\,a^2\,b\,x^4}{4}+\frac{B\,a\,b^2\,x^6}{2}+\frac{3\,C\,a^2\,b\,x^5}{5}+\frac{3\,C\,a\,b^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

```
[Out] (B*a^3*x^2)/2 + (A*b^3*x^7)/7 + (C*a^3*x^3)/3 + (B*b^3*x^8)/8 + (C*b^3*x^9)
/9 + (a^3*x^4*D)/4 + (b^3*x^10*D)/10 + A*a^3*x + (a^2*b*x^6*D)/2 + (3*a*b^2
*x^8*D)/8 + A*a^2*b*x^3 + (3*A*a*b^2*x^5)/5 + (3*B*a^2*b*x^4)/4 + (B*a*b^2*
x^6)/2 + (3*C*a^2*b*x^5)/5 + (3*C*a*b^2*x^7)/7
```

3.82 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x}\,dx$

Optimal. Leaf size=129

$$a^3Bx+\frac{3}{2}a^2Abx^2+\frac{1}{3}a^2(3bB+aD)x^3+\frac{3}{4}aAb^2x^4+\frac{3}{5}ab(bB+aD)x^5+\frac{1}{6}Ab^3x^6+\frac{1}{7}b^2(bB+3aD)x^7+\frac{1}{9}b^3Dx^9+\frac{C(}{}$$

```
[Out] a^3*B*x+3/2*a^2*A*b*x^2+1/3*a^2*(3*B*b+D*a)*x^3+3/4*a*A*b^2*x^4+3/5*a*b*(B*
b+D*a)*x^5+1/6*A*b^3*x^6+1/7*b^2*(B*b+3*D*a)*x^7+1/9*b^3*D*x^9+1/8*C*(b*x^2
+a)^4/b+a^3*A*ln(x)
```

Rubi [A]

time = 0.06, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1597, 1816}

$$a^3A\log(x)+a^3Bx+\frac{3}{2}a^2Abx^2+\frac{1}{3}a^2x^3(aD+3bB)+\frac{3}{4}aAb^2x^4+\frac{1}{7}b^2x^7(3aD+bB)+\frac{3}{5}abx^5(aD+bB)+\frac{C(a+bx^2)^4}{8b}+\frac{1}{6}Ab^3x^6+\frac{1}{9}b^3Dx^9$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] a^3*B*x + (3*a^2*A*b*x^2)/2 + (a^2*(3*b*B + a*D)*x^3)/3 + (3*a*A*b^2*x^4)/4
 + (3*a*b*(b*B + a*D)*x^5)/5 + (A*b^3*x^6)/6 + (b^2*(b*B + 3*a*D)*x^7)/7 +
(b^3*D*x^9)/9 + (C*(a + b*x^2)^4)/(8*b) + a^3*A*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3\,(A+Bx+Cx^2+Dx^3)}{x}\,dx &= \frac{C(a+bx^2)^4}{8b} + \int \frac{(a+bx^2)^3\,(A+Bx+Dx^3)}{x}\,dx \\ &= \frac{C(a+bx^2)^4}{8b} + \int \left(a^3B + \frac{a^3A}{x} + 3a^2Abx + a^2(3bB+aD)x^2 + \right. \\ &= a^3Bx + \frac{3}{2}a^2Abx^2 + \frac{1}{3}a^2(3bB+aD)x^3 + \frac{3}{4}aAb^2x^4 + \frac{3}{5}ab(bB+a\end{aligned}$$

Mathematica [A]
time = 0.04, size = 121, normalized size = 0.94

$$\frac{x(420a^3(6B+x(3C+2Dx))+126a^2bx(30A+x(20B+3x(5C+4Dx)))+18ab^2x^3(105A+2x(42B+5x(7C+6Dx)))+5b^3x^5(84A+x(72B+7x(9C+8Dx))))}{2520}+a^3A\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x,x]
```

```
[Out] (x*(420*a^3*(6*B + x*(3*C + 2*D*x)) + 126*a^2*b*x*(30*A + x*(20*B + 3*x*(5*
C + 4*D*x))) + 18*a*b^2*x^3*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x))) + 5*b^
3*x^5*(84*A + x*(72*B + 7*x*(9*C + 8*D*x)))))/2520 + a^3*A*Log[x]
```

Maple [A]
time = 0.10, size = 148, normalized size = 1.15

method	result
norman	$\left(\frac{1}{6}A\,b^3+\frac{1}{2}a\,b^2C\right)x^6+\left(\frac{1}{7}b^3B+\frac{3}{7}a\,b^2D\right)x^7+\left(\frac{3}{4}Aa\,b^2+\frac{3}{4}a^2bC\right)x^4+\left(\frac{3}{2}A\,a^2b+\frac{1}{2}a^3C\right)x^2+(B\,a$
default	$\frac{b^3Dx^9}{9}+\frac{b^3C\,x^8}{8}+\frac{B\,b^3x^7}{7}+\frac{3Da\,b^2x^7}{7}+\frac{A\,b^3x^6}{6}+\frac{Ca\,b^2x^6}{2}+\frac{3Ba\,b^2x^5}{5}+\frac{3Da^2b\,x^5}{5}+\frac{3aA\,b^2x^4}{4}+\frac{3C\,a^2b\,x^4}{4}+B\,a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b^3*D*x^9+1/8*b^3*C*x^8+1/7*B*b^3*x^7+3/7*D*a*b^2*x^7+1/6*A*b^3*x^6+1/2
*C*a*b^2*x^6+3/5*B*a*b^2*x^5+3/5*D*a^2*b*x^5+3/4*a*A*b^2*x^4+3/4*C*a^2*b*x^
4+B*a^2*b*x^3+1/3*D*a^3*x^3+3/2*a^2*A*b*x^2+1/2*C*a^3*x^2+B*a^3*x+a^3*A*ln(
x)
```

Maxima [A]
time = 0.27, size = 140, normalized size = 1.09

$$\frac{1}{9}\,Db^3x^9+\frac{1}{8}\,Cb^3x^8+\frac{1}{7}\,(3\,Dab^2+Bb^3)x^7+\frac{1}{6}\,(3\,Cab^2+Ab^3)x^6+\frac{3}{5}\,(Da^2b+Bab^2)x^5+Ba^3x+\frac{3}{4}\,(Ca^2b+Aab^2)x^4+Aa^3\log{(x)}+\frac{1}{3}\,(Da^3+3\,Ba^2b)x^3+\frac{1}{2}\,(Ca^3+3\,Aa^2b)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

```
[Out] 1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 1/7*(3*D*a*b^2 + B*b^3)*x^7 + 1/6*(3*C*a*b^
2 + A*b^3)*x^6 + 3/5*(D*a^2*b + B*a*b^2)*x^5 + B*a^3*x + 3/4*(C*a^2*b + A*a
*b^2)*x^4 + A*a^3*log(x) + 1/3*(D*a^3 + 3*B*a^2*b)*x^3 + 1/2*(C*a^3 + 3*A*a
^2*b)*x^2
```

Fricas [A]
time = 2.45, size = 140, normalized size = 1.09

$$\frac{1}{9}\,Db^3x^9+\frac{1}{8}\,Cb^3x^8+\frac{1}{7}\,(3\,Dab^2+Bb^3)x^7+\frac{1}{6}\,(3\,Cab^2+Ab^3)x^6+\frac{3}{5}\,(Da^2b+Bab^2)x^5+Ba^3x+\frac{3}{4}\,(Ca^2b+Aab^2)x^4+Aa^3\log\left(x\right)+\frac{1}{3}\,(Da^3+3\,Ba^2b)x^3+\frac{1}{2}\,(Ca^3+3\,Aa^2b)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

```
[Out] 1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 1/7*(3*D*a*b^2 + B*b^3)*x^7 + 1/6*(3*C*a*b^
2 + A*b^3)*x^6 + 3/5*(D*a^2*b + B*a*b^2)*x^5 + B*a^3*x + 3/4*(C*a^2*b + A*a
*b^2)*x^4 + A*a^3*log(x) + 1/3*(D*a^3 + 3*B*a^2*b)*x^3 + 1/2*(C*a^3 + 3*A*a
^2*b)*x^2
```

Sympy [A]
time = 0.11, size = 158, normalized size = 1.22

$$Aa^3\log\left(x\right)+Ba^3x+\frac{Cb^3x^8}{8}+\frac{Db^3x^9}{9}+x^7\left(\frac{Bb^3}{7}+\frac{3Dab^2}{7}\right)+x^6\left(\frac{Ab^3}{6}+\frac{Cab^2}{2}\right)+x^5\cdot\left(\frac{3Bab^2}{5}+\frac{3Da^2b}{5}\right)+x^4\cdot\left(\frac{3Aab^2}{4}+\frac{3Ca^2b}{4}\right)+x^3\left(Ba^2b+\frac{Da^3}{3}\right)+x^2\cdot\left(\frac{3Aa^2b}{2}+\frac{Ca^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x,x)
```

```
[Out] A*a**3*log(x) + B*a**3*x + C*b**3*x**8/8 + D*b**3*x**9/9 + x**7*(B*b**3/7 +
 3*D*a*b**2/7) + x**6*(A*b**3/6 + C*a*b**2/2) + x**5*(3*B*a*b**2/5 + 3*D*a*
*2*b/5) + x**4*(3*A*a*b**2/4 + 3*C*a**2*b/4) + x**3*(B*a**2*b + D*a**3/3) +
 x**2*(3*A*a**2*b/2 + C*a**3/2)
```

Giac [A]
time = 0.66, size = 148, normalized size = 1.15

$$\frac{1}{9}\,Db^3x^9+\frac{1}{8}\,Cb^3x^8+\frac{3}{7}\,Dab^2x^7+\frac{1}{7}\,Bb^3x^7+\frac{1}{2}\,Cab^2x^6+\frac{1}{6}\,Ab^3x^6+\frac{3}{5}\,Da^2bx^5+\frac{3}{5}\,Bab^2x^5+\frac{3}{4}\,Ca^2bx^4+\frac{3}{4}\,Aab^2x^4+\frac{1}{3}\,Da^3x^3+Ba^2bx^3+\frac{1}{2}\,Ca^3x^2+\frac{3}{2}\,Aa^2bx^2+Ba^3x+Aa^3\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

```
[Out] 1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 3/7*D*a*b^2*x^7 + 1/7*B*b^3*x^7 + 1/2*C*a*b
^2*x^6 + 1/6*A*b^3*x^6 + 3/5*D*a^2*b*x^5 + 3/5*B*a*b^2*x^5 + 3/4*C*a^2*b*x^
4 + 3/4*A*a*b^2*x^4 + 1/3*D*a^3*x^3 + B*a^2*b*x^3 + 1/2*C*a^3*x^2 + 3/2*A*a
^2*b*x^2 + B*a^3*x + A*a^3*log(abs(x))
```

Mupad [B]
time = 1.26, size = 147, normalized size = 1.14

$$\frac{A\,b^3\,x^6}{6}+\frac{C\,a^3\,x^2}{2}+\frac{B\,b^3\,x^7}{7}+\frac{C\,b^3\,x^8}{8}+A\,a^3\,\ln\left(x\right)+\frac{a^3\,x^3\,D}{3}+\frac{b^3\,x^9\,D}{9}+B\,a^3\,x+\frac{3\,a^2\,b\,x^5\,D}{5}+\frac{3\,a\,b^2\,x^7\,D}{7}+\frac{3\,A\,a^2\,b\,x^2}{2}+\frac{3\,A\,a\,b^2\,x^4}{4}+B\,a^2\,b\,x^3+\frac{3\,B\,a\,b^2\,x^5}{5}+\frac{3\,C\,a^2\,b\,x^4}{4}+\frac{C\,a\,b^2\,x^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x,x)
```

```
[Out] (A*b^3*x^6)/6 + (C*a^3*x^2)/2 + (B*b^3*x^7)/7 + (C*b^3*x^8)/8 + A*a^3*log(x
) + (a^3*x^3*D)/3 + (b^3*x^9*D)/9 + B*a^3*x + (3*a^2*b*x^5*D)/5 + (3*a*b^2*
x^7*D)/7 + (3*A*a^2*b*x^2)/2 + (3*A*a*b^2*x^4)/4 + B*a^2*b*x^3 + (3*B*a*b^2
*x^5)/5 + (3*C*a^2*b*x^4)/4 + (C*a*b^2*x^6)/2
```

3.83 $\int \frac{\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)}{x^2}\,dx$

Optimal. Leaf size=124

$$-\frac{a^3A}{x}+a^2(3Ab+aC)x+\frac{3}{2}a^2bBx^2+ab(Ab+aC)x^3+\frac{3}{4}ab^2Bx^4+\frac{1}{5}b^2(Ab+3aC)x^5+\frac{1}{6}b^3Bx^6+\frac{1}{7}b^3Cx^7+\frac{D(a+}{8b}$$

```
[Out] -a^3*A/x+a^2*(3*A*b+C*a)*x+3/2*a^2*b*B*x^2+a*b*(A*b+C*a)*x^3+3/4*a*b^2*B*x^
4+1/5*b^2*(A*b+3*C*a)*x^5+1/6*b^3*B*x^6+1/7*b^3*C*x^7+1/8*D*(b*x^2+a)^4/b+a
^3*B*ln(x)
```

Rubi [A]
time = 0.08, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1597, 1642}

$$-\frac{a^3A}{x}+a^3B\log(x)+a^2x(aC+3Ab)+\frac{3}{2}a^2bBx^2+\frac{1}{5}b^2x^5(3aC+Ab)+abx^3(aC+Ab)+\frac{3}{4}ab^2Bx^4+\frac{D\left(a+bx^2\right)^4}{8b}+\frac{1}{6}b^3Bx^6+\frac{1}{7}b^3Cx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*C)*x + (3*a^2*b*B*x^2)/2 + a*b*(A*b + a*C)*x^
3 + (3*a*b^2*B*x^4)/4 + (b^2*(A*b + 3*a*C)*x^5)/5 + (b^3*B*x^6)/6 + (b^3*C*
x^7)/7 + (D*(a + b*x^2)^4)/(8*b) + a^3*B*Log[x]
```

Rule 1597

```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{D(a+bx^2)^4}{8b} + \int \frac{(a+bx^2)^3 (A+Bx+Cx^2)}{x^2}\,dx$$
$$= \frac{D(a+bx^2)^4}{8b} + \int \left(a^2(3Ab+aC) + \frac{a^3A}{x^2} + \frac{a^3B}{x} + 3a^2bBx + 3a\right.$$
$$= -\frac{a^3A}{x} + a^2(3Ab+aC)x + \frac{3}{2}a^2bBx^2 + ab(Ab+aC)x^3 + \frac{3}{4}ab^2B$$

Mathematica [A]
time = 0.05, size = 123, normalized size = 0.99

$$a^3\left(-\frac{A}{x}+Cx+\frac{Dx^2}{2}\right)+\frac{1}{4}a^2bx(12A+x(6B+x(4C+3Dx)))+\frac{1}{20}ab^2x^3(20A+x(15B+2x(6C+5Dx)))+\frac{1}{840}b^3x^5(168A+5x(28B+3x(8C+7Dx)))+a^3B\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

```
[Out] a^3*(-(A/x) + C*x + (D*x^2)/2) + (a^2*b*x*(12*A + x*(6*B + x*(4*C + 3*D*x))
))/4 + (a*b^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))))/20 + (b^3*x^5*(168
*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))))/840 + a^3*B*Log[x]
```

Maple [A]
time = 0.11, size = 145, normalized size = 1.17

method	result
default	$\frac{b^3Dx^8}{8}+\frac{b^3Cx^7}{7}+\frac{b^3Bx^6}{6}+\frac{Dab^2x^6}{2}+\frac{Ab^3x^5}{5}+\frac{3Cab^2x^5}{5}+\frac{3ab^2Bx^4}{4}+\frac{3Da^2bx^4}{4}+Aab^2x^3+Ca^2bx^3+$
norman	$\frac{(\frac{1}{5}Ab^3+\frac{3}{5}ab^2C)x^6+(\frac{1}{6}b^3B+\frac{1}{2}ab^2D)x^7+(\frac{3}{2}Ba^2b+\frac{1}{2}a^3D)x^3+(\frac{3}{4}ab^2B+\frac{3}{4}a^2bD)x^5+(Aab^2+a^2bC)x^4+(3Aa^2b+a^3C)x^2-a^3A+}{x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*b^3*D*x^8+1/7*b^3*C*x^7+1/6*b^3*B*x^6+1/2*D*a*b^2*x^6+1/5*A*b^3*x^5+3/5
*C*a*b^2*x^5+3/4*a*b^2*B*x^4+3/4*D*a^2*b*x^4+A*a*b^2*x^3+C*a^2*b*x^3+3/2*a^
2*b*B*x^2+1/2*D*a^3*x^2+3*A*a^2*b*x+a^3*C*x+a^3*B*ln(x)-a^3*A/x
```

Maxima [A]
time = 0.28, size = 139, normalized size = 1.12

$$\frac{1}{8}Db^3x^8+\frac{1}{7}Cb^3x^7+\frac{1}{6}(3Dab^2+Bb^3)x^6+\frac{1}{5}(3Cab^2+Ab^3)x^5+\frac{3}{4}(Da^2b+Bab^2)x^4+Ba^3\log(x)+(Ca^2b+Aab^2)x^3-\frac{Aa^3}{x}+\frac{1}{2}(Da^3+3Ba^2b)x^2+(Ca^3+3Aa^2b)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

```
[Out] 1/8*D*b^3*x^8 + 1/7*C*b^3*x^7 + 1/6*(3*D*a*b^2 + B*b^3)*x^6 + 1/5*(3*C*a*b^
2 + A*b^3)*x^5 + 3/4*(D*a^2*b + B*a*b^2)*x^4 + B*a^3*log(x) + (C*a^2*b + A*
a*b^2)*x^3 - A*a^3/x + 1/2*(D*a^3 + 3*B*a^2*b)*x^2 + (C*a^3 + 3*A*a^2*b)*x
```

Fricas [A]
time = 4.51, size = 147, normalized size = 1.19

$$\frac{105\,Db^3x^9+120\,Cb^3x^8+140\,(3\,Dab^2+Bb^3)x^7+168\,(3\,Cab^2+Ab^3)x^6+630\,(Da^2b+Bab^2)x^5+840\,Ba^3x\log\left(x\right)+840\,(Ca^2b+Aab^2)x^4-840\,Aa^3+420\,(Da^3+3\,Ba^2b)x^3+840\,(Ca^3+3\,Aa^2b)x^2}{840\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

```
[Out] 1/840*(105*D*b^3*x^9 + 120*C*b^3*x^8 + 140*(3*D*a*b^2 + B*b^3)*x^7 + 168*(3
*C*a*b^2 + A*b^3)*x^6 + 630*(D*a^2*b + B*a*b^2)*x^5 + 840*B*a^3*x*log(x) +
840*(C*a^2*b + A*a*b^2)*x^4 - 840*A*a^3 + 420*(D*a^3 + 3*B*a^2*b)*x^3 + 840
*(C*a^3 + 3*A*a^2*b)*x^2)/x
```

Sympy [A]
time = 0.12, size = 150, normalized size = 1.21

$$-\frac{Aa^3}{x}+Ba^3\log\left(x\right)+\frac{Cb^3x^7}{7}+\frac{Db^3x^8}{8}+x^6\left(\frac{Bb^3}{6}+\frac{Dab^2}{2}\right)+x^5\left(\frac{Ab^3}{5}+\frac{3Cab^2}{5}\right)+x^4\cdot\left(\frac{3Bab^2}{4}+\frac{3Da^2b}{4}\right)+x^3\left(Aab^2+Ca^2b\right)+x^2\cdot\left(\frac{3Ba^2b}{2}+\frac{Da^3}{2}\right)+x\left(3Aa^2b+Ca^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

```
[Out] -A*a**3/x + B*a**3*log(x) + C*b**3*x**7/7 + D*b**3*x**8/8 + x**6*(B*b**3/6
+ D*a*b**2/2) + x**5*(A*b**3/5 + 3*C*a*b**2/5) + x**4*(3*B*a*b**2/4 + 3*D*a
**2*b/4) + x**3*(A*a*b**2 + C*a**2*b) + x**2*(3*B*a**2*b/2 + D*a**3/2) + x*
(3*A*a**2*b + C*a**3)
```

Giac [A]
time = 0.95, size = 145, normalized size = 1.17

$$\frac{1}{8}\,Db^3x^8+\frac{1}{7}\,Cb^3x^7+\frac{1}{2}\,Dab^2x^6+\frac{1}{6}\,Bb^3x^6+\frac{3}{5}\,Cab^2x^5+\frac{1}{5}\,Ab^3x^5+\frac{3}{4}\,Da^2bx^4+\frac{3}{4}\,Bab^2x^4+Ca^2bx^3+Aab^2x^3+\frac{1}{2}\,Da^3x^2+\frac{3}{2}\,Ba^2bx^2+Ca^3x+3\,Aa^2bx+Ba^3\log\left(|x|\right)-\frac{Aa^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*D*b^3*x^8 + 1/7*C*b^3*x^7 + 1/2*D*a*b^2*x^6 + 1/6*B*b^3*x^6 + 3/5*C*a*b
^2*x^5 + 1/5*A*b^3*x^5 + 3/4*D*a^2*b*x^4 + 3/4*B*a*b^2*x^4 + C*a^2*b*x^3 +
A*a*b^2*x^3 + 1/2*D*a^3*x^2 + 3/2*B*a^2*b*x^2 + C*a^3*x + 3*A*a^2*b*x + B*a
^3*log(abs(x)) - A*a^3/x
```

Mupad [B]
time = 1.18, size = 121, normalized size = 0.98

$$\frac{{\left(b\,x^2+a\right)}^4\,D}{8\,b}-\frac{A\,a^3}{x}+\frac{A\,b^3\,x^5}{5}+\frac{B\,b^3\,x^6}{6}+\frac{C\,b^3\,x^7}{7}+B\,a^3\,\ln\left(x\right)+C\,a^3\,x+3\,A\,a^2\,b\,x+A\,a\,b^2\,x^3+\frac{3\,B\,a^2\,b\,x^2}{2}+\frac{3\,B\,a\,b^2\,x^4}{4}+C\,a^2\,b\,x^3+\frac{3\,C\,a\,b^2\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

```
[Out] ((a + b*x^2)^4*D)/(8*b) - (A*a^3)/x + (A*b^3*x^5)/5 + (B*b^3*x^6)/6 + (C*b^
3*x^7)/7 + B*a^3*log(x) + C*a^3*x + 3*A*a^2*b*x + A*a*b^2*x^3 + (3*B*a^2*b*
x^2)/2 + (3*B*a*b^2*x^4)/4 + C*a^2*b*x^3 + (3*C*a*b^2*x^5)/5
```

3.84 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$

Optimal. Leaf size=135

$$-\frac{a^3A}{2x^2}-\frac{a^3B}{x}+a^2(3bB+aD)x+\frac{3}{2}ab(Ab+aC)x^2+ab(bB+aD)x^3+\frac{1}{4}b^2(Ab+3aC)x^4+\frac{1}{5}b^2(bB+3aD)x^5+\frac{1}{6}b^3C$$

```
[Out] -1/2*a^3*A/x^2-a^3*B/x+a^2*(3*B*b+D*a)*x+3/2*a*b*(A*b+C*a)*x^2+a*b*(B*b+D*a
)*x^3+1/4*b^2*(A*b+3*C*a)*x^4+1/5*b^2*(B*b+3*D*a)*x^5+1/6*b^3*C*x^6+1/7*b^3
*D*x^7+a^2*(3*A*b+C*a)*ln(x)
```

Rubi [A]
time = 0.08, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1816}

$$-\frac{a^3A}{2x^2}-\frac{a^3B}{x}+a^2\log(x)(aC+3Ab)+a^2x(aD+3bB)+\frac{1}{4}b^2x^4(3aC+Ab)+\frac{3}{2}abx^2(aC+Ab)+\frac{1}{5}b^2x^5(3aD+bB)+abx^3(aD+bB)+\frac{1}{6}b^3Cx^6+\frac{1}{7}b^3Dx^7$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -1/2*(a^3*A)/x^2 - (a^3*B)/x + a^2*(3*b*B + a*D)*x + (3*a*b*(A*b + a*C)*x^2
)/2 + a*b*(b*B + a*D)*x^3 + (b^2*(A*b + 3*a*C)*x^4)/4 + (b^2*(b*B + 3*a*D)*
x^5)/5 + (b^3*C*x^6)/6 + (b^3*D*x^7)/7 + a^2*(3*A*b + a*C)*Log[x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3}\,dx &= \int \left(a^2(3bB+aD)+\frac{a^3A}{x^3}+\frac{a^3B}{x^2}+\frac{a^2(3Ab+aC)}{x}+3ab(Ab+\right.\\ &= -\frac{a^3A}{2x^2}-\frac{a^3B}{x}+a^2(3bB+aD)x+\frac{3}{2}ab(Ab+aC)x^2+ab(bB+\end{aligned}$$

Mathematica [A]
time = 0.04, size = 124, normalized size = 0.92

$$-\frac{a^3(A+2Bx-2Dx^3)}{2x^2}+\frac{1}{2}a^2bx(6B+x(3C+2Dx))+\frac{1}{20}ab^2x^2(30A+x(20B+3x(5C+4Dx)))+\frac{1}{420}b^3x^4(105A+2x(42B+5x(7C+6Dx)))+a^2(3Ab+aC)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

```
[Out] -1/2*(a^3*(A + 2*B*x - 2*D*x^3))/x^2 + (a^2*b*x*(6*B + x*(3*C + 2*D*x)))/2
+ (a*b^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))/20 + (b^3*x^4*(105*A +
2*x*(42*B + 5*x*(7*C + 6*D*x))))/420 + a^2*(3*A*b + a*C)*Log[x]
```

Maple [A]
time = 0.10, size = 142, normalized size = 1.05

method	result
default	$\frac{b^3 D x^7}{7}+\frac{b^3 C x^6}{6}+\frac{B b^3 x^5}{5}+\frac{3 D a b^2 x^5}{5}+\frac{A b^3 x^4}{4}+\frac{3 C a b^2 x^4}{4}+B a b^2 x^3+D a^2 b x^3+\frac{3 A a b^2 x^2}{2}+\frac{3 C a^2 b x^2}{2}+3$
norman	$\frac{\left(\frac{1}{4} A b^3+\frac{3}{4} a b^2 C\right) x^6+\left(\frac{1}{5} b^3 B+\frac{3}{5} a b^2 D\right) x^7+\left(\frac{3}{2} A a b^2+\frac{3}{2} a^2 b C\right) x^4+\left(a b^2 B+a^2 b D\right) x^5+\left(3 B a^2 b+a^3 D\right) x^3-\frac{a^3 A}{2}-B a^3 x+\frac{b^3 C x^8}{6}+\frac{b^3 D x^9}{7}}{x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b^3*D*x^7+1/6*b^3*C*x^6+1/5*B*b^3*x^5+3/5*D*a*b^2*x^5+1/4*A*b^3*x^4+3/4
*C*a*b^2*x^4+B*a*b^2*x^3+D*a^2*b*x^3+3/2*A*a*b^2*x^2+3/2*C*a^2*b*x^2+3*B*a^
2*b*x+a^3*D*x-1/2*a^3*A/x^2+a^2*(3*A*b+C*a)*ln(x)-a^3*B/x
```

Maxima [A]
time = 0.27, size = 139, normalized size = 1.03

$$\frac{1}{7} D b^3 x^7+\frac{1}{6} C b^3 x^6+\frac{1}{5} \left(3 D a b^2+B b^3\right) x^5+\frac{1}{4} \left(3 C a b^2+A b^3\right) x^4+\left(D a^2 b+B a b^2\right) x^3+\frac{3}{2} \left(C a^2 b+A a b^2\right) x^2+\left(D a^3+3 B a^2 b\right) x+\left(C a^3+3 A a^2 b\right) \log \left(x\right)-\frac{2 B a^3 x+A a^3}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

```
[Out] 1/7*D*b^3*x^7 + 1/6*C*b^3*x^6 + 1/5*(3*D*a*b^2 + B*b^3)*x^5 + 1/4*(3*C*a*b^
2 + A*b^3)*x^4 + (D*a^2*b + B*a*b^2)*x^3 + 3/2*(C*a^2*b + A*a*b^2)*x^2 + (D
*a^3 + 3*B*a^2*b)*x + (C*a^3 + 3*A*a^2*b)*log(x) - 1/2*(2*B*a^3*x + A*a^3)/
x^2
```

Fricas [A]
time = 3.62, size = 147, normalized size = 1.09

$$\frac{60 D b^3 x^9+70 C b^3 x^8+84 \left(3 D a b^2+B b^3\right) x^7+105 \left(3 C a b^2+A b^3\right) x^6+420 \left(D a^2 b+B a b^2\right) x^5-420 B a^3 x+630 \left(C a^2 b+A a b^2\right) x^4-210 A a^3+420 \left(D a^3+3 B a^2 b\right) x^3+420 \left(C a^3+3 A a^2 b\right) x^2 \log \left(x\right)}{420 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

```
[Out] 1/420*(60*D*b^3*x^9 + 70*C*b^3*x^8 + 84*(3*D*a*b^2 + B*b^3)*x^7 + 105*(3*C*
a*b^2 + A*b^3)*x^6 + 420*(D*a^2*b + B*a*b^2)*x^5 - 420*B*a^3*x + 630*(C*a^2
```

```
*b + A*a*b^2)*x^4 - 210*A*a^3 + 420*(D*a^3 + 3*B*a^2*b)*x^3 + 420*(C*a^3 +
3*A*a^2*b)*x^2*log(x))/x^2
```

Sympy [A]
time = 0.21, size = 151, normalized size = 1.12

$$\frac{Cb^3x^6}{6}+\frac{Db^3x^7}{7}+a^2\cdot(3Ab+Ca)\log{(x)}+x^5\left(\frac{Bb^3}{5}+\frac{3Dab^2}{5}\right)+x^4\left(\frac{Ab^3}{4}+\frac{3Cab^2}{4}\right)+x^3\left(Bab^2+Da^2b\right)+x^2\cdot\left(\frac{3Aab^2}{2}+\frac{3Ca^2b}{2}\right)+x\left(3Ba^2b+Da^3\right)+\frac{-Aa^3-2Ba^3x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

```
[Out] C*b**3*x**6/6 + D*b**3*x**7/7 + a**2*(3*A*b + C*a)*log(x) + x**5*(B*b**3/5
+ 3*D*a*b**2/5) + x**4*(A*b**3/4 + 3*C*a*b**2/4) + x**3*(B*a*b**2 + D*a**2*
b) + x**2*(3*A*a*b**2/2 + 3*C*a**2*b/2) + x*(3*B*a**2*b + D*a**3) + (-A*a**
3 - 2*B*a**3*x)/(2*x**2)
```

Giac [A]
time = 0.68, size = 144, normalized size = 1.07

$$\frac{1}{7}\,Db^3x^7+\frac{1}{6}\,Cb^3x^6+\frac{3}{5}\,Dab^2x^5+\frac{1}{5}\,Bb^3x^5+\frac{3}{4}\,Cab^2x^4+\frac{1}{4}\,Ab^3x^4+Da^2bx^3+Bab^2x^3+\frac{3}{2}\,Ca^2bx^2+\frac{3}{2}\,Aab^2x^2+Da^3x+3\,Ba^2bx+\left(Ca^3+3\,Aa^2b\right)\log\left(|x|\right)-\frac{2\,Ba^3x+Aa^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

```
[Out] 1/7*D*b^3*x^7 + 1/6*C*b^3*x^6 + 3/5*D*a*b^2*x^5 + 1/5*B*b^3*x^5 + 3/4*C*a*b
^2*x^4 + 1/4*A*b^3*x^4 + D*a^2*b*x^3 + B*a*b^2*x^3 + 3/2*C*a^2*b*x^2 + 3/2*
A*a*b^2*x^2 + D*a^3*x + 3*B*a^2*b*x + (C*a^3 + 3*A*a^2*b)*log(abs(x)) - 1/2
*(2*B*a^3*x + A*a^3)/x^2
```

Mupad [B]
time = 1.26, size = 143, normalized size = 1.06

$$\frac{A\,b^3\,x^4}{4}-\frac{B\,a^3}{x}-\frac{A\,a^3}{2\,x^2}+\frac{B\,b^3\,x^5}{5}+\frac{C\,b^3\,x^6}{6}+C\,a^3\,\ln\left(x\right)+a^3\,x\,D+\frac{b^3\,x^7\,D}{7}+a^2\,b\,x^3\,D+\frac{3\,a\,b^2\,x^5\,D}{5}+3\,B\,a^2\,b\,x+\frac{3\,A\,a\,b^2\,x^2}{2}+B\,a\,b^2\,x^3+\frac{3\,C\,a^2\,b\,x^2}{2}+\frac{3\,C\,a\,b^2\,x^4}{4}+3\,A\,a^2\,b\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

```
[Out] (A*b^3*x^4)/4 - (B*a^3)/x - (A*a^3)/(2*x^2) + (B*b^3*x^5)/5 + (C*b^3*x^6)/6
 + C*a^3*log(x) + a^3*x*D + (b^3*x^7*D)/7 + a^2*b*x^3*D + (3*a*b^2*x^5*D)/5
 + 3*B*a^2*b*x + (3*A*a*b^2*x^2)/2 + B*a*b^2*x^3 + (3*C*a^2*b*x^2)/2 + (3*C
*a*b^2*x^4)/4 + 3*A*a^2*b*log(x)
```

3.85 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$

Optimal. Leaf size=139

$$-\frac{a^3A}{3x^3}-\frac{a^3B}{2x^2}-\frac{a^2(3Ab+aC)}{x}+3ab(Ab+aC)x+\frac{3}{2}ab(bB+aD)x^2+\frac{1}{3}b^2(Ab+3aC)x^3+\frac{1}{4}b^2(bB+3aD)x^4+\frac{1}{5}b^3Cx$$

```
[Out] -1/3*a^3*A/x^3-1/2*a^3*B/x^2-a^2*(3*A*b+C*a)/x+3*a*b*(A*b+C*a)*x+3/2*a*b*(B
*b+D*a)*x^2+1/3*b^2*(A*b+3*C*a)*x^3+1/4*b^2*(B*b+3*D*a)*x^4+1/5*b^3*C*x^5+1
/6*b^3*D*x^6+a^2*(3*B*b+D*a)*ln(x)
```

Rubi [A]
time = 0.08, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1816}

$$-\frac{a^3A}{3x^3}-\frac{a^3B}{2x^2}-\frac{a^2(aC+3Ab)}{x}+a^2\log(x)(aD+3bB)+\frac{1}{3}b^2x^3(3aC+Ab)+3abx(aC+Ab)+\frac{1}{4}b^2x^4(3aD+bB)+\frac{3}{2}abx^2(aD+bB)+\frac{1}{5}b^3Cx^5+\frac{1}{6}b^3Dx^6$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -1/3*(a^3*A)/x^3 - (a^3*B)/(2*x^2) - (a^2*(3*A*b + a*C))/x + 3*a*b*(A*b + a
*C)*x + (3*a*b*(b*B + a*D)*x^2)/2 + (b^2*(A*b + 3*a*C)*x^3)/3 + (b^2*(b*B +
 3*a*D)*x^4)/4 + (b^3*C*x^5)/5 + (b^3*D*x^6)/6 + a^2*(3*b*B + a*D)*Log[x]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4}\,dx &= \int \left(3ab(Ab+aC)+\frac{a^3A}{x^4}+\frac{a^3B}{x^3}+\frac{a^2(3Ab+aC)}{x^2}+\frac{a^2(3bB+}{x}\right.\\ &= -\frac{a^3A}{3x^3}-\frac{a^3B}{2x^2}-\frac{a^2(3Ab+aC)}{x}+3ab(Ab+aC)x+\frac{3}{2}ab(bB+a\end{aligned}$$

Mathematica [A]
time = 0.03, size = 124, normalized size = 0.89

$$-\frac{a^3(2A+3x(B+2Cx))}{6x^3}+\frac{3a^2b(-2A+x^2(2C+Dx))}{2x}+\frac{1}{4}ab^2x(12A+x(6B+x(4C+3Dx)))+\frac{1}{60}b^3x^3(20A+x(15B+2x(6C+5Dx)))+a^2(3bB+aD)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

```
[Out] -1/6*(a^3*(2*A + 3*x*(B + 2*C*x)))/x^3 + (3*a^2*b*(-2*A + x^2*(2*C + D*x)))
/(2*x) + (a*b^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))))/4 + (b^3*x^3*(20*A +
x*(15*B + 2*x*(6*C + 5*D*x))))/60 + a^2*(3*b*B + a*D)*Log[x]
```

Maple [A]
time = 0.10, size = 141, normalized size = 1.01

method	result
default	$\frac{b^{3} D x^{6}}{6}+\frac{b^{3} C \,x^{5}}{5}+\frac{B \,b^{3} x^{4}}{4}+\frac{3 D a \,b^{2} x^{4}}{4}+\frac{A \,b^{3} x^{3}}{3}+C a \,b^{2} x^{3}+\frac{3 B a \,b^{2} x^{2}}{2}+\frac{3 D a^{2} b \,x^{2}}{2}+3 A a \,b^{2} x +3 a^{2} b C x -$
norman	$\frac{\left(\frac{1}{3} A \,b^{3}+a \,b^{2} C \right) x^{6}+\left(\frac{1}{4} b^{3} B +\frac{3}{4} a \,b^{2} D \right) x^{7}+\left(\frac{3}{2} a \,b^{2} B +\frac{3}{2} a^{2} b D \right) x^{5}+\left(3 A a \,b^{2}+3 a^{2} b C \right) x^{4}+\left(-3 A \,a^{2} b -a^{3} C \right) x^{2}-\frac{a^{3} A}{3}-\frac{B \,a^{3} x}{2}+\frac{b^{3} C \,x^{8}}{5}+b}{x^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*b^3*D*x^6+1/5*b^3*C*x^5+1/4*B*b^3*x^4+3/4*D*a*b^2*x^4+1/3*A*b^3*x^3+C*a
*b^2*x^3+3/2*B*a*b^2*x^2+3/2*D*a^2*b*x^2+3*A*a*b^2*x+3*a^2*b*C*x-1/2*a^3*B/
x^2-1/3*a^3*A/x^3+a^2*(3*B*b+D*a)*ln(x)-a^2*(3*A*b+C*a)/x
```

Maxima [A]
time = 0.27, size = 142, normalized size = 1.02

$$\frac{1}{6} \, D b^{3} x^{6} + \frac{1}{5} \, C b^{3} x^{5} + \frac{1}{4} \, {\left(3 \, D a b^{2} + B b^{3}\right)} x^{4} + \frac{1}{3} \, {\left(3 \, C a b^{2} + A b^{3}\right)} x^{3} + \frac{3}{2} \, {\left(D a^{2} b + B a b^{2}\right)} x^{2} + 3 \, {\left(C a^{2} b + A a b^{2}\right)} x + {\left(D a^{3} + 3 \, B a^{2} b\right)} \log\left(x\right) - \frac{3 \, B a^{3} x + 2 \, A a^{3} + 6 \, {\left(C a^{3} + 3 \, A a^{2} b\right)} x^{2}}{6 \, x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*D*b^3*x^6 + 1/5*C*b^3*x^5 + 1/4*(3*D*a*b^2 + B*b^3)*x^4 + 1/3*(3*C*a*b^
2 + A*b^3)*x^3 + 3/2*(D*a^2*b + B*a*b^2)*x^2 + 3*(C*a^2*b + A*a*b^2)*x + (D
*a^3 + 3*B*a^2*b)*log(x) - 1/6*(3*B*a^3*x + 2*A*a^3 + 6*(C*a^3 + 3*A*a^2*b)
*x^2)/x^3
```

Fricas [A]
time = 4.22, size = 147, normalized size = 1.06

$$\frac{10 \, D b^{3} x^{9} + 12 \, C b^{3} x^{8} + 15 \, {\left(3 \, D a b^{2} + B b^{3}\right)} x^{7} + 20 \, {\left(3 \, C a b^{2} + A b^{3}\right)} x^{6} + 90 \, {\left(D a^{2} b + B a b^{2}\right)} x^{5} - 30 \, B a^{3} x + 180 \, {\left(C a^{2} b + A a b^{2}\right)} x^{4} + 60 \, {\left(D a^{3} + 3 \, B a^{2} b\right)} x^{3} \log\left(x\right) - 20 \, A a^{3} - 60 \, {\left(C a^{3} + 3 \, A a^{2} b\right)} x^{2}}{60 \, x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

```
[Out] 1/60*(10*D*b^3*x^9 + 12*C*b^3*x^8 + 15*(3*D*a*b^2 + B*b^3)*x^7 + 20*(3*C*a*
b^2 + A*b^3)*x^6 + 90*(D*a^2*b + B*a*b^2)*x^5 - 30*B*a^3*x + 180*(C*a^2*b +
```

```
 A*a*b^2)*x^4 + 60*(D*a^3 + 3*B*a^2*b)*x^3*log(x) - 20*A*a^3 - 60*(C*a^3 +
3*A*a^2*b)*x^2)/x^3
```

Sympy [A]
time = 0.50, size = 155, normalized size = 1.12

$$\frac{Cb^3x^5}{5}+\frac{Db^3x^6}{6}+a^2\cdot(3Bb+Da)\log(x)+x^4\left(\frac{Bb^3}{4}+\frac{3Dab^2}{4}\right)+x^3\left(\frac{Ab^3}{3}+Cab^2\right)+x^2\cdot\left(\frac{3Bab^2}{2}+\frac{3Da^2b}{2}\right)+x(3Aab^2+3Ca^2b)+\frac{-2Aa^3-3Ba^3x+x^2(-18Aa^2b-6Ca^3)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

```
[Out] C*b**3*x**5/5 + D*b**3*x**6/6 + a**2*(3*B*b + D*a)*log(x) + x**4*(B*b**3/4
+ 3*D*a*b**2/4) + x**3*(A*b**3/3 + C*a*b**2) + x**2*(3*B*a*b**2/2 + 3*D*a**
2*b/2) + x*(3*A*a*b**2 + 3*C*a**2*b) + (-2*A*a**3 - 3*B*a**3*x + x**2*(-18*
A*a**2*b - 6*C*a**3))/(6*x**3)
```

Giac [A]
time = 0.68, size = 146, normalized size = 1.05

$$\frac{1}{6}Db^3x^6+\frac{1}{5}Cb^3x^5+\frac{3}{4}Dab^2x^4+\frac{1}{4}Bb^3x^4+Cab^2x^3+\frac{1}{3}Ab^3x^3+\frac{3}{2}Da^2bx^2+\frac{3}{2}Bab^2x^2+3\,Ca^2bx+3\,Aab^2x+\left(Da^3+3\,Ba^2b\right)\log\left(|x|\right)-\frac{3\,Ba^3x+2\,Aa^3+6\,(Ca^3+3\,Aa^2b)x^2}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

```
[Out] 1/6*D*b^3*x^6 + 1/5*C*b^3*x^5 + 3/4*D*a*b^2*x^4 + 1/4*B*b^3*x^4 + C*a*b^2*x
^3 + 1/3*A*b^3*x^3 + 3/2*D*a^2*b*x^2 + 3/2*B*a*b^2*x^2 + 3*C*a^2*b*x + 3*A*
a*b^2*x + (D*a^3 + 3*B*a^2*b)*log(abs(x)) - 1/6*(3*B*a^3*x + 2*A*a^3 + 6*(C
*a^3 + 3*A*a^2*b)*x^2)/x^3
```

Mupad [B]
time = 1.37, size = 148, normalized size = 1.06

$$\frac{B\,b^3\,x^4}{4}-\frac{C\,a^3}{x}-\frac{B\,a^3}{2\,x^2}+\frac{C\,b^3\,x^5}{5}+\frac{b^3\,x^6\,D}{6}-\frac{A\,(a^3+9\,a^2\,b\,x^2-9\,a\,b^2\,x^4-b^3\,x^6)}{3\,x^3}+\frac{a^3\,\ln\left(x^2\right)\,D}{2}+\frac{3\,a^2\,b\,x^2\,D}{2}+3\,C\,a^2\,b\,x+\frac{3\,a\,b^2\,x^4\,D}{4}+\frac{3\,B\,a\,b^2\,x^2}{2}+C\,a\,b^2\,x^3+3\,B\,a^2\,b\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

```
[Out] (B*b^3*x^4)/4 - (C*a^3)/x - (B*a^3)/(2*x^2) + (C*b^3*x^5)/5 + (b^3*x^6*D)/6
 - (A*(a^3 - b^3*x^6 + 9*a^2*b*x^2 - 9*a*b^2*x^4))/(3*x^3) + (a^3*log(x^2)*
D)/2 + (3*a^2*b*x^2*D)/2 + 3*C*a^2*b*x + (3*a*b^2*x^4*D)/4 + (3*B*a*b^2*x^2
)/2 + C*a*b^2*x^3 + 3*B*a^2*b*log(x)
```

3.86 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$

Optimal. Leaf size=151

$$-\frac{a(Ab-aC)x}{b^3}-\frac{a(bB-aD)x^2}{2b^3}+\frac{(Ab-aC)x^3}{3b^2}+\frac{(bB-aD)x^4}{4b^2}+\frac{Cx^5}{5b}+\frac{Dx^6}{6b}+\frac{a^{3/2}(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right.}{b^{7/2}}$$

```
[Out] -a*(A*b-C*a)*x/b^3-1/2*a*(B*b-D*a)*x^2/b^3+1/3*(A*b-C*a)*x^3/b^2+1/4*(B*b-D
*a)*x^4/b^2+1/5*C*x^5/b+1/6*D*x^6/b+a^(3/2)*(A*b-C*a)*arctan(x*b^(1/2)/a^(1
/2))/b^(7/2)+1/2*a^2*(B*b-D*a)*ln(b*x^2+a)/b^4
```

Rubi [A]

time = 0.10, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1816, 649, 211, 266}

$$\frac{a^{3/2}(Ab-aC)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2(bB-aD)\log\left(a+bx^2\right)}{2b^4}-\frac{ax(Ab-aC)}{b^3}+\frac{x^3(Ab-aC)}{3b^2}-\frac{ax^2(bB-aD)}{2b^3}+\frac{x^4(bB-aD)}{4b^2}+\frac{Cx^5}{5b}+\frac{Dx^6}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] -((a*(A*b - a*C)*x)/b^3) - (a*(b*B - a*D)*x^2)/(2*b^3) + ((A*b - a*C)*x^3)/
(3*b^2) + ((b*B - a*D)*x^4)/(4*b^2) + (C*x^5)/(5*b) + (D*x^6)/(6*b) + (a^(3
/2)*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2) + (a^2*(b*B - a*D)*Log
[a + b*x^2])/(2*b^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx &= \int \left(-\frac{a(Ab-aC)}{b^3}-\frac{a(bB-aD)x}{b^3}+\frac{(Ab-aC)x^2}{b^2}+\frac{(bB-aD)x^3}{b^2}+\frac{C}{}\right. \\
&= -\frac{a(Ab-aC)x}{b^3}-\frac{a(bB-aD)x^2}{2b^3}+\frac{(Ab-aC)x^3}{3b^2}+\frac{(bB-aD)x^4}{4b^2}+\frac{Cx^5}{5b} \\
&= -\frac{a(Ab-aC)x}{b^3}-\frac{a(bB-aD)x^2}{2b^3}+\frac{(Ab-aC)x^3}{3b^2}+\frac{(bB-aD)x^4}{4b^2}+\frac{Cx^5}{5b} \\
&= -\frac{a(Ab-aC)x}{b^3}-\frac{a(bB-aD)x^2}{2b^3}+\frac{(Ab-aC)x^3}{3b^2}+\frac{(bB-aD)x^4}{4b^2}+\frac{Cx^5}{5b}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 130, normalized size = 0.86

$$\frac{bx(30a^2(2C+Dx)-5ab(12A+x(6B+x(4C+3Dx)))+b^2x^2(20A+x(15B+2x(6C+5Dx))))-60a^{3/2}\sqrt{b}\,(-Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)-30a^2(-bB+aD)\log\left(a+bx^2\right)}{60b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] (b*x*(30*a^2*(2*C + D*x) - 5*a*b*(12*A + x*(6*B + x*(4*C + 3*D*x))) + b^2*x
^2*(20*A + x*(15*B + 2*x*(6*C + 5*D*x)))) - 60*a^(3/2)*Sqrt[b]*(-(A*b) + a*
C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] - 30*a^2*(-(b*B) + a*D)*Log[a + b*x^2])/(60*
b^4)
```

Maple [A]
time = 0.10, size = 141, normalized size = 0.93

method	result
default	$-\frac{-\frac{1}{6}b^2Dx^6-\frac{1}{5}b^2Cx^5-\frac{1}{4}b^2Bx^4+\frac{1}{4}Dabx^4-\frac{1}{3}Ab^2x^3+\frac{1}{3}Cabx^3+\frac{1}{2}Babx^2-\frac{1}{2}Da^2x^2+abAx-a^2Cx}{b^3}+\frac{a^2\left(\frac{(Bb-aD)\ln\left(bx^2+a\right)}{2b}+\frac{(A}{}\right.}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(-1/6*b^2*D*x^6-1/5*b^2*C*x^5-1/4*b^2*B*x^4+1/4*D*a*b*x^4-1/3*A*b^2*
x^3+1/3*C*a*b*x^3+1/2*B*a*b*x^2-1/2*D*a^2*x^2+a*b*A*x-a^2*C*x)+a^2/b^3*(1/2
*(B*b-D*a)/b*ln(b*x^2+a)+(A*b-C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.51, size = 145, normalized size = 0.96

$$-\frac{(Ca^3 - Aa^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{10\,Db^2x^6 + 12\,Cb^2x^5 - 15\,(Dab - Bb^2)x^4 - 20\,(Cab - Ab^2)x^3 + 30\,(Da^2 - Bab)x^2 + 60\,(Ca^2 - Aab)x}{60\,b^3} - \frac{(Da^3 - Ba^2b)\log\left(bx^2 + a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(C*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/60*(10*D*b^2*x
^6 + 12*C*b^2*x^5 - 15*(D*a*b - B*b^2)*x^4 - 20*(C*a*b - A*b^2)*x^3 + 30*(D
*a^2 - B*a*b)*x^2 + 60*(C*a^2 - A*a*b)*x)/b^3 - 1/2*(D*a^3 - B*a^2*b)*log(b
*x^2 + a)/b^4
```

Fricas [A]
time = 3.15, size = 332, normalized size = 2.20

$$\left[\frac{10\,Db^3x^6 + 12\,Cb^3x^5 - 15\,(Dab^2 - Bb^3)x^4 - 20\,(Cab^2 - Ab^3)x^3 + 30\,(Da^2b - Bab^2)x^2 - 30\,(Ca^2b - Aab^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 + 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 60\,(Ca^2b - Aab^2)x - 30\,(Da^3 - Ba^2b)\log\left(bx^2 + a\right)}{60\,b^4}, \frac{10\,Db^3x^6 + 12\,Cb^3x^5 - 15\,(Dab^2 - Bb^3)x^4 - 20\,(Cab^2 - Ab^3)x^3 + 30\,(Da^2b - Bab^2)x^2 - 60\,(Ca^2b - Aab^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 60\,(Ca^2b - Aab^2)x - 30\,(Da^3 - Ba^2b)\log\left(bx^2 + a\right)}{60\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/60*(10*D*b^3*x^6 + 12*C*b^3*x^5 - 15*(D*a*b^2 - B*b^3)*x^4 - 20*(C*a*b^2
 - A*b^3)*x^3 + 30*(D*a^2*b - B*a*b^2)*x^2 - 30*(C*a^2*b - A*a*b^2)*sqrt(-a
/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 60*(C*a^2*b - A*a*b^2
)*x - 30*(D*a^3 - B*a^2*b)*log(b*x^2 + a))/b^4, 1/60*(10*D*b^3*x^6 + 12*C*b
^3*x^5 - 15*(D*a*b^2 - B*b^3)*x^4 - 20*(C*a*b^2 - A*b^3)*x^3 + 30*(D*a^2*b
- B*a*b^2)*x^2 - 60*(C*a^2*b - A*a*b^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) +
 60*(C*a^2*b - A*a*b^2)*x - 30*(D*a^3 - B*a^2*b)*log(b*x^2 + a))/b^4]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(134) = 268$.
time = 0.54, size = 316, normalized size = 2.09

$$\frac{Cx^5}{5b} + \frac{Dx^6}{6b} + x^4\left(\frac{B}{4b} - \frac{Da}{4b^2}\right) + x^3\left(\frac{A}{3b} - \frac{Ca}{3b^2}\right) + x^2\left(-\frac{Ba}{2b^2} + \frac{Da^2}{2b^3}\right) + x\left(-\frac{Aa}{b^2} + \frac{Ca^2}{b^3}\right) + \left(-\frac{a^2(-Bb + Da)}{2b^4} - \frac{\sqrt{-a^3b^9}\,(-Ab + Ca)}{2b^8}\right)\log\left(x + \frac{Ba^2b - Da^3 - 2b^4\left(-\frac{a^2(-Bb+Da)}{2b^4} - \frac{\sqrt{-a^3b^9}\,(-Ab+Ca)}{2b^8}\right)}{-Aab^2 + Ca^2b}\right) + \left(-\frac{a^2(-Bb + Da)}{2b^4} + \frac{\sqrt{-a^3b^9}\,(-Ab + Ca)}{2b^8}\right)\log\left(x + \frac{Ba^2b - Da^3 - 2b^4\left(-\frac{a^2(-Bb+Da)}{2b^4} + \frac{\sqrt{-a^3b^9}\,(-Ab+Ca)}{2b^8}\right)}{-Aab^2 + Ca^2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**5/(5*b) + D*x**6/(6*b) + x**4*(B/(4*b) - D*a/(4*b**2)) + x**3*(A/(3*b)
 - C*a/(3*b**2)) + x**2*(-B*a/(2*b**2) + D*a**2/(2*b**3)) + x*(-A*a/b**2 +
```

```
C*a**2/b**3) + (-a**2*(-B*b + D*a)/(2*b**4) - sqrt(-a**3*b**9)*(-A*b + C*a)
/(2*b**8))*log(x + (B*a**2*b - D*a**3 - 2*b**4*(-a**2*(-B*b + D*a)/(2*b**4)
 - sqrt(-a**3*b**9)*(-A*b + C*a)/(2*b**8)))/(-A*a*b**2 + C*a**2*b)) + (-a**
2*(-B*b + D*a)/(2*b**4) + sqrt(-a**3*b**9)*(-A*b + C*a)/(2*b**8))*log(x + (
B*a**2*b - D*a**3 - 2*b**4*(-a**2*(-B*b + D*a)/(2*b**4) + sqrt(-a**3*b**9)*
(-A*b + C*a)/(2*b**8)))/(-A*a*b**2 + C*a**2*b))
```

Giac [A]
time = 1.70, size = 161, normalized size = 1.07

$$-\frac{\left(Ca^{3}-Aa^{2}b\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{3}}-\frac{\left(Da^{3}-Ba^{2}b\right)\log\left(bx^{2}+a\right)}{2\,b^{4}}+\frac{10\,Db^{5}x^{6}+12\,Cb^{5}x^{5}-15\,Dab^{4}x^{4}+15\,Bb^{5}x^{4}-20\,Cab^{4}x^{3}+20\,Ab^{5}x^{3}+30\,Da^{2}b^{3}x^{2}-30\,Bab^{4}x^{2}+60\,Ca^{2}b^{3}x-60\,Aab^{4}x}{60\,b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(D*a^3 - B*a
^2*b)*log(b*x^2 + a)/b^4 + 1/60*(10*D*b^5*x^6 + 12*C*b^5*x^5 - 15*D*a*b^4*x
^4 + 15*B*b^5*x^4 - 20*C*a*b^4*x^3 + 20*A*b^5*x^3 + 30*D*a^2*b^3*x^2 - 30*B
*a*b^4*x^2 + 60*C*a^2*b^3*x - 60*A*a*b^4*x)/b^6
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

```
[Out] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

3.87 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$

Optimal. Leaf size=130

$$-\frac{a(bB-aD)x}{b^3}+\frac{(Ab-aC)x^2}{2b^2}+\frac{(bB-aD)x^3}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}+\frac{a^{3/2}(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{a(Ab-aC)\,\mathrm{l}}{2b}$$

```
[Out] -a*(B*b-D*a)*x/b^3+1/2*(A*b-C*a)*x^2/b^2+1/3*(B*b-D*a)*x^3/b^2+1/4*C*x^4/b+
1/5*D*x^5/b+a^(3/2)*(B*b-D*a)*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)-1/2*a*(A*b-
C*a)*ln(b*x^2+a)/b^3
```

Rubi [A]
time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1816, 649, 211, 266}

$$\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bB-aD)}{b^{7/2}}-\frac{a(Ab-aC)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(Ab-aC)}{2b^2}-\frac{ax(bB-aD)}{b^3}+\frac{x^3(bB-aD)}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] -((a*(b*B - a*D)*x)/b^3) + ((A*b - a*C)*x^2)/(2*b^2) + ((b*B - a*D)*x^3)/(3
*b^2) + (C*x^4)/(4*b) + (D*x^5)/(5*b) + (a^(3/2)*(b*B - a*D)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/b^(7/2) - (a*(A*b - a*C)*Log[a + b*x^2])/(2*b^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \int \left(-\frac{a(bB-aD)}{b^3} + \frac{(Ab-aC)x}{b^2} + \frac{(bB-aD)x^2}{b^2} + \frac{Cx^3}{b} + \frac{Dx^4}{b} + a^2(b\right.$$

$$= -\frac{a(bB-aD)x}{b^3} + \frac{(Ab-aC)x^2}{2b^2} + \frac{(bB-aD)x^3}{3b^2} + \frac{Cx^4}{4b} + \frac{Dx^5}{5b} + \int a^2(b$$

$$= -\frac{a(bB-aD)x}{b^3} + \frac{(Ab-aC)x^2}{2b^2} + \frac{(bB-aD)x^3}{3b^2} + \frac{Cx^4}{4b} + \frac{Dx^5}{5b} - (a(Ab$$

$$= -\frac{a(bB-aD)x}{b^3} + \frac{(Ab-aC)x^2}{2b^2} + \frac{(bB-aD)x^3}{3b^2} + \frac{Cx^4}{4b} + \frac{Dx^5}{5b} + a^{3/2}(b$$

Mathematica [A]
time = 0.07, size = 114, normalized size = 0.88

$$-\frac{a^{3/2}(-bB+aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{7/2}} + \frac{x(60a^2D-10ab(6B+x(3C+2Dx))+b^2x(30A+x(20B+3x(5C+4Dx))))+30a(-Ab+aC)\log\left(a+bx^2\right)}{60b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] -((a^(3/2)*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)) + (x*(60*a^
2*D - 10*a*b*(6*B + x*(3*C + 2*D*x)) + b^2*x*(30*A + x*(20*B + 3*x*(5*C + 4
*D*x)))) + 30*a*(-(A*b) + a*C)*Log[a + b*x^2])/(60*b^3)
```

Maple [A]
time = 0.10, size = 128, normalized size = 0.98

method	result
default	$\frac{\frac{1}{5}b^2Dx^5+\frac{1}{4}b^2Cx^4+\frac{1}{3}b^2Bx^3-\frac{1}{3}Dabx^3+\frac{1}{2}Ab^2x^2-\frac{1}{2}Cabx^2-abBx+a^2Dx}{b^3} - \frac{a\left(\frac{\left(b^2A-abC\right)\ln\left(bx^2+a\right)}{2b}+\frac{\left(-abB+a^2D\right)\arctan\left(\frac{bx}{\sqrt{a}}\right.}{\sqrt{ab}}\right.}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/5*b^2*D*x^5+1/4*b^2*C*x^4+1/3*b^2*B*x^3-1/3*D*a*b*x^3+1/2*A*b^2*x^
2-1/2*C*a*b*x^2-a*b*B*x+a^2*D*x)-a/b^3*(1/2*(A*b^2-C*a*b)/b*ln(b*x^2+a)+(-B
*a*b+D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 127, normalized size = 0.98

$$\frac{(Ca^2 - Aab)\log(bx^2 + a)}{2\,b^3} - \frac{(Da^3 - Ba^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{12\,Db^2x^5 + 15\,Cb^2x^4 - 20\,(Dab - Bb^2)x^3 - 30\,(Cab - Ab^2)x^2 + 60\,(Da^2 - Bab)x}{60\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(C*a^2 - A*a*b)*log(b*x^2 + a)/b^3 - (D*a^3 - B*a^2*b)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^3) + 1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B*b
^2)*x^3 - 30*(C*a*b - A*b^2)*x^2 + 60*(D*a^2 - B*a*b)*x)/b^3
```

Fricas [A]
time = 7.66, size = 270, normalized size = 2.08

$$\left[\frac{12\,Db^2x^5 + 15\,Cb^2x^4 - 20\,(Dab - Bb^2)x^3 - 30\,(Cab - Ab^2)x^2 + 30\,(Da^2 - Bab)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 60\,(Da^2 - Bab)x + 30\,(Ca^2 - Aab)\log(bx^2 + a)}{60\,b^3}, \frac{12\,Db^2x^5 + 15\,Cb^2x^4 - 20\,(Dab - Bb^2)x^3 - 30\,(Cab - Ab^2)x^2 - 60\,(Da^2 - Bab)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 60\,(Da^2 - Bab)x + 30\,(Ca^2 - Aab)\log(bx^2 + a)}{60\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B*b^2)*x^3 - 30*(C*a*b - A
*b^2)*x^2 + 30*(D*a^2 - B*a*b)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a
)/(b*x^2 + a)) + 60*(D*a^2 - B*a*b)*x + 30*(C*a^2 - A*a*b)*log(b*x^2 + a))/
b^3, 1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B*b^2)*x^3 - 30*(C*a*b
 - A*b^2)*x^2 - 60*(D*a^2 - B*a*b)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 60*(
D*a^2 - B*a*b)*x + 30*(C*a^2 - A*a*b)*log(b*x^2 + a))/b^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(116) = 232$.
time = 0.53, size = 274, normalized size = 2.11

$$\frac{Cx^4}{4b} + \frac{Dx^5}{5b} + x^3\left(\frac{B}{3b} - \frac{Da}{3b^2}\right) + x^2\left(\frac{A}{2b} - \frac{Ca}{2b^2}\right) + x\left(-\frac{Ba}{b^2} + \frac{Da^2}{b^3}\right) + \left(\frac{a(-Ab + Ca)}{2b^3} - \frac{\sqrt{-a^3b^7}\,(-Bb + Da)}{2b^7}\right)\log\left(x + \frac{-Aab + Ca^2 - 2b^3\left(\frac{a(-Ab+Ca)}{2b^3} - \frac{\sqrt{-a^3b^7}\,(-Bb+Da)}{2b^7}\right)}{-Bab + Da^2}\right) + \left(\frac{a(-Ab + Ca)}{2b^3} + \frac{\sqrt{-a^3b^7}\,(-Bb + Da)}{2b^7}\right)\log\left(x + \frac{-Aab + Ca^2 - 2b^3\left(\frac{a(-Ab+Ca)}{2b^3} + \frac{\sqrt{-a^3b^7}\,(-Bb+Da)}{2b^7}\right)}{-Bab + Da^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**4/(4*b) + D*x**5/(5*b) + x**3*(B/(3*b) - D*a/(3*b**2)) + x**2*(A/(2*b)
 - C*a/(2*b**2)) + x*(-B*a/b**2 + D*a**2/b**3) + (a*(-A*b + C*a)/(2*b**3) -
 sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b**7))*log(x + (-A*a*b + C*a**2 - 2*b**3*
(a*(-A*b + C*a)/(2*b**3) - sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b**7)))/(-B*a*b
 + D*a**2)) + (a*(-A*b + C*a)/(2*b**3) + sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b
**7))*log(x + (-A*a*b + C*a**2 - 2*b**3*(a*(-A*b + C*a)/(2*b**3) + sqrt(-a*
*3*b**7)*(-B*b + D*a)/(2*b**7)))/(-B*a*b + D*a**2))
```

Giac [A]
time = 0.77, size = 137, normalized size = 1.05

$$\frac{(Ca^2 - Aab)\log\left(bx^2 + a\right)}{2\,b^3} - \frac{(Da^3 - Ba^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{12\,Db^4x^5 + 15\,Cb^4x^4 - 20\,Dab^3x^3 + 20\,Bb^4x^3 - 30\,Cab^3x^2 + 30\,Ab^4x^2 + 60\,Da^2b^2x - 60\,Bab^3x}{60\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*(C*a^2 - A*a*b)*log(b*x^2 + a)/b^3 - (D*a^3 - B*a^2*b)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^3) + 1/60*(12*D*b^4*x^5 + 15*C*b^4*x^4 - 20*D*a*b^3*x^3
+ 20*B*b^4*x^3 - 30*C*a*b^3*x^2 + 30*A*b^4*x^2 + 60*D*a^2*b^2*x - 60*B*a*b^
3*x)/b^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

```
[Out] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

3.88 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$

Optimal. Leaf size=111

$$\frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}-\frac{\sqrt{a}\,(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{a(bB-aD)\log\left(a+bx^2\right)}{2b^3}$$

```
[Out] (A*b-C*a)*x/b^2+1/2*(B*b-D*a)*x^2/b^2+1/3*C*x^3/b+1/4*D*x^4/b-1/2*a*(B*b-D*
a)*ln(b*x^2+a)/b^3-(A*b-C*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(5/2)
```

Rubi [A]
time = 0.08, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1816, 649, 211, 266}

$$-\frac{\sqrt{a}\,(Ab-aC)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(Ab-aC)}{b^2}-\frac{a(bB-aD)\log\left(a+bx^2\right)}{2b^3}+\frac{x^2(bB-aD)}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] ((A*b - a*C)*x)/b^2 + ((b*B - a*D)*x^2)/(2*b^2) + (C*x^3)/(3*b) + (D*x^4)/(
4*b) - (Sqrt[a]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) - (a*(b*B
- a*D)*Log[a + b*x^2])/(2*b^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx &= \int \left(\frac{Ab-aC}{b^2}+\frac{(bB-aD)x}{b^2}+\frac{Cx^2}{b}+\frac{Dx^3}{b}-\frac{a(Ab-aC)+a(bB-a}{b^2\left(a+bx^2\right)}\right.\\ &= \frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}-\frac{\int \frac{a(Ab-aC)+a(bB-aD)x}{a+bx^2}\,dx}{b^2}\\ &= \frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}-\frac{(a(Ab-aC))\int \frac{1}{a+bx^2}\,dx}{b^2}\\ &= \frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}-\frac{\sqrt{a}\,(Ab-aC)\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{b^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 95, normalized size = 0.86

$$\frac{bx(12Ab-6a(2C+Dx)+bx(6B+4Cx+3Dx^2))+12\sqrt{a}\,\sqrt{b}\,(-Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+6a(-bB+aD)\log\left(a+bx^2\right)}{12b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] (b*x*(12*A*b - 6*a*(2*C + D*x) + b*x*(6*B + 4*C*x + 3*D*x^2)) + 12*Sqrt[a]*
Sqrt[b]*(-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 6*a*(-(b*B) + a*D)*Log
[a + b*x^2])/(12*b^3)
```

Maple [A]
time = 0.10, size = 95, normalized size = 0.86

method	result	size
default	$\frac{\frac{1}{4}bDx^4+\frac{1}{3}bC\,x^3+\frac{1}{2}bB\,x^2-\frac{1}{2}Da\,x^2+Abx-aCx}{b^2}-\frac{a\left(\frac{(Bb-aD)\ln\left(b\,x^2+a\right)}{2b}+\frac{(Ab-aC)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b^2}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/4*b*D*x^4+1/3*b*C*x^3+1/2*b*B*x^2-1/2*D*a*x^2+A*b*x-a*C*x)-a/b^2*(
1/2*(B*b-D*a)/b*ln(b*x^2+a)+(A*b-C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 98, normalized size = 0.88

$$\frac{(Ca^2 - Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{3\,Dbx^4 + 4\,Cbx^3 - 6\,(Da - Bb)x^2 - 12\,(Ca - Ab)x}{12\,b^2} + \frac{(Da^2 - Bab)\log\left(bx^2 + a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (C*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/12*(3*D*b*x^4 + 4
*C*b*x^3 - 6*(D*a - B*b)*x^2 - 12*(C*a - A*b)*x)/b^2 + 1/2*(D*a^2 - B*a*b)*
log(b*x^2 + a)/b^3
```

Fricas [A]
time = 8.50, size = 238, normalized size = 2.14

$$\left[\frac{3\,Db^2x^4 + 4\,Cb^2x^3 - 6\,(Dab - Bb^2)x^2 - 6\,(Cab - Ab^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) - 12\,(Cab - Ab^2)x + 6\,(Da^2 - Bab)\log\left(bx^2 + a\right)}{12\,b^3}, \frac{3\,Db^2x^4 + 4\,Cb^2x^3 - 6\,(Dab - Bb^2)x^2 + 12\,(Cab - Ab^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) - 12\,(Cab - Ab^2)x + 6\,(Da^2 - Bab)\log\left(bx^2 + a\right)}{12\,b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(3*D*b^2*x^4 + 4*C*b^2*x^3 - 6*(D*a*b - B*b^2)*x^2 - 6*(C*a*b - A*b^2
)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 12*(C*a*b -
A*b^2)*x + 6*(D*a^2 - B*a*b)*log(b*x^2 + a))/b^3, 1/12*(3*D*b^2*x^4 + 4*C*b
^2*x^3 - 6*(D*a*b - B*b^2)*x^2 + 12*(C*a*b - A*b^2)*sqrt(a/b)*arctan(b*x*sq
rt(a/b)/a) - 12*(C*a*b - A*b^2)*x + 6*(D*a^2 - B*a*b)*log(b*x^2 + a))/b^3]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(97) = 194$.
time = 0.48, size = 245, normalized size = 2.21

$$\frac{Cx^3}{3b} + \frac{Dx^4}{4b} + x^2\left(\frac{B}{2b} - \frac{Da}{2b^2}\right) + x\left(\frac{A}{b} - \frac{Ca}{b^2}\right) + \left(\frac{a(-Bb + Da)}{2b^3} - \frac{\sqrt{-ab^7}\,(-Ab + Ca)}{2b^6}\right)\log\left(x + \frac{Bab - Da^2 + 2b^3\left(\frac{a(-Bb+Da)}{2b^3} - \frac{\sqrt{-ab^7}\,(-Ab+Ca)}{2b^6}\right)}{-Ab^2 + Cab}\right) + \left(\frac{a(-Bb + Da)}{2b^3} + \frac{\sqrt{-ab^7}\,(-Ab + Ca)}{2b^6}\right)\log\left(x + \frac{Bab - Da^2 + 2b^3\left(\frac{a(-Bb+Da)}{2b^3} + \frac{\sqrt{-ab^7}\,(-Ab+Ca)}{2b^6}\right)}{-Ab^2 + Cab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**3/(3*b) + D*x**4/(4*b) + x**2*(B/(2*b) - D*a/(2*b**2)) + x*(A/b - C*a/
b**2) + (a*(-B*b + D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*b + C*a)/(2*b**6))*log
(x + (B*a*b - D*a**2 + 2*b**3*(a*(-B*b + D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*
b + C*a)/(2*b**6)))/(-A*b**2 + C*a*b)) + (a*(-B*b + D*a)/(2*b**3) + sqrt(-a
*b**7)*(-A*b + C*a)/(2*b**6))*log(x + (B*a*b - D*a**2 + 2*b**3*(a*(-B*b + D
*a)/(2*b**3) + sqrt(-a*b**7)*(-A*b + C*a)/(2*b**6)))/(-A*b**2 + C*a*b))
```

Giac [**A**]
time = 1.14, size = 112, normalized size = 1.01

$$\frac{\left(Ca^2 - Aab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2} + \frac{\left(Da^2 - Bab\right)\log\left(bx^2 + a\right)}{2\,b^3} + \frac{3\,Db^3x^4 + 4\,Cb^3x^3 - 6\,Dab^2x^2 + 6\,Bb^3x^2 - 12\,Cab^2x + 12\,Ab^3x}{12\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (C*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(D*a^2 - B*a*b)
*log(b*x^2 + a)/b^3 + 1/12*(3*D*b^3*x^4 + 4*C*b^3*x^3 - 6*D*a*b^2*x^2 + 6*B
*b^3*x^2 - 12*C*a*b^2*x + 12*A*b^3*x)/b^4
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

```
[Out] int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

3.89 $\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx$

Optimal. Leaf size=92

$$\frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}-\frac{\sqrt{a}\,(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{(Ab-aC)\log\left(a+bx^2\right)}{2b^2}$$

```
[Out] (B*b-D*a)*x/b^2+1/2*C*x^2/b+1/3*D*x^3/b+1/2*(A*b-C*a)*ln(b*x^2+a)/b^2-(B*b-
D*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(5/2)
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1816, 649, 211, 266}

$$\frac{(Ab-aC)\log\left(a+bx^2\right)}{2b^2}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bB-aD)}{b^{5/2}}+\frac{x(bB-aD)}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] ((b*B - a*D)*x)/b^2 + (C*x^2)/(2*b) + (D*x^3)/(3*b) - (Sqrt[a]*(b*B - a*D)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + ((A*b - a*C)*Log[a + b*x^2])/(2*b^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

`&& PolyQ[Pq, x] && IGtQ[p, -2]`

Rubi steps

$$\begin{aligned}\int \frac{x\left(A+Bx+Cx^2+Dx^3\right)}{a+bx^2}\,dx &= \int \left(\frac{bB-aD}{b^2}+\frac{Cx}{b}+\frac{Dx^2}{b}-\frac{a(bB-aD)-b(Ab-aC)x}{b^2\left(a+bx^2\right)}\right)dx\\ &= \frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}-\frac{\int \frac{a(bB-aD)-b(Ab-aC)x}{a+bx^2}\,dx}{b^2}\\ &= \frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}+\frac{(Ab-aC)\int \frac{x}{a+bx^2}\,dx}{b}-\frac{(a(bB-aD))\int \frac{}{a-}}{b^2}\\ &= \frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}-\frac{\sqrt{a}\,(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{(Ab-aC}{}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 81, normalized size = 0.88

$$\frac{\sqrt{a}\,(-bB+aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{x(6bB-6aD+bx(3C+2Dx))+3(Ab-aC)\log\left(a+bx^2\right)}{6b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

```
[Out] (Sqrt[a]*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + (x*(6*b*B -
6*a*D + b*x*(3*C + 2*D*x)) + 3*(A*b - a*C)*Log[a + b*x^2])/(6*b^2)
```

Maple [A]
time = 0.10, size = 85, normalized size = 0.92

method	result	size
default	$\frac{\frac{1}{3}bDx^3+\frac{1}{2}bC\,x^2+bBx-aDx}{b^2}+\frac{\frac{\left(b^2A-abC\right)\ln\left(b\,x^2+a\right)}{2b}+\frac{\left(-abB+a^2D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{b^2}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/3*b*D*x^3+1/2*b*C*x^2+b*B*x-a*D*x)+1/b^2*(1/2*(A*b^2-C*a*b)/b*ln(b
*x^2+a)+(-B*a*b+D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.52, size = 82, normalized size = 0.89

$$-\frac{(Ca-Ab)\log\left(bx^2+a\right)}{2\,b^2}+\frac{(Da^2-Bab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{2\,Dbx^3+3\,Cbx^2-6\,(Da-Bb)x}{6\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*(C*a - A*b)*log(b*x^2 + a)/b^2 + (D*a^2 - B*a*b)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*b^2) + 1/6*(2*D*b*x^3 + 3*C*b*x^2 - 6*(D*a - B*b)*x)/b^2
```

Fricas [A]
time = 5.07, size = 180, normalized size = 1.96

$$\left[\frac{2\,Dbx^3+3\,Cbx^2+3\,(Da-Bb)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6\,(Da-Bb)x-3\,(Ca-Ab)\log\left(bx^2+a\right)}{6\,b^2},\frac{2\,Dbx^3+3\,Cbx^2+6\,(Da-Bb)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-6\,(Da-Bb)x-3\,(Ca-Ab)\log\left(bx^2+a\right)}{6\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*D*b*x^3 + 3*C*b*x^2 + 3*(D*a - B*b)*sqrt(-a/b)*log((b*x^2 + 2*b*x*s
qrt(-a/b) - a)/(b*x^2 + a)) - 6*(D*a - B*b)*x - 3*(C*a - A*b)*log(b*x^2 + a
))/b^2, 1/6*(2*D*b*x^3 + 3*C*b*x^2 + 6*(D*a - B*b)*sqrt(a/b)*arctan(b*x*sqr
t(a/b)/a) - 6*(D*a - B*b)*x - 3*(C*a - A*b)*log(b*x^2 + a))/b^2]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(80) = 160$.
time = 0.46, size = 211, normalized size = 2.29

$$\frac{Cx^2}{2b}+\frac{Dx^3}{3b}+x\left(\frac{B}{b}-\frac{Da}{b^2}\right)+\left(-\frac{-Ab+Ca}{2b^2}-\frac{\sqrt{-ab^5}\,(-Bb+Da)}{2b^5}\right)\log\left(x+\frac{-Ab+Ca+2b^2\left(-\frac{-Ab+Ca}{2b^2}-\frac{\sqrt{-ab^5}\,(-Bb+Da)}{2b^5}\right)}{-Bb+Da}\right)+\left(-\frac{-Ab+Ca}{2b^2}+\frac{\sqrt{-ab^5}\,(-Bb+Da)}{2b^5}\right)\log\left(x+\frac{-Ab+Ca+2b^2\left(-\frac{-Ab+Ca}{2b^2}+\frac{\sqrt{-ab^5}\,(-Bb+Da)}{2b^5}\right)}{-Bb+Da}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x**2/(2*b) + D*x**3/(3*b) + x*(B/b - D*a/b**2) + (-(-A*b + C*a)/(2*b**2)
- sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5))*log(x + (-A*b + C*a + 2*b**2*(-(-A*b
 + C*a)/(2*b**2) - sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5)))/(-B*b + D*a)) + (-
(-A*b + C*a)/(2*b**2) + sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5))*log(x + (-A*b
+ C*a + 2*b**2*(-(-A*b + C*a)/(2*b**2) + sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5
)))/(-B*b + D*a))
```

Giac [**A**]
time = 1.80, size = 88, normalized size = 0.96

$$-\frac{(Ca-Ab)\log\left(bx^{2}+a\right)}{2\,b^{2}}+\frac{(Da^{2}-Bab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{2}}+\frac{2\,Db^{2}x^{3}+3\,Cb^{2}x^{2}-6\,Dabx+6\,Bb^{2}x}{6\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(C*a - A*b)*log(b*x^2 + a)/b^2 + (D*a^2 - B*a*b)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*b^2) + 1/6*(2*D*b^2*x^3 + 3*C*b^2*x^2 - 6*D*a*b*x + 6*B*b^2*x)/
b^3
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(A+B\,x+C\,x^{2}+x^{3}\,D\right)}{b\,x^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

```
[Out] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

3.90 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx$

Optimal. Leaf size=73

$$\frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}$$

```
[Out] C*x/b+1/2*D*x^2/b+1/2*(B*b-D*a)*ln(b*x^2+a)/b^2+(A*b-C*a)*arctan(x*b^(1/2)/
a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1824, 649, 211, 266}

$$\frac{(Ab-aC)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}+\frac{Cx}{b}+\frac{Dx^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

```
[Out] (C*x)/b + (D*x^2)/(2*b) + ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a
]*b^(3/2)) + ((b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx &= \int \left(\frac{C}{b}+\frac{Dx}{b}+\frac{Ab-aC+(bB-aD)x}{b\,(a+bx^2)}\right)dx \\
&= \frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{\int \frac{Ab-aC+(bB-aD)x}{a+bx^2}\,dx}{b} \\
&= \frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{(Ab-aC)\int \frac{1}{a+bx^2}\,dx}{b}+\frac{(bB-aD)\int \frac{x}{a+bx^2}\,dx}{b} \\
&= \frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 68, normalized size = 0.93

$$\frac{bx(2C+Dx)+\frac{2\sqrt{b}\,(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}+(bB-aD)\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

```
[Out] (b*x*(2*C + D*x) + (2*Sqrt[b]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt
[a] + (b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

Maple [A]
time = 0.10, size = 65, normalized size = 0.89

method	result	size
default	$\frac{\frac{1}{2}Dx^2+Cx}{b}+\frac{\frac{(Bb-aD)\ln\left(b\,x^2+a\right)}{2b}+\frac{(Ab-aC)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{b}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(1/2*D*x^2+C*x)+1/b*(1/2*(B*b-D*a)/b*ln(b*x^2+a)+(A*b-C*a)/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 64, normalized size = 0.88

$$-\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{Dx^2+2\,Cx}{2\,b}-\frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/2*(D*x^2 + 2*C*x)/b -
1/2*(D*a - B*b)*log(b*x^2 + a)/b^2
```

Fricas [A]
time = 6.74, size = 157, normalized size = 2.15

$$\left[\frac{Dabx^2+2\,Cabx+(Ca-Ab)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-(Da^2-Bab)\log\left(bx^2+a\right)}{2\,ab^2},\ \frac{Dabx^2+2\,Cabx-2\,(Ca-Ab)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-(Da^2-Bab)\log\left(bx^2+a\right)}{2\,ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(D*a*b*x^2 + 2*C*a*b*x + (C*a - A*b)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a
*b)*x - a)/(b*x^2 + a)) - (D*a^2 - B*a*b)*log(b*x^2 + a))/(a*b^2), 1/2*(D*a
*b*x^2 + 2*C*a*b*x - 2*(C*a - A*b)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (D*a^2
 - B*a*b)*log(b*x^2 + a))/(a*b^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(65) = 130.
time = 0.42, size = 219, normalized size = 3.00

$$\frac{Cx}{b}+\frac{Dx^2}{2b}+\left(-\frac{-Bb+Da}{2b^2}-\frac{\sqrt{-ab^5}\,(-Ab+Ca)}{2ab^4}\right)\log\left(x+\frac{Bab-Da^2-2ab^2\left(-\frac{-Bb+Da}{2b^2}-\frac{\sqrt{-ab^5}\,(-Ab+Ca)}{2ab^4}\right)}{-Ab^2+Cab}\right)+\left(-\frac{-Bb+Da}{2b^2}+\frac{\sqrt{-ab^5}\,(-Ab+Ca)}{2ab^4}\right)\log\left(x+\frac{Bab-Da^2-2ab^2\left(-\frac{-Bb+Da}{2b^2}+\frac{\sqrt{-ab^5}\,(-Ab+Ca)}{2ab^4}\right)}{-Ab^2+Cab}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

```
[Out] C*x/b + D*x**2/(2*b) + (-(-B*b + D*a)/(2*b**2) - sqrt(-a*b**5)*(-A*b + C*a)
/(2*a*b**4))*log(x + (B*a*b - D*a**2 - 2*a*b**2*(-(-B*b + D*a)/(2*b**2) - s
qrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4)))/(-A*b**2 + C*a*b)) + (-(-B*b + D*a)/
(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4))*log(x + (B*a*b - D*a**2 -
 2*a*b**2*(-(-B*b + D*a)/(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4)))
/(-A*b**2 + C*a*b))
```

Giac [A]
time = 1.02, size = 66, normalized size = 0.90

$$-\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,b^2}+\frac{Dbx^2+2\,Cbx}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*(D*a - B*b)*log(b*x^
2 + a)/b^2 + 1/2*(D*b*x^2 + 2*C*b*x)/b^2
```

Mupad [B]
time = 1.42, size = 79, normalized size = 1.08

$$\frac{B\,\ln\left(b\,x^2+a\right)}{2\,b}-\frac{\left(a\,\ln\left(b\,x^2+a\right)-b\,x^2\right)\,D}{2\,b^2}+\frac{C\,x}{b}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}-\frac{C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2),x)
```

```
[Out] (B*log(a + b*x^2))/(2*b) - ((a*log(a + b*x^2) - b*x^2)*D)/(2*b^2) + (C*x)/b
 + (A*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2)) - (C*a^(1/2)*atan((b^(1/
2)*x)/a^(1/2)))/b^(3/2)
```

3.91 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx$

Optimal. Leaf size=72

$$\frac{Dx}{b}+\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{A\log(x)}{a}-\frac{(Ab-aC)\log\left(a+bx^2\right)}{2ab}$$

```
[Out] D*x/b+A*ln(x)/a-1/2*(A*b-C*a)*ln(b*x^2+a)/a/b+(B*b-D*a)*arctan(x*b^(1/2)/a^
(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1816, 649, 211, 266}

$$-\frac{(Ab-aC)\log\left(a+bx^2\right)}{2ab}+\frac{A\log(x)}{a}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bB-aD)}{\sqrt{a}\,b^{3/2}}+\frac{Dx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)),x]
```

```
[Out] (D*x)/b + ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) + (A*
Log[x])/a - ((A*b - a*C)*Log[a + b*x^2])/(2*a*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)}\,dx &= \int \left(\frac{D}{b}+\frac{A}{ax}+\frac{a(bB-aD)-b(Ab-aC)x}{ab\left(a+bx^2\right)}\right)dx \\
&= \frac{Dx}{b}+\frac{A\log(x)}{a}+\frac{\int \frac{a(bB-aD)-b(Ab-aC)x}{a+bx^2}\,dx}{ab} \\
&= \frac{Dx}{b}+\frac{A\log(x)}{a}-\frac{(Ab-aC)\int \frac{x}{a+bx^2}\,dx}{a}+\frac{(bB-aD)\int \frac{1}{a+bx^2}\,dx}{b} \\
&= \frac{Dx}{b}+\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{A\log(x)}{a}-\frac{(Ab-aC)\log\left(a+bx^2\right)}{2ab}
\end{aligned}
$$

Mathematica [**A**]
time = 0.04, size = 73, normalized size = 1.01

$$
\frac{Dx}{b}-\frac{(-bB+aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}+\frac{A\log(x)}{a}+\frac{(-Ab+aC)\log\left(a+bx^2\right)}{2ab}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)),x]
```

```
[Out] (D*x)/b - ((-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) +
(A*Log[x])/a + ((-(A*b) + a*C)*Log[a + b*x^2])/(2*a*b)
```

Maple [**A**]
time = 0.10, size = 73, normalized size = 1.01

method	result	size
default	$\frac{Dx}{b}+\frac{\frac{\left(-b^{2}A+abC\right)\ln\left(b\,x^{2}+a\right)}{2b}+\frac{\left(abB-a^{2}D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{ba}+\frac{A\ln(x)}{a}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] D*x/b+1/b/a*(1/2*(-A*b^2+C*a*b)/b*ln(b*x^2+a)+(B*a*b-D*a^2)/(a*b)^(1/2)*arc
tan(b*x/(a*b)^(1/2)))+A*ln(x)/a
```

Maxima [**A**]
time = 0.49, size = 65, normalized size = 0.90

$$\frac{Dx}{b}+\frac{A\log\left(x\right)}{a}-\frac{\left(Da-Bb\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{\left(Ca-Ab\right)\log\left(bx^{2}+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] D*x/b + A*log(x)/a - (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/2*
(C*a - A*b)*log(b*x^2 + a)/(a*b)
```

Fricas [**A**]
time = 6.23, size = 158, normalized size = 2.19

$$\left[\frac{2\,Dabx+2\,Ab^{2}\log\left(x\right)-\left(Da-Bb\right)\sqrt{-ab}\,\log\left(\frac{bx^{2}+2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)+\left(Cab-Ab^{2}\right)\log\left(bx^{2}+a\right)}{2\,ab^{2}},\frac{2\,Dabx+2\,Ab^{2}\log\left(x\right)-2\,\left(Da-Bb\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(Cab-Ab^{2}\right)\log\left(bx^{2}+a\right)}{2\,ab^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*D*a*b*x + 2*A*b^2*log(x) - (D*a - B*b)*sqrt(-a*b)*log((b*x^2 + 2*sq
rt(-a*b)*x - a)/(b*x^2 + a)) + (C*a*b - A*b^2)*log(b*x^2 + a))/(a*b^2), 1/2
*(2*D*a*b*x + 2*A*b^2*log(x) - 2*(D*a - B*b)*sqrt(a*b)*arctan(sqrt(a*b)*x/a
) + (C*a*b - A*b^2)*log(b*x^2 + a))/(a*b^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.80, size = 66, normalized size = 0.92

$$\frac{Dx}{b}+\frac{A\log\left(|x|\right)}{a}-\frac{\left(Da-Bb\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}+\frac{\left(Ca-Ab\right)\log\left(bx^{2}+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="giac")
```

```
[Out] D*x/b + A*log(abs(x))/a - (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) +
 1/2*(C*a - A*b)*log(b*x^2 + a)/(a*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,x+C\,x^2+x^3\,D}{x\,\left(b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)),x)
```

```
[Out] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)), x)
```

3.92 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx$

Optimal. Leaf size=76

$$-\frac{A}{ax}-\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{B\log(x)}{a}-\frac{(bB-aD)\log\left(a+bx^2\right)}{2ab}$$

```
[Out] -A/a/x+B*ln(x)/a-1/2*(B*b-D*a)*ln(b*x^2+a)/a/b-(A*b-C*a)*arctan(x*b^(1/2)/a
^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1816, 649, 211, 266}

$$-\frac{(Ab-aC)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}-\frac{(bB-aD)\log\left(a+bx^2\right)}{2ab}+\frac{B\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(A/(a*x)) - ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]) +
(B*Log[x])/a - ((b*B - a*D)*Log[a + b*x^2])/(2*a*b)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)}\,dx &= \int \left(\frac{A}{ax^2}+\frac{B}{ax}+\frac{-Ab+aC-(bB-aD)x}{a\left(a+bx^2\right)}\right)dx\\
&= -\frac{A}{ax}+\frac{B\log(x)}{a}+\frac{\int \frac{-Ab+aC-(bB-aD)x}{a+bx^2}\,dx}{a}\\
&= -\frac{A}{ax}+\frac{B\log(x)}{a}+\frac{(-Ab+aC)\int \frac{1}{a+bx^2}\,dx}{a}+\frac{(-bB+aD)\int \frac{x}{a+bx^2}\,dx}{a}\\
&= -\frac{A}{ax}-\frac{(Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{B\log(x)}{a}-\frac{(bB-aD)\log\left(a+bx^2\right)}{2ab}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 75, normalized size = 0.99

$$-\frac{A}{ax}+\frac{(-Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{B\log(x)}{a}+\frac{(-bB+aD)\log\left(a+bx^2\right)}{2ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(A/(a*x)) + ((-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b])
 + (B*Log[x])/a + ((-(b*B) + a*D)*Log[a + b*x^2])/(2*a*b)
```

Maple [A]
time = 0.10, size = 67, normalized size = 0.88

method	result	size
default	$\frac{\frac{(-Bb+aD)\ln\left(b\,x^2+a\right)}{2b}+\frac{(-Ab+aC)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{a}-\frac{A}{ax}+\frac{B\ln(x)}{a}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/2*(-B*b+D*a)/b*ln(b*x^2+a)+(-A*b+C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(
1/2)))-A/a/x+B*ln(x)/a
```

Maxima [**A**]
time = 0.48, size = 67, normalized size = 0.88

$$\frac{B\log\left(x\right)}{a}+\frac{\left(Ca-Ab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{\left(Da-Bb\right)\log\left(bx^{2}+a\right)}{2\,ab}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] B*log(x)/a + (C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*(D*a - B
*b)*log(b*x^2 + a)/(a*b) - A/(a*x)
```

Fricas [**A**]
time = 9.50, size = 165, normalized size = 2.17

$$\left[\frac{2\,Babx\log\left(x\right)+\left(Ca-Ab\right)\sqrt{-ab}\,x\log\left(\frac{bx^{2}+2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)-2\,Aab+\left(Da^{2}-Bab\right)x\log\left(bx^{2}+a\right)}{2\,a^{2}bx},\frac{2\,Babx\log\left(x\right)+2\left(Ca-Ab\right)\sqrt{ab}\,x\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-2\,Aab+\left(Da^{2}-Bab\right)x\log\left(bx^{2}+a\right)}{2\,a^{2}bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*a*b*x*log(x) + (C*a - A*b)*sqrt(-a*b)*x*log((b*x^2 + 2*sqrt(-a*b)
*x - a)/(b*x^2 + a)) - 2*A*a*b + (D*a^2 - B*a*b)*x*log(b*x^2 + a))/(a^2*b*x
), 1/2*(2*B*a*b*x*log(x) + 2*(C*a - A*b)*sqrt(a*b)*x*arctan(sqrt(a*b)*x/a)
- 2*A*a*b + (D*a^2 - B*a*b)*x*log(b*x^2 + a))/(a^2*b*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.69, size = 68, normalized size = 0.89

$$\frac{B\log\left(|x|\right)}{a}+\frac{\left(Ca-Ab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}+\frac{\left(Da-Bb\right)\log\left(bx^{2}+a\right)}{2\,ab}-\frac{A}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] B*log(abs(x))/a + (C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*(D*
a - B*b)*log(b*x^2 + a)/(a*b) - A/(a*x)
```

Mupad [B]
time = 1.21, size = 78, normalized size = 1.03

$$\frac{\ln\left(b\,x^2+a\right)\,D}{2\,b}-\frac{A}{a\,x}-\frac{B\,\left(\ln\left(b\,x^2+a\right)-2\,\ln\left(x\right)\right)}{2\,a}-\frac{A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}+\frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)),x)
```

```
[Out] (log(a + b*x^2)*D)/(2*b) - A/(a*x) - (B*(log(a + b*x^2) - 2*log(x)))/(2*a)
- (A*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2) + (C*atan((b^(1/2)*x)/a^(1/
2)))/(a^(1/2)*b^(1/2))
```

3.93 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=92

$$-\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{(Ab-aC)\log\left(a+bx^2\right)}{2a^2}$$

```
[Out] -1/2*A/a/x^2-B/a/x-(A*b-C*a)*ln(x)/a^2+1/2*(A*b-C*a)*ln(b*x^2+a)/a^2-(B*b-D
*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1816, 649, 211, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bB-aD)}{a^{3/2}\sqrt{b}}+\frac{(Ab-aC)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{A}{2ax^2}-\frac{B}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)),x]
```

```
[Out] -1/2*A/(a*x^2) - B/(a*x) - ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/
2)*Sqrt[b]) - ((A*b - a*C)*Log[x])/a^2 + ((A*b - a*C)*Log[a + b*x^2])/(2*a^
2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)}\,dx &= \int \left(\frac{A}{ax^3}+\frac{B}{ax^2}+\frac{-Ab+aC}{a^2x}+\frac{-a(bB-aD)+b(Ab-aC)x}{a^2\left(a+bx^2\right)}\right)\,dx \\
&= -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{\int \frac{-a(bB-aD)+b(Ab-aC)x}{a+bx^2}\,dx}{a^2} \\
&= -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{(b(Ab-aC))\int \frac{x}{a+bx^2}\,dx}{a^2}-\frac{(bB-aD)\int}{a} \\
&= -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{(Ab-aC)\log(x)}{a^2}+\frac{(Ab-aC)}{2}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 84, normalized size = 0.91

$$
\frac{-\frac{aA}{x^2}-\frac{2aB}{x}+\frac{2\sqrt{a}\,(-bB+aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}+2(-Ab+aC)\log(x)+(Ab-aC)\log\left(a+bx^2\right)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)),x]
```

```
[Out] (-((a*A)/x^2) - (2*a*B)/x + (2*Sqrt[a]*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/Sqrt[b] + 2*(-(A*b) + a*C)*Log[x] + (A*b - a*C)*Log[a + b*x^2])/(2*
a^2)
```

Maple [A]
time = 0.11, size = 89, normalized size = 0.97

method	result	size
default	$\frac{\frac{\left(b^{2} A-abC\right)\ln\left(b\,x^{2}+a\right)}{2b}+\frac{\left(-abB+a^{2} D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{a^{2}}-\frac{A}{2a\,x^{2}}-\frac{B}{ax}+\frac{(-Ab+aC)\ln(x)}{a^{2}}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*(A*b^2-C*a*b)/b*ln(b*x^2+a)+(-B*a*b+D*a^2)/(a*b)^(1/2)*arctan(b*
x/(a*b)^(1/2)))-1/2*A/a/x^2-B/a/x+1/a^2*(-A*b+C*a)*ln(x)
```

Maxima [A]
time = 0.48, size = 76, normalized size = 0.83

$$\frac{(Da - Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a} - \frac{(Ca - Ab)\log\left(bx^2 + a\right)}{2\,a^2} + \frac{(Ca - Ab)\log\left(x\right)}{a^2} - \frac{2\,Bx + A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*(C*a - A*b)*log(b*x^2
 + a)/a^2 + (C*a - A*b)*log(x)/a^2 - 1/2*(2*B*x + A)/(a*x^2)
```

Fricas [A]
time = 3.19, size = 205, normalized size = 2.23

$$\left[-\frac{(Da - Bb)\sqrt{-ab}\,x^2\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right) + 2\,Babx + (Cab - Ab^2)x^2\log\left(bx^2 + a\right) - 2\,(Cab - Ab^2)x^2\log\left(x\right) + Aab}{2\,a^2bx^2}, \frac{2\,(Da - Bb)\sqrt{ab}\,x^2\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) - 2\,Babx - (Cab - Ab^2)x^2\log\left(bx^2 + a\right) + 2\,(Cab - Ab^2)x^2\log\left(x\right) - Aab}{2\,a^2bx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*((D*a - B*b)*sqrt(-a*b)*x^2*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)) + 2*B*a*b*x + (C*a*b - A*b^2)*x^2*log(b*x^2 + a) - 2*(C*a*b - A*b^2)*x
^2*log(x) + A*a*b)/(a^2*b*x^2), 1/2*(2*(D*a - B*b)*sqrt(a*b)*x^2*arctan(sqr
t(a*b)*x/a) - 2*B*a*b*x - (C*a*b - A*b^2)*x^2*log(b*x^2 + a) + 2*(C*a*b - A
*b^2)*x^2*log(x) - A*a*b)/(a^2*b*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.92, size = 80, normalized size = 0.87

$$\frac{(Da - Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a} - \frac{(Ca - Ab)\log\left(bx^2 + a\right)}{2\,a^2} + \frac{(Ca - Ab)\log\left(|x|\right)}{a^2} - \frac{2\,Bax + Aa}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*(C*a - A*b)*log(b*x^2
 + a)/a^2 + (C*a - A*b)*log(abs(x))/a^2 - 1/2*(2*B*a*x + A*a)/(a^2*x^2)
```

Mupad [B]
time = 1.30, size = 97, normalized size = 1.05

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,D}{\sqrt{a}\,\sqrt{b}}-\frac{B}{a\,x}-\frac{C\,\left(\ln\left(b\,x^2+a\right)-2\,\ln\left(x\right)\right)}{2\,a}-\frac{A}{2\,a\,x^2}+\frac{A\,b\,\ln\left(b\,x^2+a\right)}{2\,a^2}-\frac{A\,b\,\ln\left(x\right)}{a^2}-\frac{B\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*D)/(a^(1/2)*b^(1/2)) - B/(a*x) - (C*(log(a + b*x
^2) - 2*log(x)))/(2*a) - A/(2*a*x^2) + (A*b*log(a + b*x^2))/(2*a^2) - (A*b*
log(x))/a^2 - (B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

3.94 $\int \frac{x^4\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=176

$$\frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2}+\frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\sqrt{a}\,(3Ab-5aC}{2}$$

```
[Out] 1/2*(3*A*b-5*C*a)*x/b^3+1/2*(2*B*b-3*D*a)*x^2/b^3-1/6*(3*A*b-5*C*a)*x^3/a/b
^2+1/4*D*x^4/b^2-1/2*x^4*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)-1/2*a*(2*B
*b-3*D*a)*ln(b*x^2+a)/b^4-1/2*(3*A*b-5*C*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/
2)/b^(7/2)
```

Rubi [A]
time = 0.19, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1818, 1816, 649, 211, 266}

$$-\frac{\sqrt{a}\,(3Ab-5aC)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{x(3Ab-5aC)}{2b^3}-\frac{x^3(3Ab-5aC)}{6ab^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}-\frac{a(2bB-3aD)\log\left(a+bx^2\right)}{2b^4}+\frac{x^2(2bB-3aD)}{2b^3}+\frac{Dx^4}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] ((3*A*b - 5*a*C)*x)/(2*b^3) + ((2*b*B - 3*a*D)*x^2)/(2*b^3) - ((3*A*b - 5*a
*C)*x^3)/(6*a*b^2) + (D*x^4)/(4*b^2) - (x^4*(a*(B - (a*D)/b) - (A*b - a*C)*
x))/(2*a*b*(a + b*x^2)) - (Sqrt[a]*(3*A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[
a]])/(2*b^(7/2)) - (a*(2*b*B - 3*a*D)*Log[a + b*x^2])/(2*b^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx &= -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int \frac{x^3\left(-4a\left(B-\frac{aD}{b}\right)+(3Ab-5aC)x-2aDx^2\right)}{a+bx^2}\,dx}{2ab} \\
&= -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}-\frac{\int\left(-\frac{a(3Ab-5aC)}{b^2}-\frac{2a(2bB-3aD)x}{b^2}+\frac{(3Ab}{}\right.}{} \\
&= \frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2}+\frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\right.\right.}{2} \\
&= \frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2}+\frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\right.\right.}{2} \\
&= \frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2}+\frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\right.\right.}{2}
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 139, normalized size = 0.79

$$
\frac{12b(Ab-2aC)x+6b(bB-2aD)x^2+4b^2Cx^3+3b^2Dx^4+\frac{6a\left(a^2D+Ab^2x-ab(B+Cx)\right)}{a+bx^2}+6\sqrt{a}\,\sqrt{b}\,(-3Ab+5aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)+6a(-2bB+3aD)\log\left(a+bx^2\right)}{12b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (12*b*(A*b - 2*a*C)*x + 6*b*(b*B - 2*a*D)*x^2 + 4*b^2*C*x^3 + 3*b^2*D*x^4 +
 (6*a*(a^2*D + A*b^2*x - a*b*(B + C*x)))/(a + b*x^2) + 6*Sqrt[a]*Sqrt[b]*(-
```

```
3*A*b + 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 6*a*(-2*b*B + 3*a*D)*Log[a + b
*x^2])/(12*b^4)
```

Maple [A]
time = 0.11, size = 134, normalized size = 0.76

method	result
default	$\frac{\frac{1}{4}bDx^4+\frac{1}{3}bC\,x^3+\frac{1}{2}bB\,x^2-Da\,x^2+Abx-2aCx}{b^3}-\frac{a\left(\frac{\left(-\frac{Ab}{2}+\frac{aC}{2}\right)x+\frac{a(Bb-aD)}{2b}}{b\,x^2+a}+\frac{(4Bb-6aD)\ln\left(b\,x^2+a\right)}{4b}+\frac{(3Ab-5aC)\arctan\left(\frac{bx}{\sqrt{ab}}\right.}{2\sqrt{ab}}\right.}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/4*b*D*x^4+1/3*b*C*x^3+1/2*b*B*x^2-D*a*x^2+A*b*x-2*a*C*x)-a/b^3*(((
-1/2*A*b+1/2*a*C)*x+1/2*a*(B*b-D*a)/b)/(b*x^2+a)+1/4*(4*B*b-6*D*a)/b*ln(b*x
^2+a)+1/2*(3*A*b-5*C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 150, normalized size = 0.85

$$\frac{Da^3-Ba^2b-(Ca^2b-Aab^2)x}{2\,(b^5x^2+ab^4)}+\frac{(5\,Ca^2-3\,Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}+\frac{3\,Dbx^4+4\,Cbx^3-6\,(2\,Da-Bb)x^2-12\,(2\,Ca-Ab)x}{12\,b^3}+\frac{(3\,Da^2-2\,Bab)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(D*a^3 - B*a^2*b - (C*a^2*b - A*a*b^2)*x)/(b^5*x^2 + a*b^4) + 1/2*(5*C*
a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/12*(3*D*b*x^4 + 4*
C*b*x^3 - 6*(2*D*a - B*b)*x^2 - 12*(2*C*a - A*b)*x)/b^3 + 1/2*(3*D*a^2 - 2*
B*a*b)*log(b*x^2 + a)/b^4
```

Fricas [A]
time = 2.39, size = 468, normalized size = 2.66

$$\left[\frac{3\,Db^3x^6+4\,Cb^3x^5-3\,(3\,Dab^2-2\,Bb^3)x^4+6\,Da^3-6\,Ba^2b-4\,(5\,Cab^2-3\,Ab^3)x^3-6\,(2\,Da^2b-Bab^2)x^2-3\,(5\,Ca^2b-3\,Aab^2+(5\,Cab^2-3\,Ab^3)x^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6\,(5\,Ca^2b-3\,Aab^2)x+6\,(3\,Da^3-2\,Ba^2b+(3\,Da^2b-2\,Bab^2)x^2)\log\left(bx^2+a\right)}{12\,(b^5x^2+ab^4)},\frac{3\,Db^3x^6+4\,Cb^3x^5-3\,(3\,Dab^2-2\,Bb^3)x^4+6\,Da^3-6\,Ba^2b-4\,(5\,Cab^2-3\,Ab^3)x^3-6\,(2\,Da^2b-Bab^2)x^2+6\,(5\,Ca^2b-3\,Aab^2+(5\,Cab^2-3\,Ab^3)x^2)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-6\,(5\,Ca^2b-3\,Aab^2)x+6\,(3\,Da^3-2\,Ba^2b+(3\,Da^2b-2\,Bab^2)x^2)\log\left(bx^2+a\right)}{12\,(b^5x^2+ab^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(3*D*b^3*x^6 + 4*C*b^3*x^5 - 3*(3*D*a*b^2 - 2*B*b^3)*x^4 + 6*D*a^3 -
6*B*a^2*b - 4*(5*C*a*b^2 - 3*A*b^3)*x^3 - 6*(2*D*a^2*b - B*a*b^2)*x^2 - 3*(
5*C*a^2*b - 3*A*a*b^2 + (5*C*a*b^2 - 3*A*b^3)*x^2)*sqrt(-a/b)*log((b*x^2 -
2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 6*(5*C*a^2*b - 3*A*a*b^2)*x + 6*(3*D*a
```

```
^3 - 2*B*a^2*b + (3*D*a^2*b - 2*B*a*b^2)*x^2)*log(b*x^2 + a))/(b^5*x^2 + a*
b^4), 1/12*(3*D*b^3*x^6 + 4*C*b^3*x^5 - 3*(3*D*a*b^2 - 2*B*b^3)*x^4 + 6*D*a
^3 - 6*B*a^2*b - 4*(5*C*a*b^2 - 3*A*b^3)*x^3 - 6*(2*D*a^2*b - B*a*b^2)*x^2
+ 6*(5*C*a^2*b - 3*A*a*b^2 + (5*C*a*b^2 - 3*A*b^3)*x^2)*sqrt(a/b)*arctan(b*
x*sqrt(a/b)/a) - 6*(5*C*a^2*b - 3*A*a*b^2)*x + 6*(3*D*a^3 - 2*B*a^2*b + (3*
D*a^2*b - 2*B*a*b^2)*x^2)*log(b*x^2 + a))/(b^5*x^2 + a*b^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(160) = 320$.
time = 1.93, size = 335, normalized size = 1.90

$$\frac{Cx^3}{3b^2} + \frac{Dx^4}{4b^2} + x^2\left(\frac{B}{2b^2} - \frac{Da}{b^3}\right) + x\left(\frac{A}{b^2} - \frac{2Ca}{b^3}\right) + \left(\frac{a(-2Bb+3Da)}{2b^4} - \frac{\sqrt{-ab^9}\,(-3Ab+5Ca)}{4b^8}\right)\log\left(x + \frac{4Bab - 6Da^2 + 4b^4\left(\frac{a(-2Bb+3Da)}{2b^4} - \frac{\sqrt{-ab^9}\,(-3Ab+5Ca)}{4b^8}\right)}{-3Ab^2 + 5Cab}\right) + \left(\frac{a(-2Bb+3Da)}{2b^4} + \frac{\sqrt{-ab^9}\,(-3Ab+5Ca)}{4b^8}\right)\log\left(x + \frac{4Bab - 6Da^2 + 4b^4\left(\frac{a(-2Bb+3Da)}{2b^4} + \frac{\sqrt{-ab^9}\,(-3Ab+5Ca)}{4b^8}\right)}{-3Ab^2 + 5Cab}\right) + \frac{-Ba^2b + Da^3 + x(Aab^2 - Ca^2b)}{2ab^4 + 2b^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x**3/(3*b**2) + D*x**4/(4*b**2) + x**2*(B/(2*b**2) - D*a/b**3) + x*(A/b**
2 - 2*C*a/b**3) + (a*(-2*B*b + 3*D*a)/(2*b**4) - sqrt(-a*b**9)*(-3*A*b + 5*
C*a)/(4*b**8))*log(x + (4*B*a*b - 6*D*a**2 + 4*b**4*(a*(-2*B*b + 3*D*a)/(2*
b**4) - sqrt(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8)))/(-3*A*b**2 + 5*C*a*b)) +
(a*(-2*B*b + 3*D*a)/(2*b**4) + sqrt(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8))*log
(x + (4*B*a*b - 6*D*a**2 + 4*b**4*(a*(-2*B*b + 3*D*a)/(2*b**4) + sqrt(-a*b*
*9)*(-3*A*b + 5*C*a)/(4*b**8)))/(-3*A*b**2 + 5*C*a*b)) + (-B*a**2*b + D*a**
3 + x*(A*a*b**2 - C*a**2*b))/(2*a*b**4 + 2*b**5*x**2)
```

Giac [A]
time = 0.87, size = 159, normalized size = 0.90

$$\frac{(5\,Ca^2 - 3\,Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3} + \frac{(3\,Da^2 - 2\,Bab)\log\left(bx^2 + a\right)}{2\,b^4} + \frac{Da^3 - Ba^2b - (Ca^2b - Aab^2)x}{2\,(bx^2 + a)b^4} + \frac{3\,Db^6x^4 + 4\,Cb^6x^3 - 12\,Dab^5x^2 + 6\,Bb^6x^2 - 24\,Cab^5x + 12\,Ab^6x}{12\,b^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*C*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*(3*D*a^
2 - 2*B*a*b)*log(b*x^2 + a)/b^4 + 1/2*(D*a^3 - B*a^2*b - (C*a^2*b - A*a*b^2
)*x)/((b*x^2 + a)*b^4) + 1/12*(3*D*b^6*x^4 + 4*C*b^6*x^3 - 12*D*a*b^5*x^2 +
 6*B*b^6*x^2 - 24*C*a*b^5*x + 12*A*b^6*x)/b^8
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

```
[Out] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

3.95 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=154

$$\frac{(3bB-5aD)x}{2b^3}-\frac{(Ab-2aC)x^2}{2ab^2}+\frac{Dx^3}{3b^2}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}-\frac{\sqrt{a}\,(3bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+$$

```
[Out] 1/2*(3*B*b-5*D*a)*x/b^3-1/2*(A*b-2*C*a)*x^2/a/b^2+1/3*D*x^3/b^2-1/2*x^3*(a*
(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)+1/2*(A*b-2*C*a)*ln(b*x^2+a)/b^3-1/2*(3
*B*b-5*D*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(7/2)
```

Rubi [A]
time = 0.17, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1818, 1816, 649, 211, 266}

$$\frac{(Ab-2aC)\log(a+bx^2)}{2b^3}-\frac{x^2(Ab-2aC)}{2ab^2}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\,(a+bx^2)}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(3bB-5aD)}{2b^{7/2}}+\frac{x(3bB-5aD)}{2b^3}+\frac{Dx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] ((3*b*B - 5*a*D)*x)/(2*b^3) - ((A*b - 2*a*C)*x^2)/(2*a*b^2) + (D*x^3)/(3*b^
2) - (x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)) - (Sqrt[a]
*(3*b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2)) + ((A*b - 2*a*C)*
Log[a + b*x^2])/(2*b^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx &= -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}-\frac{\int \frac{x^2\left(-3a\left(B-\frac{aD}{b}\right)+2(Ab-2aC)x-2aDx^2\right)}{a+bx^2}\,dx}{2ab} \\
&= -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}-\frac{\int \left(-\frac{a(3bB-5aD)}{b^2}+\frac{2(Ab-2aC)x}{b}-\frac{2aDx^2}{b}\right.}{2ab} \\
&= \frac{(3bB-5aD)x}{2b^3}-\frac{(Ab-2aC)x^2}{2ab^2}+\frac{Dx^3}{3b^2}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right.}{2ab\,(a+bx^2)} \\
&= \frac{(3bB-5aD)x}{2b^3}-\frac{(Ab-2aC)x^2}{2ab^2}+\frac{Dx^3}{3b^2}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right.}{2ab\,(a+bx^2)} \\
&= \frac{(3bB-5aD)x}{2b^3}-\frac{(Ab-2aC)x^2}{2ab^2}+\frac{Dx^3}{3b^2}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right.}{2ab\,(a+bx^2)}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 128, normalized size = 0.83

$$\frac{(bB-2aD)x}{b^3}+\frac{Cx^2}{2b^2}+\frac{Dx^3}{3b^2}+\frac{a(Ab+bBx-a(C+Dx))}{2b^3\,(a+bx^2)}+\frac{\sqrt{a}\,(-3bB+5aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{(Ab-2aC)\log\left(a+bx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] ((b*B - 2*a*D)*x)/b^3 + (C*x^2)/(2*b^2) + (D*x^3)/(3*b^2) + (a*(A*b + b*B*x
 - a*(C + D*x)))/(2*b^3*(a + b*x^2)) + (Sqrt[a]*(-3*b*B + 5*a*D)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*b^(7/2)) + ((A*b - 2*a*C)*Log[a + b*x^2])/(2*b^3)
```

Maple [A]
time = 0.11, size = 124, normalized size = 0.81

method	result	siz
default	$\frac{\frac{1}{3}bD x^{3}+\frac{1}{2}bC\,x^{2}+bBx-2aDx}{b^{3}}+\frac{\frac{\left(\frac{1}{2}abB-\frac{1}{2}a^{2}D\right)x+\frac{a(Ab-aC)}{2}}{b\,x^{2}+a}+\frac{\left(2b^{2}A-4abC\right)\ln\left(b\,x^{2}+a\right)}{4b}+\frac{\left(-3abB+5a^{2}D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{b^{3}}$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/3*b*D*x^3+1/2*b*C*x^2+b*B*x-2*a*D*x)+1/b^3*(((1/2*a*b*B-1/2*a^2*D)
*x+1/2*a*(A*b-C*a))/(b*x^2+a)+1/4*(2*A*b^2-4*C*a*b)/b*ln(b*x^2+a)+1/2*(-3*B
*a*b+5*D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.51, size = 127, normalized size = 0.82

$$-\frac{Ca^{2}-Aab+(Da^{2}-Bab)x}{2\,(b^{4}x^{2}+ab^{3})}-\frac{(2\,Ca-Ab)\log\left(bx^{2}+a\right)}{2\,b^{3}}+\frac{(5\,Da^{2}-3\,Bab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{3}}+\frac{2\,Dbx^{3}+3\,Cbx^{2}-6\,(2\,Da-Bb)x}{6\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/(b^4*x^2 + a*b^3) - 1/2*(2*C*a - A
*b)*log(b*x^2 + a)/b^3 + 1/2*(5*D*a^2 - 3*B*a*b)*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*b^3) + 1/6*(2*D*b*x^3 + 3*C*b*x^2 - 6*(2*D*a - B*b)*x)/b^3
```

Fricas [A]
time = 3.63, size = 372, normalized size = 2.42

$$\left[\frac{4\,Db^{2}x^{5}+6\,Cb^{2}x^{4}+6\,Cabx^{2}-4\,(5\,Dab-3\,Bb^{2})x^{3}-6\,Ca^{2}+6\,Aab+3\,(5\,Da^{2}-3\,Bab+(5\,Dab-3\,Bb^{2})x^{2})\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)-6\,(5\,Da^{2}-3\,Bab)x-6\,(2\,Ca^{2}-Aab+(2\,Cab-Ab^{2})x^{2})\log\left(bx^{2}+a\right)}{12\,(b^{4}x^{2}+ab^{3})},\ \frac{2\,Db^{2}x^{5}+3\,Cb^{2}x^{4}+3\,Cabx^{2}-2\,(5\,Dab-3\,Bb^{2})x^{3}-3\,Ca^{2}+3\,Aab+3\,(5\,Da^{2}-3\,Bab+(5\,Dab-3\,Bb^{2})x^{2})\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-3\,(5\,Da^{2}-3\,Bab)x-3\,(2\,Ca^{2}-Aab+(2\,Cab-Ab^{2})x^{2})\log\left(bx^{2}+a\right)}{6\,(b^{4}x^{2}+ab^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*D*b^2*x^5 + 6*C*b^2*x^4 + 6*C*a*b*x^2 - 4*(5*D*a*b - 3*B*b^2)*x^3
- 6*C*a^2 + 6*A*a*b + 3*(5*D*a^2 - 3*B*a*b + (5*D*a*b - 3*B*b^2)*x^2)*sqrt(
-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 6*(5*D*a^2 - 3*B*a*
b)*x - 6*(2*C*a^2 - A*a*b + (2*C*a*b - A*b^2)*x^2)*log(b*x^2 + a))/(b^4*x^2
 + a*b^3), 1/6*(2*D*b^2*x^5 + 3*C*b^2*x^4 + 3*C*a*b*x^2 - 2*(5*D*a*b - 3*B*
b^2)*x^3 - 3*C*a^2 + 3*A*a*b + 3*(5*D*a^2 - 3*B*a*b + (5*D*a*b - 3*B*b^2)*x
^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*(5*D*a^2 - 3*B*a*b)*x - 3*(2*C*a^
2 - A*a*b + (2*C*a*b - A*b^2)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(134) = 268$.
time = 1.82, size = 289, normalized size = 1.88

$$\frac{Cx^2}{2b^2}+\frac{Dx^3}{3b^2}+x\left(\frac{B}{b^2}-\frac{2Da}{b^3}\right)+\left(-\frac{-Ab+2Ca}{2b^3}-\frac{\sqrt{-ab^7}\,(-3Bb+5Da)}{4b^7}\right)\log\left(x+\frac{-2Ab+4Ca+4b^3\left(-\frac{-Ab+2Ca}{2b^3}-\frac{\sqrt{-ab^7}\,(-3Bb+5Da)}{4b^7}\right)}{-3Bb+5Da}\right)+\left(-\frac{-Ab+2Ca}{2b^3}+\frac{\sqrt{-ab^7}\,(-3Bb+5Da)}{4b^7}\right)\log\left(x+\frac{-2Ab+4Ca+4b^3\left(-\frac{-Ab+2Ca}{2b^3}+\frac{\sqrt{-ab^7}\,(-3Bb+5Da)}{4b^7}\right)}{-3Bb+5Da}\right)+\frac{Aab-Ca^2+x(Bab-Da^2)}{2ab^3+2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x**2/(2*b**2) + D*x**3/(3*b**2) + x*(B/b**2 - 2*D*a/b**3) + (-(-A*b + 2*C
*a)/(2*b**3) - sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4*b**7))*log(x + (-2*A*b + 4
*C*a + 4*b**3*(-(-A*b + 2*C*a)/(2*b**3) - sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4
*b**7)))/(-3*B*b + 5*D*a)) + (-(-A*b + 2*C*a)/(2*b**3) + sqrt(-a*b**7)*(-3*
B*b + 5*D*a)/(4*b**7))*log(x + (-2*A*b + 4*C*a + 4*b**3*(-(-A*b + 2*C*a)/(2
*b**3) + sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4*b**7)))/(-3*B*b + 5*D*a)) + (A*a
*b - C*a**2 + x*(B*a*b - D*a**2))/(2*a*b**3 + 2*b**4*x**2)
```

Giac [A]
time = 0.77, size = 131, normalized size = 0.85

$$-\frac{(2\,Ca-Ab)\log\left(bx^2+a\right)}{2\,b^3}+\frac{\left(5\,Da^2-3\,Bab\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^3}-\frac{Ca^2-Aab+\left(Da^2-Bab\right)x}{2\,(bx^2+a)b^3}+\frac{2\,Db^4x^3+3\,Cb^4x^2-12\,Dab^3x+6\,Bb^4x}{6\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*C*a - A*b)*log(b*x^2 + a)/b^3 + 1/2*(5*D*a^2 - 3*B*a*b)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/((b*x^
2 + a)*b^3) + 1/6*(2*D*b^4*x^3 + 3*C*b^4*x^2 - 12*D*a*b^3*x + 6*B*b^4*x)/b^
6
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

```
[Out] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

3.96 $\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=134

$$-\frac{(Ab-3aC)x}{2ab^2}+\frac{Dx^2}{2b^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{(Ab-3aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}+\frac{(bB-2aD)\log(a+}{2b^3}$$

```
[Out] -1/2*(A*b-3*C*a)*x/a/b^2+1/2*D*x^2/b^2-1/2*x^2*(a*(B-a*D/b)-(A*b-C*a)*x)/a/
b/(b*x^2+a)+1/2*(B*b-2*D*a)*ln(b*x^2+a)/b^3+1/2*(A*b-3*C*a)*arctan(x*b^(1/2
)/a^(1/2))/b^(5/2)/a^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1818, 1816, 649, 211, 266}

$$\frac{(Ab-3aC)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}-\frac{x(Ab-3aC)}{2ab^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-2aD)\log\left(a+bx^2\right)}{2b^3}+\frac{Dx^2}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] -1/2*((A*b - 3*a*C)*x)/(a*b^2) + (D*x^2)/(2*b^2) - (x^2*(a*(B - (a*D)/b) -
(A*b - a*C)*x))/(2*a*b*(a + b*x^2)) + ((A*b - 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(2*Sqrt[a]*b^(5/2)) + ((b*B - 2*a*D)*Log[a + b*x^2])/(2*b^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx &= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}-\frac{\int \frac{x\left(-2a\left(B-\frac{aD}{b}\right)+(Ab-3aC)x-2aDx^2\right)}{a+bx^2}\,dx}{2ab}\\
&= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}-\frac{\int \left(A-\frac{3aC}{b}-\frac{2aDx}{b}-\frac{a(Ab-3aC)+2a(bB}{b(a+bx^2)}\right.}{2ab}\\
&= -\frac{(Ab-3aC)x}{2ab^2}+\frac{Dx^2}{2b^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}+\frac{\int \frac{a(Ab-3aC)+2}{a+}}{2a}\\
&= -\frac{(Ab-3aC)x}{2ab^2}+\frac{Dx^2}{2b^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}+\frac{(Ab-3aC)}{2b^2}\\
&= -\frac{(Ab-3aC)x}{2ab^2}+\frac{Dx^2}{2b^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}+\frac{(Ab-3aC)}{2\sqrt{}}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 100, normalized size = 0.75

$$\frac{2bCx+bDx^2+\frac{-a^2D-Ab^2x+ab(B+Cx)}{a+bx^2}+\frac{\sqrt{b}\,(Ab-3aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}+(bB-2aD)\log\left(a+bx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (2*b*C*x + b*D*x^2 + (-(a^2*D) - A*b^2*x + a*b*(B + C*x))/(a + b*x^2) + (Sq
rt[b]*(A*b - 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (b*B - 2*a*D)*Lo
g[a + b*x^2])/(2*b^3)
```

Maple [A]
time = 0.10, size = 103, normalized size = 0.77

method	result	size
default	$\frac{\frac{1}{2} D x^2+C x}{b^2}+\frac{\frac{\left(-\frac{A b}{2}+\frac{a C}{2}\right) x+\frac{a (B b-a D)}{2 b}}{b\,x^2+a}+\frac{(2 B b-4 a D) \ln\left(b\,x^2+a\right)}{4 b}+\frac{(A b-3 a C) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}}{b^2}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^2*(1/2*D*x^2+C*x)+1/b^2*(((-1/2*A*b+1/2*a*C)*x+1/2*a*(B*b-D*a)/b)/(b*x^
2+a)+1/4*(2*B*b-4*D*a)/b*ln(b*x^2+a)+1/2*(A*b-3*C*a)/(a*b)^(1/2)*arctan(b*x
/(a*b)^(1/2)))
```

Maxima [A]
time = 0.51, size = 108, normalized size = 0.81

$$-\frac{Da^2-Bab-(Cab-Ab^2)x}{2\left(b^4x^2+ab^3\right)}-\frac{(3\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{Dx^2+2\,Cx}{2\,b^2}-\frac{(2\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(D*a^2 - B*a*b - (C*a*b - A*b^2)*x)/(b^4*x^2 + a*b^3) - 1/2*(3*C*a - A
*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(D*x^2 + 2*C*x)/b^2 - 1/2*(
2*D*a - B*b)*log(b*x^2 + a)/b^3
```

Fricas [A]
time = 4.27, size = 357, normalized size = 2.66

$$\left[\frac{2\,Dab^2x^4+4\,Cab^2x^3+2\,Da^2bx^2-2\,Da^3+2\,Ba^2b+\left(3\,Ca^2-Aab+\left(3\,Cab-Ab^2\right)x^2\right)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\left(3\,Ca^2b-Aab^2\right)x-2\left(2\,Da^3-Ba^2b+\left(2\,Da^2b-Bab^2\right)x^2\right)\log\left(bx^2+a\right)}{4\left(ab^4x^2+a^2b^3\right)},\ \frac{Dab^2x^4+2\,Cab^2x^3+Da^2bx^2-Da^3+Ba^2b-\left(3\,Ca^2-Aab+\left(3\,Cab-Ab^2\right)x^2\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(3\,Ca^2b-Aab^2\right)x-\left(2\,Da^3-Ba^2b+\left(2\,Da^2b-Bab^2\right)x^2\right)\log\left(bx^2+a\right)}{2\left(ab^4x^2+a^2b^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*D*a*b^2*x^4 + 4*C*a*b^2*x^3 + 2*D*a^2*b*x^2 - 2*D*a^3 + 2*B*a^2*b +
 (3*C*a^2 - A*a*b + (3*C*a*b - A*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-
a*b)*x - a)/(b*x^2 + a)) + 2*(3*C*a^2*b - A*a*b^2)*x - 2*(2*D*a^3 - B*a^2*b
 + (2*D*a^2*b - B*a*b^2)*x^2)*log(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3), 1/2*(D
```

```
*a*b^2*x^4 + 2*C*a*b^2*x^3 + D*a^2*b*x^2 - D*a^3 + B*a^2*b - (3*C*a^2 - A*a
*b + (3*C*a*b - A*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*C*a^2*b -
A*a*b^2)*x - (2*D*a^3 - B*a^2*b + (2*D*a^2*b - B*a*b^2)*x^2)*log(b*x^2 + a)
)/(a*b^4*x^2 + a^2*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(116) = 232$.
time = 1.70, size = 284, normalized size = 2.12

$$\frac{Cx}{b^2}+\frac{Dx^2}{2b^2}+\left(-\frac{-Bb+2Da}{2b^3}-\frac{\sqrt{-ab^7}\,(-Ab+3Ca)}{4ab^6}\right)\log\left(x+\frac{2Bab-4Da^2-4ab^3\left(-\frac{-Bb+2Da}{2b^3}-\frac{\sqrt{-ab^7}\,(-Ab+3Ca)}{4ab^6}\right)}{-Ab^2+3Cab}\right)+\left(-\frac{-Bb+2Da}{2b^3}+\frac{\sqrt{-ab^7}\,(-Ab+3Ca)}{4ab^6}\right)\log\left(x+\frac{2Bab-4Da^2-4ab^3\left(-\frac{-Bb+2Da}{2b^3}+\frac{\sqrt{-ab^7}\,(-Ab+3Ca)}{4ab^6}\right)}{-Ab^2+3Cab}\right)+\frac{Bab-Da^2+x(-Ab^2+Cab)}{2ab^3+2b^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] C*x/b**2 + D*x**2/(2*b**2) + (-(-B*b + 2*D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*
b + 3*C*a)/(4*a*b**6))*log(x + (2*B*a*b - 4*D*a**2 - 4*a*b**3*(-(-B*b + 2*D
*a)/(2*b**3) - sqrt(-a*b**7)*(-A*b + 3*C*a)/(4*a*b**6)))/(-A*b**2 + 3*C*a*b
)) + (-(-B*b + 2*D*a)/(2*b**3) + sqrt(-a*b**7)*(-A*b + 3*C*a)/(4*a*b**6))*l
og(x + (2*B*a*b - 4*D*a**2 - 4*a*b**3*(-(-B*b + 2*D*a)/(2*b**3) + sqrt(-a*b
**7)*(-A*b + 3*C*a)/(4*a*b**6)))/(-A*b**2 + 3*C*a*b)) + (B*a*b - D*a**2 + x
*(-A*b**2 + C*a*b))/(2*a*b**3 + 2*b**4*x**2)
```

Giac [A]
time = 0.76, size = 111, normalized size = 0.83

$$-\frac{(3\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}-\frac{(2\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^3}+\frac{Db^2x^2+2\,Cb^2x}{2\,b^4}-\frac{Da^2-Bab-(Cab-Ab^2)x}{2\,(bx^2+a)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/2*(2*D*a - B*b
)*log(b*x^2 + a)/b^3 + 1/2*(D*b^2*x^2 + 2*C*b^2*x)/b^4 - 1/2*(D*a^2 - B*a*b
 - (C*a*b - A*b^2)*x)/((b*x^2 + a)*b^3)
```

Mupad [B]
time = 1.29, size = 152, normalized size = 1.13

$$\frac{B\ln\left(b\,x^2+a\right)}{2\,b^2}+\frac{x^2\,D}{2\,b^2}+\frac{C\,x}{b^2}-\frac{a^2\,D}{2\,b^3\,\left(b\,x^2+a\right)}+\frac{B\,a}{2\,b^2\,\left(b\,x^2+a\right)}-\frac{A\,x}{2\,b\,\left(b\,x^2+a\right)}+\frac{C\,a\,x}{2\,\left(b^3\,x^2+a\,b^2\right)}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}-\frac{3\,C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{5/2}}-\frac{a\,\ln\left(b\,x^2+a\right)\,D}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

```
[Out] (B*log(a + b*x^2))/(2*b^2) + (x^2*D)/(2*b^2) + (C*x)/b^2 - (a^2*D)/(2*b^3*(
a + b*x^2)) + (B*a)/(2*b^2*(a + b*x^2)) - (A*x)/(2*b*(a + b*x^2)) + (C*a*x)
/(2*(a*b^2 + b^3*x^2)) + (A*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*b^(3/2))
- (3*C*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*b^(5/2)) - (a*log(a + b*x^2)*D
)/b^3
```

3.97 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=101

$$\frac{Dx}{b^2}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\left(a+bx^2\right)}+\frac{(bB-3aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}+\frac{C\log\left(a+bx^2\right)}{2b^2}$$

```
[Out] D*x/b^2-1/2*x*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)+1/2*C*ln(b*x^2+a)/b^2
+1/2*(B*b-3*D*a)*arctan(x*b^(1/2)/a^(1/2))/b^(5/2)/a^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.192$, Rules used = {1818, 1824, 649, 211, 266}

$$-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bB-3aD)}{2\sqrt{a}\,b^{5/2}}+\frac{C\log\left(a+bx^2\right)}{2b^2}+\frac{Dx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (D*x)/b^2 - (x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(2*a*b*(a + b*x^2)) + ((b
*B - 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2)) + (C*Log[a + b
*x^2])/(2*b^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx &= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}-\frac{\int \frac{-a\left(B-\frac{aD}{b}\right)-2aCx-2aDx^2}{a+bx^2}\,dx}{2ab}\\
&= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}-\frac{\int \left(-\frac{2aD}{b}-\frac{a(bB-3aD)+2abCx}{b(a+bx^2)}\right)\,dx}{2ab}\\
&= \frac{Dx}{b^2}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}+\frac{\int \frac{a(bB-3aD)+2abCx}{a+bx^2}\,dx}{2ab^2}\\
&= \frac{Dx}{b^2}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}+\frac{C\int \frac{x}{a+bx^2}\,dx}{b}+\frac{(bB-3aD)\int \frac{1}{a+b}}{2b^2}\\
&= \frac{Dx}{b^2}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}+\frac{(bB-3aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}+\frac{C\,\mathrm{l}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 92, normalized size = 0.91

$$
\frac{Dx}{b^2}+\frac{-Ab+aC-bBx+aDx}{2b^2\,(a+bx^2)}-\frac{(-bB+3aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{5/2}}+\frac{C\log\left(a+bx^2\right)}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

```
[Out] (D*x)/b^2 + (-(A*b) + a*C - b*B*x + a*D*x)/(2*b^2*(a + b*x^2)) - ((-(b*B) +
 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2)) + (C*Log[a + b*x^2
])/(2*b^2)
```

Maple [A]
time = 0.10, size = 78, normalized size = 0.77

method	result	size
default	$\frac{Dx}{b^{2}}+\frac{\frac{\left(-\frac{Bb}{2}+\frac{aD}{2}\right)x-\frac{Ab}{2}+\frac{aC}{2}}{b\,x^{2}+a}+\frac{C\ln\left(b\,x^{2}+a\right)}{2}+\frac{\left(Bb-3aD\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{b^{2}}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] D*x/b^2+1/b^2*(((-1/2*B*b+1/2*a*D)*x-1/2*A*b+1/2*a*C)/(b*x^2+a)+1/2*C*ln(b*
x^2+a)+1/2*(B*b-3*D*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.50, size = 84, normalized size = 0.83

$$\frac{Ca-Ab+(Da-Bb)x}{2\left(b^{3}x^{2}+ab^{2}\right)}+\frac{Dx}{b^{2}}+\frac{C\log\left(bx^{2}+a\right)}{2\,b^{2}}-\frac{\left(3\,Da-Bb\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(C*a - A*b + (D*a - B*b)*x)/(b^3*x^2 + a*b^2) + D*x/b^2 + 1/2*C*log(b*x
^2 + a)/b^2 - 1/2*(3*D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2)
```

Fricas [A]
time = 2.16, size = 287, normalized size = 2.84

$$\left[\frac{4\,Dab^{2}x^{3}+2\,Ca^{2}b-2\,Aab^{2}-\left(3\,Da^{2}-Bab+\left(3\,Dab-Bb^{2}\right)x^{2}\right)\sqrt{-ab}\,\log\left(\frac{bx^{2}+2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)+2\left(3\,Da^{2}b-Bab^{2}\right)x+2\left(Cab^{2}x^{2}+Ca^{2}b\right)\log\left(bx^{2}+a\right)}{4\left(ab^{4}x^{2}+a^{2}b^{3}\right)},\ \frac{2\,Dab^{2}x^{3}+Ca^{2}b-Aab^{2}-\left(3\,Da^{2}-Bab+\left(3\,Dab-Bb^{2}\right)x^{2}\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(3\,Da^{2}b-Bab^{2}\right)x+\left(Cab^{2}x^{2}+Ca^{2}b\right)\log\left(bx^{2}+a\right)}{2\left(ab^{4}x^{2}+a^{2}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*D*a*b^2*x^3 + 2*C*a^2*b - 2*A*a*b^2 - (3*D*a^2 - B*a*b + (3*D*a*b -
 B*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(
3*D*a^2*b - B*a*b^2)*x + 2*(C*a*b^2*x^2 + C*a^2*b)*log(b*x^2 + a))/(a*b^4*x
^2 + a^2*b^3), 1/2*(2*D*a*b^2*x^3 + C*a^2*b - A*a*b^2 - (3*D*a^2 - B*a*b +
(3*D*a*b - B*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*D*a^2*b - B*a*b
^2)*x + (C*a*b^2*x^2 + C*a^2*b)*log(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(87) = 174$.
time = 1.36, size = 212, normalized size = 2.10

$$\frac{Dx}{b^{2}}+\left(\frac{C}{2b^{2}}-\frac{\sqrt{-ab^{5}}\left(-Bb+3Da\right)}{4ab^{5}}\right)\log\left(x+\frac{2Ca-4ab^{2}\left(\frac{C}{2b^{2}}-\frac{\sqrt{-ab^{5}}\left(-Bb+3Da\right)}{4ab^{5}}\right)}{-Bb+3Da}\right)+\left(\frac{C}{2b^{2}}+\frac{\sqrt{-ab^{5}}\left(-Bb+3Da\right)}{4ab^{5}}\right)\log\left(x+\frac{2Ca-4ab^{2}\left(\frac{C}{2b^{2}}+\frac{\sqrt{-ab^{5}}\left(-Bb+3Da\right)}{4ab^{5}}\right)}{-Bb+3Da}\right)+\frac{-Ab+Ca+x(-Bb+Da)}{2ab^{2}+2b^{3}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] D*x/b**2 + (C/(2*b**2) - sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5))*log(x + (
2*C*a - 4*a*b**2*(C/(2*b**2) - sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5)))/(-
B*b + 3*D*a)) + (C/(2*b**2) + sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5))*log(
x + (2*C*a - 4*a*b**2*(C/(2*b**2) + sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5)
))/(-B*b + 3*D*a)) + (-A*b + C*a + x*(-B*b + D*a))/(2*a*b**2 + 2*b**3*x**2)
```

Giac [A]
time = 0.63, size = 81, normalized size = 0.80

$$\frac{Dx}{b^2}+\frac{C\log\left(bx^2+a\right)}{2\,b^2}-\frac{\left(3\,Da-Bb\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^2}+\frac{Ca-Ab+\left(Da-Bb\right)x}{2\,\left(bx^2+a\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] D*x/b^2 + 1/2*C*log(b*x^2 + a)/b^2 - 1/2*(3*D*a - B*b)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^2) + 1/2*(C*a - A*b + (D*a - B*b)*x)/((b*x^2 + a)*b^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

```
[Out] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

3.98 $\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=93

$$\frac{-a\left(B-\frac{aD}{b}\right)+(Ab-aC)x}{2ab\,(a+bx^2)}+\frac{(Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{D\log\left(a+bx^2\right)}{2b^2}$$

```
[Out] 1/2*(-a*(B-a*D/b)+(A*b-C*a)*x)/a/b/(b*x^2+a)+1/2*(A*b+C*a)*arctan(x*b^(1/2)
/a^(1/2))/a^(3/2)/b^(3/2)+1/2*D*ln(b*x^2+a)/b^2
```

Rubi [A]
time = 0.04, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1828, 649, 211, 266}

$$\frac{(aC+Ab)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\,(a+bx^2)}+\frac{D\log\left(a+bx^2\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

```
[Out] -1/2*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)) + ((A*b + a*C)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + (D*Log[a + b*x^2])/(2*b^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
```

```
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx &= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{2ab\,(a+bx^2)}-\frac{\int \frac{-\frac{Ab+aC}{b}-\frac{2aDx}{b}}{a+bx^2}\,dx}{2a}\\
&= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{2ab\,(a+bx^2)}+\frac{(Ab+aC)\int \frac{1}{a+bx^2}\,dx}{2ab}+\frac{D\int \frac{x}{a+bx^2}\,dx}{b}\\
&= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{2ab\,(a+bx^2)}+\frac{(Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{D\log\left(a+bx^2\right)}{2b^2}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 83, normalized size = 0.89

$$
\frac{\frac{a^2D+Ab^2x-ab(B+Cx)}{a(a+bx^2)}+\frac{\sqrt{b}\,(Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}+D\log\left(a+bx^2\right)}{2b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

```
[Out] ((a^2*D + A*b^2*x - a*b*(B + C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b + a*C)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) + D*Log[a + b*x^2])/(2*b^2)
```

Maple [A]
time = 0.10, size = 88, normalized size = 0.95

method	result	size
default	$\frac{\frac{(Ab-aC)x}{2ab}-\frac{Bb-aD}{2b^2}}{b\,x^2+a}+\frac{\frac{aD\ln\left(b\,x^2+a\right)}{b}+\frac{(Ab+aC)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{2ba}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*(A*b-C*a)/a/b*x-1/2*(B*b-D*a)/b^2)/(b*x^2+a)+1/2/b/a*(a*D/b*ln(b*x^2+a
)+(A*b+C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [**A**]
time = 0.56, size = 89, normalized size = 0.96

$$\frac{Da^2 - Bab - (Cab - Ab^2)x}{2\left(ab^3x^2 + a^2b^2\right)} + \frac{D\log\left(bx^2 + a\right)}{2\,b^2} + \frac{(Ca + Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(D*a^2 - B*a*b - (C*a*b - A*b^2)*x)/(a*b^3*x^2 + a^2*b^2) + 1/2*D*log(b
*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b)
```

Fricas [**A**]
time = 3.57, size = 257, normalized size = 2.76

$$\left[\frac{2\,Da^3 - 2\,Ba^2b - (Ca^2 + Aab + (Cab + Ab^2)x^2)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right) - 2\,(Ca^2b - Aab^2)x + 2\,(Da^2bx^2 + Da^3)\log\left(bx^2 + a\right)}{4\,(a^2b^3x^2 + a^3b^2)}, \frac{Da^3 - Ba^2b + (Ca^2 + Aab + (Cab + Ab^2)x^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) - (Ca^2b - Aab^2)x + (Da^2bx^2 + Da^3)\log\left(bx^2 + a\right)}{2\,(a^2b^3x^2 + a^3b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*D*a^3 - 2*B*a^2*b - (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(-a*b
)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(C*a^2*b - A*a*b^2)*x +
 2*(D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a))/(a^2*b^3*x^2 + a^3*b^2), 1/2*(D*a^
3 - B*a^2*b + (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a
*b)*x/a) - (C*a^2*b - A*a*b^2)*x + (D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a))/(a
^2*b^3*x^2 + a^3*b^2)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(78) = 156$.
time = 1.06, size = 233, normalized size = 2.51

$$\left(\frac{D}{2b^2} - \frac{\sqrt{-a^3b^5}\,(Ab + Ca)}{4a^3b^4}\right)\log\left(x + \frac{-2Da^2 + 4a^2b^2\left(\frac{D}{2b^2} - \frac{\sqrt{-a^3b^5}\,(Ab+Ca)}{4a^3b^4}\right)}{Ab^2 + Cab}\right) + \left(\frac{D}{2b^2} + \frac{\sqrt{-a^3b^5}\,(Ab + Ca)}{4a^3b^4}\right)\log\left(x + \frac{-2Da^2 + 4a^2b^2\left(\frac{D}{2b^2} + \frac{\sqrt{-a^3b^5}\,(Ab+Ca)}{4a^3b^4}\right)}{Ab^2 + Cab}\right) + \frac{-Bab + Da^2 + x(Ab^2 - Cab)}{2a^2b^2 + 2ab^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

```
[Out] (D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4))*log(x + (-2*D*a**
2 + 4*a**2*b**2*(D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4)))/
(A*b**2 + C*a*b)) + (D/(2*b**2) + sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4
))*log(x + (-2*D*a**2 + 4*a**2*b**2*(D/(2*b**2) + sqrt(-a**3*b**5)*(A*b + C
```

```
*a)/(4*a**3*b**4)))/(A*b**2 + C*a*b)) + (-B*a*b + D*a**2 + x*(A*b**2 - C*a*
b))/(2*a**2*b**2 + 2*a*b**3*x**2)
```

Giac [A]
time = 1.67, size = 88, normalized size = 0.95

$$\frac{D\log\left(bx^2+a\right)}{2\,b^2}+\frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab}-\frac{(Ca-Ab)x-\frac{Da^2-Bab}{b}}{2\,(bx^2+a)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*D*log(b*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a*b) - 1/2*((C*a - A*b)*x - (D*a^2 - B*a*b)/b)/((b*x^2 + a)*a*b)
```

Mupad [B]
time = 1.32, size = 110, normalized size = 1.18

$$\frac{\left(\ln\left(b\,x^2+a\right)+\frac{a}{b\,x^2+a}\right)\,D}{2\,b^2}-\frac{B}{2\,b\,\left(b\,x^2+a\right)}+\frac{A\,x}{2\,a\,\left(b\,x^2+a\right)}-\frac{C\,x}{2\,b\,\left(b\,x^2+a\right)}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}+\frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^2,x)
```

```
[Out] ((log(a + b*x^2) + a/(a + b*x^2))*D)/(2*b^2) - B/(2*b*(a + b*x^2)) + (A*x)/
(2*a*(a + b*x^2)) - (C*x)/(2*b*(a + b*x^2)) + (A*atan((b^(1/2)*x)/a^(1/2)))
/(2*a^(3/2)*b^(1/2)) + (C*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*b^(3/2))
```

3.99 $\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=95

$$\frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}+\frac{(bB+aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{A\log(x)}{a^2}-\frac{A\log\left(a+bx^2\right)}{2a^2}$$

```
[Out] 1/2*(A*b-a*C+(B*b-D*a)*x)/a/b/(b*x^2+a)+1/2*(B*b+D*a)*arctan(x*b^(1/2)/a^(1
/2))/a^(3/2)/b^(3/2)+A*ln(x)/a^2-1/2*A*ln(b*x^2+a)/a^2
```

Rubi [A]
time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1819, 815, 649, 211, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(aD+bB)}{2a^{3/2}b^{3/2}}-\frac{A\log\left(a+bx^2\right)}{2a^2}+\frac{A\log(x)}{a^2}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2),x]
```

```
[Out] (A*b - a*C + (b*B - a*D)*x)/(2*a*b*(a + b*x^2)) + ((b*B + a*D)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2])/
(2*a^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx &= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}-\frac{\int \frac{-2A-\frac{(bB+aD)x}{b}}{x(a+bx^2)}\,dx}{2a}\\
&= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}-\frac{\int \left(-\frac{2A}{ax}+\frac{-abB-a^2D+2Ab^2x}{ab(a+bx^2)}\right)dx}{2a}\\
&= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}+\frac{A\log(x)}{a^2}-\frac{\int \frac{-abB-a^2D+2Ab^2x}{a+bx^2}\,dx}{2a^2b}\\
&= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}+\frac{A\log(x)}{a^2}-\frac{(Ab)\int \frac{x}{a+bx^2}\,dx}{a^2}+\frac{(bB+aD)\int \frac{1}{a+bx^2}}{2ab}\\
&= \frac{Ab-aC+(bB-aD)x}{2ab\left(a+bx^2\right)}+\frac{(bB+aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{A\log(x)}{a^2}-\frac{A\log(a}{2a}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 85, normalized size = 0.89

$$
\frac{\frac{a(Ab+bBx-a(C+Dx))}{b(a+bx^2)}+\frac{\sqrt{a}\,(bB+aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}+2A\log(x)-A\log\left(a+bx^2\right)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2),x]
```

```
[Out] ((a*(A*b + b*B*x - a*(C + D*x)))/(b*(a + b*x^2)) + (Sqrt[a]*(b*B + a*D)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) + 2*A*Log[x] - A*Log[a + b*x^2])/(2*a^2)
```

Maple [A]
time = 0.11, size = 99, normalized size = 1.04

method	result	size
default	$-\frac{\frac{-\frac{a(Bb-aD)x}{2b}-\frac{a(Ab-aC)}{2b}}{b\,x^{2}+a}+\frac{bA\ln\left(b\,x^{2}+a\right)+\frac{\left(-abB-a^{2}D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{2b}}{a^{2}}+\frac{A\ln\left(x\right)}{a^{2}}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*((-1/2*a*(B*b-D*a)/b*x-1/2*a*(A*b-C*a)/b)/(b*x^2+a)+1/2/b*(b*A*ln(b*
x^2+a)+(-B*a*b-D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))+A*ln(x)/a^2
```

Maxima [A]
time = 0.55, size = 87, normalized size = 0.92

$$-\frac{Ca-Ab+(Da-Bb)x}{2\,(ab^{2}x^{2}+a^{2}b)}-\frac{A\log\left(bx^{2}+a\right)}{2\,a^{2}}+\frac{A\log\left(x\right)}{a^{2}}+\frac{(Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(C*a - A*b + (D*a - B*b)*x)/(a*b^2*x^2 + a^2*b) - 1/2*A*log(b*x^2 + a)
/a^2 + A*log(x)/a^2 + 1/2*(D*a + B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b)
```

Fricas [A]
time = 5.37, size = 296, normalized size = 3.12

$$\left[-\frac{2\,Ca^{2}b-2\,Aab^{2}+(Da^{2}+Bab+(Dab+Bb^{2})x^{2})\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)+2\,(Da^{2}b-Bab^{2})x+2\,(Ab^{3}x^{2}+Aab^{2})\log\left(bx^{2}+a\right)-4\,(Ab^{3}x^{2}+Aab^{2})\log\left(x\right)}{4\,(a^{2}b^{3}x^{2}+a^{3}b^{2})},-\frac{Ca^{2}b-Aab^{2}-(Da^{2}+Bab+(Dab+Bb^{2})x^{2})\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+(Da^{2}b-Bab^{2})x+(Ab^{3}x^{2}+Aab^{2})\log\left(bx^{2}+a\right)-2\,(Ab^{3}x^{2}+Aab^{2})\log\left(x\right)}{2\,(a^{2}b^{3}x^{2}+a^{3}b^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*C*a^2*b - 2*A*a*b^2 + (D*a^2 + B*a*b + (D*a*b + B*b^2)*x^2)*sqrt(-
a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(D*a^2*b - B*a*b^2)*
x + 2*(A*b^3*x^2 + A*a*b^2)*log(b*x^2 + a) - 4*(A*b^3*x^2 + A*a*b^2)*log(x)
)/(a^2*b^3*x^2 + a^3*b^2), -1/2*(C*a^2*b - A*a*b^2 - (D*a^2 + B*a*b + (D*a*
b + B*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (D*a^2*b - B*a*b^2)*x + (
A*b^3*x^2 + A*a*b^2)*log(b*x^2 + a) - 2*(A*b^3*x^2 + A*a*b^2)*log(x))/(a^2*
b^3*x^2 + a^3*b^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.36, size = 93, normalized size = 0.98

$$-\frac{A\log\left(bx^{2}+a\right)}{2\,a^{2}}+\frac{A\log\left(|x|\right)}{a^{2}}+\frac{(Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab}-\frac{Ca^{2}-Aab+(Da^{2}-Bab)x}{2\,(bx^{2}+a)a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*A*log(b*x^2 + a)/a^2 + A*log(abs(x))/a^2 + 1/2*(D*a + B*b)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*a*b) - 1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/((b*x^
2 + a)*a^2*b)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,x+C\,x^{2}+x^{3}\,D}{x\,{\left(b\,x^{2}+a\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^2),x)
```

```
[Out] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^2), x)
```

3.100 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{A}{a^2x}+\frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)}-\frac{(3Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}+\frac{B\log(x)}{a^2}-\frac{B\log\left(a+bx^2\right)}{2a^2}$$

```
[Out] -A/a^2/x+1/2*(b*B-a*D-b*(A*b/a-C)*x)/a/b/(b*x^2+a)+B*ln(x)/a^2-1/2*B*ln(b*x
^2+a)/a^2-1/2*(3*A*b-C*a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1819, 1816, 649, 211, 266}

$$-\frac{(3Ab-aC)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{A}{a^2x}-\frac{B\log\left(a+bx^2\right)}{2a^2}+\frac{B\log(x)}{a^2}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) + (b*B - a*D - b*((A*b)/a - C)*x)/(2*a*b*(a + b*x^2)) - ((3*A*
b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) + (B*Log[x])/a^2
- (B*Log[a + b*x^2])/(2*a^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx &= \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)}-\frac{\int \frac{-2A-2Bx+\left(\frac{Ab}{a}-C\right)x^2}{x^2\left(a+bx^2\right)}\,dx}{2a}\\
&= \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)}-\frac{\int \left(-\frac{2A}{ax^2}-\frac{2B}{ax}+\frac{3Ab-aC+2bBx}{a\left(a+bx^2\right)}\right)dx}{2a}\\
&= -\frac{A}{a^2x}+\frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)}+\frac{B\log(x)}{a^2}-\frac{\int \frac{3Ab-aC+2bBx}{a+bx^2}\,dx}{2a^2}\\
&= -\frac{A}{a^2x}+\frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)}+\frac{B\log(x)}{a^2}-\frac{(bB)\int \frac{x}{a+bx^2}\,dx}{a^2}-(3Ab-a\\
&= -\frac{A}{a^2x}+\frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)}-\frac{(3Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}+\frac{B\log(x)}{a^2}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 110, normalized size = 1.00

$$
-\frac{A}{a^2x}+\frac{abB-a^2D-Ab^2x+abCx}{2a^2b\left(a+bx^2\right)}+\frac{(-3Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}+\frac{B\log(x)}{a^2}-\frac{B\log\left(a+bx^2\right)}{2a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(A/(a^2*x)) + (a*b*B - a^2*D - A*b^2*x + a*b*C*x)/(2*a^2*b*(a + b*x^2)) +
((-3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) + (B*Log[x
])/a^2 - (B*Log[a + b*x^2])/(2*a^2)
```

Maple [A]
time = 0.10, size = 96, normalized size = 0.87

method	result	size
default	$-\frac{\frac{\left(\frac{Ab}{2}-\frac{aC}{2}\right)x-\frac{a(Bb-aD)}{2b}}{b\,x^{2}+a}+\frac{B\ln\left(b\,x^{2}+a\right)}{2}+\frac{(3Ab-aC)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^{2}}-\frac{A}{a^{2}x}+\frac{B\ln(x)}{a^{2}}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*(((1/2*A*b-1/2*a*C)*x-1/2*a*(B*b-D*a)/b)/(b*x^2+a)+1/2*B*ln(b*x^2+a)
+1/2*(3*A*b-C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-A/a^2/x+B*ln(x)/a^2
```

Maxima [A]
time = 0.52, size = 105, normalized size = 0.95

$$-\frac{2\,Aab-(Cab-3\,Ab^{2})x^{2}+(Da^{2}-Bab)x}{2\,(a^{2}b^{2}x^{3}+a^{3}bx)}-\frac{B\log\left(bx^{2}+a\right)}{2\,a^{2}}+\frac{B\log\left(x\right)}{a^{2}}+\frac{(Ca-3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*A*a*b - (C*a*b - 3*A*b^2)*x^2 + (D*a^2 - B*a*b)*x)/(a^2*b^2*x^3 + a
^3*b*x) - 1/2*B*log(b*x^2 + a)/a^2 + B*log(x)/a^2 + 1/2*(C*a - 3*A*b)*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^2)
```

Fricas [A]
time = 6.39, size = 336, normalized size = 3.05

$$\left[-\frac{4\,Aa^{2}b-2\,(Ca^{2}b-3\,Aab^{2})x^{2}-((Cab-3\,Ab^{2})x^{3}+(Ca^{2}-3\,Aab)x)\sqrt{-ab}\,\log\left(\frac{bx^{2}+2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)+2\,(Da^{3}-Ba^{2}b)x+2\,(Bab^{2}x^{3}+Ba^{2}bx)\log\left(bx^{2}+a\right)-4\,(Bab^{2}x^{3}+Ba^{2}bx)\log\left(x\right)}{4\,(a^{3}b^{2}x^{3}+a^{4}bx)},\,-\frac{2\,Aa^{2}b-(Ca^{2}b-3\,Aab^{2})x^{2}-((Cab-3\,Ab^{2})x^{3}+(Ca^{2}-3\,Aab)x)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+(Da^{3}-Ba^{2}b)x+(Bab^{2}x^{3}+Ba^{2}bx)\log\left(bx^{2}+a\right)-2\,(Bab^{2}x^{3}+Ba^{2}bx)\log\left(x\right)}{2\,(a^{3}b^{2}x^{3}+a^{4}bx)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*A*a^2*b - 2*(C*a^2*b - 3*A*a*b^2)*x^2 - ((C*a*b - 3*A*b^2)*x^3 + (
C*a^2 - 3*A*a*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) + 2*(D*a^3 - B*a^2*b)*x + 2*(B*a*b^2*x^3 + B*a^2*b*x)*log(b*x^2 + a) - 4*
(B*a*b^2*x^3 + B*a^2*b*x)*log(x))/(a^3*b^2*x^3 + a^4*b*x), -1/2*(2*A*a^2*b
- (C*a^2*b - 3*A*a*b^2)*x^2 - ((C*a*b - 3*A*b^2)*x^3 + (C*a^2 - 3*A*a*b)*x)
*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (D*a^3 - B*a^2*b)*x + (B*a*b^2*x^3 + B*a
^2*b*x)*log(b*x^2 + a) - 2*(B*a*b^2*x^3 + B*a^2*b*x)*log(x))/(a^3*b^2*x^3 +
 a^4*b*x)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.17, size = 103, normalized size = 0.94

$$-\frac{B\log\left(bx^{2}+a\right)}{2\,a^{2}}+\frac{B\log\left(|x|\right)}{a^{2}}+\frac{(Ca-3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{2}}+\frac{Cabx^{2}-3\,Ab^{2}x^{2}-Da^{2}x+Babx-2\,Aab}{2\,(bx^{3}+ax)a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*B*log(b*x^2 + a)/a^2 + B*log(abs(x))/a^2 + 1/2*(C*a - 3*A*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/2*(C*a*b*x^2 - 3*A*b^2*x^2 - D*a^2*x + B*a
*b*x - 2*A*a*b)/((b*x^3 + a*x)*a^2*b)
```

Mupad [**B**]
time = 1.41, size = 133, normalized size = 1.21

$$\frac{B}{2\,a\,\left(b\,x^{2}+a\right)}-\frac{\frac{A}{a}+\frac{3\,A\,b\,x^{2}}{2\,a^{2}}}{b\,x^{3}+a\,x}-\frac{B\,\ln\left(b\,x^{2}+a\right)}{2\,a^{2}}+\frac{B\,\ln\left(x\right)}{a^{2}}-\frac{D}{2\,b\,\left(b\,x^{2}+a\right)}+\frac{C\,x}{2\,a\,\left(b\,x^{2}+a\right)}-\frac{3\,A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}+\frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] B/(2*a*(a + b*x^2)) - (A/a + (3*A*b*x^2)/(2*a^2))/(a*x + b*x^3) - (B*log(a
+ b*x^2))/(2*a^2) + (B*log(x))/a^2 - D/(2*b*(a + b*x^2)) + (C*x)/(2*a*(a +
b*x^2)) - (3*A*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(5/2)) + (C*atan((b^
(1/2)*x)/a^(1/2)))/(2*a^(3/2)*b^(1/2))
```

3.101 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=135

$$-\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{(3bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{(2Ab-aC)\log(x)}{a^3}+\frac{(2Ab-aC)\log(}{2a^3}$$

```
[Out] -1/2*A/a^2/x^2-B/a^2/x+1/2*(-A*b/a+C-(b*B/a-D)*x)/a/(b*x^2+a)-(2*A*b-C*a)*l
n(x)/a^3+1/2*(2*A*b-C*a)*ln(b*x^2+a)/a^3-1/2*(3*B*b-D*a)*arctan(x*b^(1/2)/a
^(1/2))/a^(5/2)/b^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1819, 1816, 649, 211, 266}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(3bB-aD)}{2a^{5/2}\sqrt{b}}+\frac{(2Ab-aC)\log\left(a+bx^2\right)}{2a^3}-\frac{\log(x)(2Ab-aC)}{a^3}-\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] -1/2*A/(a^2*x^2) - B/(a^2*x) - ((A*b)/a - C + ((b*B)/a - D)*x)/(2*a*(a + b*
x^2)) - ((3*b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) - (
(2*A*b - a*C)*Log[x])/a^3 + ((2*A*b - a*C)*Log[a + b*x^2])/(2*a^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx &= -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{\int \frac{-2A-2Bx+2\left(\frac{Ab}{a}-C\right)x^2+\left(\frac{bB}{a}-D\right)x^3}{x^3\left(a+bx^2\right)}\,dx}{2a}\\
&= -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{\int \left(-\frac{2A}{ax^3}-\frac{2B}{ax^2}-\frac{2(-2Ab+aC)}{a^2x}+\frac{a(3bB-aD)-2b(2Ab-aC}{a^2\left(a+bx^2\right)}\right.}{2a}\\
&= -\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{(2Ab-aC)\log(x)}{a^3}-\frac{\int \frac{a(3bB-aD)-}{a+}}{2}\\
&= -\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{(2Ab-aC)\log(x)}{a^3}+\frac{(b(2Ab-aC}{a}\\
&= -\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{(3bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-(2A
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 112, normalized size = 0.83

$$\frac{-\frac{aA}{x^2}-\frac{2aB}{x}+\frac{a(-Ab-bBx+a(C+Dx))}{a+bx^2}+\frac{\sqrt{a}\,(-3bB+aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}+2(-2Ab+aC)\log(x)+(2Ab-aC)\log\left(a+bx^2\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2),x]
```

```
[Out] (-((a*A)/x^2) - (2*a*B)/x + (a*(-(A*b) - b*B*x + a*(C + D*x)))/(a + b*x^2)
+ (Sqrt[a]*(-3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + 2*(-2*A*b
+ a*C)*Log[x] + (2*A*b - a*C)*Log[a + b*x^2])/(2*a^3)
```

Maple [A]
time = 0.10, size = 127, normalized size = 0.94

method	result
default	$\frac{\frac{\left(-\frac{1}{2}abB+\frac{1}{2}a^{2}D\right)x-\frac{a(Ab-aC)}{2}}{b\,x^{2}+a}+\frac{\left(4b^{2}A-2abC\right)\ln\left(b\,x^{2}+a\right)}{4b}+\frac{\left(-3abB+a^{2}D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^{3}}-\frac{A}{2a^{2}x^{2}}-\frac{B}{a^{2}x}+\frac{(-2Ab+aC)\ln(x)}{a^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(((-1/2*a*b*B+1/2*a^2*D)*x-1/2*a*(A*b-C*a))/(b*x^2+a)+1/4*(4*A*b^2-2*
C*a*b)/b*ln(b*x^2+a)+1/2*(-3*B*a*b+D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2
)))-1/2*A/a^2/x^2-B/a^2/x+(-2*A*b+C*a)/a^3*ln(x)
```

Maxima [A]
time = 0.56, size = 117, normalized size = 0.87

$$\frac{(Da-3\,Bb)x^{3}-2\,Bax+(Ca-2\,Ab)x^{2}-Aa}{2\,(a^{2}bx^{4}+a^{3}x^{2})}+\frac{(Da-3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{2}}-\frac{(Ca-2\,Ab)\log\left(bx^{2}+a\right)}{2\,a^{3}}+\frac{(Ca-2\,Ab)\log\left(x\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((D*a - 3*B*b)*x^3 - 2*B*a*x + (C*a - 2*A*b)*x^2 - A*a)/(a^2*b*x^4 + a^
3*x^2) + 1/2*(D*a - 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(C*a
 - 2*A*b)*log(b*x^2 + a)/a^3 + (C*a - 2*A*b)*log(x)/a^3
```

Fricas [A]
time = 7.44, size = 441, normalized size = 3.27

$$\left[-\frac{4\,Ba^{2}bx+2\,Aa^{2}b-2\,(Da^{2}b-3\,Bab^{2})x^{3}-2\,(Ca^{2}b-2\,Aab^{2})x^{2}+((Dab-3\,Bb^{2})x^{4}+(Da^{2}-3\,Bab)x^{2})\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)+2\,((Cab^{2}-2\,Ab^{3})x^{4}+(Ca^{2}b-2\,Aab^{2})x^{2})\log\left(bx^{2}+a\right)-4\,((Cab^{2}-2\,Ab^{3})x^{4}+(Ca^{2}b-2\,Aab^{2})x^{2})\log\left(x\right)}{4\,(a^{3}b^{2}x^{4}+a^{4}bx^{2})},-\frac{2\,Ba^{2}bx+Aa^{2}b-(Da^{2}b-3\,Bab^{2})x^{3}-(Ca^{2}b-2\,Aab^{2})x^{2}-((Dab-3\,Bb^{2})x^{4}+(Da^{2}-3\,Bab)x^{2})\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+((Cab^{2}-2\,Ab^{3})x^{4}+(Ca^{2}b-2\,Aab^{2})x^{2})\log\left(bx^{2}+a\right)-2\,((Cab^{2}-2\,Ab^{3})x^{4}+(Ca^{2}b-2\,Aab^{2})x^{2})\log\left(x\right)}{2\,(a^{3}b^{2}x^{4}+a^{4}bx^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*B*a^2*b*x + 2*A*a^2*b - 2*(D*a^2*b - 3*B*a*b^2)*x^3 - 2*(C*a^2*b -
 2*A*a*b^2)*x^2 + ((D*a*b - 3*B*b^2)*x^4 + (D*a^2 - 3*B*a*b)*x^2)*sqrt(-a*b
)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*((C*a*b^2 - 2*A*b^3)*x^
4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(b*x^2 + a) - 4*((C*a*b^2 - 2*A*b^3)*x^4
+ (C*a^2*b - 2*A*a*b^2)*x^2)*log(x))/(a^3*b^2*x^4 + a^4*b*x^2), -1/2*(2*B*a
^2*b*x + A*a^2*b - (D*a^2*b - 3*B*a*b^2)*x^3 - (C*a^2*b - 2*A*a*b^2)*x^2 -
((D*a*b - 3*B*b^2)*x^4 + (D*a^2 - 3*B*a*b)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*
x/a) + ((C*a*b^2 - 2*A*b^3)*x^4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(b*x^2 + a)
```

```
 - 2*((C*a*b^2 - 2*A*b^3)*x^4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(x))/(a^3*b^2
*x^4 + a^4*b*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.53, size = 126, normalized size = 0.93

$$\frac{(Da-3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2}-\frac{(Ca-2\,Ab)\log\left(bx^2+a\right)}{2\,a^3}+\frac{(Ca-2\,Ab)\log\left(|x|\right)}{a^3}-\frac{2\,Ba^2x-(Da^2-3\,Bab)x^3+Aa^2-(Ca^2-2\,Aab)x^2}{2\,(bx^2+a)a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(D*a - 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(C*a - 2*A*b)
*log(b*x^2 + a)/a^3 + (C*a - 2*A*b)*log(abs(x))/a^3 - 1/2*(2*B*a^2*x - (D*a
^2 - 3*B*a*b)*x^3 + A*a^2 - (C*a^2 - 2*A*a*b)*x^2)/((b*x^2 + a)*a^3*x^2)
```

Mupad [**B**]
time = 1.35, size = 158, normalized size = 1.17

$$\frac{C}{2\,a\,\left(b\,x^2+a\right)}-\frac{\frac{A}{2\,a}+\frac{A\,b\,x^2}{a^2}}{b\,x^4+a\,x^2}-\frac{\frac{B}{a}+\frac{3\,B\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{C\,\ln\left(b\,x^2+a\right)}{2\,a^2}+\frac{C\,\ln\left(x\right)}{a^2}+\frac{A\,b\,\ln\left(b\,x^2+a\right)}{a^3}-\frac{2\,A\,b\,\ln\left(x\right)}{a^3}+\frac{x\,D\,{}_2\mathrm{F}_1\left(\frac{1}{2},2;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{a^2}-\frac{3\,B\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)^2),x)
```

```
[Out] C/(2*a*(a + b*x^2)) - (A/(2*a) + (A*b*x^2)/a^2)/(a*x^2 + b*x^4) - (B/a + (3
*B*b*x^2)/(2*a^2))/(a*x + b*x^3) - (C*log(a + b*x^2))/(2*a^2) + (C*log(x))/
a^2 + (A*b*log(a + b*x^2))/a^3 - (2*A*b*log(x))/a^3 + (x*D*hypergeom([1/2,
2], 3/2, -(b*x^2)/a))/a^2 - (3*B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(5
/2))
```

3.102 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=185

$$-\frac{3(Ab-5aC)x}{8ab^3}-\frac{(bB-3aD)x^2}{2ab^3}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}+\frac{x^3(Ab-5aC+4(bB-2aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{3(Ab}{}$$

```
[Out] -3/8*(A*b-5*C*a)*x/a/b^3-1/2*(B*b-3*D*a)*x^2/a/b^3-1/4*x^4*(a*(B-a*D/b)-(A*
b-C*a)*x)/a/b/(b*x^2+a)^2+1/8*x^3*(A*b-5*a*C+4*(B*b-2*D*a)*x)/a/b^2/(b*x^2+
a)+1/2*(B*b-3*D*a)*ln(b*x^2+a)/b^4+3/8*(A*b-5*C*a)*arctan(x*b^(1/2)/a^(1/2)
)/b^(7/2)/a^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1818, 815, 649, 211, 266}

$$\frac{3(Ab-5aC)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}-\frac{3x(Ab-5aC)}{8ab^3}+\frac{x^3(4x(bB-2aD)-5aC+Ab)}{8ab^2\left(a+bx^2\right)}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}+\frac{(bB-3aD)\log\left(a+bx^2\right)}{2b^4}-\frac{x^2(bB-3aD)}{2ab^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (-3*(A*b - 5*a*C)*x)/(8*a*b^3) - ((b*B - 3*a*D)*x^2)/(2*a*b^3) - (x^4*(a*(B
 - (a*D)/b) - (A*b - a*C)*x))/(4*a*b*(a + b*x^2)^2) + (x^3*(A*b - 5*a*C + 4
*(b*B - 2*a*D)*x))/(8*a*b^2*(a + b*x^2)) + (3*(A*b - 5*a*C)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + ((b*B - 3*a*D)*Log[a + b*x^2])/(2*b^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx &= -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2}-\frac{\int \frac{x^3\left(-4a\left(B-\frac{aD}{b}\right)+(Ab-5aC)x-4aDx^2\right)}{(a+bx^2)^2}\,dx}{4ab}\\
&= -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2}+\frac{x^3(Ab-5aC+4(bB-2aD)x)}{8ab^2\,(a+bx^2)}+\int\\
&= -\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2}+\frac{x^3(Ab-5aC+4(bB-2aD)x)}{8ab^2\,(a+bx^2)}+\int\\
&= -\frac{3(Ab-5aC)x}{8ab^3}-\frac{(bB-3aD)x^2}{2ab^3}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2}+x\\
&= -\frac{3(Ab-5aC)x}{8ab^3}-\frac{(bB-3aD)x^2}{2ab^3}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2}+x\\
&= -\frac{3(Ab-5aC)x}{8ab^3}-\frac{(bB-3aD)x^2}{2ab^3}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2}+x
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 139, normalized size = 0.75

$$
\frac{8bCx+4bDx^2+\frac{8abB-12a^2D-5Ab^2x+9abCx}{a+bx^2}+\frac{2a\left(a^2D+Ab^2x-ab(B+Cx)\right)}{(a+bx^2)^2}+\frac{3\sqrt{b}\,(Ab-5aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}}+4(bB-3aD)\log\left(a+bx^2\right)}{8b^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (8*b*C*x + 4*b*D*x^2 + (8*a*b*B - 12*a^2*D - 5*A*b^2*x + 9*a*b*C*x)/(a + b*
x^2) + (2*a*(a^2*D + A*b^2*x - a*b*(B + C*x)))/(a + b*x^2)^2 + (3*Sqrt[b]*(
A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + 4*(b*B - 3*a*D)*Log[a +
 b*x^2])/(8*b^4)
```

Maple [A]
time = 0.10, size = 140, normalized size = 0.76

method	result
default	$\frac{\frac{1}{2}Dx^2+Cx}{b^3}+\frac{\frac{\left(-\frac{5}{8}b^2A+\frac{9}{8}abC\right)x^3+\left(abB-\frac{3}{2}a^2D\right)x^2-\frac{a(3Ab-7aC)x}{8}+\frac{a^2(3Bb-5aD)}{4b}}{\left(bx^2+a\right)^2}+\frac{(8Bb-24aD)\ln\left(bx^2+a\right)}{16b}+\frac{(3Ab-15aC)\arctan\left(\right.}{8\sqrt{ab}}}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/2*D*x^2+C*x)+1/b^3*(((-5/8*b^2*A+9/8*a*b*C)*x^3+(a*b*B-3/2*a^2*D)*
x^2-1/8*a*(3*A*b-7*C*a)*x+1/4*a^2*(3*B*b-5*D*a)/b)/(b*x^2+a)^2+1/16*(8*B*b-
24*D*a)/b*ln(b*x^2+a)+1/8*(3*A*b-15*C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)
))
```

Maxima [A]
time = 0.59, size = 165, normalized size = 0.89

$$-\frac{10\,Da^3-6\,Ba^2b-(9\,Cab^2-5\,Ab^3)x^3+4\,(3\,Da^2b-2\,Bab^2)x^2-(7\,Ca^2b-3\,Aab^2)x}{8\,(b^6x^4+2\,ab^5x^2+a^2b^4)}-\frac{3\,(5\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}+\frac{Dx^2+2\,Cx}{2\,b^3}-\frac{(3\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(10*D*a^3 - 6*B*a^2*b - (9*C*a*b^2 - 5*A*b^3)*x^3 + 4*(3*D*a^2*b - 2*B
*a*b^2)*x^2 - (7*C*a^2*b - 3*A*a*b^2)*x)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4)
- 3/8*(5*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*(D*x^2 + 2*
C*x)/b^3 - 1/2*(3*D*a - B*b)*log(b*x^2 + a)/b^4
```

Fricas [A]
time = 16.49, size = 574, normalized size = 3.10

$$\left[\frac{8\,Dab^3x^6+16\,Cab^3x^5+16\,Da^2b^2x^4-20\,Da^4+12\,Ba^3b+10\,(5\,Ca^2b^2-Aab^3)x^3-16\,(Da^3b-Ba^2b^2)x^2+3\,((5\,Cab^2-Ab^3)x^4+5\,Ca^3-Aa^2b+2\,(5\,Ca^2b-Aab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+6\,(5\,Ca^3b-Aa^2b^2)x-8\,(3\,Da^4-Ba^3b+(3\,Da^2b^2-Bab^3)x^4+2\,(3\,Da^3b-Ba^2b^2)x^2)\log\left(bx^2+a\right)}{16\,(ab^6x^4+2\,a^2b^5x^2+a^3b^4)},\ \frac{4\,Dab^3x^6+8\,Cab^3x^5+8\,Da^2b^2x^4-10\,Da^4+6\,Ba^3b+5\,(5\,Ca^2b^2-Aab^3)x^3-8\,(Da^3b-Ba^2b^2)x^2-3\,((5\,Cab^2-Ab^3)x^4+5\,Ca^3-Aa^2b+2\,(5\,Ca^2b-Aab^2)x^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+3\,(5\,Ca^3b-Aa^2b^2)x-4\,(3\,Da^4-Ba^3b+(3\,Da^2b^2-Bab^3)x^4+2\,(3\,Da^3b-Ba^2b^2)x^2)\log\left(bx^2+a\right)}{8\,(ab^6x^4+2\,a^2b^5x^2+a^3b^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*D*a*b^3*x^6 + 16*C*a*b^3*x^5 + 16*D*a^2*b^2*x^4 - 20*D*a^4 + 12*B*
a^3*b + 10*(5*C*a^2*b^2 - A*a*b^3)*x^3 - 16*(D*a^3*b - B*a^2*b^2)*x^2 + 3*(
```

```
(5*C*a*b^2 - A*b^3)*x^4 + 5*C*a^3 - A*a^2*b + 2*(5*C*a^2*b - A*a*b^2)*x^2)*
sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(5*C*a^3*b - A
*a^2*b^2)*x - 8*(3*D*a^4 - B*a^3*b + (3*D*a^2*b^2 - B*a*b^3)*x^4 + 2*(3*D*a
^3*b - B*a^2*b^2)*x^2)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4
), 1/8*(4*D*a*b^3*x^6 + 8*C*a*b^3*x^5 + 8*D*a^2*b^2*x^4 - 10*D*a^4 + 6*B*a^
3*b + 5*(5*C*a^2*b^2 - A*a*b^3)*x^3 - 8*(D*a^3*b - B*a^2*b^2)*x^2 - 3*((5*C
*a*b^2 - A*b^3)*x^4 + 5*C*a^3 - A*a^2*b + 2*(5*C*a^2*b - A*a*b^2)*x^2)*sqrt
(a*b)*arctan(sqrt(a*b)*x/a) + 3*(5*C*a^3*b - A*a^2*b^2)*x - 4*(3*D*a^4 - B*
a^3*b + (3*D*a^2*b^2 - B*a*b^3)*x^4 + 2*(3*D*a^3*b - B*a^2*b^2)*x^2)*log(b*
x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(172) = 344$.
time = 97.10, size = 357, normalized size = 1.93

$$\frac{Cx}{b^3}+\frac{Dx^2}{2b^3}+\left(-\frac{-Bb+3Da}{2b^4}-\frac{3\sqrt{-ab^9}\,(-Ab+5Ca)}{16ab^8}\right)\log\left(x+\frac{8Bab-24Da^2-16ab^4\left(-\frac{-Bb+3Da}{2b^4}-\frac{3\sqrt{-ab^9}\,(-Ab+5Ca)}{16ab^8}\right)}{-3Ab^2+15Cab}\right)+\left(-\frac{-Bb+3Da}{2b^4}+\frac{3\sqrt{-ab^9}\,(-Ab+5Ca)}{16ab^8}\right)\log\left(x+\frac{8Bab-24Da^2-16ab^4\left(-\frac{-Bb+3Da}{2b^4}+\frac{3\sqrt{-ab^9}\,(-Ab+5Ca)}{16ab^8}\right)}{-3Ab^2+15Cab}\right)+\frac{6Ba^2b-10Da^3+x^3(-5Ab^3+9Cab^2)+x^2\cdot(8Bab^2-12Da^2b)+x(-3Aab^2+7Ca^2b)}{8a^2b^4+16ab^5x^2+8b^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] C*x/b**3 + D*x**2/(2*b**3) + (-(-B*b + 3*D*a)/(2*b**4) - 3*sqrt(-a*b**9)*(-
A*b + 5*C*a)/(16*a*b**8))*log(x + (8*B*a*b - 24*D*a**2 - 16*a*b**4*(-(-B*b
+ 3*D*a)/(2*b**4) - 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/(16*a*b**8)))/(-3*A*b**2
 + 15*C*a*b)) + (-(-B*b + 3*D*a)/(2*b**4) + 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/
(16*a*b**8))*log(x + (8*B*a*b - 24*D*a**2 - 16*a*b**4*(-(-B*b + 3*D*a)/(2*b
**4) + 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/(16*a*b**8)))/(-3*A*b**2 + 15*C*a*b))
 + (6*B*a**2*b - 10*D*a**3 + x**3*(-5*A*b**3 + 9*C*a*b**2) + x**2*(8*B*a*b*
*2 - 12*D*a**2*b) + x*(-3*A*a*b**2 + 7*C*a**2*b))/(8*a**2*b**4 + 16*a*b**5*
x**2 + 8*b**6*x**4)
```

Giac [A]
time = 1.26, size = 157, normalized size = 0.85

$$-\frac{3\,(5\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3}-\frac{(3\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^4}+\frac{Db^3x^2+2\,Cb^3x}{2\,b^6}-\frac{10\,Da^3-6\,Ba^2b-(9\,Cab^2-5\,Ab^3)x^3+4\,(3\,Da^2b-2\,Bab^2)x^2-(7\,Ca^2b-3\,Aab^2)x}{8\,(bx^2+a)^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3/8*(5*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(3*D*a - B*b
)*log(b*x^2 + a)/b^4 + 1/2*(D*b^3*x^2 + 2*C*b^3*x)/b^6 - 1/8*(10*D*a^3 - 6*
B*a^2*b - (9*C*a*b^2 - 5*A*b^3)*x^3 + 4*(3*D*a^2*b - 2*B*a*b^2)*x^2 - (7*C*
a^2*b - 3*A*a*b^2)*x)/((b*x^2 + a)^2*b^4)
```

Mupad [B]
time = 1.56, size = 232, normalized size = 1.25

$$\frac{\frac{7\,C\,a^2\,x}{8}+\frac{9\,C\,b\,a\,x^3}{8}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}-\frac{\frac{5\,A\,x^3}{8\,b}+\frac{3\,A\,a\,x}{8\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\frac{3\,B\,a^2}{4\,b^3}+\frac{B\,a\,x^2}{b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{D\,\left(3\,a\,\ln\left(b\,x^2+a\right)-b\,x^2+\frac{3\,a^2}{b\,x^2+a}-\frac{a^3}{2\,{\left(b\,x^2+a\right)}^2}\right)}{2\,b^4}+\frac{B\,\ln\left(b\,x^2+a\right)}{2\,b^3}+\frac{C\,x}{b^3}+\frac{3\,A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,b^{5/2}}-\frac{15\,C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

```
[Out] ((7*C*a^2*x)/8 + (9*C*a*b*x^3)/8)/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) - ((5*A
*x^3)/(8*b) + (3*A*a*x)/(8*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((3*B*a^2)/(
4*b^3) + (B*a*x^2)/b^2)/(a^2 + b^2*x^4 + 2*a*b*x^2) - (D*(3*a*log(a + b*x^2
) - b*x^2 + (3*a^2)/(a + b*x^2) - a^3/(2*(a + b*x^2)^2)))/(2*b^4) + (B*log(
a + b*x^2))/(2*b^3) + (C*x)/b^3 + (3*A*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(1/2
)*b^(5/2)) - (15*C*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*b^(7/2))
```

3.103 $\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=155

$$-\frac{3(bB-5aD)x}{8ab^3}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-(3bB-7aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{3(bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}$$

```
[Out] -3/8*(B*b-5*D*a)*x/a/b^3-1/4*x^3*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)^2-
1/8*x^2*(4*a*C-(3*B*b-7*D*a)*x)/a/b^2/(b*x^2+a)+1/2*C*ln(b*x^2+a)/b^3+3/8*(
B*b-5*D*a)*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1818, 788, 649, 211, 266}

$$-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}+\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bB-5aD)}{8\sqrt{a}\,b^{7/2}}-\frac{3x(bB-5aD)}{8ab^3}+\frac{C\log\left(a+bx^2\right)}{2b^3}-\frac{x^2(4aC-x(3bB-7aD))}{8ab^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (-3*(b*B - 5*a*D)*x)/(8*a*b^3) - (x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(4
*a*b*(a + b*x^2)^2) - (x^2*(4*a*C - (3*b*B - 7*a*D)*x))/(8*a*b^2*(a + b*x^2
)) + (3*(b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + (C
*Log[a + b*x^2])/(2*b^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 788

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Sym
bol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + c*(e*f + d*g)*x
)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{\left(a+bx^2\right)^3}\,dx &= -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{\int \frac{x^2\left(-3a\left(B-\frac{aD}{b}\right)-4aCx-4aDx^2\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}\\
&= -\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-(3bB-7aD)x)}{8ab^2\left(a+bx^2\right)}+\int \frac{x(8a}{}\\
&= -\frac{3(bB-5aD)x}{8ab^3}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-(3bB-}{8ab^2(a+b}\\
&= -\frac{3(bB-5aD)x}{8ab^3}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-(3bB-}{8ab^2(a+b}\\
&= -\frac{3(bB-5aD)x}{8ab^3}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x^2(4aC-(3bB-}{8ab^2(a+b}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 126, normalized size = 0.81

$$\frac{Dx}{b^3}+\frac{-4Ab+8aC-5bBx+9aDx}{8b^3\left(a+bx^2\right)}+\frac{a(Ab+bBx-a(C+Dx))}{4b^3\left(a+bx^2\right)^2}+\frac{3(bB-5aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{7/2}}+\frac{C\log\left(a+bx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] (D*x)/b^3 + (-4*A*b + 8*a*C - 5*b*B*x + 9*a*D*x)/(8*b^3*(a + b*x^2)) + (a*(
A*b + b*B*x - a*(C + D*x)))/(4*b^3*(a + b*x^2)^2) + (3*(b*B - 5*a*D)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + (C*Log[a + b*x^2])/(2*b^3)
```

Maple [A]
time = 0.11, size = 115, normalized size = 0.74

method	result	size
default	$\frac{Dx}{b^{3}}+\frac{\frac{\left(-\frac{5}{8}b^{2}B+\frac{9}{8}abD\right)x^{3}+\left(-\frac{1}{2}b^{2}A+abC\right)x^{2}-\frac{a(3Bb-7aD)x}{8}-\frac{abA}{4}+\frac{3a^{2}C}{4}}{\left(b\,x^{2}+a\right)^{2}}+\frac{C\ln\left(b\,x^{2}+a\right)}{2}+\frac{(3Bb-15aD)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{b^{3}}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] D/b^3*x+1/b^3*(((-5/8*b^2*B+9/8*a*b*D)*x^3+(-1/2*b^2*A+a*b*C)*x^2-1/8*a*(3*
B*b-7*D*a)*x-1/4*a*b*A+3/4*a^2*C)/(b*x^2+a)^2+1/2*C*ln(b*x^2+a)+1/8*(3*B*b-
15*D*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.54, size = 136, normalized size = 0.88

$$\frac{(9\,Dab-5\,Bb^{2})x^{3}+6\,Ca^{2}-2\,Aab+4\,(2\,Cab-Ab^{2})x^{2}+(7\,Da^{2}-3\,Bab)x}{8\,(b^{5}x^{4}+2\,ab^{4}x^{2}+a^{2}b^{3})}+\frac{Dx}{b^{3}}+\frac{C\log\left(bx^{2}+a\right)}{2\,b^{3}}-\frac{3\,(5\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((9*D*a*b - 5*B*b^2)*x^3 + 6*C*a^2 - 2*A*a*b + 4*(2*C*a*b - A*b^2)*x^2
+ (7*D*a^2 - 3*B*a*b)*x)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) + D*x/b^3 + 1/2*
C*log(b*x^2 + a)/b^3 - 3/8*(5*D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b
^3)
```

Fricas [A]
time = 8.01, size = 480, normalized size = 3.10

$$\left[\frac{16\,Dab^{3}x^{5}+12\,Ca^{3}b-4\,Aa^{2}b^{2}+10\,(5\,Da^{2}b^{2}-Bab^{3})x^{3}+8\,(2\,Ca^{2}b^{2}-Aab^{3})x^{2}-3\,((5\,Dab^{2}-Bb^{3})x^{4}+5\,Da^{3}-Ba^{2}b+2\,(5\,Da^{2}b-Bab^{2})x^{2})\sqrt{-ab}\,\log\left(\frac{bx^{2}+2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)+6\,(5\,Da^{3}b-Ba^{2}b^{2})x+8\,(Cab^{3}x^{4}+2\,Ca^{2}b^{2}x^{2}+Ca^{3}b)\log\left(bx^{2}+a\right)}{16\,(ab^{6}x^{4}+2\,a^{2}b^{5}x^{2}+a^{3}b^{4})},\ \frac{8\,Dab^{3}x^{5}+6\,Ca^{3}b-2\,Aa^{2}b^{2}+5\,(5\,Da^{2}b^{2}-Bab^{3})x^{3}+4\,(2\,Ca^{2}b^{2}-Aab^{3})x^{2}-3\,((5\,Dab^{2}-Bb^{3})x^{4}+5\,Da^{3}-Ba^{2}b+2\,(5\,Da^{2}b-Bab^{2})x^{2})\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+3\,(5\,Da^{3}b-Ba^{2}b^{2})x+4\,(Cab^{3}x^{4}+2\,Ca^{2}b^{2}x^{2}+Ca^{3}b)\log\left(bx^{2}+a\right)}{8\,(ab^{6}x^{4}+2\,a^{2}b^{5}x^{2}+a^{3}b^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*D*a*b^3*x^5 + 12*C*a^3*b - 4*A*a^2*b^2 + 10*(5*D*a^2*b^2 - B*a*b^
3)*x^3 + 8*(2*C*a^2*b^2 - A*a*b^3)*x^2 - 3*((5*D*a*b^2 - B*b^3)*x^4 + 5*D*a
^3 - B*a^2*b + 2*(5*D*a^2*b - B*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(
-a*b)*x - a)/(b*x^2 + a)) + 6*(5*D*a^3*b - B*a^2*b^2)*x + 8*(C*a*b^3*x^4 +
2*C*a^2*b^2*x^2 + C*a^3*b)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3
*b^4), 1/8*(8*D*a*b^3*x^5 + 6*C*a^3*b - 2*A*a^2*b^2 + 5*(5*D*a^2*b^2 - B*a*
b^3)*x^3 + 4*(2*C*a^2*b^2 - A*a*b^3)*x^2 - 3*((5*D*a*b^2 - B*b^3)*x^4 + 5*D
```

```
*a^3 - B*a^2*b + 2*(5*D*a^2*b - B*a*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/
a) + 3*(5*D*a^3*b - B*a^2*b^2)*x + 4*(C*a*b^3*x^4 + 2*C*a^2*b^2*x^2 + C*a^3
*b)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(139) = 278$.
time = 86.67, size = 282, normalized size = 1.82

$$\frac{Dx}{b^3} + \left(\frac{C}{2b^3} - \frac{3\sqrt{-ab^7}\,(-Bb+5Da)}{16ab^7}\right)\log\left(x + \frac{8Ca - 16ab^3\left(\frac{C}{2b^3} - \frac{3\sqrt{-ab^7}\,(-Bb+5Da)}{16ab^7}\right)}{-3Bb+15Da}\right) + \left(\frac{C}{2b^3} + \frac{3\sqrt{-ab^7}\,(-Bb+5Da)}{16ab^7}\right)\log\left(x + \frac{8Ca - 16ab^3\left(\frac{C}{2b^3} + \frac{3\sqrt{-ab^7}\,(-Bb+5Da)}{16ab^7}\right)}{-3Bb+15Da}\right) + \frac{-2Aab + 6Ca^2 + x^3(-5Bb^2+9Dab) + x^2(-4Ab^2+8Cab) + x(-3Bab+7Da^2)}{8a^2b^3 + 16ab^4x^2 + 8b^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] D*x/b**3 + (C/(2*b**3) - 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*a*b**7))*log(x
+ (8*C*a - 16*a*b**3*(C/(2*b**3) - 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*a*b**
7)))/(-3*B*b + 15*D*a)) + (C/(2*b**3) + 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*
a*b**7))*log(x + (8*C*a - 16*a*b**3*(C/(2*b**3) + 3*sqrt(-a*b**7)*(-B*b + 5
*D*a)/(16*a*b**7)))/(-3*B*b + 15*D*a)) + (-2*A*a*b + 6*C*a**2 + x**3*(-5*B*
b**2 + 9*D*a*b) + x**2*(-4*A*b**2 + 8*C*a*b) + x*(-3*B*a*b + 7*D*a**2))/(8*
a**2*b**3 + 16*a*b**4*x**2 + 8*b**5*x**4)
```

Giac [A]
time = 1.35, size = 122, normalized size = 0.79

$$\frac{Dx}{b^3} + \frac{C\log\left(bx^2+a\right)}{2\,b^3} - \frac{3\,(5\,Da - Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^3} + \frac{(9\,Dab - 5\,Bb^2)x^3 + 6\,Ca^2 - 2\,Aab + 4\,(2\,Cab - Ab^2)x^2 + (7\,Da^2 - 3\,Bab)x}{8\,(bx^2+a)^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] D*x/b^3 + 1/2*C*log(b*x^2 + a)/b^3 - 3/8*(5*D*a - B*b)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^3) + 1/8*((9*D*a*b - 5*B*b^2)*x^3 + 6*C*a^2 - 2*A*a*b + 4*(2
*C*a*b - A*b^2)*x^2 + (7*D*a^2 - 3*B*a*b)*x)/((b*x^2 + a)^2*b^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

```
[Out] int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3, x)
```

3.104 $\int \frac{x^2\left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=136

$$-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\left(a+bx^2\right)}+\frac{(Ab+3aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}+\frac{D\log\left(a+\right.}{2b^3}$$

```
[Out] -1/4*x^2*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)^2-1/8*x*(A*b+3*a*C-2*(B*b-
3*D*a)*x)/a/b^2/(b*x^2+a)+1/8*(A*b+3*C*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)
/b^(5/2)+1/2*D*ln(b*x^2+a)/b^3
```

Rubi [A]
time = 0.11, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1818, 649, 211, 266}

$$\frac{(3aC+Ab)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{8ab^2\left(a+bx^2\right)}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}+\frac{D\log\left(a+bx^2\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] -1/4*(x^2*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)^2) - (x*(A*b
+ 3*a*C - 2*(b*B - 3*a*D)*x))/(8*a*b^2*(a + b*x^2)) + ((A*b + 3*a*C)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)) + (D*Log[a + b*x^2])/(2*b^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx &= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2}-\frac{\int \frac{x\left(-2a\left(B-\frac{aD}{b}\right)-(Ab+3aC)x-4aDx^2\right)}{(a+bx^2)^2}\,dx}{4ab}\\ &= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2}-\frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\,(a+bx^2)}+\int\\ &= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2}-\frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\,(a+bx^2)}+(\\ &= -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2}-\frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\,(a+bx^2)}+(\end{aligned}$$

Mathematica [A]
time = 0.07, size = 122, normalized size = 0.90

$$\frac{\frac{-2a^2D-2Ab^2x+2ab(B+Cx)}{(a+bx^2)^2}+\frac{8a^2D+Ab^2x-ab(4B+5Cx)}{a(a+bx^2)}+\frac{\sqrt{b}\,(Ab+3aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}+4D\log\left(a+bx^2\right)}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] ((-2*a^2*D - 2*A*b^2*x + 2*a*b*(B + C*x))/(a + b*x^2)^2 + (8*a^2*D + A*b^2*
x - a*b*(4*B + 5*C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b + 3*a*C)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/a^(3/2) + 4*D*Log[a + b*x^2])/(8*b^3)
```

Maple [A]
time = 0.11, size = 123, normalized size = 0.90

method	result	size

default	$\frac{\frac{(Ab-5aC)x^3}{8ab}-\frac{(Bb-2aD)x^2}{2b^2}-\frac{(Ab+3aC)x}{8b^2}-\frac{a(Bb-3aD)}{4b^3}}{(b\,x^2+a)^2}+\frac{\frac{4aD\ln\left(b\,x^2+a\right)}{b}+\frac{(Ab+3aC)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{8a\,b^2}$	123

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(A*b-5*C*a)/a/b*x^3-1/2*(B*b-2*D*a)/b^2*x^2-1/8*(A*b+3*C*a)/b^2*x-1/4*
a*(B*b-3*D*a)/b^3)/(b*x^2+a)^2+1/8/a/b^2*(4*a*D/b*ln(b*x^2+a)+(A*b+3*C*a)/(
a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.53, size = 146, normalized size = 1.07

$$\frac{6\,Da^3-2\,Ba^2b-(5\,Cab^2-Ab^3)x^3+4\,(2\,Da^2b-Bab^2)x^2-(3\,Ca^2b+Aab^2)x}{8\,(ab^5x^4+2\,a^2b^4x^2+a^3b^3)}+\frac{D\log\left(bx^2+a\right)}{2\,b^3}+\frac{(3\,Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(6*D*a^3 - 2*B*a^2*b - (5*C*a*b^2 - A*b^3)*x^3 + 4*(2*D*a^2*b - B*a*b^2
)*x^2 - (3*C*a^2*b + A*a*b^2)*x)/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3) + 1/
2*D*log(b*x^2 + a)/b^3 + 1/8*(3*C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a*b^2)
```

Fricas [A]
time = 4.87, size = 447, normalized size = 3.29

$$\left[\frac{12\,Da^4-4\,Ba^3b-2\,(5\,Ca^2b^2-Aab^3)x^3+8\,(2\,Da^3b-Ba^2b^2)x^2-((3\,Cab^2+Ab^3)x^4+3\,Ca^3+Aa^2b+2\,(3\,Ca^2b+Aab^2)x^2)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\,(3\,Ca^3b+Aa^2b^2)x+8\,(Da^2b^2x^4+2\,Da^3bx^2+Da^4)\log\left(bx^2+a\right)}{16\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)},\ \frac{6\,Da^4-2\,Ba^3b-(5\,Ca^2b^2-Aab^3)x^3+4\,(2\,Da^3b-Ba^2b^2)x^2+((3\,Cab^2+Ab^3)x^4+3\,Ca^3+Aa^2b+2\,(3\,Ca^2b+Aab^2)x^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-(3\,Ca^3b+Aa^2b^2)x+4\,(Da^2b^2x^4+2\,Da^3bx^2+Da^4)\log\left(bx^2+a\right)}{8\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(12*D*a^4 - 4*B*a^3*b - 2*(5*C*a^2*b^2 - A*a*b^3)*x^3 + 8*(2*D*a^3*b
- B*a^2*b^2)*x^2 - ((3*C*a*b^2 + A*b^3)*x^4 + 3*C*a^3 + A*a^2*b + 2*(3*C*a^
2*b + A*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) - 2*(3*C*a^3*b + A*a^2*b^2)*x + 8*(D*a^2*b^2*x^4 + 2*D*a^3*b*x^2 + D*a^4)
*log(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3), 1/8*(6*D*a^4 - 2*
B*a^3*b - (5*C*a^2*b^2 - A*a*b^3)*x^3 + 4*(2*D*a^3*b - B*a^2*b^2)*x^2 + ((3
*C*a*b^2 + A*b^3)*x^4 + 3*C*a^3 + A*a^2*b + 2*(3*C*a^2*b + A*a*b^2)*x^2)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a) - (3*C*a^3*b + A*a^2*b^2)*x + 4*(D*a^2*b^2*x^
4 + 2*D*a^3*b*x^2 + D*a^4)*log(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a
^4*b^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(119) = 238$.
time = 74.15, size = 304, normalized size = 2.24

$$\left(\frac{D}{2b^3} - \frac{\sqrt{-a^3b^7}\,(Ab+3Ca)}{16a^3b^6}\right)\log\left(x + \frac{-8Da^2 + 16a^2b^3\left(\frac{D}{2b^3} - \frac{\sqrt{-a^3b^7}\,(Ab+3Ca)}{16a^3b^6}\right)}{Ab^2 + 3Cab}\right) + \left(\frac{D}{2b^3} + \frac{\sqrt{-a^3b^7}\,(Ab+3Ca)}{16a^3b^6}\right)\log\left(x + \frac{-8Da^2 + 16a^2b^3\left(\frac{D}{2b^3} + \frac{\sqrt{-a^3b^7}\,(Ab+3Ca)}{16a^3b^6}\right)}{Ab^2 + 3Cab}\right) + \frac{-2Ba^2b + 6Da^3 + x^3(Ab^3 - 5Cab^2) + x^2(-4Bab^2 + 8Da^2b) + x(-Aab^2 - 3Ca^2b)}{8a^3b^3 + 16a^2b^4x^2 + 8ab^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] (D/(2*b**3) - sqrt(-a**3*b**7)*(A*b + 3*C*a)/(16*a**3*b**6))*log(x + (-8*D*
a**2 + 16*a**2*b**3*(D/(2*b**3) - sqrt(-a**3*b**7)*(A*b + 3*C*a)/(16*a**3*b
**6)))/(A*b**2 + 3*C*a*b)) + (D/(2*b**3) + sqrt(-a**3*b**7)*(A*b + 3*C*a)/(
16*a**3*b**6))*log(x + (-8*D*a**2 + 16*a**2*b**3*(D/(2*b**3) + sqrt(-a**3*b
**7)*(A*b + 3*C*a)/(16*a**3*b**6)))/(A*b**2 + 3*C*a*b)) + (-2*B*a**2*b + 6*
D*a**3 + x**3*(A*b**3 - 5*C*a*b**2) + x**2*(-4*B*a*b**2 + 8*D*a**2*b) + x*(
-A*a*b**2 - 3*C*a**2*b))/(8*a**3*b**3 + 16*a**2*b**4*x**2 + 8*a*b**5*x**4)
```

Giac [A]
time = 1.13, size = 128, normalized size = 0.94

$$\frac{D\log\left(bx^2 + a\right)}{2\,b^3} + \frac{(3\,Ca + Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^2} - \frac{(5\,Cab - Ab^2)x^3 - 4\,(2\,Da^2 - Bab)x^2 + (3\,Ca^2 + Aab)x - \frac{2\,(3\,Da^3 - Ba^2b)}{b}}{8\,(bx^2 + a)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*D*log(b*x^2 + a)/b^3 + 1/8*(3*C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*
b)*a*b^2) - 1/8*((5*C*a*b - A*b^2)*x^3 - 4*(2*D*a^2 - B*a*b)*x^2 + (3*C*a^2
 + A*a*b)*x - 2*(3*D*a^3 - B*a^2*b)/b)/((b*x^2 + a)^2*a*b^2)
```

Mupad [B]
time = 1.39, size = 195, normalized size = 1.43

$$\frac{\frac{A\,x^3}{8\,a} - \frac{A\,x}{8\,b}}{a^2 + 2\,a\,b\,x^2 + b^2\,x^4} - \frac{\frac{B\,x^2}{2\,b} + \frac{B\,a}{4\,b^2}}{a^2 + 2\,a\,b\,x^2 + b^2\,x^4} - \frac{\frac{5\,C\,x^3}{8\,b} + \frac{3\,C\,a\,x}{8\,b^2}}{a^2 + 2\,a\,b\,x^2 + b^2\,x^4} + \frac{D\left(\ln\left(b\,x^2 + a\right) + \frac{2\,a}{b\,x^2 + a} - \frac{a^2}{2\,\left(b\,x^2 + a\right)^2}\right)}{2\,b^3} + \frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}} + \frac{3\,C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

```
[Out] ((A*x^3)/(8*a) - (A*x)/(8*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) - ((B*x^2)/(2*b)
+ (B*a)/(4*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - ((5*C*x^3)/(8*b) + (3*C*a*x)
/(8*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (D*(log(a + b*x^2) + (2*a)/(a + b*x
^2) - a^2/(2*(a + b*x^2)^2)))/(2*b^3) + (A*atan((b^(1/2)*x)/a^(1/2)))/(8*a^
(3/2)*b^(3/2)) + (3*C*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*b^(5/2))
```

3.105 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx$

Optimal. Leaf size=119

$$-\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\left(a+bx^2\right)^2}-\frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\left(a+bx^2\right)}+\frac{(bB+3aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}$$

```
[Out] -1/4*x*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)^2+1/8*(-2*A*b-2*a*C+(B*b-5*D
*a)*x)/a/b^2/(b*x^2+a)+1/8*(B*b+3*D*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^
(5/2)
```

Rubi [A]

time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1818, 1828, 12, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(3aD+bB)}{8a^{3/2}b^{5/2}}-\frac{2(aC+Ab)-x(bB-5aD)}{8ab^2\left(a+bx^2\right)}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] -1/4*(x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)^2) - (2*(A*b +
a*C) - (b*B - 5*a*D)*x)/(8*a*b^2*(a + b*x^2)) + ((b*B + 3*a*D)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
```

```
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx &= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2}-\frac{\int \frac{-a\left(B-\frac{aD}{b}\right)-2(Ab+aC)x-4aDx^2}{(a+bx^2)^2}\,dx}{4ab}\\
&= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2}-\frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\,(a+bx^2)}+\\
&= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2}-\frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\,(a+bx^2)}+\\
&= -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2}-\frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\,(a+bx^2)}+
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 99, normalized size = 0.83

$$\frac{\frac{\sqrt{b}\,(b^2Bx^3-a^2(2C+3Dx)-ab(2A+x(B+4Cx+5Dx^2)))}{a(a+bx^2)^2}+\frac{(bB+3aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[b]*(b^2*B*x^3 - a^2*(2*C + 3*D*x) - a*b*(2*A + x*(B + 4*C*x + 5*D*x^
2))))/(a*(a + b*x^2)^2) + ((b*B + 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/
2))/(8*b^(5/2))
```

Maple [A]
time = 0.10, size = 97, normalized size = 0.82

method	result	size
default	$\frac{\frac{(Bb-5aD)x^{3}}{8ab}-\frac{C\,x^{2}}{2b}-\frac{(Bb+3aD)x}{8b^{2}}-\frac{Ab+aC}{4b^{2}}}{\left(b\,x^{2}+a\right)^{2}}+\frac{(Bb+3aD)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8b^{2}a\sqrt{ab}}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(B*b-5*D*a)/a/b*x^3-1/2*C*x^2/b-1/8*(B*b+3*D*a)/b^2*x-1/4*(A*b+C*a)/b^
2)/(b*x^2+a)^2+1/8*(B*b+3*D*a)/b^2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.54, size = 111, normalized size = 0.93

$$-\frac{4\,Cabx^{2}+\left(5\,Dab-Bb^{2}\right)x^{3}+2\,Ca^{2}+2\,Aab+\left(3\,Da^{2}+Bab\right)x}{8\,\left(ab^{4}x^{4}+2\,a^{2}b^{3}x^{2}+a^{3}b^{2}\right)}+\frac{\left(3\,Da+Bb\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*C*a*b*x^2 + (5*D*a*b - B*b^2)*x^3 + 2*C*a^2 + 2*A*a*b + (3*D*a^2 +
B*a*b)*x)/(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2) + 1/8*(3*D*a + B*b)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

Fricas [A]
time = 2.52, size = 357, normalized size = 3.00

$$\left[-\frac{8\,Ca^{2}b^{2}x^{2}+4\,Ca^{3}b+4\,Aa^{2}b^{2}+2\,\left(5\,Da^{2}b^{2}-Bab^{3}\right)x^{3}+\left(\left(3\,Dab^{2}+Bb^{3}\right)x^{4}+3\,Da^{3}+Ba^{2}b+2\,\left(3\,Da^{2}b+Bab^{2}\right)x^{2}\right)\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)+2\,\left(3\,Da^{3}b+Ba^{2}b^{2}\right)x}{16\,\left(a^{2}b^{5}x^{4}+2\,a^{3}b^{4}x^{2}+a^{4}b^{3}\right)},-\frac{4\,Ca^{2}b^{2}x^{2}+2\,Ca^{3}b+2\,Aa^{2}b^{2}+\left(5\,Da^{2}b^{2}-Bab^{3}\right)x^{3}-\left(\left(3\,Dab^{2}+Bb^{3}\right)x^{4}+3\,Da^{3}+Ba^{2}b+2\,\left(3\,Da^{2}b+Bab^{2}\right)x^{2}\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(3\,Da^{3}b+Ba^{2}b^{2}\right)x}{8\,\left(a^{2}b^{5}x^{4}+2\,a^{3}b^{4}x^{2}+a^{4}b^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(8*C*a^2*b^2*x^2 + 4*C*a^3*b + 4*A*a^2*b^2 + 2*(5*D*a^2*b^2 - B*a*b^
3)*x^3 + ((3*D*a*b^2 + B*b^3)*x^4 + 3*D*a^3 + B*a^2*b + 2*(3*D*a^2*b + B*a*
b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(3*D
*a^3*b + B*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3), -1/8*(4*C*a
^2*b^2*x^2 + 2*C*a^3*b + 2*A*a^2*b^2 + (5*D*a^2*b^2 - B*a*b^3)*x^3 - ((3*D*
a*b^2 + B*b^3)*x^4 + 3*D*a^3 + B*a^2*b + 2*(3*D*a^2*b + B*a*b^2)*x^2)*sqrt(
a*b)*arctan(sqrt(a*b)*x/a) + (3*D*a^3*b + B*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a^
3*b^4*x^2 + a^4*b^3)]
```

Sympy [A]
time = 8.49, size = 178, normalized size = 1.50

$$-\frac{\sqrt{-\frac{1}{a^{3}b^{5}}}\,\left(Bb+3Da\right)\log\left(-a^{2}b^{2}\sqrt{-\frac{1}{a^{3}b^{5}}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^{3}b^{5}}}\,\left(Bb+3Da\right)\log\left(a^{2}b^{2}\sqrt{-\frac{1}{a^{3}b^{5}}}+x\right)}{16}+\frac{-2Aab-2Ca^{2}-4Cabx^{2}+x^{3}\left(Bb^{2}-5Dab\right)+x\left(-Bab-3Da^{2}\right)}{8a^{3}b^{2}+16a^{2}b^{3}x^{2}+8ab^{4}x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**3*b**5))*(B*b + 3*D*a)*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x
)/16 + sqrt(-1/(a**3*b**5))*(B*b + 3*D*a)*log(a**2*b**2*sqrt(-1/(a**3*b**5)
) + x)/16 + (-2*A*a*b - 2*C*a**2 - 4*C*a*b*x**2 + x**3*(B*b**2 - 5*D*a*b) +
 x*(-B*a*b - 3*D*a**2))/(8*a**3*b**2 + 16*a**2*b**3*x**2 + 8*a*b**4*x**4)
```

Giac [**A**]
time = 1.88, size = 97, normalized size = 0.82

$$\frac{(3\,Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^2}-\frac{5\,Dabx^3-Bb^2x^3+4\,Cabx^2+3\,Da^2x+Babx+2\,Ca^2+2\,Aab}{8\,(bx^2+a)^2ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*D*a + B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/8*(5*D*a*b*x^
3 - B*b^2*x^3 + 4*C*a*b*x^2 + 3*D*a^2*x + B*a*b*x + 2*C*a^2 + 2*A*a*b)/((b*
x^2 + a)^2*a*b^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

```
[Out] int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3, x)
```

3.106 $\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^3} \, dx$

Optimal. Leaf size=116

$$\frac{-a\left(B-\frac{aD}{b}\right)+(Ab-aC)x}{4ab\left(a+bx^2\right)^2}-\frac{4a^2D-b(3Ab+aC)x}{8a^2b^2\left(a+bx^2\right)}+\frac{(3Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

```
[Out] 1/4*(-a*(B-a*D/b)+(A*b-C*a)*x)/a/b/(b*x^2+a)^2+1/8*(-4*a^2*D+b*(3*A*b+C*a)*
x)/a^2/b^2/(b*x^2+a)+1/8*(3*A*b+C*a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(3
/2)
```

Rubi [A]

time = 0.05, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1828, 653, 211}

$$\frac{(aC+3Ab)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}-\frac{4a^2D-bx(aC+3Ab)}{8a^2b^2\left(a+bx^2\right)}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

```
[Out] -1/4*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)^2) - (4*a^2*D - b*(
3*A*b + a*C)*x)/(8*a^2*b^2*(a + b*x^2)) + ((3*A*b + a*C)*ArcTan[(Sqrt[b]*x)
/Sqrt[a]])/(8*a^(5/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
```

```
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^3}\,dx &= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{4ab\left(a+bx^2\right)^2}-\frac{\int \frac{-3A-\frac{aC}{b}-\frac{4aDx}{b}}{\left(a+bx^2\right)^2}\,dx}{4a}\\
&= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{4ab\left(a+bx^2\right)^2}-\frac{4a^2D-b(3Ab+aC)x}{8a^2b^2\left(a+bx^2\right)}+\frac{(3Ab+aC)\int \frac{1}{a+bx}}{8a^2b}\\
&= -\frac{a\left(B-\frac{aD}{b}\right)-(Ab-aC)x}{4ab\left(a+bx^2\right)^2}-\frac{4a^2D-b(3Ab+aC)x}{8a^2b^2\left(a+bx^2\right)}+\frac{(3Ab+aC)\tan^{-1}}{8a^{5/2}b^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 104, normalized size = 0.90

$$
\frac{\frac{\sqrt{a}\left(-2a^3D+3Ab^3x^3+ab^2x\left(5A+Cx^2\right)-a^2b(2B+x(C+4Dx))\right)}{\left(a+bx^2\right)^2}+\sqrt{b}\,(3Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

```
[Out] ((Sqrt[a]*(-2*a^3*D + 3*A*b^3*x^3 + a*b^2*x*(5*A + C*x^2) - a^2*b*(2*B + x*
(C + 4*D*x))))/(a + b*x^2)^2 + Sqrt[b]*(3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(8*a^(5/2)*b^2)
```

Maple [A]
time = 0.14, size = 98, normalized size = 0.84

method	result	size
default	$\frac{\frac{(3Ab+aC)x^3}{8a^2}-\frac{Dx^2}{2b}+\frac{(5Ab-aC)x}{8ab}-\frac{Bb+aD}{4b^2}}{\left(b\,x^2+a\right)^2}+\frac{(3Ab+aC)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2b\sqrt{ab}}$	98

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*A*b+C*a)/a^2*x^3-1/2*D*x^2/b+1/8*(5*A*b-C*a)/a/b*x-1/4*(B*b+D*a)/b^
2)/(b*x^2+a)^2+1/8*(3*A*b+C*a)/a^2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.52, size = 122, normalized size = 1.05

$$-\frac{4\,Da^2bx^2+2\,Da^3+2\,Ba^2b-(Cab^2+3\,Ab^3)x^3+(Ca^2b-5\,Aab^2)x}{8\,(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2)}+\frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(4*D*a^2*b*x^2 + 2*D*a^3 + 2*B*a^2*b - (C*a*b^2 + 3*A*b^3)*x^3 + (C*a^
2*b - 5*A*a*b^2)*x)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2) + 1/8*(C*a + 3*
A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

Fricas [A]
time = 5.31, size = 346, normalized size = 2.98

$$\left[-\frac{8\,Da^3bx^2+4\,Da^4+4\,Ba^3b-2\,(Ca^2b^2+3\,Aab^3)x^3+((Cab^2+3\,Ab^3)x^4+Ca^3+3\,Aa^2b+2\,(Ca^2b+3\,Aab^2)x^2)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+2\,(Ca^3b-5\,Aa^2b^2)x}{16\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)},-\frac{4\,Da^3bx^2+2\,Da^4+2\,Ba^3b-(Ca^2b^2+3\,Aab^3)x^3-((Cab^2+3\,Ab^3)x^4+Ca^3+3\,Aa^2b+2\,(Ca^2b+3\,Aab^2)x^2)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+(Ca^3b-5\,Aa^2b^2)x}{8\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(8*D*a^3*b*x^2 + 4*D*a^4 + 4*B*a^3*b - 2*(C*a^2*b^2 + 3*A*a*b^3)*x^3
 + ((C*a*b^2 + 3*A*b^3)*x^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a*b^2)*x
^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(C*a^3*b -
 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5*b^2), -1/8*(4*D*a^3*b*x
^2 + 2*D*a^4 + 2*B*a^3*b - (C*a^2*b^2 + 3*A*a*b^3)*x^3 - ((C*a*b^2 + 3*A*b^
3)*x^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a*b^2)*x^2)*sqrt(a*b)*arctan(
sqrt(a*b)*x/a) + (C*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 +
a^5*b^2)]
```

Sympy [A]
time = 3.77, size = 184, normalized size = 1.59

$$-\frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ca)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ca)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}+\frac{-2Ba^2b-2Da^3-4Da^2bx^2+x^3\cdot(3Ab^3+Cab^2)+x(5Aab^2-Ca^2b)}{8a^4b^2+16a^3b^3x^2+8a^2b^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/1
6 + sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)
/16 + (-2*B*a**2*b - 2*D*a**3 - 4*D*a**2*b*x**2 + x**3*(3*A*b**3 + C*a*b**2
) + x*(5*A*a*b**2 - C*a**2*b))/(8*a**4*b**2 + 16*a**3*b**3*x**2 + 8*a**2*b*
*4*x**4)
```

Giac [A]
time = 1.60, size = 106, normalized size = 0.91

$$\frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b}+\frac{Cab^2x^3+3\,Ab^3x^3-4\,Da^2bx^2-Ca^2bx+5\,Aab^2x-2\,Da^3-2\,Ba^2b}{8\,(bx^2+a)^2a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(C*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(C*a*b^2*x^
3 + 3*A*b^3*x^3 - 4*D*a^2*b*x^2 - C*a^2*b*x + 5*A*a*b^2*x - 2*D*a^3 - 2*B*a
^2*b)/((b*x^2 + a)^2*a^2*b^2)
```

Mupad [B]
time = 1.33, size = 163, normalized size = 1.41

$$\frac{\frac{C\,x^3}{8\,a}-\frac{C\,x}{8\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\frac{5\,A\,x}{8\,a}+\frac{3\,A\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{B}{4\,b\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}-\frac{\left(2\,b\,x^2+a\right)\,D}{4\,b^2\,{\left(b\,x^2+a\right)}^2}+\frac{3\,A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}+\frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^3,x)
```

```
[Out] ((C*x^3)/(8*a) - (C*x)/(8*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((5*A*x)/(8*a)
+ (3*A*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - B/(4*b*(a^2 + b^2*x^4
+ 2*a*b*x^2)) - ((a + 2*b*x^2)*D)/(4*b^2*(a + b*x^2)^2) + (3*A*atan((b^(1/2
)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2)) + (C*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(3/
2)*b^(3/2))
```

3.107 $\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=130

$$\frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2}+\frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)}+\frac{(3bB+aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{A\log(x)}{a^3}-\frac{A\log\left(a+bx^2\right)}{2a^3}$$

```
[Out] 1/4*(A*b-a*C+(B*b-D*a)*x)/a/b/(b*x^2+a)^2+1/8*(4*A*b+(3*B*b+D*a)*x)/a^2/b/(
b*x^2+a)+1/8*(3*B*b+D*a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)+A*ln(x)/
a^3-1/2*A*ln(b*x^2+a)/a^3
```

Rubi [A]
time = 0.09, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1819, 837, 815, 649, 211, 266}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(aD+3bB)}{8a^{5/2}b^{3/2}}-\frac{A\log\left(a+bx^2\right)}{2a^3}+\frac{A\log(x)}{a^3}+\frac{x(aD+3bB)+4Ab}{8a^2b\left(a+bx^2\right)}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3),x]
```

```
[Out] (A*b - a*C + (b*B - a*D)*x)/(4*a*b*(a + b*x^2)^2) + (4*A*b + (3*b*B + a*D)*
x)/(8*a^2*b*(a + b*x^2)) + ((3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(5/2)*b^(3/2)) + (A*Log[x])/a^3 - (A*Log[a + b*x^2])/(2*a^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx &= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2}-\frac{\int \frac{-4A-\frac{(3bB+aD)x}{b}}{x\left(a+bx^2\right)^2}\,dx}{4a}\\
&= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2}+\frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)}+\frac{\int \frac{8aAb+a(3bB+aD)x}{x\left(a+bx^2\right)}\,dx}{8a^3b}\\
&= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2}+\frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)}+\frac{\int \left(\frac{8Ab}{x}+\frac{3abB+a^2D-8Ab^2x}{a+bx^2}\right.}{8a^3b}\\
&= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2}+\frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)}+\frac{A\log(x)}{a^3}+\frac{\int \frac{3abB+a^2D-8}{a+bx^2}}{8a^3b}\\
&= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2}+\frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)}+\frac{A\log(x)}{a^3}-\frac{(Ab)\int \frac{x}{a+bx^2}}{a^3}\\
&= \frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2}+\frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)}+\frac{(3bB+aD)\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.}{8a^{5/2}b^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 117, normalized size = 0.90

$$\frac{\frac{a(4Ab+3bBx+aDx)}{b(a+bx^2)}+\frac{2a^2(Ab+bBx-a(C+Dx))}{b(a+bx^2)^2}+\frac{\sqrt{a}\,(3bB+aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}+8A\log(x)-4A\log\left(a+bx^2\right)}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3),x]
```

```
[Out] ((a*(4*A*b + 3*b*B*x + a*D*x))/(b*(a + b*x^2)) + (2*a^2*(A*b + b*B*x - a*(C
 + D*x)))/(b*(a + b*x^2)^2) + (Sqrt[a]*(3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/b^(3/2) + 8*A*Log[x] - 4*A*Log[a + b*x^2])/(8*a^3)
```

Maple [A]
time = 0.10, size = 130, normalized size = 1.00

method	result	size
default	$-\frac{\frac{\left(-\frac{3}{8}abB-\frac{1}{8}a^2D\right)x^3-\frac{aAb\,x^2}{2}-\frac{a^2(5Bb-aD)x}{8b}-\frac{a^2(3Ab-aC)}{4b}}{\left(b\,x^2+a\right)^2}+\frac{4bA\ln\left(b\,x^2+a\right)+\frac{\left(-3abB-a^2D\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{8b}}{a^3}+\frac{A\ln(x)}{a^3}$	130

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*(((-3/8*a*b*B-1/8*a^2*D)*x^3-1/2*a*A*b*x^2-1/8*a^2*(5*B*b-D*a)/b*x-1
/4*a^2*(3*A*b-C*a)/b)/(b*x^2+a)^2+1/8/b*(4*b*A*ln(b*x^2+a)+(-3*B*a*b-D*a^2)
/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))+A*ln(x)/a^3
```

Maxima [A]
time = 0.50, size = 133, normalized size = 1.02

$$\frac{4\,Ab^2x^2+(Dab+3\,Bb^2)x^3-2\,Ca^2+6\,Aab-(Da^2-5\,Bab)x}{8\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)}-\frac{A\log\left(bx^2+a\right)}{2\,a^3}+\frac{A\log\left(x\right)}{a^3}+\frac{(Da+3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*A*b^2*x^2 + (D*a*b + 3*B*b^2)*x^3 - 2*C*a^2 + 6*A*a*b - (D*a^2 - 5*B
*a*b)*x)/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b) - 1/2*A*log(b*x^2 + a)/a^3 +
 A*log(x)/a^3 + 1/8*(D*a + 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(114) = 228.
time = 3.16, size = 488, normalized size = 3.75

$$\left[\frac{8\,Aab^3x^2 - 4\,Ca^3b + 12\,Aa^2b^2 + 2\,(Da^2b^2 + 3\,Bab^3)x^3 - ((Dab^2 + 3\,Bb^3)x^4 + Da^3 + 3\,Ba^2b + 2\,(Da^2b + 3\,Bab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2 - 2\sqrt{-ab}\,x - a}{bx^2 + a}\right) - 2\,(Da^3b - 5\,Ba^2b^2)x - 8\,(Ab^4x^4 + 2\,Aab^3x^2 + Aa^2b^2)\log\left(bx^2 + a\right) + 16\,(Ab^4x^4 + 2\,Aab^3x^2 + Aa^2b^2)\log\left(x\right)}{16\,(a^3b^4x^4 + 2\,a^4b^3x^2 + a^5b^2)}, \frac{4\,Aab^3x^2 - 2\,Ca^3b + 6\,Aa^2b^2 + (Da^2b^2 + 3\,Bab^3)x^3 + ((Dab^2 + 3\,Bb^3)x^4 + Da^3 + 3\,Ba^2b + 2\,(Da^2b + 3\,Bab^2)x^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) - (Da^3b - 5\,Ba^2b^2)x - 4\,(Ab^4x^4 + 2\,Aab^3x^2 + Aa^2b^2)\log\left(bx^2 + a\right) + 8\,(Ab^4x^4 + 2\,Aab^3x^2 + Aa^2b^2)\log\left(x\right)}{8\,(a^3b^4x^4 + 2\,a^4b^3x^2 + a^5b^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*A*a*b^3*x^2 - 4*C*a^3*b + 12*A*a^2*b^2 + 2*(D*a^2*b^2 + 3*B*a*b^3)
*x^3 - ((D*a*b^2 + 3*B*b^3)*x^4 + D*a^3 + 3*B*a^2*b + 2*(D*a^2*b + 3*B*a*b^
2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(D*a^3
*b - 5*B*a^2*b^2)*x - 8*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(b*x^2 +
 a) + 16*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(x))/(a^3*b^4*x^4 + 2*a
^4*b^3*x^2 + a^5*b^2), 1/8*(4*A*a*b^3*x^2 - 2*C*a^3*b + 6*A*a^2*b^2 + (D*a^
2*b^2 + 3*B*a*b^3)*x^3 + ((D*a*b^2 + 3*B*b^3)*x^4 + D*a^3 + 3*B*a^2*b + 2*(
D*a^2*b + 3*B*a*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (D*a^3*b - 5*B*
a^2*b^2)*x - 4*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(b*x^2 + a) + 8*(
A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(x))/(a^3*b^4*x^4 + 2*a^4*b^3*x^2
 + a^5*b^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.05, size = 128, normalized size = 0.98

$$-\frac{A\log\left(bx^2 + a\right)}{2\,a^3} + \frac{A\log\left(|x|\right)}{a^3} + \frac{(Da + 3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b} + \frac{4\,Aab^2x^2 - 2\,Ca^3 + 6\,Aa^2b + (Da^2b + 3\,Bab^2)x^3 - (Da^3 - 5\,Ba^2b)x}{8\,(bx^2 + a)^2a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*A*log(b*x^2 + a)/a^3 + A*log(abs(x))/a^3 + 1/8*(D*a + 3*B*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(4*A*a*b^2*x^2 - 2*C*a^3 + 6*A*a^2*b +
 (D*a^2*b + 3*B*a*b^2)*x^3 - (D*a^3 - 5*B*a^2*b)*x)/((b*x^2 + a)^2*a^3*b)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,x + C\,x^2 + x^3\,D}{x\,{\left(b\,x^2 + a\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^3),x)
[Out] int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^3), x)
```

3.108 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=144

$$-\frac{A}{a^3x}+\frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2}+\frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)}-\frac{3(5Ab-aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}+\frac{B\log(x)}{a^3}-\frac{B\log(a+}{2a^3}$$

```
[Out] -A/a^3/x+1/4*(b*B-a*D-b*(A*b/a-C)*x)/a/b/(b*x^2+a)^2+1/8*(4*B-(7*A*b/a-3*C)
*x)/a^2/(b*x^2+a)+B*ln(x)/a^3-1/2*B*ln(b*x^2+a)/a^3-3/8*(5*A*b-C*a)*arctan(
x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1819, 1816, 649, 211, 266}

$$-\frac{3(5Ab-aC)\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-\frac{A}{a^3x}-\frac{B\log\left(a+bx^2\right)}{2a^3}+\frac{B\log(x)}{a^3}+\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{8a^2\left(a+bx^2\right)}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) + (b*B - a*D - b*((A*b)/a - C)*x)/(4*a*b*(a + b*x^2)^2) + (4*B
 - ((7*A*b)/a - 3*C)*x)/(8*a^2*(a + b*x^2)) - (3*(5*A*b - a*C)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) + (B*Log[x])/a^3 - (B*Log[a + b*x^2])/
(2*a^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx = \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} - \frac{\int \frac{-4A-4Bx+3\left(\frac{Ab}{a}-C\right)x^2}{x^2\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} + \frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)} + \frac{\int \frac{8A+8Bx-\left(\frac{7Ab}{a}-3C\right)x^2}{x^2\left(a+bx^2\right)}\,dx}{8a^2}$$

$$= \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} + \frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)} + \frac{\int \left(\frac{8A}{ax^2}+\frac{8B}{ax}+\frac{-15Ab+3aC-}{a\left(a+bx^2\right)}\right.}{8a^2}$$

$$= -\frac{A}{a^3x} + \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} + \frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)} + \frac{B\log(x)}{a^3} + \frac{\int \frac{-15}{}}{}$$

$$= -\frac{A}{a^3x} + \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} + \frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)} + \frac{B\log(x)}{a^3} - \frac{(bB)}{}$$

$$= -\frac{A}{a^3x} + \frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab\left(a+bx^2\right)^2} + \frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2\left(a+bx^2\right)} - \frac{3(5Ab-aC)\tan^{}}{8a^{7/2}\sqrt{b}}$$

Mathematica [A]

time = 0.07, size = 141, normalized size = 0.98

$$-\frac{A}{a^3x} + \frac{abB-a^2D-Ab^2x+abCx}{4a^2b\left(a+bx^2\right)^2} + \frac{4aB-7Abx+3aCx}{8a^3\left(a+bx^2\right)} + \frac{3(-5Ab+aC)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}} + \frac{B\log(x)}{a^3} - \frac{B\log\left(a+bx^2\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(A/(a^3*x)) + (a*b*B - a^2*D - A*b^2*x + a*b*C*x)/(4*a^2*b*(a + b*x^2)^2)
+ (4*a*B - 7*A*b*x + 3*a*C*x)/(8*a^3*(a + b*x^2)) + (3*(-5*A*b + a*C)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) + (B*Log[x])/a^3 - (B*Log[a + b
*x^2])/(2*a^3)
```

Maple [A]
time = 0.11, size = 125, normalized size = 0.87

method	result	size
default	$-\frac{\frac{\left(\frac{7}{8}b^{2}A-\frac{3}{8}abC\right)x^{3}-\frac{Bab\,x^{2}}{2}+\frac{a(9Ab-5aC)x}{8}-\frac{a^{2}(3Bb-aD)}{4b}}{\left(b\,x^{2}+a\right)^{2}}+\frac{B\ln\left(b\,x^{2}+a\right)}{2}+\frac{(15Ab-3aC)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{a^{3}}-\frac{A}{a^{3}x}+\frac{B\ln(x)}{a^{3}}$	125

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*((((7/8*b^2*A-3/8*a*b*C)*x^3-1/2*B*a*b*x^2+1/8*a*(9*A*b-5*C*a)*x-1/4*
a^2*(3*B*b-D*a)/b)/(b*x^2+a)^2+1/2*B*ln(b*x^2+a)+1/8*(15*A*b-3*C*a)/(a*b)^(
1/2)*arctan(b*x/(a*b)^(1/2)))-A/a^3/x+B*ln(x)/a^3
```

Maxima [A]
time = 0.51, size = 152, normalized size = 1.06

$$\frac{4\,Bab^{2}x^{3}+3\,(Cab^{2}-5\,Ab^{3})x^{4}-8\,Aa^{2}b+5\,(Ca^{2}b-5\,Aab^{2})x^{2}-2\,(Da^{3}-3\,Ba^{2}b)x}{8\,(a^{3}b^{3}x^{5}+2\,a^{4}b^{2}x^{3}+a^{5}bx)}-\frac{B\log\left(bx^{2}+a\right)}{2\,a^{3}}+\frac{B\log\left(x\right)}{a^{3}}+\frac{3\,(Ca-5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*B*a*b^2*x^3 + 3*(C*a*b^2 - 5*A*b^3)*x^4 - 8*A*a^2*b + 5*(C*a^2*b - 5
*A*a*b^2)*x^2 - 2*(D*a^3 - 3*B*a^2*b)*x)/(a^3*b^3*x^5 + 2*a^4*b^2*x^3 + a^5
*b*x) - 1/2*B*log(b*x^2 + a)/a^3 + B*log(x)/a^3 + 3/8*(C*a - 5*A*b)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(124) = 248$.
time = 5.99, size = 524, normalized size = 3.64

$$\left[\frac{8\,Ba^{2}b^{2}x^{3}-16\,Aa^{3}b+6\,(Ca^{2}b^{2}-5\,Aab^{3})x^{4}+10\,(Ca^{3}b-5\,Aa^{2}b^{2})x^{2}+3\,((Cab^{2}-5\,Ab^{3})x^{5}+2\,(Ca^{2}b-5\,Aab^{2})x^{3}+(Ca^{3}-5\,Aa^{2}b)x)\sqrt{-ab}\,\log\left(\frac{bx^{2}+2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)-4\,(Da^{4}-3\,Ba^{3}b)x-8\,(Bab^{3}x^{5}+2\,Ba^{2}b^{2}x^{3}+Ba^{3}bx)\log\left(bx^{2}+a\right)+16\,(Bab^{3}x^{5}+2\,Ba^{2}b^{2}x^{3}+Ba^{3}bx)\log\left(x\right)}{16\,(a^{4}b^{3}x^{5}+2\,a^{5}b^{2}x^{3}+a^{6}bx)},\ \frac{4\,Ba^{2}b^{2}x^{3}-8\,Aa^{3}b+3\,(Ca^{2}b^{2}-5\,Aab^{3})x^{4}+5\,(Ca^{3}b-5\,Aa^{2}b^{2})x^{2}+3\,((Cab^{2}-5\,Ab^{3})x^{5}+2\,(Ca^{2}b-5\,Aab^{2})x^{3}+(Ca^{3}-5\,Aa^{2}b)x)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-2\,(Da^{4}-3\,Ba^{3}b)x-4\,(Bab^{3}x^{5}+2\,Ba^{2}b^{2}x^{3}+Ba^{3}bx)\log\left(bx^{2}+a\right)+8\,(Bab^{3}x^{5}+2\,Ba^{2}b^{2}x^{3}+Ba^{3}bx)\log\left(x\right)}{8\,(a^{4}b^{3}x^{5}+2\,a^{5}b^{2}x^{3}+a^{6}bx)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*B*a^2*b^2*x^3 - 16*A*a^3*b + 6*(C*a^2*b^2 - 5*A*a*b^3)*x^4 + 10*(C
*a^3*b - 5*A*a^2*b^2)*x^2 + 3*((C*a*b^2 - 5*A*b^3)*x^5 + 2*(C*a^2*b - 5*A*a
```

```
*b^2)*x^3 + (C*a^3 - 5*A*a^2*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x -
 a)/(b*x^2 + a)) - 4*(D*a^4 - 3*B*a^3*b)*x - 8*(B*a*b^3*x^5 + 2*B*a^2*b^2*x
^3 + B*a^3*b*x)*log(b*x^2 + a) + 16*(B*a*b^3*x^5 + 2*B*a^2*b^2*x^3 + B*a^3*
b*x)*log(x))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x), 1/8*(4*B*a^2*b^2*x^3
- 8*A*a^3*b + 3*(C*a^2*b^2 - 5*A*a*b^3)*x^4 + 5*(C*a^3*b - 5*A*a^2*b^2)*x^2
 + 3*((C*a*b^2 - 5*A*b^3)*x^5 + 2*(C*a^2*b - 5*A*a*b^2)*x^3 + (C*a^3 - 5*A*
a^2*b)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 2*(D*a^4 - 3*B*a^3*b)*x - 4*(B*
a*b^3*x^5 + 2*B*a^2*b^2*x^3 + B*a^3*b*x)*log(b*x^2 + a) + 8*(B*a*b^3*x^5 +
2*B*a^2*b^2*x^3 + B*a^3*b*x)*log(x))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x
)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.69, size = 141, normalized size = 0.98

$$-\frac{B\log\left(bx^{2}+a\right)}{2\,a^{3}}+\frac{B\log\left(|x|\right)}{a^{3}}+\frac{3\,(Ca-5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{3}}+\frac{4\,Bab^{2}x^{3}+3\,(Cab^{2}-5\,Ab^{3})x^{4}-8\,Aa^{2}b+5\,(Ca^{2}b-5\,Aab^{2})x^{2}-2\,(Da^{3}-3\,Ba^{2}b)x}{8\,\left(bx^{2}+a\right)^{2}a^{3}bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*B*log(b*x^2 + a)/a^3 + B*log(abs(x))/a^3 + 3/8*(C*a - 5*A*b)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/8*(4*B*a*b^2*x^3 + 3*(C*a*b^2 - 5*A*b^3)*x
^4 - 8*A*a^2*b + 5*(C*a^2*b - 5*A*a*b^2)*x^2 - 2*(D*a^3 - 3*B*a^2*b)*x)/((b
*x^2 + a)^2*a^3*b*x)
```

Mupad [**B**]
time = 1.40, size = 202, normalized size = 1.40

$$\frac{\frac{3\,B}{4\,a}+\frac{B\,b\,x^{2}}{2\,a^{2}}}{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}+\frac{\frac{5\,C\,x}{8\,a}+\frac{3\,C\,b\,x^{3}}{8\,a^{2}}}{a^{2}+2\,a\,b\,x^{2}+b^{2}\,x^{4}}-\frac{\frac{A}{a}+\frac{25\,A\,b\,x^{2}}{8\,a^{2}}+\frac{15\,A\,b^{2}\,x^{4}}{8\,a^{3}}}{a^{2}\,x+2\,a\,b\,x^{3}+b^{2}\,x^{5}}-\frac{D}{4\,b\,\left(b\,x^{2}+a\right)^{2}}-\frac{B\,\ln\left(b\,x^{2}+a\right)}{2\,a^{3}}+\frac{B\,\ln\left(x\right)}{a^{3}}-\frac{15\,A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}}+\frac{3\,C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)^3),x)
```

```
[Out] ((3*B)/(4*a) + (B*b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((5*C*x)/(8
*a) + (3*C*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (A/a + (25*A*b*x^2
)/(8*a^2) + (15*A*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5 + 2*a*b*x^3) - D/(4*b*
(a + b*x^2)^2) - (B*log(a + b*x^2))/(2*a^3) + (B*log(x))/a^3 - (15*A*b^(1/2
)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2)) + (3*C*atan((b^(1/2)*x)/a^(1/2)))/
(8*a^(5/2)*b^(1/2))
```

3.109 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=174

$$-\frac{A}{2a^3x^2}-\frac{B}{a^3x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)}-\frac{3(5bB-aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}-(3A$$

```
[Out] -1/2*A/a^3/x^2-B/a^3/x+1/4*(-A*b/a+C-(b*B/a-D)*x)/a/(b*x^2+a)^2+1/8*(-8*A*b
+4*a*C-(7*B*b-3*D*a)*x)/a^3/(b*x^2+a)-(3*A*b-C*a)*ln(x)/a^4+1/2*(3*A*b-C*a)
*ln(b*x^2+a)/a^4-3/8*(5*B*b-D*a)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1819, 1816, 649, 211, 266}

$$-\frac{3\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(5bB-aD)}{8a^{7/2}\sqrt{b}}+\frac{(3Ab-aC)\log\left(a+bx^2\right)}{2a^4}-\frac{\log(x)(3Ab-aC)}{a^4}-\frac{4(2Ab-aC)+x(7bB-3aD)}{8a^3\left(a+bx^2\right)}-\frac{A}{2a^3x^2}-\frac{B}{a^3x}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{4a\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] -1/2*A/(a^3*x^2) - B/(a^3*x) - ((A*b)/a - C + ((b*B)/a - D)*x)/(4*a*(a + b*
x^2)^2) - (4*(2*A*b - a*C) + (7*b*B - 3*a*D)*x)/(8*a^3*(a + b*x^2)) - (3*(5
*b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) - ((3*A*b - a*
C)*Log[x])/a^4 + ((3*A*b - a*C)*Log[a + b*x^2])/(2*a^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx &= -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{-4A-4Bx+4\left(\frac{Ab}{a}-C\right)x^2+3\left(\frac{bB}{a}-D\right)x^3}{x^3\left(a+bx^2\right)^2}\,dx}{4a}\\
&= -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)}+\frac{\int \frac{8A+8Bx-8\left(\frac{2Ab}{a}-\right.}{x^3(}}{}\\
&= -\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)}+\frac{\int \left(\frac{8A}{ax^3}+\frac{8B}{ax^2}+\right.}{}\\
&= -\frac{A}{2a^3x^2}-\frac{B}{a^3x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)}-\\
&= -\frac{A}{2a^3x^2}-\frac{B}{a^3x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)}-\\
&= -\frac{A}{2a^3x^2}-\frac{B}{a^3x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a\left(a+bx^2\right)^2}-\frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3\left(a+bx^2\right)}-
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 147, normalized size = 0.84

$$
\frac{-\frac{4aA}{x^2}-\frac{8aB}{x}+\frac{a(-8Ab+4aC-7bBx+3aDx)}{a+bx^2}+\frac{2a^2(-Ab-bBx+a(C+Dx))}{\left(a+bx^2\right)^2}+\frac{3\sqrt{a}\,(-5bB+aD)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{b}}+8(-3Ab+aC)\log(x)+4(3Ab-aC)\log\left(a+bx^2\right)}{8a^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3),x]
```

```
[Out] ((-4*a*A)/x^2 - (8*a*B)/x + (a*(-8*A*b + 4*a*C - 7*b*B*x + 3*a*D*x))/(a + b
*x^2) + (2*a^2*(-(A*b) - b*B*x + a*(C + D*x)))/(a + b*x^2)^2 + (3*Sqrt[a]*(
-5*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + 8*(-3*A*b + a*C)*Log[x
] + 4*(3*A*b - a*C)*Log[a + b*x^2])/(8*a^4)
```

Maple [A]
time = 0.10, size = 169, normalized size = 0.97

method	result
default	$\frac{\frac{\left(-\frac{7}{8}a\,b^{2}B+\frac{3}{8}a^{2}bD\right)x^{3}+\left(-Aa\,b^{2}+\frac{1}{2}a^{2}bC\right)x^{2}-\frac{a^{2}(9Bb-5aD)x}{8}-\frac{5A\,a^{2}b}{4}+\frac{3a^{3}C}{4}}{\left(b\,x^{2}+a\right)^{2}}+\frac{\left(24b^{2}A-8abC\right)\ln\left(b\,x^{2}+a\right)}{16b}+\frac{\left(-15abB+3a^{2}D\right)\arctan\left(\frac{b}{\sqrt{a}}\right.}{8\sqrt{ab}}}{a^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(((-7/8*a*b^2*B+3/8*a^2*b*D)*x^3+(-A*a*b^2+1/2*a^2*b*C)*x^2-1/8*a^2*(
9*B*b-5*D*a)*x-5/4*A*a^2*b+3/4*a^3*C)/(b*x^2+a)^2+1/16*(24*A*b^2-8*C*a*b)/b
*ln(b*x^2+a)+1/8*(-15*B*a*b+3*D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1
/2*A/a^3/x^2-B/a^3/x+(-3*A*b+C*a)/a^4*ln(x)
```

Maxima [A]
time = 0.51, size = 172, normalized size = 0.99

$$\frac{3\,(Dab-5\,Bb^{2})x^{5}+4\,(Cab-3\,Ab^{2})x^{4}-8\,Ba^{2}x+5\,(Da^{2}-5\,Bab)x^{3}-4\,Aa^{2}+6\,(Ca^{2}-3\,Aab)x^{2}}{8\,(a^{3}b^{2}x^{6}+2\,a^{4}bx^{4}+a^{5}x^{2})}+\frac{3\,(Da-5\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{3}}-\frac{(Ca-3\,Ab)\log\left(bx^{2}+a\right)}{2\,a^{4}}+\frac{(Ca-3\,Ab)\log\left(x\right)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(D*a*b - 5*B*b^2)*x^5 + 4*(C*a*b - 3*A*b^2)*x^4 - 8*B*a^2*x + 5*(D*a
^2 - 5*B*a*b)*x^3 - 4*A*a^2 + 6*(C*a^2 - 3*A*a*b)*x^2)/(a^3*b^2*x^6 + 2*a^4
*b*x^4 + a^5*x^2) + 3/8*(D*a - 5*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
 - 1/2*(C*a - 3*A*b)*log(b*x^2 + a)/a^4 + (C*a - 3*A*b)*log(x)/a^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(147) = 294.
time = 3.87, size = 696, normalized size = 4.00

$$\left[-\frac{16\,Ba^{3}bx-6\,(Da^{2}b^{2}-5\,Bab^{3})x^{5}+8\,Aa^{3}b-8\,(Ca^{2}b^{2}-3\,Aab^{3})x^{4}-10\,(Da^{3}b-5\,Ba^{2}b^{2})x^{3}-12\,(Ca^{3}b-3\,Aa^{2}b^{2})x^{2}+3\,((Dab^{2}-5\,Bb^{3})x^{6}+2\,(Da^{2}b-5\,Bab^{2})x^{4}+(Da^{3}-5\,Ba^{2}b)x^{2})\sqrt{-ab}\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)+8\,((Cab^{3}-3\,Ab^{4})x^{6}+2\,(Ca^{2}b^{2}-3\,Aab^{3})x^{4}+(Ca^{3}b-3\,Aa^{2}b^{2})x^{2})\log\left(bx^{2}+a\right)-16\,((Cab^{3}-3\,Ab^{4})x^{6}+2\,(Ca^{2}b^{2}-3\,Aab^{3})x^{4}+(Ca^{3}b-3\,Aa^{2}b^{2})x^{2})\log\left(x\right)}{16\,(a^{4}b^{3}x^{6}+2\,a^{5}b^{2}x^{4}+a^{6}bx^{2})},\ -\frac{8\,Ba^{3}bx-3\,(D\ldots}{\ldots}\right]$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*B*a^3*b*x - 6*(D*a^2*b^2 - 5*B*a*b^3)*x^5 + 8*A*a^3*b - 8*(C*a^2
*b^2 - 3*A*a*b^3)*x^4 - 10*(D*a^3*b - 5*B*a^2*b^2)*x^3 - 12*(C*a^3*b - 3*A*
a^2*b^2)*x^2 + 3*((D*a*b^2 - 5*B*b^3)*x^6 + 2*(D*a^2*b - 5*B*a*b^2)*x^4 + (
D*a^3 - 5*B*a^2*b)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2
+ a)) + 8*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C*a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a^3
*b - 3*A*a^2*b^2)*x^2)*log(b*x^2 + a) - 16*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C*
a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a^3*b - 3*A*a^2*b^2)*x^2)*log(x))/(a^4*b^3*x^
6 + 2*a^5*b^2*x^4 + a^6*b*x^2), -1/8*(8*B*a^3*b*x - 3*(D*a^2*b^2 - 5*B*a*b^
3)*x^5 + 4*A*a^3*b - 4*(C*a^2*b^2 - 3*A*a*b^3)*x^4 - 5*(D*a^3*b - 5*B*a^2*b
^2)*x^3 - 6*(C*a^3*b - 3*A*a^2*b^2)*x^2 - 3*((D*a*b^2 - 5*B*b^3)*x^6 + 2*(D
*a^2*b - 5*B*a*b^2)*x^4 + (D*a^3 - 5*B*a^2*b)*x^2)*sqrt(a*b)*arctan(sqrt(a*
b)*x/a) + 4*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C*a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a
^3*b - 3*A*a^2*b^2)*x^2)*log(b*x^2 + a) - 8*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C
*a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a^3*b - 3*A*a^2*b^2)*x^2)*log(x))/(a^4*b^3*x
^6 + 2*a^5*b^2*x^4 + a^6*b*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.13, size = 162, normalized size = 0.93

$$\frac{3\,(Da-5\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3}-\frac{(Ca-3\,Ab)\log\left(bx^2+a\right)}{2\,a^4}+\frac{(Ca-3\,Ab)\log\left(|x|\right)}{a^4}+\frac{3\,Dabx^5-15\,Bb^2x^5+4\,Cabx^4-12\,Ab^2x^4+5\,Da^2x^3-25\,Babx^3+6\,Ca^2x^2-18\,Aabx^2-8\,Ba^2x-4\,Aa^2}{8\,(bx^3+ax)^2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*(D*a - 5*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/2*(C*a - 3*A*b)
*log(b*x^2 + a)/a^4 + (C*a - 3*A*b)*log(abs(x))/a^4 + 1/8*(3*D*a*b*x^5 - 15
*B*b^2*x^5 + 4*C*a*b*x^4 - 12*A*b^2*x^4 + 5*D*a^2*x^3 - 25*B*a*b*x^3 + 6*C*
a^2*x^2 - 18*A*a*b*x^2 - 8*B*a^2*x - 4*A*a^2)/((b*x^3 + a*x)^2*a^3)
```

Mupad [**B**]
time = 1.46, size = 229, normalized size = 1.32

$$\frac{\frac{3\,C}{4\,a}+\frac{C\,b\,x^2}{2\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\frac{A}{2\,a}+\frac{9\,A\,b\,x^2}{4\,a^2}+\frac{3\,A\,b^2\,x^4}{2\,a^3}}{a^2\,x^2+2\,a\,b\,x^4+b^2\,x^6}-\frac{\frac{B}{a}+\frac{25\,B\,b\,x^2}{8\,a^2}+\frac{15\,B\,b^2\,x^4}{8\,a^3}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5}-\frac{C\,\ln\left(b\,x^2+a\right)}{2\,a^3}+\frac{C\,\ln\left(x\right)}{a^3}+\frac{3\,A\,b\,\ln\left(b\,x^2+a\right)}{2\,a^4}-\frac{3\,A\,b\,\ln\left(x\right)}{a^4}+\frac{x\,D\,{}_2\mathrm{F}_1\left(\frac{1}{2},3;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{a^3}-\frac{15\,B\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)^3),x)
```

```
[Out] ((3*C)/(4*a) + (C*b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (A/(2*a) +
(9*A*b*x^2)/(4*a^2) + (3*A*b^2*x^4)/(2*a^3))/(a^2*x^2 + b^2*x^6 + 2*a*b*x^4
) - (B/a + (25*B*b*x^2)/(8*a^2) + (15*B*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5
+ 2*a*b*x^3) - (C*log(a + b*x^2))/(2*a^3) + (C*log(x))/a^3 + (3*A*b*log(a +
 b*x^2))/(2*a^4) - (3*A*b*log(x))/a^4 + (x*D*hypergeom([1/2, 3], 3/2, -(b*x
^2)/a))/a^3 - (15*B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2))
```

3.110 $\int \frac{-x+4x^3}{\left(5+x^2\right)^2}\,dx$

Optimal. Leaf size=20

$$\frac{21}{2\left(5+x^2\right)}+2\log\left(5+x^2\right)$$

```
[Out] 21/2/(x^2+5)+2*ln(x^2+5)
```

Rubi [A]

time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1607, 455, 45}

$$\frac{21}{2\left(x^2+5\right)}+2\log\left(x^2+5\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + 4*x^3)/(5 + x^2)^2,x]
```

```
[Out] 21/(2*(5 + x^2)) + 2*Log[5 + x^2]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{-x+4x^3}{(5+x^2)^2}\,dx &= \int \frac{x(-1+4x^2)}{(5+x^2)^2}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{-1+4x}{(5+x)^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{21}{(5+x)^2}+\frac{4}{5+x}\right)\,dx,x,x^2\right)\\ &= \frac{21}{2\,(5+x^2)}+2\log\left(5+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$\frac{21}{2\,(5+x^2)}+2\log\left(5+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + 4*x^3)/(5 + x^2)^2,x]
```

```
[Out] 21/(2*(5 + x^2)) + 2*Log[5 + x^2]
```

Maple [A]
time = 0.12, size = 19, normalized size = 0.95

method	result	size
default	$\frac{21}{2(x^2+5)}+2\ln\left(x^2+5\right)$	19
norman	$\frac{21}{2(x^2+5)}+2\ln\left(x^2+5\right)$	19
risch	$\frac{21}{2(x^2+5)}+2\ln\left(x^2+5\right)$	19
meijerg	$-\frac{21x^2}{50\left(1+\frac{x^2}{5}\right)}+2\ln\left(1+\frac{x^2}{5}\right)$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-x)/(x^2+5)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 21/2/(x^2+5)+2*ln(x^2+5)
```

Maxima [A]
time = 0.29, size = 18, normalized size = 0.90

$$\frac{21}{2\,(x^2+5)}+2\,\log\left(x^2+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="maxima")
```

```
[Out] 21/2/(x^2 + 5) + 2*log(x^2 + 5)
```

Fricas [**A**]
time = 2.04, size = 24, normalized size = 1.20

$$\frac{4\,(x^2+5)\log{(x^2+5)}+21}{2\,(x^2+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*(x^2 + 5)*log(x^2 + 5) + 21)/(x^2 + 5)
```

Sympy [**A**]
time = 0.03, size = 15, normalized size = 0.75

$$2\log{\left(x^2+5\right)}+\frac{21}{2x^2+10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-x)/(x**2+5)**2,x)
```

```
[Out] 2*log(x**2 + 5) + 21/(2*x**2 + 10)
```

Giac [**A**]
time = 1.61, size = 25, normalized size = 1.25

$$-\frac{4\,x^2-1}{2\,(x^2+5)}+2\,\log\left(x^2+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*x^2 - 1)/(x^2 + 5) + 2*log(x^2 + 5)
```

Mupad [**B**]
time = 0.91, size = 20, normalized size = 1.00

$$2\,\ln\left(x^2+5\right)+\frac{21}{2\,\left(x^2+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 4*x^3)/(x^2 + 5)^2,x)
```

```
[Out] 2*log(x^2 + 5) + 21/(2*(x^2 + 5))
```

3.111 $\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx$

Optimal. Leaf size=23

$$\sqrt{-2+x^2}+\frac{1}{3}\left(-2+x^2\right)^{3/2}$$

```
[Out] 1/3*(x^2-2)^(3/2)+(x^2-2)^(1/2)
```

Rubi [A]

time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1607, 455, 45}

$$\frac{1}{3}\left(x^2-2\right)^{3/2}+\sqrt{x^2-2}$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)/Sqrt[-2 + x^2],x]
```

```
[Out] Sqrt[-2 + x^2] + (-2 + x^2)^(3/2)/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx &= \int \frac{x(-1+x^2)}{\sqrt{-2+x^2}}\,dx\\ &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{-1+x}{\sqrt{-2+x}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\mathrm{Subst}\left(\int \left(\frac{1}{\sqrt{-2+x}}+\sqrt{-2+x}\right)\,dx,x,x^2\right)\\ &= \sqrt{-2+x^2}+\frac{1}{3}\left(-2+x^2\right)^{3/2}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 18, normalized size = 0.78

$$\frac{1}{3}\sqrt{-2+x^2}\,\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)/Sqrt[-2 + x^2],x]
```

```
[Out] (Sqrt[-2 + x^2]*(1 + x^2))/3
```

Maple [**A**]
time = 0.14, size = 23, normalized size = 1.00

method	result
gosper	$\frac{\left(x^2+1\right)\sqrt{x^2-2}}{3}$
risch	$\frac{\left(x^2+1\right)\sqrt{x^2-2}}{3}$
trager	$\left(\frac{x^2}{3}+\frac{1}{3}\right)\sqrt{x^2-2}$
default	$\frac{x^2\sqrt{x^2-2}}{3}+\frac{\sqrt{x^2-2}}{3}$
meijerg	$\frac{\sqrt{2}\,\sqrt{-\mathrm{signum}\left(-1+\frac{x^2}{2}\right)}\,\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\,\left(2x^2+8\right)\sqrt{1-\frac{x^2}{2}}}{6}\right)}{\sqrt{\pi}\,\sqrt{\mathrm{signum}\left(-1+\frac{x^2}{2}\right)}}+\frac{\sqrt{2}\,\sqrt{-\mathrm{signum}\left(-1+\frac{x^2}{2}\right)}\,\left(-2\sqrt{\pi}\right.}{2\sqrt{\pi}\,\sqrt{\mathrm{signum}\left(-1+\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x)/(x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^2*(x^2-2)^(1/2)+1/3*(x^2-2)^(1/2)
```

Maxima [A]
time = 0.29, size = 22, normalized size = 0.96

$$\frac{1}{3}\sqrt{x^2-2}\,x^2+\frac{1}{3}\sqrt{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="maxima")
[Out] 1/3*sqrt(x^2 - 2)*x^2 + 1/3*sqrt(x^2 - 2)
```

Fricas [A]
time = 1.36, size = 14, normalized size = 0.61

$$\frac{1}{3}\left(x^2+1\right)\sqrt{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="fricas")
[Out] 1/3*(x^2 + 1)*sqrt(x^2 - 2)
```

Sympy [A]
time = 0.10, size = 22, normalized size = 0.96

$$\frac{x^2\sqrt{x^2-2}}{3}+\frac{\sqrt{x^2-2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x)/(x**2-2)**(1/2),x)
[Out] x**2*sqrt(x**2 - 2)/3 + sqrt(x**2 - 2)/3
```

Giac [A]
time = 2.83, size = 17, normalized size = 0.74

$$\frac{1}{3}\left(x^2-2\right)^{\frac{3}{2}}+\sqrt{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="giac")
[Out] 1/3*(x^2 - 2)^(3/2) + sqrt(x^2 - 2)
```

Mupad [B]
time = 0.10, size = 14, normalized size = 0.61

$$\frac{\left(x^2+1\right)\,\sqrt{x^2-2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - x^3)/(x^2 - 2)^(1/2),x)
[Out] ((x^2 + 1)*(x^2 - 2)^(1/2))/3
```

3.112 $\int \frac{-x^2+2x^4}{1+2x^2}\,dx$

Optimal. Leaf size=25

$$-x+\frac{x^3}{3}+\frac{\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

```
[Out] -x+1/3*x^3+1/2*arctan(x*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1607, 470, 327, 209}

$$\frac{\text{ArcTan}\left(\sqrt{2}\,x\right)}{\sqrt{2}}+\frac{x^3}{3}-x$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + 2*x^4)/(1 + 2*x^2),x]
```

```
[Out] -x + x^3/3 + ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{-x^2+2x^4}{1+2x^2}\,dx &= \int \frac{x^2(-1+2x^2)}{1+2x^2}\,dx\\ &= \frac{x^3}{3}-2\int \frac{x^2}{1+2x^2}\,dx\\ &= -x+\frac{x^3}{3}+\int \frac{1}{1+2x^2}\,dx\\ &= -x+\frac{x^3}{3}+\frac{\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$-x+\frac{x^3}{3}+\frac{\tan^{-1}\left(\sqrt{2}\,x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + 2*x^4)/(1 + 2*x^2),x]
```

```
[Out] -x + x^3/3 + ArcTan[Sqrt[2]*x]/Sqrt[2]
```

Maple [A]
time = 0.12, size = 21, normalized size = 0.84

method	result	size
default	$-x+\frac{x^3}{3}+\frac{\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}$	21
risch	$-x+\frac{x^3}{3}+\frac{\arctan\left(x\sqrt{2}\right)\sqrt{2}}{2}$	21
meijerg	$\frac{\sqrt{2}\left(-\frac{2x\sqrt{2}\left(-10x^2+15\right)}{15}+2\arctan\left(x\sqrt{2}\right)\right)}{8}-\frac{\sqrt{2}\left(2x\sqrt{2}-2\arctan\left(x\sqrt{2}\right)\right)}{8}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4-x^2)/(2*x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -x+1/3*x^3+1/2*arctan(x*2^(1/2))*2^(1/2)
```

Maxima [A]
time = 0.48, size = 20, normalized size = 0.80

$$\frac{1}{3}\,x^3 + \frac{1}{2}\,\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

Fricas [A]
time = 1.78, size = 20, normalized size = 0.80

$$\frac{1}{3}\,x^3 + \frac{1}{2}\,\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

Sympy [A]
time = 0.03, size = 20, normalized size = 0.80

$$\frac{x^3}{3} - x + \frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4-x**2)/(2*x**2+1),x)
```

```
[Out] x**3/3 - x + sqrt(2)*atan(sqrt(2)*x)/2
```

Giac [A]
time = 2.24, size = 20, normalized size = 0.80

$$\frac{1}{3}\,x^3 + \frac{1}{2}\,\sqrt{2}\,\arctan\left(\sqrt{2}\,x\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

Mupad [B]
time = 0.04, size = 20, normalized size = 0.80

$$\frac{\sqrt{2}\ \operatorname{atan}\left(\sqrt{2}\ x\right)}{2} - x + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - 2*x^4)/(2*x^2 + 1),x)
```

```
[Out] (2^(1/2)*atan(2^(1/2)*x))/2 - x + x^3/3
```

3.113 $\int \frac{x^3+x^4}{1+x^2}\,dx$

Optimal. Leaf size=30

$$-x+\frac{x^2}{2}+\frac{x^3}{3}+\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)$$

```
[Out] -x+1/2*x^2+1/3*x^3+arctan(x)-1/2*ln(x^2+1)
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1607, 815, 649, 209, 266}

$$\text{ArcTan}(x)+\frac{x^3}{3}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-x$$

Antiderivative was successfully verified.

```
[In] Int[(x^3 + x^4)/(1 + x^2),x]
```

```
[Out] -x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3+x^4}{1+x^2}\,dx &= \int \frac{x^3(1+x)}{1+x^2}\,dx\\ &= \int \left(-1+x+x^2+\frac{1-x}{1+x^2}\right)dx\\ &= -x+\frac{x^2}{2}+\frac{x^3}{3}+\int \frac{1-x}{1+x^2}\,dx\\ &= -x+\frac{x^2}{2}+\frac{x^3}{3}+\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\ &= -x+\frac{x^2}{2}+\frac{x^3}{3}+\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 30, normalized size = 1.00

$$-x+\frac{x^2}{2}+\frac{x^3}{3}+\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3 + x^4)/(1 + x^2),x]
[Out] -x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2
```

Maple [A]
time = 0.12, size = 25, normalized size = 0.83

method	result	size
default	$-x+\frac{x^2}{2}+\frac{x^3}{3}+\arctan(x)-\frac{\ln(x^2+1)}{2}$	25
risch	$-x+\frac{x^2}{2}+\frac{x^3}{3}+\arctan(x)-\frac{\ln(x^2+1)}{2}$	25
meijerg	$-\frac{x(-5x^2+15)}{15}+\arctan(x)+\frac{x^2}{2}-\frac{\ln(x^2+1)}{2}$	27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^3)/(x^2+1),x,method=_RETURNVERBOSE)
[Out] -x+1/2*x^2+1/3*x^3+arctan(x)-1/2*ln(x^2+1)
```

Maxima [A]
time = 0.50, size = 24, normalized size = 0.80

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - x + \arctan(x) - \frac{1}{2}\log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3)/(x^2+1),x, algorithm="maxima")
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

Fricas [A]
time = 1.25, size = 24, normalized size = 0.80

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - x + \arctan(x) - \frac{1}{2}\log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3)/(x^2+1),x, algorithm="fricas")
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

Sympy [A]
time = 0.03, size = 22, normalized size = 0.73

$$\frac{x^3}{3} + \frac{x^2}{2} - x - \frac{\log(x^2 + 1)}{2} + \operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**3)/(x**2+1),x)
[Out] x**3/3 + x**2/2 - x - log(x**2 + 1)/2 + atan(x)
```

Giac [A]
time = 1.96, size = 24, normalized size = 0.80

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 - x + \arctan(x) - \frac{1}{2}\log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3)/(x^2+1),x, algorithm="giac")
[Out] 1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

Mupad [B]
time = 0.03, size = 24, normalized size = 0.80

$$\operatorname{atan}(x) - \frac{\ln(x^2 + 1)}{2} - x + \frac{x^2}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + x^4)/(x^2 + 1),x)
[Out] atan(x) - log(x^2 + 1)/2 - x + x^2/2 + x^3/3
```

3.114 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$

Optimal. Leaf size=210

$$\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{b^6}-\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^5}{5b^4}+\frac{(b^2d-abe+}{7b^3}$$

```
[Out] a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^6-1/3*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*
c)*x^3/b^5+1/5*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^5/b^4+1/7*(a^2*f-a*b*e+b^2*
d)*x^7/b^3+1/9*(-a*f+b*e)*x^9/b^2+1/11*f*x^11/b-a^(5/2)*(-a^3*f+a^2*b*e-a*b
^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(13/2)
```

Rubi [A]
time = 0.10, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1816, 211}

$$\frac{x^7(a^2f-abe+b^2d)}{7b^3}+\frac{a^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{b^6}-\frac{ax^3(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5}+\frac{x^5(a^3(-f)+a^2be-ab^2d+b^3c)}{5b^4}-\frac{a^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{b^{13/2}}+\frac{x^9(be-af)}{9b^2}+\frac{fx^{11}}{11b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^5)/(5*b
^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e - a*f)*x^9)/(9*b^2) + (
f*x^11)/(11*b) - (a^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/b^(13/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = \int \left(\frac{a^2(b^3c-ab^2d+a^2be-a^3f)}{b^6} - \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^2}{b^5} + (b^3c -\right.$$

$$= \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{b^6} - \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^5} + (b^3c-a$$

$$= \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{b^6} - \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^5} + (b^3c-a$$

Mathematica [A]
time = 0.10, size = 210, normalized size = 1.00

$$-\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)\,x}{b^6}+\frac{a(-b^3c+ab^2d-a^2be+a^3f)\,x^3}{3b^5}+\frac{(b^3c-ab^2d+a^2be-a^3f)\,x^5}{5b^4}+\frac{(b^2d-abe+a^2f)\,x^7}{7b^3}+\frac{(be-af)x^9}{9b^2}+\frac{fx^{11}}{11b}+\frac{a^{5/2}(-b^3c+ab^2d-a^2be+a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] -((a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^6) + (a*(-(b^3*c) + a*b^
2*d - a^2*b*e + a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e - a*f)*x^9)/(9
*b^2) + (f*x^11)/(11*b) + (a^(5/2)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(13/2)
```

Maple [A]
time = 0.15, size = 233, normalized size = 1.11

method	result
default	$-\frac{-\frac{1}{11}f\,x^{11}b^5+\frac{1}{9}a\,b^4f\,x^9-\frac{1}{9}b^5e\,x^9-\frac{1}{7}a^2b^3f\,x^7+\frac{1}{7}a\,b^4e\,x^7-\frac{1}{7}b^5d\,x^7+\frac{1}{5}a^3b^2f\,x^5-\frac{1}{5}a^2b^3e\,x^5+\frac{1}{5}a\,b^4d\,x^5-\frac{1}{5}b^5c\,x^5-\frac{1}{3}a^4bf\,x^3+\frac{1}{3}a^3b^2}{b^6}$
risch	$-\frac{\sqrt{-ab}\;a^3\ln\left(\sqrt{-ab}\;x+a\right)d}{2b^5}+\frac{\sqrt{-ab}\;a^2\ln\left(\sqrt{-ab}\;x+a\right)c}{2b^4}+\frac{f\,x^{11}}{11b}+\frac{\sqrt{-ab}\;a^5\ln\left(-\sqrt{-ab}\;x+a\right)f}{2b^7}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^6*(-1/11*f*x^11*b^5+1/9*a*b^4*f*x^9-1/9*b^5*e*x^9-1/7*a^2*b^3*f*x^7+1/
7*a*b^4*e*x^7-1/7*b^5*d*x^7+1/5*a^3*b^2*f*x^5-1/5*a^2*b^3*e*x^5+1/5*a*b^4*d
*x^5-1/5*b^5*c*x^5-1/3*a^4*b*f*x^3+1/3*a^3*b^2*e*x^3-1/3*a^2*b^3*d*x^3+1/3*
a*b^4*c*x^3+a^5*f*x-a^4*b*e*x+a^3*b^2*d*x-a^2*b^3*c*x)+a^3*(a^3*f-a^2*b*e+a
*b^2*d-b^3*c)/b^6/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]

time = 0.51, size = 219, normalized size = 1.04

$$-\frac{(a^3b^3c - a^4b^2d - a^6f + a^5be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\, b^6} + \frac{315\, b^5fx^{11} - 385\,(ab^4f - b^5e)x^9 + 495\,(b^5d + a^2b^3f - ab^4e)x^7 + 693\,(b^5c - ab^4d - a^3b^2f + a^2b^3e)x^5 - 1155\,(ab^4c - a^2b^3d - a^4bf + a^3b^2e)x^3 + 3465\,(a^2b^3c - a^3b^2d - a^5f + a^4be)x}{3465\, b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(a^3*b^3*c - a^4*b^2*d - a^6*f + a^5*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^6) + 1/3465*(315*b^5*f*x^11 - 385*(a*b^4*f - b^5*e)*x^9 + 495*(b^5*d + a
^2*b^3*f - a*b^4*e)*x^7 + 693*(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*x^5
 - 1155*(a*b^4*c - a^2*b^3*d - a^4*b*f + a^3*b^2*e)*x^3 + 3465*(a^2*b^3*c -
 a^3*b^2*d - a^5*f + a^4*b*e)*x)/b^6
```

Fricas [A]

time = 3.80, size = 482, normalized size = 2.30

$$\left[\frac{630\, b^5fx^{11} - 770\, ab^4fx^9 + 990\,(b^5d + a^2b^3f)x^7 + 1386\,(b^5c - ab^4d - a^3b^2f)x^5 - 2310\,(ab^4c - a^2b^3d - a^4bf)x^3 + 3465\,(a^2b^3c - a^3b^2d - a^5f + a^4be)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2 - 2\,bx\sqrt{-\frac{a}{b}} - a}{bx^2 + a}\right) + 6930\,(a^2b^3c - a^3b^2d - a^5f)x + 22\,(35\, b^5x^9 - 45\, ab^4x^7 + 63\, a^2b^3x^5 - 105\, a^3b^2x^3 + 315\, a^4bx)e}{6930\, b^6}, \frac{315\, b^5fx^{11} - 385\, ab^4fx^9 + 495\,(b^5d + a^2b^3f)x^7 + 693\,(b^5c - ab^4d - a^3b^2f)x^5 - 1155\,(ab^4c - a^2b^3d - a^4bf)x^3 - 3465\,(a^2b^3c - a^3b^2d - a^5f + a^4be)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right) + 3465\,(a^2b^3c - a^3b^2d - a^5f)x + 11\,(35\, b^5x^9 - 45\, ab^4x^7 + 63\, a^2b^3x^5 - 105\, a^3b^2x^3 + 315\, a^4bx)e}{3465\, b^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6930*(630*b^5*f*x^11 - 770*a*b^4*f*x^9 + 990*(b^5*d + a^2*b^3*f)*x^7 + 1
386*(b^5*c - a*b^4*d - a^3*b^2*f)*x^5 - 2310*(a*b^4*c - a^2*b^3*d - a^4*b*f
)*x^3 + 3465*(a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*sqrt(-a/b)*log((b*x^
2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 6930*(a^2*b^3*c - a^3*b^2*d - a^5*
f)*x + 22*(35*b^5*x^9 - 45*a*b^4*x^7 + 63*a^2*b^3*x^5 - 105*a^3*b^2*x^3 + 3
15*a^4*b*x)*e)/b^6, 1/3465*(315*b^5*f*x^11 - 385*a*b^4*f*x^9 + 495*(b^5*d +
 a^2*b^3*f)*x^7 + 693*(b^5*c - a*b^4*d - a^3*b^2*f)*x^5 - 1155*(a*b^4*c - a
^2*b^3*d - a^4*b*f)*x^3 - 3465*(a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*sq
rt(a/b)*arctan(b*x*sqrt(a/b)/a) + 3465*(a^2*b^3*c - a^3*b^2*d - a^5*f)*x +
11*(35*b^5*x^9 - 45*a*b^4*x^7 + 63*a^2*b^3*x^5 - 105*a^3*b^2*x^3 + 315*a^4*
b*x)*e)/b^6]
```

Sympy [A]

time = 0.46, size = 384, normalized size = 1.83

$$x^9\left(-\frac{af}{9b^2} + \frac{e}{9b}\right) + x^7\left(\frac{a^2f}{7b^3} - \frac{ae}{7b^2} + \frac{d}{7b}\right) + x^5\left(-\frac{a^3f}{5b^4} + \frac{a^2e}{5b^3} - \frac{ad}{5b^2} + \frac{c}{5b}\right) + x^3\left(\frac{a^4f}{3b^5} - \frac{a^3e}{3b^4} + \frac{a^2d}{3b^3} - \frac{ac}{3b^2}\right) + x\left(-\frac{a^5f}{b^6} + \frac{a^4e}{b^5} - \frac{a^3d}{b^4} + \frac{a^2c}{b^3}\right) - \frac{\sqrt{-\frac{a^5}{b^{13}}}\,(a^3f - a^2be + ab^2d - b^3c)\log\left(-\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}\,(a^3f - a^2be + ab^2d - b^3c)}{a^5f - a^4be + a^3b^2d - a^2b^3c} + x\right)}{2} + \frac{\sqrt{-\frac{a^5}{b^{13}}}\,(a^3f - a^2be + ab^2d - b^3c)\log\left(\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}\,(a^3f - a^2be + ab^2d - b^3c)}{a^5f - a^4be + a^3b^2d - a^2b^3c} + x\right)}{2} + \frac{fx^{11}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**9*(-a*f/(9*b**2) + e/(9*b)) + x**7*(a**2*f/(7*b**3) - a*e/(7*b**2) + d/(
7*b)) + x**5*(-a**3*f/(5*b**4) + a**2*e/(5*b**3) - a*d/(5*b**2) + c/(5*b))
```

```
+ x**3*(a**4*f/(3*b**5) - a**3*e/(3*b**4) + a**2*d/(3*b**3) - a*c/(3*b**2))
 + x*(-a**5*f/b**6 + a**4*e/b**5 - a**3*d/b**4 + a**2*c/b**3) - sqrt(-a**5/
b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**6*sqrt(-a**5/b**13)*
(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**5*f - a**4*b*e + a**3*b**2*d -
a**2*b**3*c) + x)/2 + sqrt(-a**5/b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**
3*c)*log(b**6*sqrt(-a**5/b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a*
*5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c) + x)/2 + f*x**11/(11*b)
```

Giac [**A**]
time = 1.07, size = 250, normalized size = 1.19

$$-\frac{\left(a^{3} b^{3} c - a^{4} b^{2} d - a^{6} f + a^{5} b e\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^{6}} + \frac{315 \, b^{10} f x^{11} - 385 \, a b^{9} f x^{9} + 385 \, b^{10} x^{9} e + 495 \, b^{10} d x^{7} + 495 \, a^{2} b^{8} f x^{7} - 495 \, a b^{9} x^{7} e + 693 \, b^{10} c x^{5} - 693 \, a b^{9} d x^{5} - 693 \, a^{3} b^{7} f x^{5} + 693 \, a^{2} b^{8} x^{5} e - 1155 \, a b^{9} c x^{3} + 1155 \, a^{2} b^{8} d x^{3} + 1155 \, a^{4} b^{6} f x^{3} - 1155 \, a^{3} b^{7} x^{3} e + 3465 \, a^{2} b^{8} c x - 3465 \, a^{3} b^{7} d x - 3465 \, a^{5} b^{5} f x + 3465 \, a^{4} b^{6} x e}{3465 \, b^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a^3*b^3*c - a^4*b^2*d - a^6*f + a^5*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^6) + 1/3465*(315*b^10*f*x^11 - 385*a*b^9*f*x^9 + 385*b^10*x^9*e + 495*b^
10*d*x^7 + 495*a^2*b^8*f*x^7 - 495*a*b^9*x^7*e + 693*b^10*c*x^5 - 693*a*b^9
*d*x^5 - 693*a^3*b^7*f*x^5 + 693*a^2*b^8*x^5*e - 1155*a*b^9*c*x^3 + 1155*a^
2*b^8*d*x^3 + 1155*a^4*b^6*f*x^3 - 1155*a^3*b^7*x^3*e + 3465*a^2*b^8*c*x -
3465*a^3*b^7*d*x - 3465*a^5*b^5*f*x + 3465*a^4*b^6*x*e)/b^11
```

Mupad [**B**]
time = 0.93, size = 289, normalized size = 1.38

$$x^9\,\left(\frac{e}{9\,b}-\frac{a\,f}{9\,b^2}\right)+x^7\,\left(\frac{d}{7\,b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{7\,b}\right)+x^5\,\left(\frac{c}{5\,b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{5\,b}\right)+\frac{f\,x^{11}}{11\,b}+\frac{a^{5/2}\,\mathrm{atan}\left(\frac{a^{5/2}\,\sqrt{b}\,x\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{f\,a^6-e\,a^5\,b+d\,a^4\,b^2-c\,a^3\,b^3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{b^{13/2}}-\frac{a\,x^3\,\left(\frac{c}{b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{3\,b}+\frac{a^2\,x\,\left(\frac{c}{b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x)
```

```
[Out] x^9*(e/(9*b) - (a*f)/(9*b^2)) + x^7*(d/(7*b) - (a*(e/b - (a*f)/b^2))/(7*b))
 + x^5*(c/(5*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(5*b)) + (f*x^11)/(11
*b) + (a^(5/2)*atan((a^(5/2)*b^(1/2)*x*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))
/(a^6*f - a^3*b^3*c + a^4*b^2*d - a^5*b*e))*(b^3*c - a^3*f - a*b^2*d + a^2*
b*e))/b^(13/2) - (a*x^3*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/(3*b
) + (a^2*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/b^2
```

3.115 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$

Optimal. Leaf size=172

$$-\frac{a\left(b^3c-ab^2d+a^2be-a^3f\right)x}{b^5}+\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x^3}{3b^4}+\frac{\left(b^2d-abe+a^2f\right)x^5}{5b^3}+\frac{(be-af)x^7}{7b^2}+\frac{fx^9}{9b}+$$

```
[Out] -a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^5+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*
x^3/b^4+1/5*(a^2*f-a*b*e+b^2*d)*x^5/b^3+1/7*(-a*f+b*e)*x^7/b^2+1/9*f*x^9/b+
a^(3/2)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)
```

Rubi [A]
time = 0.08, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$,
Rules used = {1816, 211}

$$\frac{x^5(a^2f-abe+b^2d)}{5b^3}-\frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{b^5}+\frac{x^3(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^4}+\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{b^{11/2}}+\frac{x^7(be-af)}{7b^2}+\frac{fx^9}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b*e + a^2*f)*x^5)/(5*b^3) + ((b*e -
 a*f)*x^7)/(7*b^2) + (f*x^9)/(9*b) + (a^(3/2)*(b^3*c - a*b^2*d + a^2*b*e -
a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = \int \left(-\frac{a(b^3c-ab^2d+a^2be-a^3f)}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^2}{b^4} + (b^2d -\right.$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^4} + (b^2d - a$$

$$= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^4} + (b^2d - a$$

Mathematica [A]
time = 0.08, size = 162, normalized size = 0.94

$$\frac{x(315a^4f-105a^3b(3e+fx^2)+21a^2b^2(15d+5ex^2+3fx^4)-3ab^3(105c+35dx^2+21ex^4+15fx^6)+b^4x^2(105c+63dx^2+45ex^4+35fx^6))}{315b^5} - \frac{a^{3/2}(-b^3c+ab^2d-a^2be+a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (x*(315*a^4*f - 105*a^3*b*(3*e + f*x^2) + 21*a^2*b^2*(15*d + 5*e*x^2 + 3*f*
x^4) - 3*a*b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6) + b^4*x^2*(105*c +
63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5) - (a^(3/2)*(-(b^3*c) + a*b^2*d
- a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

Maple [A]
time = 0.15, size = 185, normalized size = 1.08

method	result
default	$\frac{\frac{1}{9}fx^9b^4-\frac{1}{7}ab^3fx^7+\frac{1}{7}b^4ex^7+\frac{1}{5}a^2b^2fx^5-\frac{1}{5}ab^3ex^5+\frac{1}{5}b^4dx^5-\frac{1}{3}a^3bfx^3+\frac{1}{3}a^2b^2ex^3-\frac{1}{3}ab^3dx^3+\frac{1}{3}b^4cx^3+a^4fx-a^3bex+a^2b^2dx-a}{b^5}$
risch	$\frac{fx^9}{9b}-\frac{afx^7}{7b^2}+\frac{ex^7}{7b}+\frac{a^2fx^5}{5b^3}-\frac{aex^5}{5b^2}+\frac{dx^5}{5b}-\frac{a^3fx^3}{3b^4}+\frac{a^2ex^3}{3b^3}-\frac{adx^3}{3b^2}+\frac{cx^3}{3b}+\frac{a^4fx}{b^5}-\frac{a^3ex}{b^4}+\frac{a^2dx}{b^3}-\frac{acx}{b^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/9*f*x^9*b^4-1/7*a*b^3*f*x^7+1/7*b^4*e*x^7+1/5*a^2*b^2*f*x^5-1/5*a*
b^3*e*x^5+1/5*b^4*d*x^5-1/3*a^3*b*f*x^3+1/3*a^2*b^2*e*x^3-1/3*a*b^3*d*x^3+1
/3*b^4*c*x^3+a^4*f*x-a^3*b*e*x+a^2*b^2*d*x-a*b^3*c*x)-a^2*(a^3*f-a^2*b*e+a*
b^2*d-b^3*c)/b^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.51, size = 177, normalized size = 1.03

$$\frac{(a^2b^3c-a^3b^2d-a^5f+a^4be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^5} + \frac{35\,b^4fx^9-45\,(ab^3f-b^4e)x^7+63\,(b^4d+a^2b^2f-ab^3e)x^5+105\,(b^4c-ab^3d-a^3bf+a^2b^2e)x^3-315\,(ab^3c-a^2b^2d-a^4f+a^3be)x}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
b^5) + 1/315*(35*b^4*f*x^9 - 45*(a*b^3*f - b^4*e)*x^7 + 63*(b^4*d + a^2*b^2
*f - a*b^3*e)*x^5 + 105*(b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*x^3 - 315*(
a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*x)/b^5
```

Fricas [**A**]
time = 2.87, size = 392, normalized size = 2.28

$$\left[\frac{70\,b^{4}fx^{9}-90\,ab^{3}fx^{7}+126\,(b^{4}d+a^{2}b^{2}f)x^{5}+210\,(b^{4}c-ab^{3}d-a^{3}bf)x^{3}+315\,(ab^{3}c-a^{2}b^{2}d-a^{4}f+a^{3}be)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^{2}+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)-630\,(ab^{3}c-a^{2}b^{2}d-a^{4}f)x+6\,(15\,b^{4}x^{7}-21\,ab^{3}x^{5}+35\,a^{2}b^{2}x^{3}-105\,a^{3}bx)e}{630\,b^{5}},\ \frac{35\,b^{4}fx^{9}-45\,ab^{3}fx^{7}+63\,(b^{4}d+a^{2}b^{2}f)x^{5}+105\,(b^{4}c-ab^{3}d-a^{3}bf)x^{3}+315\,(ab^{3}c-a^{2}b^{2}d-a^{4}f+a^{3}be)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-315\,(ab^{3}c-a^{2}b^{2}d-a^{4}f)x+3\,(15\,b^{4}x^{7}-21\,ab^{3}x^{5}+35\,a^{2}b^{2}x^{3}-105\,a^{3}bx)e}{315\,b^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/630*(70*b^4*f*x^9 - 90*a*b^3*f*x^7 + 126*(b^4*d + a^2*b^2*f)*x^5 + 210*(
b^4*c - a*b^3*d - a^3*b*f)*x^3 + 315*(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e
)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 630*(a*b^3*c
 - a^2*b^2*d - a^4*f)*x + 6*(15*b^4*x^7 - 21*a*b^3*x^5 + 35*a^2*b^2*x^3 - 1
05*a^3*b*x)*e)/b^5, 1/315*(35*b^4*f*x^9 - 45*a*b^3*f*x^7 + 63*(b^4*d + a^2*
b^2*f)*x^5 + 105*(b^4*c - a*b^3*d - a^3*b*f)*x^3 + 315*(a*b^3*c - a^2*b^2*d
 - a^4*f + a^3*b*e)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 315*(a*b^3*c - a^2*
b^2*d - a^4*f)*x + 3*(15*b^4*x^7 - 21*a*b^3*x^5 + 35*a^2*b^2*x^3 - 105*a^3*
b*x)*e)/b^5]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(167) = 334$.
time = 0.43, size = 337, normalized size = 1.96

$$x^{7}\left(-\frac{af}{7b^{2}}+\frac{e}{7b}\right)+x^{5}\left(\frac{a^{2}f}{5b^{3}}-\frac{ae}{5b^{2}}+\frac{d}{5b}\right)+x^{3}\left(-\frac{a^{3}f}{3b^{4}}+\frac{a^{2}e}{3b^{3}}-\frac{ad}{3b^{2}}+\frac{c}{3b}\right)+x\left(\frac{a^{4}f}{b^{5}}-\frac{a^{3}e}{b^{4}}+\frac{a^{2}d}{b^{3}}-\frac{ac}{b^{2}}\right)+\frac{\sqrt{-\frac{a^{3}}{b^{11}}}\,(a^{3}f-a^{2}be+ab^{2}d-b^{3}c)\log\left(-\frac{b^{5}\sqrt{-\frac{a^{3}}{b^{11}}}\,(a^{3}f-a^{2}be+ab^{2}d-b^{3}c)}{a^{4}f-a^{3}be+a^{2}b^{2}d-ab^{3}c}+x\right)}{2}-\frac{\sqrt{-\frac{a^{3}}{b^{11}}}\,(a^{3}f-a^{2}be+ab^{2}d-b^{3}c)\log\left(\frac{b^{5}\sqrt{-\frac{a^{3}}{b^{11}}}\,(a^{3}f-a^{2}be+ab^{2}d-b^{3}c)}{a^{4}f-a^{3}be+a^{2}b^{2}d-ab^{3}c}+x\right)}{2}+\frac{fx^{9}}{9b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**7*(-a*f/(7*b**2) + e/(7*b)) + x**5*(a**2*f/(5*b**3) - a*e/(5*b**2) + d/(
5*b)) + x**3*(-a**3*f/(3*b**4) + a**2*e/(3*b**3) - a*d/(3*b**2) + c/(3*b))
+ x*(a**4*f/b**5 - a**3*e/b**4 + a**2*d/b**3 - a*c/b**2) + sqrt(-a**3/b**11
)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**5*sqrt(-a**3/b**11)*(a**3
*f - a**2*b*e + a*b**2*d - b**3*c)/(a**4*f - a**3*b*e + a**2*b**2*d - a*b**
3*c) + x)/2 - sqrt(-a**3/b**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log
(b**5*sqrt(-a**3/b**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**4*f - a
**3*b*e + a**2*b**2*d - a*b**3*c) + x)/2 + f*x**9/(9*b)
```

Giac [A]

time = 0.96, size = 200, normalized size = 1.16

$$\frac{\left(a^{2} b^{3} c - a^{3} b^{2} d - a^{5} f + a^{4} b e\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{\sqrt{a b}\, b^{5}} + \frac{35 \, b^{8} f x^{9} - 45 \, a b^{7} f x^{7} + 45 \, b^{8} x^{7} e + 63 \, b^{8} d x^{5} + 63 \, a^{2} b^{6} f x^{5} - 63 \, a b^{7} x^{5} e + 105 \, b^{8} c x^{3} - 105 \, a b^{7} d x^{3} - 105 \, a^{3} b^{5} f x^{3} + 105 \, a^{2} b^{6} x^{3} e - 315 \, a b^{7} c x + 315 \, a^{2} b^{6} d x + 315 \, a^{4} b^{4} f x - 315 \, a^{3} b^{5} x e}{315 \, b^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
b^5) + 1/315*(35*b^8*f*x^9 - 45*a*b^7*f*x^7 + 45*b^8*x^7*e + 63*b^8*d*x^5 +
 63*a^2*b^6*f*x^5 - 63*a*b^7*x^5*e + 105*b^8*c*x^3 - 105*a*b^7*d*x^3 - 105*
a^3*b^5*f*x^3 + 105*a^2*b^6*x^3*e - 315*a*b^7*c*x + 315*a^2*b^6*d*x + 315*a
^4*b^4*f*x - 315*a^3*b^5*x*e)/b^9
```

Mupad [B]

time = 0.94, size = 243, normalized size = 1.41

$$x^7\,\left(\frac{e}{7\,b}-\frac{a\,f}{7\,b^2}\right)+x^5\,\left(\frac{d}{5\,b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{5\,b}\right)+x^3\,\left(\frac{c}{3\,b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{3\,b}\right)+\frac{f\,x^9}{9\,b}-\frac{a\,x\,\left(\frac{c}{b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{b}-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{f\,a^5-e\,a^4\,b+d\,a^3\,b^2-c\,a^2\,b^3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x)
```

```
[Out] x^7*(e/(7*b) - (a*f)/(7*b^2)) + x^5*(d/(5*b) - (a*(e/b - (a*f)/b^2))/(5*b))
 + x^3*(c/(3*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(3*b)) + (f*x^9)/(9*b
) - (a*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/b - (a^(3/2)*atan((
a^(3/2)*b^(1/2)*x*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^5*f - a^2*b^3*c +
 a^3*b^2*d - a^4*b*e))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/b^(11/2)
```

3.116 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$

Optimal. Leaf size=136

$$\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4}+\frac{(b^2d-abe+a^2f)\,x^3}{3b^3}+\frac{(be-af)x^5}{5b^2}+\frac{fx^7}{7b}-\frac{\sqrt{a}\,(b^3c-ab^2d+a^2be-a^3f)\tan}{b^{9/2}}$$

```
[Out] (-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^4+1/3*(a^2*f-a*b*e+b^2*d)*x^3/b^3+1/5*(-
a*f+b*e)*x^5/b^2+1/7*f*x^7/b-(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x*b^(1/2
)/a^(1/2))*a^(1/2)/b^(9/2)
```

Rubi [A]
time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1816, 211}

$$\frac{x^3(a^2f-abe+b^2d)}{3b^3}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{b^{9/2}}+\frac{x(a^3(-f)+a^2be-ab^2d+b^3c)}{b^4}+\frac{x^5(be-af)}{5b^2}+\frac{fx^7}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a^2*f)*x^3)
/(3*b^3) + ((b*e - a*f)*x^5)/(5*b^2) + (f*x^7)/(7*b) - (Sqrt[a]*(b^3*c - a*
b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = \int \left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^4} + \frac{(b^2d-abe+a^2f)\,x^2}{b^3} + \frac{(be-af)x^4}{b^2} + \frac{fx^6}{b}\right.$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4} + \frac{(b^2d-abe+a^2f)\,x^3}{3b^3} + \frac{(be-af)x^5}{5b^2} + \frac{fx^7}{7b}$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4} + \frac{(b^2d-abe+a^2f)\,x^3}{3b^3} + \frac{(be-af)x^5}{5b^2} + \frac{fx^7}{7b}$$

Mathematica [A]
time = 0.06, size = 128, normalized size = 0.94

$$\frac{x(-105a^3f+35a^2b(3e+fx^2)-7ab^2(15d+5ex^2+3fx^4)+b^3(105c+35dx^2+21ex^4+15fx^6))}{105b^4} + \frac{\sqrt{a}\,(-b^3c+ab^2d-a^2be+a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

```
[Out] (x*(-105*a^3*f + 35*a^2*b*(3*e + f*x^2) - 7*a*b^2*(15*d + 5*e*x^2 + 3*f*x^4
) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^4) + (Sqrt[a]*(-(
b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

Maple [A]
time = 0.14, size = 137, normalized size = 1.01

method	result
default	$-\frac{-\frac{1}{7}f\,x^7b^3+\frac{1}{5}a\,b^2f\,x^5-\frac{1}{5}b^3e\,x^5-\frac{1}{3}a^2bf\,x^3+\frac{1}{3}a\,b^2e\,x^3-\frac{1}{3}b^3d\,x^3+a^3fx-a^2bex+a\,b^2dx-b^3cx}{b^4}+\frac{a(a^3f-a^2be+a\,b^2d-b^3c)\arctan(}{b^4\sqrt{ab}}$
risch	$\frac{f\,x^7}{7b}-\frac{af\,x^5}{5b^2}+\frac{e\,x^5}{5b}+\frac{a^2f\,x^3}{3b^3}-\frac{ae\,x^3}{3b^2}+\frac{d\,x^3}{3b}-\frac{a^3fx}{b^4}+\frac{a^2ex}{b^3}-\frac{adx}{b^2}+\frac{cx}{b}+\frac{\sqrt{-ab}\,\ln\left(-\sqrt{-ab}\,x+a\right)a^3f}{2b^5}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/7*f*x^7*b^3+1/5*a*b^2*f*x^5-1/5*b^3*e*x^5-1/3*a^2*b*f*x^3+1/3*a*
b^2*e*x^3-1/3*b^3*d*x^3+a^3*f*x-a^2*b*e*x+a*b^2*d*x-b^3*c*x)+a*(a^3*f-a^2*b
*e+a*b^2*d-b^3*c)/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.53, size = 137, normalized size = 1.01

$$-\frac{(ab^3c-a^2b^2d-a^4f+a^3be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^4} + \frac{15\,b^3fx^7-21\,(ab^2f-b^3e)x^5+35\,(b^3d+a^2bf-ab^2e)x^3+105\,(b^3c-ab^2d-a^3f+a^2be)x}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b
^4) + 1/105*(15*b^3*f*x^7 - 21*(a*b^2*f - b^3*e)*x^5 + 35*(b^3*d + a^2*b*f
- a*b^2*e)*x^3 + 105*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*x)/b^4
```

Fricas [**A**]
time = 3.65, size = 304, normalized size = 2.24

$$\left[\frac{30\,b^{3}fx^{7}-42\,ab^{2}fx^{5}+70\,(b^{3}d+a^{2}bf)x^{3}+105\,(b^{3}c-ab^{2}d-a^{3}f+a^{2}be)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^{2}-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^{2}+a}\right)+210\,(b^{3}c-ab^{2}d-a^{3}f)x+14\,(3\,b^{3}x^{5}-5\,ab^{2}x^{3}+15\,a^{2}bx)e}{210\,b^{4}}, \frac{15\,b^{3}fx^{7}-21\,ab^{2}fx^{5}+35\,(b^{3}d+a^{2}bf)x^{3}-105\,(b^{3}c-ab^{2}d-a^{3}f+a^{2}be)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+105\,(b^{3}c-ab^{2}d-a^{3}f)x+7\,(3\,b^{3}x^{5}-5\,ab^{2}x^{3}+15\,a^{2}bx)e}{105\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b^3*f*x^7 - 42*a*b^2*f*x^5 + 70*(b^3*d + a^2*b*f)*x^3 + 105*(b^3
*c - a*b^2*d - a^3*f + a^2*b*e)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) -
a)/(b*x^2 + a)) + 210*(b^3*c - a*b^2*d - a^3*f)*x + 14*(3*b^3*x^5 - 5*a*b^2
*x^3 + 15*a^2*b*x)*e)/b^4, 1/105*(15*b^3*f*x^7 - 21*a*b^2*f*x^5 + 35*(b^3*d
 + a^2*b*f)*x^3 - 105*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*sqrt(a/b)*arctan(
b*x*sqrt(a/b)/a) + 105*(b^3*c - a*b^2*d - a^3*f)*x + 7*(3*b^3*x^5 - 5*a*b^2
*x^3 + 15*a^2*b*x)*e)/b^4]
```

Sympy [**A**]
time = 0.40, size = 185, normalized size = 1.36

$$x^{5}\left(-\frac{af}{5b^{2}}+\frac{e}{5b}\right)+x^{3}\left(\frac{a^{2}f}{3b^{3}}-\frac{ae}{3b^{2}}+\frac{d}{3b}\right)+x\left(-\frac{a^{3}f}{b^{4}}+\frac{a^{2}e}{b^{3}}-\frac{ad}{b^{2}}+\frac{c}{b}\right)-\frac{\sqrt{-\frac{a}{b^{9}}}\,(a^{3}f-a^{2}be+ab^{2}d-b^{3}c)\log\left(-b^{4}\sqrt{-\frac{a}{b^{9}}}+x\right)}{2}+\frac{\sqrt{-\frac{a}{b^{9}}}\,(a^{3}f-a^{2}be+ab^{2}d-b^{3}c)\log\left(b^{4}\sqrt{-\frac{a}{b^{9}}}+x\right)}{2}+\frac{fx^{7}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**5*(-a*f/(5*b**2) + e/(5*b)) + x**3*(a**2*f/(3*b**3) - a*e/(3*b**2) + d/(
3*b)) + x*(-a**3*f/b**4 + a**2*e/b**3 - a*d/b**2 + c/b) - sqrt(-a/b**9)*(a*
*3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**4*sqrt(-a/b**9) + x)/2 + sqrt(
-a/b**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(b**4*sqrt(-a/b**9) + x
)/2 + f*x**7/(7*b)
```

Giac [**A**]
time = 1.55, size = 152, normalized size = 1.12

$$-\frac{(ab^{3}c-a^{2}b^{2}d-a^{4}f+a^{3}be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^{4}}+\frac{15\,b^{6}fx^{7}-21\,ab^{5}fx^{5}+21\,b^{6}x^{5}e+35\,b^{6}dx^{3}+35\,a^{2}b^{4}fx^{3}-35\,ab^{5}x^{3}e+105\,b^{6}cx-105\,ab^{5}dx-105\,a^{3}b^{3}fx+105\,a^{2}b^{4}xe}{105\,b^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b
^4) + 1/105*(15*b^6*f*x^7 - 21*a*b^5*f*x^5 + 21*b^6*x^5*e + 35*b^6*d*x^3 +
35*a^2*b^4*f*x^3 - 35*a*b^5*x^3*e + 105*b^6*c*x - 105*a*b^5*d*x - 105*a^3*b
^3*f*x + 105*a^2*b^4*x*e)/b^7
```

Mupad [B]
time = 0.91, size = 193, normalized size = 1.42

$$x^5\left(\frac{e}{5\,b}-\frac{a\,f}{5\,b^2}\right)+x^3\left(\frac{d}{3\,b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{3\,b}\right)+x\left(\frac{c}{b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)+\frac{f\,x^7}{7\,b}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{f\,a^4-e\,a^3\,b+d\,a^2\,b^2-c\,a\,b^3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x)
```

```
[Out] x^5*(e/(5*b) - (a*f)/(5*b^2)) + x^3*(d/(3*b) - (a*(e/b - (a*f)/b^2))/(3*b))
 + x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b) + (f*x^7)/(7*b) + (a^(1/
2)*atan((a^(1/2)*b^(1/2)*x*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^4*f + a^
2*b^2*d - a*b^3*c - a^3*b*e))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/b^(9/2)
```

3.117 $\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx$

Optimal. Leaf size=100

$$\frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^3}{3b^2}+\frac{fx^5}{5b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}$$

```
[Out] (a^2*f-a*b*e+b^2*d)*x/b^3+1/3*(-a*f+b*e)*x^3/b^2+1/5*f*x^5/b+(-a^3*f+a^2*b*
e-a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1824, 211}

$$\frac{x(a^2f-abe+b^2d)}{b^3}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{\sqrt{a}\,b^{7/2}}+\frac{x^3(be-af)}{3b^2}+\frac{fx^5}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^3)/(3*b^2) + (f*x^5)/(5*b)
 + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[
a]*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx &= \int \left(\frac{b^2d-abe+a^2f}{b^3}+\frac{(be-af)x^2}{b^2}+\frac{fx^4}{b}+\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\,(a+bx^2)}\right)dx\\ &= \frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^3}{3b^2}+\frac{fx^5}{5b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\int \frac{1}{a+b}}{b^3}\\ &= \frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^3}{3b^2}+\frac{fx^5}{5b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}}{\sqrt{a}\,b^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 98, normalized size = 0.98

$$\frac{x(15a^2f-5ab(3e+fx^2)+b^2(15d+5ex^2+3fx^4))}{15b^3}+\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x]
```

```
[Out] (x*(15*a^2*f - 5*a*b*(3*e + f*x^2) + b^2*(15*d + 5*e*x^2 + 3*f*x^4)))/(15*b
^3) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sq
rt[a]*b^(7/2))
```

Maple [A]
time = 0.14, size = 94, normalized size = 0.94

method	result
default	$\frac{\frac{1}{5}f\,x^5b^2-\frac{1}{3}abf\,x^3+\frac{1}{3}b^2e\,x^3+a^2fx-abex+b^2dx}{b^3}+\frac{(-a^3f+a^2be-a\,b^2d+b^3c)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{b^3\sqrt{ab}}$
risch	$\frac{f\,x^5}{5b}-\frac{af\,x^3}{3b^2}+\frac{e\,x^3}{3b}+\frac{a^2fx}{b^3}-\frac{aex}{b^2}+\frac{dx}{b}-\frac{\ln\left(bx-\sqrt{-ab}\right)a^3f}{2b^3\sqrt{-ab}}+\frac{\ln\left(bx-\sqrt{-ab}\right)a^2e}{2b^2\sqrt{-ab}}-\frac{\ln\left(bx-\sqrt{-ab}\right)ad}{2b\sqrt{-ab}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^3*(1/5*f*x^5*b^2-1/3*a*b*f*x^3+1/3*b^2*e*x^3+a^2*f*x-a*b*e*x+b^2*d*x)+(
-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [A]
time = 0.52, size = 97, normalized size = 0.97

$$\frac{(b^3c-ab^2d-a^3f+a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3}+\frac{3\,b^2fx^5-5\,(abf-b^2e)x^3+15\,(b^2d+a^2f-abe)x}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) +
 1/15*(3*b^2*f*x^5 - 5*(a*b*f - b^2*e)*x^3 + 15*(b^2*d + a^2*f - a*b*e)*x)/
b^3
```

Fricas [A]
time = 4.57, size = 246, normalized size = 2.46

$$\left[\frac{6\,ab^3fx^5 - 10\,a^2b^2fx^3 - 15\,(b^3c - ab^2d - a^3f + a^2be)\sqrt{-ab}\,\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right) + 30\,(ab^3d + a^3bf)x + 10\,(ab^3x^3 - 3\,a^2b^2x)e}{30\,ab^4}, \frac{3\,ab^3fx^5 - 5\,a^2b^2fx^3 + 15\,(b^3c - ab^2d - a^3f + a^2be)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) + 15\,(ab^3d + a^3bf)x + 5\,(ab^3x^3 - 3\,a^2b^2x)e}{15\,ab^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/30*(6*a*b^3*f*x^5 - 10*a^2*b^2*f*x^3 - 15*(b^3*c - a*b^2*d - a^3*f + a^2
*b*e)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 30*(a*b^3*
d + a^3*b*f)*x + 10*(a*b^3*x^3 - 3*a^2*b^2*x)*e)/(a*b^4), 1/15*(3*a*b^3*f*x
^5 - 5*a^2*b^2*f*x^3 + 15*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*sqrt(a*b)*arc
tan(sqrt(a*b)*x/a) + 15*(a*b^3*d + a^3*b*f)*x + 5*(a*b^3*x^3 - 3*a^2*b^2*x)
*e)/(a*b^4)]
```

Sympy [A]
time = 0.36, size = 160, normalized size = 1.60

$$x^3\left(-\frac{af}{3b^2} + \frac{e}{3b}\right) + x\left(\frac{a^2f}{b^3} - \frac{ae}{b^2} + \frac{d}{b}\right) + \frac{\sqrt{-\frac{1}{ab^7}}\,(a^3f - a^2be + ab^2d - b^3c)\log\left(-ab^3\sqrt{-\frac{1}{ab^7}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^7}}\,(a^3f - a^2be + ab^2d - b^3c)\log\left(ab^3\sqrt{-\frac{1}{ab^7}} + x\right)}{2} + \frac{fx^5}{5b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

```
[Out] x**3*(-a*f/(3*b**2) + e/(3*b)) + x*(a**2*f/b**3 - a*e/b**2 + d/b) + sqrt(-1
/(a*b**7))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a*b**3*sqrt(-1/(a*b
**7)) + x)/2 - sqrt(-1/(a*b**7))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*lo
g(a*b**3*sqrt(-1/(a*b**7)) + x)/2 + f*x**5/(5*b)
```

Giac [A]
time = 1.19, size = 106, normalized size = 1.06

$$\frac{(b^3c - ab^2d - a^3f + a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^3} + \frac{3\,b^4fx^5 - 5\,ab^3fx^3 + 5\,b^4x^3e + 15\,b^4dx + 15\,a^2b^2fx - 15\,ab^3xe}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) +
 1/15*(3*b^4*f*x^5 - 5*a*b^3*f*x^3 + 5*b^4*x^3*e + 15*b^4*d*x + 15*a^2*b^2*
f*x - 15*a*b^3*x*e)/b^5
```

Mupad [B]
time = 0.94, size = 96, normalized size = 0.96

$$x^3\left(\frac{e}{3\,b} - \frac{a\,f}{3\,b^2}\right) + x\left(\frac{d}{b} - \frac{a\left(\frac{e}{b} - \frac{a\,f}{b^2}\right)}{b}\right) + \frac{f\,x^5}{5\,b} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3 + e\,a^2\,b - d\,a\,b^2 + c\,b^3\right)}{\sqrt{a}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x)
```

```
[Out] x^3*(e/(3*b) - (a*f)/(3*b^2)) + x*(d/b - (a*(e/b - (a*f)/b^2))/b) + (f*x^5)
/(5*b) + (atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a
^(1/2)*b^(7/2))
```

3.118 $\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=84

$$-\frac{c}{ax}+\frac{(be-af)x}{b^2}+\frac{fx^3}{3b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}$$

```
[Out] -c/a/x+(-a*f+b*e)*x/b^2+1/3*f*x^3/b-(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x
*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

Rubi [A]

time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1816, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{3/2}b^{5/2}}+\frac{x(be-af)}{b^2}-\frac{c}{ax}+\frac{fx^3}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x)/b^2 + (f*x^3)/(3*b) - ((b^3*c - a*b^2*d + a^2*
b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)}\,dx &= \int \left(\frac{be-af}{b^2}+\frac{c}{ax^2}+\frac{fx^2}{b}+\frac{-b^3c+ab^2d-a^2be+a^3f}{ab^2\left(a+bx^2\right)}\right)dx\\
&= -\frac{c}{ax}+\frac{(be-af)x}{b^2}+\frac{fx^3}{3b}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{1}{a+bx^2}\,dx}{ab^2}\\
&= -\frac{c}{ax}+\frac{(be-af)x}{b^2}+\frac{fx^3}{3b}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 83, normalized size = 0.99

$$-\frac{c}{ax}+\frac{(be-af)x}{b^2}+\frac{fx^3}{3b}+\frac{(-b^3c+ab^2d-a^2be+a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x]
```

```
[Out] -(c/(a*x)) + ((b*e - a*f)*x)/b^2 + (f*x^3)/(3*b) + ((-(b^3*c) + a*b^2*d - a
^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

Maple [A]
time = 0.14, size = 79, normalized size = 0.94

method	result
default	$-\frac{-\frac{1}{3} f\, x^{3} b+a f x-b e x}{b^{2}}+\frac{\left(a^{3} f-a^{2} b e+a\, b^{2} d-b^{3} c\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{a\, b^{2} \sqrt{a b}}-\frac{c}{a x}$
risch	$\frac{f\, x^{3}}{3 b}-\frac{a f x}{b^{2}}+\frac{e x}{b}-\frac{c}{a x}-\frac{a^{2} \ln\left(-\sqrt{-a b}\, x+a\right) f}{2 b^{2} \sqrt{-a b}}+\frac{a \ln\left(-\sqrt{-a b}\, x+a\right) e}{2 b \sqrt{-a b}}-\frac{\ln\left(-\sqrt{-a b}\, x+a\right) d}{2 \sqrt{-a b}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2*(-1/3*f*x^3*b+a*f*x-b*e*x)+1/a/b^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/(a*
b)^(1/2)*arctan(b*x/(a*b)^(1/2))-c/a/x
```

Maxima [A]
time = 0.52, size = 82, normalized size = 0.98

$$\frac{bfx^3-3\,(af-be)x}{3\,b^2}-\frac{c}{ax}-\frac{(b^3c-ab^2d-a^3f+a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(b*f*x^3 - 3*(a*f - b*e)*x)/b^2 - c/(a*x) - (b^3*c - a*b^2*d - a^3*f +
a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

Fricas [A]
time = 3.53, size = 223, normalized size = 2.65

$$\left[\frac{2\,a^2b^2fx^4-6\,a^3bfx^2+6\,a^2b^2x^2e-6\,ab^3c-3\,(a^2bxe+(b^3c-ab^2d-a^3f)x)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{6\,a^2b^3x},\frac{a^2b^2fx^4-3\,a^3bfx^2+3\,a^2b^2x^2e-3\,ab^3c-3\,(a^2bxe+(b^3c-ab^2d-a^3f)x)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{3\,a^2b^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*a^2*b^2*f*x^4 - 6*a^3*b*f*x^2 + 6*a^2*b^2*x^2*e - 6*a*b^3*c - 3*(a^
2*b*x*e + (b^3*c - a*b^2*d - a^3*f)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)
*x - a)/(b*x^2 + a)))/(a^2*b^3*x), 1/3*(a^2*b^2*f*x^4 - 3*a^3*b*f*x^2 + 3*a
^2*b^2*x^2*e - 3*a*b^3*c - 3*(a^2*b*x*e + (b^3*c - a*b^2*d - a^3*f)*x)*sqrt
(a*b)*arctan(sqrt(a*b)*x/a))/(a^2*b^3*x)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(71) = 142.

time = 0.51, size = 150, normalized size = 1.79

$$x\left(-\frac{af}{b^2}+\frac{e}{b}\right)-\frac{\sqrt{-\frac{1}{a^3b^5}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^3b^5}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{2}+\frac{fx^3}{3b}-\frac{c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a),x)
```

```
[Out] x*(-a*f/b**2 + e/b) - sqrt(-1/(a**3*b**5))*(a**3*f - a**2*b*e + a*b**2*d -
b**3*c)*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/2 + sqrt(-1/(a**3*b**5))*(
a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**2*b**2*sqrt(-1/(a**3*b**5)) +
 x)/2 + f*x**3/(3*b) - c/(a*x)
```

Giac [**A**]

time = 0.77, size = 86, normalized size = 1.02

$$-\frac{c}{ax}-\frac{(b^3c-ab^2d-a^3f+a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab^2}+\frac{b^2fx^3-3\,abfx+3\,b^2xe}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -c/(a*x) - (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*a*b^2) + 1/3*(b^2*f*x^3 - 3*a*b*f*x + 3*b^2*x*e)/b^3
```

Mupad [**B**]

time = 1.07, size = 76, normalized size = 0.90

$$x\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)-\frac{c}{a\,x}+\frac{f\,x^3}{3\,b}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{3/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x)
```

```
[Out] x*(e/b - (a*f)/b^2) - c/(a*x) + (f*x^3)/(3*b) - (atan((b^(1/2)*x)/a^(1/2))*
(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^(3/2)*b^(5/2))
```

3.119 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx$

Optimal. Leaf size=82

$$-\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{fx}{b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}$$

```
[Out] -1/3*c/a/x^3+(-a*d+b*c)/a^2/x+f*x/b+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x
*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

Rubi [A]

time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{1816, 211\}$

$$\frac{bc-ad}{a^2x}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{a^{5/2}b^{3/2}}-\frac{c}{3ax^3}+\frac{fx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*c/(a*x^3) + (b*c - a*d)/(a^2*x) + (f*x)/b + ((b^3*c - a*b^2*d + a^2*b*
e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx &= \int \left(\frac{f}{b}+\frac{c}{ax^4}+\frac{-bc+ad}{a^2x^2}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^2b(a+bx^2)}\right)dx\\ &= -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{fx}{b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\int \frac{1}{a+bx^2}\,dx}{a^2b}\\ &= -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{fx}{b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}\end{aligned}$$

Mathematica [A]

time = 0.06, size = 83, normalized size = 1.01

$$-\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{fx}{b}-\frac{(-b^3c+ab^2d-a^2be+a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x]
```

```
[Out] -1/3*c/(a*x^3) + (b*c - a*d)/(a^2*x) + (f*x)/b - ((-(b^3*c) + a*b^2*d - a^2
*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

Maple [A]

time = 0.13, size = 79, normalized size = 0.96

method	result
default	$\frac{fx}{b}+\frac{(-a^3f+a^2be-a\,b^2d+b^3c)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^2b\sqrt{ab}}-\frac{c}{3a\,x^3}-\frac{ad-bc}{a^2x}$
risch	$\frac{fx}{b}+\frac{-\frac{b(ad-bc)x^2}{a^2}-\frac{bc}{3a}}{b\,x^3}+\frac{\sum_{\_R=\operatorname{RootOf}\left(a^5\_Z^2b+a^6f^2-2a^5bef+2a^4b^2df+a^4b^2e^2-2a^3b^3cf-2a^3b^3de+2a^2b^4ce+a^2b^4d^2-2acd\,b^5+c^2b^6\right)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] f*x/b+1/a^2/b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(
1/2))-1/3*c/a/x^3-(a*d-b*c)/a^2/x
```

Maxima [A]

time = 0.50, size = 80, normalized size = 0.98

$$\frac{fx}{b}+\frac{(b^3c-ab^2d-a^3f+a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2b}+\frac{3\,(bc-ad)x^2-ac}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] f*x/b + (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^2*b) + 1/3*(3*(b*c - a*d)*x^2 - a*c)/(a^2*x^3)
```

Fricas [A]

time = 6.34, size = 228, normalized size = 2.78

$$\left[\frac{6\,a^3bfx^4-2\,a^2b^2c+6\,(ab^3c-a^2b^2d)x^2-3\,(a^2bx^3e+(b^3c-ab^2d-a^3f)x^3)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{6\,a^3b^2x^3},\ \frac{3\,a^3bfx^4-a^2b^2c+3\,(ab^3c-a^2b^2d)x^2+3\,(a^2bx^3e+(b^3c-ab^2d-a^3f)x^3)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{3\,a^3b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(6*a^3*b*f*x^4 - 2*a^2*b^2*c + 6*(a*b^3*c - a^2*b^2*d)*x^2 - 3*(a^2*b*
x^3*e + (b^3*c - a*b^2*d - a^3*f)*x^3)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)
*x - a)/(b*x^2 + a)))/(a^3*b^2*x^3), 1/3*(3*a^3*b*f*x^4 - a^2*b^2*c + 3*(a*
b^3*c - a^2*b^2*d)*x^2 + 3*(a^2*b*x^3*e + (b^3*c - a*b^2*d - a^3*f)*x^3)*sq
rt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^2*x^3)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(71) = 142$.
time = 1.05, size = 151, normalized size = 1.84

$$\frac{\sqrt{-\frac{1}{a^5b^3}}\,(a^3f - a^2be + ab^2d - b^3c)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{a^5b^3}}\,(a^3f - a^2be + ab^2d - b^3c)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}} + x\right)}{2} + \frac{fx}{b} + \frac{-ac + x^2(-3ad + 3bc)}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a),x)
```

```
[Out] sqrt(-1/(a**5*b**3))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**3*b*sq
rt(-1/(a**5*b**3)) + x)/2 - sqrt(-1/(a**5*b**3))*(a**3*f - a**2*b*e + a*b**
2*d - b**3*c)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)/2 + f*x/b + (-a*c + x**2
*(-3*a*d + 3*b*c))/(3*a**2*x**3)
```

Giac [**A**]
time = 1.06, size = 81, normalized size = 0.99

$$\frac{fx}{b} + \frac{(b^3c - ab^2d - a^3f + a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2b} + \frac{3\,bcx^2 - 3\,adx^2 - ac}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="giac")
```

```
[Out] f*x/b + (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^2*b) + 1/3*(3*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^2*x^3)
```

Mupad [**B**]
time = 0.11, size = 80, normalized size = 0.98

$$\frac{f\,x}{b} - \frac{\frac{b\,c}{3\,a} + \frac{b\,x^2\,(a\,d-b\,c)}{a^2}}{b\,x^3} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3 + e\,a^2\,b - d\,a\,b^2 + c\,b^3\right)}{a^{5/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x)
```

```
[Out] (f*x)/b - ((b*c)/(3*a) + (b*x^2*(a*d - b*c))/a^2)/(b*x^3) + (atan((b^(1/2)*
x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^(5/2)*b^(3/2))
```

3.120 $\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)}\,dx$

Optimal. Leaf size=104

$$-\frac{c}{5ax^5}+\frac{bc-ad}{3a^2x^3}-\frac{b^2c-abd+a^2e}{a^3x}-\frac{(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}\sqrt{b}}$$

```
[Out] -1/5*c/a/x^5+1/3*(-a*d+b*c)/a^2/x^3+(-a^2*e+a*b*d-b^2*c)/a^3/x-(-a^3*f+a^2*
b*e-a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1816, 211}

$$\frac{bc-ad}{3a^2x^3}-\frac{a^2e-abd+b^2c}{a^3x}-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{a^{7/2}\sqrt{b}}-\frac{c}{5ax^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x]
```

```
[Out] -1/5*c/(a*x^5) + (b*c - a*d)/(3*a^2*x^3) - (b^2*c - a*b*d + a^2*e)/(a^3*x)
- ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7/2
)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)}\,dx &= \int \left(\frac{c}{ax^6}+\frac{-bc+ad}{a^2x^4}+\frac{b^2c-abd+a^2e}{a^3x^2}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^3\left(a+bx^2\right)}\right)\,dx\\ &= -\frac{c}{5ax^5}+\frac{bc-ad}{3a^2x^3}-\frac{b^2c-abd+a^2e}{a^3x}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\int \frac{1}{a+bx^2}\,dx}{a^3}\\ &= -\frac{c}{5ax^5}+\frac{bc-ad}{3a^2x^3}-\frac{b^2c-abd+a^2e}{a^3x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right.}{a^{7/2}\sqrt{b}}\end{aligned}$$

Mathematica [**A**]
time = 0.06, size = 103, normalized size = 0.99

$$-\frac{c}{5ax^5}+\frac{bc-ad}{3a^2x^3}+\frac{-b^2c+abd-a^2e}{a^3x}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{7/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x]
```

```
[Out] -1/5*c/(a*x^5) + (b*c - a*d)/(3*a^2*x^3) + (-(b^2*c) + a*b*d - a^2*e)/(a^3*
x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
a^(7/2)*Sqrt[b])
```

Maple [**A**]
time = 0.16, size = 94, normalized size = 0.90

method	result
default	$\frac{\left(a^3f-a^2be+a\,b^2d-b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^3\sqrt{ab}}-\frac{c}{5a\,x^5}-\frac{ad-bc}{3a^2x^3}-\frac{a^2e-abd+b^2c}{a^3x}$
risch	$\frac{-\frac{\left(a^2e-abd+b^2c\right)x^4}{a^3}-\frac{(ad-bc)x^2}{3a^2}-\frac{c}{5a}}{x^5}-\frac{\ln\left(-\sqrt{-ab}\,x+a\right)f}{2\sqrt{-ab}}+\frac{\ln\left(-\sqrt{-ab}\,x+a\right)be}{2\sqrt{-ab}\,a}-\frac{\ln\left(-\sqrt{-ab}\,x+a\right)b^2d}{2\sqrt{-ab}\,a^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] (a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/5*c
/a/x^5-1/3*(a*d-b*c)/a^2/x^3-(a^2*e-a*b*d+b^2*c)/a^3/x
```

Maxima [**A**]
time = 0.51, size = 99, normalized size = 0.95

$$-\frac{\left(b^3c-ab^2d-a^3f+a^2be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}-\frac{15\,\left(b^2c-abd+a^2e\right)x^4+3\,a^2c-5\,\left(abc-a^2d\right)x^2}{15\,a^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
- 1/15*(15*(b^2*c - a*b*d + a^2*e)*x^4 + 3*a^2*c - 5*(a*b*c - a^2*d)*x^2)/(
a^3*x^5)
```

Fricas [A]
time = 9.33, size = 268, normalized size = 2.58

$$\left[-\frac{30\,a^{3}bx^{4}e + 6\,a^{3}bc + 30\,(ab^{3}c - a^{2}b^{2}d)x^{4} - 10\,(a^{2}b^{2}c - a^{3}bd)x^{2} + 15\,(a^{2}bx^{5}e + (b^{3}c - ab^{2}d - a^{3}f)x^{5})\sqrt{-ab}\,\log\left(\frac{bx^{2}+2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)}{30\,a^{4}bx^{5}}, -\frac{15\,a^{3}bx^{4}e + 3\,a^{3}bc + 15\,(ab^{3}c - a^{2}b^{2}d)x^{4} - 5\,(a^{2}b^{2}c - a^{3}bd)x^{2} + 15\,(a^{2}bx^{5}e + (b^{3}c - ab^{2}d - a^{3}f)x^{5})\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{15\,a^{4}bx^{5}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/30*(30*a^3*b*x^4*e + 6*a^3*b*c + 30*(a*b^3*c - a^2*b^2*d)*x^4 - 10*(a^2
*b^2*c - a^3*b*d)*x^2 + 15*(a^2*b*x^5*e + (b^3*c - a*b^2*d - a^3*f)*x^5)*sq
rt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b*x^5), -1/15*
(15*a^3*b*x^4*e + 3*a^3*b*c + 15*(a*b^3*c - a^2*b^2*d)*x^4 - 5*(a^2*b^2*c -
 a^3*b*d)*x^2 + 15*(a^2*b*x^5*e + (b^3*c - a*b^2*d - a^3*f)*x^5)*sqrt(a*b)*
arctan(sqrt(a*b)*x/a))/(a^4*b*x^5)]
```

Sympy [A]
time = 2.53, size = 167, normalized size = 1.61

$$-\frac{\sqrt{-\frac{1}{a^{7}b}}\,(a^{3}f - a^{2}be + ab^{2}d - b^{3}c)\log\left(-a^{4}\sqrt{-\frac{1}{a^{7}b}} + x\right)}{2} + \frac{\sqrt{-\frac{1}{a^{7}b}}\,(a^{3}f - a^{2}be + ab^{2}d - b^{3}c)\log\left(a^{4}\sqrt{-\frac{1}{a^{7}b}} + x\right)}{2} + \frac{-3a^{2}c + x^{4}(-15a^{2}e + 15abd - 15b^{2}c) + x^{2}(-5a^{2}d + 5abc)}{15a^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a**7*b))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**4*sqrt(-
1/(a**7*b)) + x)/2 + sqrt(-1/(a**7*b))*(a**3*f - a**2*b*e + a*b**2*d - b**3
*c)*log(a**4*sqrt(-1/(a**7*b)) + x)/2 + (-3*a**2*c + x**4*(-15*a**2*e + 15*
a*b*d - 15*b**2*c) + x**2*(-5*a**2*d + 5*a*b*c))/(15*a**3*x**5)
```

Giac [A]
time = 1.31, size = 105, normalized size = 1.01

$$-\frac{(b^{3}c - ab^{2}d - a^{3}f + a^{2}be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{3}} - \frac{15\,b^{2}cx^{4} - 15\,abdx^{4} + 15\,a^{2}x^{4}e - 5\,abcx^{2} + 5\,a^{2}dx^{2} + 3\,a^{2}c}{15\,a^{3}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
- 1/15*(15*b^2*c*x^4 - 15*a*b*d*x^4 + 15*a^2*x^4*e - 5*a*b*c*x^2 + 5*a^2*d*
x^2 + 3*a^2*c)/(a^3*x^5)
```

Mupad [B]
time = 1.20, size = 94, normalized size = 0.90

$$-\frac{\frac{c}{5\,a}+\frac{x^2\,\left(a\,d-b\,c\right)}{3\,a^2}+\frac{x^4\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{a^3}}{x^5}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{7/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x)
```

```
[Out] - (c/(5*a) + (x^2*(a*d - b*c))/(3*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/a^3)
/x^5 - (atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^(
7/2)*b^(1/2))
```

3.121 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=137

$$-\frac{c}{7ax^7}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}+\frac{\sqrt{b}\,\left(b^3c-ab^2d+a^2be-a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}$$

```
[Out] -1/7*c/a/x^7+1/5*(-a*d+b*c)/a^2/x^5+1/3*(-a^2*e+a*b*d-b^2*c)/a^3/x^3+(-a^3*
f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x*b^(1
/2)/a^(1/2))*b^(1/2)/a^(9/2)
```

Rubi [A]
time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1816, 211}

$$\frac{bc-ad}{5a^2x^5}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{9/2}}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^4x}-\frac{c}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x]
```

```
[Out] -1/7*c/(a*x^7) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/(3*a^3*x
^3) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) + (Sqrt[b]*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)}\,dx = \int \left(\frac{c}{ax^8}+\frac{-bc+ad}{a^2x^6}+\frac{b^2c-abd+a^2e}{a^3x^4}+\frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^2}-\frac{b(-b^3c}{}\right.$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}+\frac{(b(b^3c-ab^2}{}$$

$$= -\frac{c}{7ax^7}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}+\frac{\sqrt{b}\,(b^3c-a}{}$$

Mathematica [A]
time = 0.08, size = 139, normalized size = 1.01

$$-\frac{c}{7ax^7}+\frac{bc-ad}{5a^2x^5}+\frac{-b^2c+abd-a^2e}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x}-\frac{\sqrt{b}\left(-b^3c+ab^2d-a^2be+a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x]
```

```
[Out] -1/7*c/(a*x^7) + (b*c - a*d)/(5*a^2*x^5) + (-(b^2*c) + a*b*d - a^2*e)/(3*a^
3*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) - (Sqrt[b]*(-(b^3*c) +
 a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

Maple [A]
time = 0.13, size = 129, normalized size = 0.94

method	result
default	$-\frac{b\left(a^3f-a^2be+a\,b^2d-b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^4\sqrt{ab}}-\frac{c}{7a\,x^7}-\frac{ad-bc}{5a^2x^5}-\frac{a^2e-abd+b^2c}{3a^3x^3}-\frac{a^3f-a^2be+a\,b^2d-b^3c}{a^4x}$
risch	$\frac{-\frac{\left(a^3f-a^2be+a\,b^2d-b^3c\right)x^6}{a^4}-\frac{\left(a^2e-abd+b^2c\right)x^4}{3a^3}-\frac{(ad-bc)x^2}{5a^2}-\frac{c}{7a}}{x^7}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^9\_Z^2+a^6b\,f^2-2a^5b^2ef+2a^4b^3df+a^4b^3e^2-2a^3b^4cf-\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -b*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1/
7*c/a/x^7-1/5*(a*d-b*c)/a^2/x^5-1/3*(a^2*e-a*b*d+b^2*c)/a^3/x^3-(a^3*f-a^2*
b*e+a*b^2*d-b^3*c)/a^4/x
```

Maxima [A]
time = 0.50, size = 137, normalized size = 1.00

$$\frac{\left(b^4c-ab^3d-a^3bf+a^2b^2e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}+\frac{105\left(b^3c-ab^2d-a^3f+a^2be\right)x^6-35\left(ab^2c-a^2bd+a^3e\right)x^4-15\,a^3c+21\left(a^2bc-a^3d\right)x^2}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^
4) + 1/105*(105*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*x^6 - 35*(a*b^2*c - a^2
*b*d + a^3*e)*x^4 - 15*a^3*c + 21*(a^2*b*c - a^3*d)*x^2)/(a^4*x^7)
```

Fricas [**A**]
time = 5.80, size = 326, normalized size = 2.38

$$\left[\frac{210\,(b^3c-ab^2d-a^3f)x^6-70\,(ab^2c-a^2bd)x^4-30\,a^3c+42\,(a^2bc-a^3d)x^2+105\,(a^2bx^7e+(b^3c-ab^2d-a^3f)x^7)\sqrt{-\frac{b}{a}}\,\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+70\,(3\,a^2bx^6-a^3x^4)e}{210\,a^4x^7},\ \frac{105\,(b^3c-ab^2d-a^3f)x^6-35\,(ab^2c-a^2bd)x^4-15\,a^3c+21\,(a^2bc-a^3d)x^2+105\,(a^2bx^7e+(b^3c-ab^2d-a^3f)x^7)\sqrt{\frac{b}{a}}\,\arctan\left(x\sqrt{\frac{b}{a}}\right)+35\,(3\,a^2bx^6-a^3x^4)e}{105\,a^4x^7}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/210*(210*(b^3*c - a*b^2*d - a^3*f)*x^6 - 70*(a*b^2*c - a^2*b*d)*x^4 - 30
*a^3*c + 42*(a^2*b*c - a^3*d)*x^2 + 105*(a^2*b*x^7*e + (b^3*c - a*b^2*d - a
^3*f)*x^7)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 70*
(3*a^2*b*x^6 - a^3*x^4)*e)/(a^4*x^7), 1/105*(105*(b^3*c - a*b^2*d - a^3*f)*
x^6 - 35*(a*b^2*c - a^2*b*d)*x^4 - 15*a^3*c + 21*(a^2*b*c - a^3*d)*x^2 + 10
5*(a^2*b*x^7*e + (b^3*c - a*b^2*d - a^3*f)*x^7)*sqrt(b/a)*arctan(x*sqrt(b/a
)) + 35*(3*a^2*b*x^6 - a^3*x^4)*e)/(a^4*x^7)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(128) = 256$.
time = 6.66, size = 301, normalized size = 2.20

$$\frac{\sqrt{-\frac{b}{a^9}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(-\frac{a^5\sqrt{-\frac{b}{a^9}}\,(a^3f-a^2be+ab^2d-b^3c)}{a^3bf-a^2b^2e+ab^3d-b^4c}+x\right)}{2}-\frac{\sqrt{-\frac{b}{a^9}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(\frac{a^5\sqrt{-\frac{b}{a^9}}\,(a^3f-a^2be+ab^2d-b^3c)}{a^3bf-a^2b^2e+ab^3d-b^4c}+x\right)}{2}+\frac{-15a^3c+x^6(-105a^3f+105a^2be-105ab^2d+105b^3c)+x^4(-35a^3e+35a^2bd-35ab^2c)+x^2(-21a^3d+21a^2bc)}{105a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a),x)
```

```
[Out] sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**5*sqrt(-b/a**
9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b*f - a**2*b**2*e + a*b**3
*d - b**4*c) + x)/2 - sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)
*log(a**5*sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b*f -
 a**2*b**2*e + a*b**3*d - b**4*c) + x)/2 + (-15*a**3*c + x**6*(-105*a**3*f
+ 105*a**2*b*e - 105*a*b**2*d + 105*b**3*c) + x**4*(-35*a**3*e + 35*a**2*b*
d - 35*a*b**2*c) + x**2*(-21*a**3*d + 21*a**2*b*c))/(105*a**4*x**7)
```

Giac [**A**]
time = 1.75, size = 151, normalized size = 1.10

$$\frac{(b^4c-ab^3d-a^3bf+a^2b^2e)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^4}+\frac{105\,b^3cx^6-105\,ab^2dx^6-105\,a^3fx^6+105\,a^2bx^6e-35\,ab^2cx^4+35\,a^2bdx^4-35\,a^3x^4e+21\,a^2bcx^2-21\,a^3dx^2-15\,a^3c}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^
4) + 1/105*(105*b^3*c*x^6 - 105*a*b^2*d*x^6 - 105*a^3*f*x^6 + 105*a^2*b*x^6
*e - 35*a*b^2*c*x^4 + 35*a^2*b*d*x^4 - 35*a^3*x^4*e + 21*a^2*b*c*x^2 - 21*a
^3*d*x^2 - 15*a^3*c)/(a^4*x^7)
```

Mupad [B]
time = 0.98, size = 127, normalized size = 0.93

$$\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{9/2}}-\frac{\frac{c}{7\,a}-\frac{x^6\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^4}+\frac{x^2\,\left(a\,d-b\,c\right)}{5\,a^2}+\frac{x^4\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{3\,a^3}}{x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x)
```

```
[Out] (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^(
9/2) - (c/(7*a) - (x^6*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^4 + (x^2*(a*d
 - b*c))/(5*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/(3*a^3))/x^7
```

3.122 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)}\,dx$

Optimal. Leaf size=175

$$-\frac{c}{9ax^9}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{5a^3x^5}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{a^5x}-\frac{b^{3/2}(b^3c-ab^2d}{}$$

```
[Out] -1/9*c/a/x^9+1/7*(-a*d+b*c)/a^2/x^7+1/5*(-a^2*e+a*b*d-b^2*c)/a^3/x^5+1/3*(-
a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^3-b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^5/x
-b^(3/2)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(11/2)
```

Rubi [A]
time = 0.10, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1816, 211}

$$\frac{bc-ad}{7a^2x^7}-\frac{a^2e-abd+b^2c}{5a^3x^5}-\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{a^{11/2}}-\frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{a^5x}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^4x^3}-\frac{c}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x]
```

```
[Out] -1/9*c/(a*x^9) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e)/(5*a^3*x
^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) - (b*(b^3*c - a*b^2*d
 + a^2*b*e - a^3*f))/(a^5*x) - (b^(3/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(11/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)}\,dx = \int \left(\frac{c}{ax^{10}} + \frac{-bc+ad}{a^2x^8} + \frac{b^2c-abd+a^2e}{a^3x^6} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^4} - b(-b^3$$

$$= -\frac{c}{9ax^9} + \frac{bc-ad}{7a^2x^7} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3} - b(b^3c-ab^2$$

$$= -\frac{c}{9ax^9} + \frac{bc-ad}{7a^2x^7} - \frac{b^2c-abd+a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3} - b(b^3c-ab^2$$

Mathematica [A]
time = 0.10, size = 174, normalized size = 0.99

$$-\frac{c}{9ax^9} + \frac{bc-ad}{7a^2x^7} + \frac{-b^2c+abd-a^2e}{5a^3x^5} + \frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3} + \frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^5x} + \frac{b^{3/2}(-b^3c+ab^2d-a^2be+a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x]
```

```
[Out] -1/9*c/(a*x^9) + (b*c - a*d)/(7*a^2*x^7) + (-(b^2*c) + a*b*d - a^2*e)/(5*a^
3*x^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) + (b*(-(b^3*c) + a
*b^2*d - a^2*b*e + a^3*f))/(a^5*x) + (b^(3/2)*(-(b^3*c) + a*b^2*d - a^2*b*e
 + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(11/2)
```

Maple [A]
time = 0.13, size = 163, normalized size = 0.93

method	result
default	$\frac{b^2\left(a^3f-a^2be+a\,b^2d-b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^5\sqrt{ab}} - \frac{c}{9a\,x^9} - \frac{ad-bc}{7a^2x^7} - \frac{a^2e-abd+b^2c}{5a^3x^5} - \frac{a^3f-a^2be+a\,b^2d-b^3c}{3a^4x^3} + \frac{b(a^3f-a^2be+a\,b^2}{a^5x}$
risch	$\frac{\frac{b\left(a^3f-a^2be+a\,b^2d-b^3c\right)x^8}{a^5} - \frac{\left(a^3f-a^2be+a\,b^2d-b^3c\right)x^6}{3a^4} - \frac{\left(a^2e-abd+b^2c\right)x^4}{5a^3} - \frac{(ad-bc)x^2}{7a^2} - \frac{c}{9a}}{x^9} + \Big(\_R{=}\mathrm{RootOf}\left(a^{11}\_Z^2+a^6b^3f^2-2a^5b^4ef\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] b^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-1
/9*c/a/x^9-1/7*(a*d-b*c)/a^2/x^7-1/5*(a^2*e-a*b*d+b^2*c)/a^3/x^5-1/3*(a^3*f
-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^3+b*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5/x
```

Maxima [A]
time = 0.49, size = 179, normalized size = 1.02

$$-\frac{(b^5c-ab^4d-a^3b^2f+a^2b^3e)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^5} - \frac{315\,(b^4c-ab^3d-a^3bf+a^2b^2e)x^8-105\,(ab^3c-a^2b^2d-a^4f+a^3be)x^6+35\,a^4c+63\,(a^2b^2c-a^3bd+a^4e)x^4-45\,(a^3bc-a^4d)x^2}{315\,a^5x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] -(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^5) - 1/315*(315*(b^4*c - a*b^3*d - a^3*b*f + a^2*b^2*e)*x^8 - 105*(a*b^3
*c - a^2*b^2*d - a^4*f + a^3*b*e)*x^6 + 35*a^4*c + 63*(a^2*b^2*c - a^3*b*d
+ a^4*e)*x^4 - 45*(a^3*b*c - a^4*d)*x^2)/(a^5*x^9)
```

Fricas [**A**]
time = 6.66, size = 414, normalized size = 2.37

$$\left[-\frac{630\,(b^4c-ab^3d-a^3bf)x^8-210\,(ab^3c-a^2b^2d-a^4f)x^6+70\,a^4c+126\,(a^2b^2c-a^3bd)x^4-90\,(a^3bc-a^4d)x^2-315\,(a^2b^2x^9e+(b^4c-ab^3d-a^3bf)x^9)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+42\,(15\,a^2b^2x^8-5\,a^3bx^6+3\,a^4x^4)e}{630\,a^5x^9},\ -\frac{315\,(b^4c-ab^3d-a^3bf)x^8-105\,(ab^3c-a^2b^2d-a^4f)x^6+35\,a^4c+63\,(a^2b^2c-a^3bd)x^4-45\,(a^3bc-a^4d)x^2+315\,(a^2b^2x^9e+(b^4c-ab^3d-a^3bf)x^9)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+21\,(15\,a^2b^2x^8-5\,a^3bx^6+3\,a^4x^4)e}{315\,a^5x^9}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/630*(630*(b^4*c - a*b^3*d - a^3*b*f)*x^8 - 210*(a*b^3*c - a^2*b^2*d - a
^4*f)*x^6 + 70*a^4*c + 126*(a^2*b^2*c - a^3*b*d)*x^4 - 90*(a^3*b*c - a^4*d)
*x^2 - 315*(a^2*b^2*x^9*e + (b^4*c - a*b^3*d - a^3*b*f)*x^9)*sqrt(-b/a)*log
((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 42*(15*a^2*b^2*x^8 - 5*a^3*b
*x^6 + 3*a^4*x^4)*e)/(a^5*x^9), -1/315*(315*(b^4*c - a*b^3*d - a^3*b*f)*x^8
 - 105*(a*b^3*c - a^2*b^2*d - a^4*f)*x^6 + 35*a^4*c + 63*(a^2*b^2*c - a^3*b
*d)*x^4 - 45*(a^3*b*c - a^4*d)*x^2 + 315*(a^2*b^2*x^9*e + (b^4*c - a*b^3*d
- a^3*b*f)*x^9)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 21*(15*a^2*b^2*x^8 - 5*a^3*
b*x^6 + 3*a^4*x^4)*e)/(a^5*x^9)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(167) = 334$.
time = 25.30, size = 354, normalized size = 2.02

$$-\frac{\sqrt{-\frac{b^3}{a^{11}}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(-\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}\,(a^3f-a^2be+ab^2d-b^3c)}{a^3b^2f-a^2b^3e+ab^4d-b^5c}+x\right)}{2}+\frac{\sqrt{-\frac{b^3}{a^{11}}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}\,(a^3f-a^2be+ab^2d-b^3c)}{a^3b^2f-a^2b^3e+ab^4d-b^5c}+x\right)}{2}+\frac{-35a^4c+x^8\cdot(315a^3bf-315a^2b^2e+315ab^3d-315b^4c)+x^6(-105a^4f+105a^3be-105a^2b^2d+105ab^3c)+x^4(-63a^4e+63a^3bd-63a^2b^2c)+x^2(-45a^4d+45a^3bc)}{315a^5x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a),x)
```

```
[Out] -sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**6*sqrt(-
b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b**2*f - a**2*b**
3*e + a*b**4*d - b**5*c) + x)/2 + sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*
b**2*d - b**3*c)*log(a**6*sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d -
 b**3*c)/(a**3*b**2*f - a**2*b**3*e + a*b**4*d - b**5*c) + x)/2 + (-35*a**4
*c + x**8*(315*a**3*b*f - 315*a**2*b**2*e + 315*a*b**3*d - 315*b**4*c) + x*
*6*(-105*a**4*f + 105*a**3*b*e - 105*a**2*b**2*d + 105*a*b**3*c) + x**4*(-6
3*a**4*e + 63*a**3*b*d - 63*a**2*b**2*c) + x**2*(-45*a**4*d + 45*a**3*b*c))
/(315*a**5*x**9)
```

Giac [A]
time = 1.55, size = 201, normalized size = 1.15

$$-\frac{\left(b^{5}c-ab^{4}d-a^{3}b^{2}f+a^{2}b^{3}e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{5}}-\frac{315\,b^{4}cx^{8}-315\,ab^{3}dx^{8}-315\,a^{3}bfx^{8}+315\,a^{2}b^{2}x^{8}e-105\,ab^{3}cx^{6}+105\,a^{2}b^{2}dx^{6}+105\,a^{4}fx^{6}-105\,a^{3}bx^{6}e+63\,a^{2}b^{2}cx^{4}-63\,a^{3}bdx^{4}+63\,a^{4}x^{4}e-45\,a^{3}bcx^{2}+45\,a^{4}dx^{2}+35\,a^{4}c}{315\,a^{5}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="giac")
```

```
[Out] -(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^5) - 1/315*(315*b^4*c*x^8 - 315*a*b^3*d*x^8 - 315*a^3*b*f*x^8 + 315*a^2*
b^2*x^8*e - 105*a*b^3*c*x^6 + 105*a^2*b^2*d*x^6 + 105*a^4*f*x^6 - 105*a^3*b
*x^6*e + 63*a^2*b^2*c*x^4 - 63*a^3*b*d*x^4 + 63*a^4*x^4*e - 45*a^3*b*c*x^2
+ 45*a^4*d*x^2 + 35*a^4*c)/(a^5*x^9)
```

Mupad [B]
time = 1.02, size = 161, normalized size = 0.92

$$-\frac{\frac{c}{9\,a}-\frac{x^{6}\,\left(-f\,a^{3}+e\,a^{2}\,b-d\,a\,b^{2}+c\,b^{3}\right)}{3\,a^{4}}+\frac{x^{2}\,\left(a\,d-b\,c\right)}{7\,a^{2}}+\frac{x^{4}\,\left(e\,a^{2}-d\,a\,b+c\,b^{2}\right)}{5\,a^{3}}+\frac{b\,x^{8}\,\left(-f\,a^{3}+e\,a^{2}\,b-d\,a\,b^{2}+c\,b^{3}\right)}{a^{5}}}{x^{9}}-\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^{3}+e\,a^{2}\,b-d\,a\,b^{2}+c\,b^{3}\right)}{a^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x)
```

```
[Out] - (c/(9*a) - (x^6*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^4) + (x^2*(a*d
- b*c))/(7*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/(5*a^3) + (b*x^8*(b^3*c - a
^3*f - a*b^2*d + a^2*b*e))/a^5)/x^9 - (b^(3/2)*atan((b^(1/2)*x)/a^(1/2))*(b
^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^(11/2)
```

3.123 $\int \frac{c+dx^2+ex^4+fx^6}{x^{12}\left(a+bx^2\right)}\,dx$

Optimal. Leaf size=211

$$-\frac{c}{11ax^{11}}+\frac{bc-ad}{9a^2x^9}-\frac{b^2c-abd+a^2e}{7a^3x^7}+\frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{3a^5x^3}+\frac{b^2(b^3c-ab^2d}{a}$$

```
[Out] -1/11*c/a/x^11+1/9*(-a*d+b*c)/a^2/x^9+1/7*(-a^2*e+a*b*d-b^2*c)/a^3/x^7+1/5*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^5-1/3*b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
/a^5/x^3+b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^6/x+b^(5/2)*(-a^3*f+a^2*b*e-a
*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(13/2)
```

Rubi [A]
time = 0.12, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1816, 211}

$$\frac{bc-ad}{9a^2x^9}-\frac{a^2e-abd+b^2c}{7a^3x^7}+\frac{b^{5/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{a^{13/2}}+\frac{b^2(a^3(-f)+a^2be-ab^2d+b^3c)}{a^6x}-\frac{b(a^3(-f)+a^2be-ab^2d+b^3c)}{3a^5x^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{5a^4x^5}-\frac{c}{11ax^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x]
```

```
[Out] -1/11*c/(a*x^11) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(7*a^3
*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) - (b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f))/(3*a^5*x^3) + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*
f))/(a^6*x) + (b^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/a^(13/2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}\left(a+bx^2\right)}\,dx = \int \left(\frac{c}{ax^{12}} + \frac{-bc+ad}{a^2x^{10}} + \frac{b^2c-abd+a^2e}{a^3x^8} + \frac{-b^3c+ab^2d-a^2be+a^3f}{a^4x^6} - \right.$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{9a^2x^9} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} -$$

$$= -\frac{c}{11ax^{11}} + \frac{bc-ad}{9a^2x^9} - \frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5} -$$

Mathematica [A]
time = 0.12, size = 211, normalized size = 1.00

$$-\frac{c}{11ax^{11}}+\frac{bc-ad}{9a^2x^9}-\frac{b^2c-abd+a^2e}{7a^3x^7}+\frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5}+\frac{b(-b^3c+ab^2d-a^2be+a^3f)}{3a^5x^3}+\frac{b^2(b^3c-ab^2d+a^2be-a^3f)}{a^6x}+\frac{b^{5/2}(b^3c-ab^2d+a^2be-a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x]
```

```
[Out] -1/11*c/(a*x^11) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(7*a^3
*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) + (b*(-(b^3*c) + a*
b^2*d - a^2*b*e + a^3*f))/(3*a^5*x^3) + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a
^3*f))/(a^6*x) + (b^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/a^(13/2)
```

Maple [A]
time = 0.13, size = 201, normalized size = 0.95

method	result
default	$-\frac{b^3\left(a^3f-a^2be+a\,b^2d-b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^6\sqrt{ab}}-\frac{c}{11a\,x^{11}}-\frac{ad-bc}{9a^2x^9}-\frac{a^2e-abd+b^2c}{7a^3x^7}-\frac{a^3f-a^2be+a\,b^2d-b^3c}{5a^4x^5}-$
risch	$\frac{-\frac{b^2\left(a^3f-a^2be+a\,b^2d-b^3c\right)x^{10}}{a^6}+\frac{b\left(a^3f-a^2be+a\,b^2d-b^3c\right)x^8}{3a^5}-\frac{\left(a^3f-a^2be+a\,b^2d-b^3c\right)x^6}{5a^4}-\frac{\left(a^2e-abd+b^2c\right)x^4}{7a^3}-\frac{(ad-bc)x^2}{9a^2}-\frac{c}{11a}}{x^{11}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -b^3*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^6/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-
1/11*c/a/x^11-1/9*(a*d-b*c)/a^2/x^9-1/7*(a^2*e-a*b*d+b^2*c)/a^3/x^7-1/5*(a^
3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^5-b^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^6/x+
1/3*b*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5/x^3
```

Maxima [A]
time = 0.50, size = 219, normalized size = 1.04

$$\frac{(b^6c-ab^5d-a^3b^3f+a^2b^4e)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^6}+\frac{3465\,(b^5c-ab^4d-a^3b^2f+a^2b^3e)x^{10}-1155\,(ab^4c-a^2b^3d-a^4bf+a^3b^2e)x^8+693\,(a^2b^3c-a^3b^2d-a^5f+a^4be)x^6-315\,a^5c-495\,(a^3b^2c-a^4bd+a^5e)x^4+385\,(a^4bc-a^5d)x^2}{3465\,a^6x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] (b^6*c - a*b^5*d - a^3*b^3*f + a^2*b^4*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
a^6) + 1/3465*(3465*(b^5*c - a*b^4*d - a^3*b^2*f + a^2*b^3*e)*x^10 - 1155*(
a*b^4*c - a^2*b^3*d - a^4*b*f + a^3*b^2*e)*x^8 + 693*(a^2*b^3*c - a^3*b^2*d
 - a^5*f + a^4*b*e)*x^6 - 315*a^5*c - 495*(a^3*b^2*c - a^4*b*d + a^5*e)*x^4
 + 385*(a^4*b*c - a^5*d)*x^2)/(a^6*x^11)
```

Fricas [A]
time = 4.18, size = 504, normalized size = 2.39

$$\left[\frac{6930\,(b^5c-ab^4d-a^3b^2f)x^{10}-2310\,(ab^4c-a^2b^3d-a^4bf)x^8+1386\,(a^2b^3c-a^3b^2d-a^5f)x^6-630\,a^5c-990\,(a^3b^2c-a^4bd)x^4+770\,(a^4bc-a^5d)x^2+3465\,(a^2b^3x^{11}e+(b^5c-ab^4d-a^3b^2f)x^{11})\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+66\,(105\,a^2b^3x^{10}-35\,a^3b^2x^8+21\,a^4bx^6-15\,a^5x^4)e}{6930\,a^6x^{11}},\frac{3465\,(b^5c-ab^4d-a^3b^2f)x^{10}-1155\,(ab^4c-a^2b^3d-a^4bf)x^8+693\,(a^2b^3c-a^3b^2d-a^5f)x^6-315\,a^5c-495\,(a^3b^2c-a^4bd)x^4+385\,(a^4bc-a^5d)x^2+3465\,(a^2b^3x^{11}e+(b^5c-ab^4d-a^3b^2f)x^{11})\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+33\,(105\,a^2b^3x^{10}-35\,a^3b^2x^8+21\,a^4bx^6-15\,a^5x^4)e}{3465\,a^6x^{11}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6930*(6930*(b^5*c - a*b^4*d - a^3*b^2*f)*x^10 - 2310*(a*b^4*c - a^2*b^3*
d - a^4*b*f)*x^8 + 1386*(a^2*b^3*c - a^3*b^2*d - a^5*f)*x^6 - 630*a^5*c - 9
90*(a^3*b^2*c - a^4*b*d)*x^4 + 770*(a^4*b*c - a^5*d)*x^2 + 3465*(a^2*b^3*x^
11*e + (b^5*c - a*b^4*d - a^3*b^2*f)*x^11)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sq
rt(-b/a) - a)/(b*x^2 + a)) + 66*(105*a^2*b^3*x^10 - 35*a^3*b^2*x^8 + 21*a^4
*b*x^6 - 15*a^5*x^4)*e)/(a^6*x^11), 1/3465*(3465*(b^5*c - a*b^4*d - a^3*b^2
*f)*x^10 - 1155*(a*b^4*c - a^2*b^3*d - a^4*b*f)*x^8 + 693*(a^2*b^3*c - a^3*
b^2*d - a^5*f)*x^6 - 315*a^5*c - 495*(a^3*b^2*c - a^4*b*d)*x^4 + 385*(a^4*b
*c - a^5*d)*x^2 + 3465*(a^2*b^3*x^11*e + (b^5*c - a*b^4*d - a^3*b^2*f)*x^11
)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 33*(105*a^2*b^3*x^10 - 35*a^3*b^2*x^8 + 2
1*a^4*b*x^6 - 15*a^5*x^4)*e)/(a^6*x^11)]
```

Sympy [A]
time = 40.34, size = 398, normalized size = 1.89

$$\frac{\sqrt{-\frac{b^5}{a^{13}}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(-\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}\,(a^3f-a^2be+ab^2d-b^3c)}{a^3b^3f-a^2b^4e+ab^5d-b^6c}+x\right)}{2}-\frac{\sqrt{-\frac{b^5}{a^{13}}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}\,(a^3f-a^2be+ab^2d-b^3c)}{a^3b^3f-a^2b^4e+ab^5d-b^6c}+x\right)}{2}+\frac{-315a^5c+x^{10}(-3465a^3b^2f+3465a^2b^3e-3465ab^4d+3465b^5c)+x^8\cdot(1155a^4bf-1155a^3b^2e+1155a^2b^3d-1155ab^4c)+x^6(-693a^5f+693a^4be-693a^3b^2d+693a^2b^3c)+x^4(-495a^5e+495a^4bd-495a^3b^2c)+x^2(-385a^5d+385a^4bc)}{3465a^6x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**12/(b*x**2+a),x)
```

```
[Out] sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**7*sqrt(-b
**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b**3*f - a**2*b**4
*e + a*b**5*d - b**6*c) + x)/2 - sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b
```

```
**2*d - b**3*c)*log(a**7*sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d -
b**3*c)/(a**3*b**3*f - a**2*b**4*e + a*b**5*d - b**6*c) + x)/2 + (-315*a**5
*c + x**10*(-3465*a**3*b**2*f + 3465*a**2*b**3*e - 3465*a*b**4*d + 3465*b**
5*c) + x**8*(1155*a**4*b*f - 1155*a**3*b**2*e + 1155*a**2*b**3*d - 1155*a*b
**4*c) + x**6*(-693*a**5*f + 693*a**4*b*e - 693*a**3*b**2*d + 693*a**2*b**3
*c) + x**4*(-495*a**5*e + 495*a**4*b*d - 495*a**3*b**2*c) + x**2*(-385*a**5
*d + 385*a**4*b*c))/(3465*a**6*x**11)
```

Giac [A]
time = 3.08, size = 249, normalized size = 1.18

$$\frac{(b^6c-ab^5d-a^3b^3f+a^2b^4e)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^6}+\frac{3465\,b^5cx^{10}-3465\,ab^4dx^{10}-3465\,a^3b^2fx^{10}+3465\,a^2b^3x^{10}e-1155\,ab^4cx^8+1155\,a^2b^3dx^8+1155\,a^4bfx^8-1155\,a^3b^2x^8e+693\,a^2b^3cx^6-693\,a^3b^2dx^6-693\,a^5fx^6+693\,a^4bx^6e-495\,a^3b^2cx^4+495\,a^4bdx^4-495\,a^5x^4e+385\,a^4bcx^2-385\,a^5dx^2-315\,a^5c}{3465\,a^6x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="giac")
```

```
[Out] (b^6*c - a*b^5*d - a^3*b^3*f + a^2*b^4*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
a^6) + 1/3465*(3465*b^5*c*x^10 - 3465*a*b^4*d*x^10 - 3465*a^3*b^2*f*x^10 +
3465*a^2*b^3*x^10*e - 1155*a*b^4*c*x^8 + 1155*a^2*b^3*d*x^8 + 1155*a^4*b*f*
x^8 - 1155*a^3*b^2*x^8*e + 693*a^2*b^3*c*x^6 - 693*a^3*b^2*d*x^6 - 693*a^5*
f*x^6 + 693*a^4*b*x^6*e - 495*a^3*b^2*c*x^4 + 495*a^4*b*d*x^4 - 495*a^5*x^4
*e + 385*a^4*b*c*x^2 - 385*a^5*d*x^2 - 315*a^5*c)/(a^6*x^11)
```

Mupad [B]
time = 0.99, size = 197, normalized size = 0.93

$$\frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{13/2}}-\frac{\frac{c}{11\,a}-\frac{x^6\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{5\,a^4}+\frac{x^2\,\left(a\,d-b\,c\right)}{9\,a^2}+\frac{x^4\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{7\,a^3}+\frac{b\,x^8\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^5}-\frac{b^2\,x^{10}\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^6}}{x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x)
```

```
[Out] (b^(5/2)*atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^(
13/2) - (c/(11*a) - (x^6*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(5*a^4) + (x^
2*(a*d - b*c))/(9*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/(7*a^3) + (b*x^8*(b^
3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^5) - (b^2*x^10*(b^3*c - a^3*f - a*b^
2*d + a^2*b*e))/a^6)/x^11
```

3.124 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=240

$$-\frac{a(5b^3c-7ab^2d+9a^2be-11a^3f)\,x}{2b^6}+\frac{(5b^3c-7ab^2d+9a^2be-11a^3f)\,x^3}{6b^5}-\frac{(5b^3c-7ab^2d+9a^2be-11a^3f)}{10ab^4}$$

```
[Out] -1/2*a*(-11*a^3*f+9*a^2*b*e-7*a*b^2*d+5*b^3*c)*x/b^6+1/6*(-11*a^3*f+9*a^2*b
*e-7*a*b^2*d+5*b^3*c)*x^3/b^5-1/10*(-11*a^3*f+9*a^2*b*e-7*a*b^2*d+5*b^3*c)*
x^5/a/b^4+1/7*(-2*a*f+b*e)*x^7/b^3+1/9*f*x^9/b^2+1/2*(c-a*(a^2*f-a*b*e+b^2*
d)/b^3)*x^7/a/(b*x^2+a)+1/2*a^(3/2)*(-11*a^3*f+9*a^2*b*e-7*a*b^2*d+5*b^3*c)
*arctan(x*b^(1/2)/a^(1/2))/b^(13/2)
```

Rubi [A]
time = 0.19, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1818, 1599, 1275, 211}

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a\,(a+bx^2)}-\frac{ax(-11a^3f+9a^2be-7ab^2d+5b^3c)}{2b^6}+\frac{x^3(-11a^3f+9a^2be-7ab^2d+5b^3c)}{6b^5}-\frac{x^5(-11a^3f+9a^2be-7ab^2d+5b^3c)}{10ab^4}+\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(-11a^3f+9a^2be-7ab^2d+5b^3c)}{2b^{13/2}}+\frac{x^7(be-2af)}{7b^3}+\frac{fx^9}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] -1/2*(a*(5*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x)/b^6 + ((5*b^3*c - 7
*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x^3)/(6*b^5) - ((5*b^3*c - 7*a*b^2*d + 9*a
^2*b*e - 11*a^3*f)*x^5)/(10*a*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^9)/
(9*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^7)/(2*a*(a + b*x^2)) + (
a^(3/2)*(5*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(2*b^(13/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{2a\,(a+bx^2)} - \frac{\int \frac{x^5\left(\left(5bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}\right)x-2a\left(e-\frac{af}{b}\right)x^3-2afx^5\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{2a\,(a+bx^2)} - \frac{\int \frac{x^6\left(5bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}-2a\left(e-\frac{af}{b}\right)x^2-2afx^4\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{2a\,(a+bx^2)} - \frac{\int \left(\frac{a^2(5b^3c-7ab^2d+9a^2be-11a^3f)}{b^5} - \frac{a(5b^3c-7ab^2d+9a^2be-1}{b^4}\right.}{}$$

$$= -\frac{a(5b^3c-7ab^2d+9a^2be-11a^3f)\,x}{2b^6} + \frac{(5b^3c-7ab^2d+9a^2be-11a^3f)\,x^3}{6b^5}$$

$$= -\frac{a(5b^3c-7ab^2d+9a^2be-11a^3f)\,x}{2b^6} + \frac{(5b^3c-7ab^2d+9a^2be-11a^3f)\,x^3}{6b^5}$$

Mathematica [A]
time = 0.09, size = 227, normalized size = 0.95

$$\frac{a(-2b^3c+3ab^2d-4a^2be+5a^3f)\,x}{b^6}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^3}{3b^5}+\frac{(b^2d-2abe+3a^2f)\,x^5}{5b^4}+\frac{(be-2af)x^7}{7b^3}+\frac{fx^9}{9b^2}-\frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\,x}{2b^6\,(a+bx^2)}-\frac{a^{3/2}(-5b^3c+7ab^2d-9a^2be+11a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] (a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x)/b^6 + ((b^3*c - 2*a*b^2*
d + 3*a^2*b*e - 4*a^3*f)*x^3)/(3*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^5)/(
5*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^9)/(9*b^2) - ((a^2*b^3*c - a^3*
b^2*d + a^4*b*e - a^5*f)*x)/(2*b^6*(a + b*x^2)) - (a^(3/2)*(-5*b^3*c + 7*a*
b^2*d - 9*a^2*b*e + 11*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(13/2))
```

Maple [A]

time = 0.15, size = 229, normalized size = 0.95

method	result
default	$\frac{\frac{1}{9}f\,x^9b^4-\frac{2}{7}a\,b^3f\,x^7+\frac{1}{7}b^4e\,x^7+\frac{3}{5}a^2b^2f\,x^5-\frac{2}{5}a\,b^3e\,x^5+\frac{1}{5}b^4d\,x^5-\frac{4}{3}a^3bf\,x^3+a^2b^2e\,x^3-\frac{2}{3}a\,b^3d\,x^3+\frac{1}{3}b^4c\,x^3+5a^4fx-4a^3bex+3a^2b^2dx}{b^6}$
risch	$\frac{f\,x^9}{9b^2}-\frac{2af\,x^7}{7b^3}+\frac{e\,x^7}{7b^2}+\frac{3a^2f\,x^5}{5b^4}-\frac{2ae\,x^5}{5b^3}+\frac{d\,x^5}{5b^2}-\frac{4a^3f\,x^3}{3b^5}+\frac{a^2e\,x^3}{b^4}-\frac{2ad\,x^3}{3b^3}+\frac{c\,x^3}{3b^2}+\frac{5a^4fx}{b^6}-\frac{4a^3ex}{b^5}+\frac{3a^2dx}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^6*(1/9*f*x^9*b^4-2/7*a*b^3*f*x^7+1/7*b^4*e*x^7+3/5*a^2*b^2*f*x^5-2/5*a*
b^3*e*x^5+1/5*b^4*d*x^5-4/3*a^3*b*f*x^3+a^2*b^2*e*x^3-2/3*a*b^3*d*x^3+1/3*b
^4*c*x^3+5*a^4*f*x-4*a^3*b*e*x+3*a^2*b^2*d*x-2*a*b^3*c*x)-a^2/b^6*((-1/2*a^
3*f+1/2*a^2*b*e-1/2*a*b^2*d+1/2*b^3*c)*x/(b*x^2+a)+1/2*(11*a^3*f-9*a^2*b*e+
7*a*b^2*d-5*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]

time = 0.52, size = 234, normalized size = 0.98

$$-\frac{(a^2b^3c-a^3b^2d-a^5f+a^4be)x}{2\,(b^7x^2+ab^6)}+\frac{(5\,a^2b^3c-7\,a^3b^2d-11\,a^5f+9\,a^4be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^6}+\frac{35\,b^4fx^9-45\,(2\,ab^3f-b^4e)x^7+63\,(b^4d+3\,a^2b^2f-2\,ab^3e)x^5+105\,(b^4c-2\,ab^3d-4\,a^3bf+3\,a^2b^2e)x^3-315\,(2\,ab^3c-3\,a^2b^2d-5\,a^4f+4\,a^3be)x}{315\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*x/(b^7*x^2 + a*b^6) + 1/2*(5
*a^2*b^3*c - 7*a^3*b^2*d - 11*a^5*f + 9*a^4*b*e)*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*b^6) + 1/315*(35*b^4*f*x^9 - 45*(2*a*b^3*f - b^4*e)*x^7 + 63*(b^4*d
+ 3*a^2*b^2*f - 2*a*b^3*e)*x^5 + 105*(b^4*c - 2*a*b^3*d - 4*a^3*b*f + 3*a^2
*b^2*e)*x^3 - 315*(2*a*b^3*c - 3*a^2*b^2*d - 5*a^4*f + 4*a^3*b*e)*x)/b^6
```

Fricas [A]

time = 5.45, size = 600, normalized size = 2.50

$$\left[\frac{140\,b^5fx^{11}-220\,ab^4fx^9+36\,(7\,b^5d+11\,a^2b^3f)x^7+84\,(5\,b^5c-7\,ab^4d-11\,a^3b^2f)x^5-420\,(5\,ab^4c-7\,a^2b^3d-11\,a^4bf)x^3+315\,(5\,a^2b^3c-7\,a^3b^2d-11\,a^5f+(5\,ab^4c-7\,a^2b^3d-11\,a^4bf)x^2+9\,(a^3b^2x^2+a^4b)e)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-630\,(5\,a^2b^3c-7\,a^3b^2d-11\,a^5f)x+18\,(10\,b^5x^9-18\,ab^4x^7+42\,a^2b^3x^5-210\,a^3b^2x^3-315\,a^4bx)e}{1260\,(b^7x^2+ab^6)},\ \frac{70\,b^5fx^{11}-110\,ab^4fx^9+18\,(7\,b^5d+11\,a^2b^3f)x^7+42\,(5\,b^5c-7\,ab^4d-11\,a^3b^2f)x^5-210\,(5\,ab^4c-7\,a^2b^3d-11\,a^4bf)x^3+315\,(5\,a^2b^3c-7\,a^3b^2d-11\,a^5f+(5\,ab^4c-7\,a^2b^3d-11\,a^4bf)x^2+9\,(a^3b^2x^2+a^4b)e)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)-315\,(5\,a^2b^3c-7\,a^3b^2d-11\,a^5f)x+9\,(10\,b^5x^9-18\,ab^4x^7+42\,a^2b^3x^5-210\,a^3b^2x^3-315\,a^4bx)e}{630\,(b^7x^2+ab^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/1260*(140*b^5*f*x^11 - 220*a*b^4*f*x^9 + 36*(7*b^5*d + 11*a^2*b^3*f)*x^7
 + 84*(5*b^5*c - 7*a*b^4*d - 11*a^3*b^2*f)*x^5 - 420*(5*a*b^4*c - 7*a^2*b^3
*d - 11*a^4*b*f)*x^3 + 315*(5*a^2*b^3*c - 7*a^3*b^2*d - 11*a^5*f + (5*a*b^4
*c - 7*a^2*b^3*d - 11*a^4*b*f)*x^2 + 9*(a^3*b^2*x^2 + a^4*b)*e)*sqrt(-a/b)*
log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 630*(5*a^2*b^3*c - 7*a^3*
b^2*d - 11*a^5*f)*x + 18*(10*b^5*x^9 - 18*a*b^4*x^7 + 42*a^2*b^3*x^5 - 210*
a^3*b^2*x^3 - 315*a^4*b*x)*e)/(b^7*x^2 + a*b^6), 1/630*(70*b^5*f*x^11 - 110
*a*b^4*f*x^9 + 18*(7*b^5*d + 11*a^2*b^3*f)*x^7 + 42*(5*b^5*c - 7*a*b^4*d -
11*a^3*b^2*f)*x^5 - 210*(5*a*b^4*c - 7*a^2*b^3*d - 11*a^4*b*f)*x^3 + 315*(5
*a^2*b^3*c - 7*a^3*b^2*d - 11*a^5*f + (5*a*b^4*c - 7*a^2*b^3*d - 11*a^4*b*f
)*x^2 + 9*(a^3*b^2*x^2 + a^4*b)*e)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 315*
(5*a^2*b^3*c - 7*a^3*b^2*d - 11*a^5*f)*x + 9*(10*b^5*x^9 - 18*a*b^4*x^7 + 4
2*a^2*b^3*x^5 - 210*a^3*b^2*x^3 - 315*a^4*b*x)*e)/(b^7*x^2 + a*b^6)]
```

Sympy [A]
time = 1.21, size = 444, normalized size = 1.85

$$x^{7}\left(-\frac{2af}{7b^{3}}+\frac{e}{7b^{2}}\right)+x^{5}\cdot\left(\frac{3a^{2}f}{5b^{4}}-\frac{2ae}{5b^{3}}+\frac{d}{5b^{2}}\right)+x^{3}\left(-\frac{4a^{3}f}{3b^{5}}+\frac{a^{2}e}{b^{4}}-\frac{2ad}{3b^{3}}+\frac{c}{3b^{2}}\right)+x\left(\frac{5a^{4}f}{b^{6}}-\frac{4a^{3}e}{b^{5}}+\frac{3a^{2}d}{b^{4}}-\frac{2ac}{b^{3}}\right)+\frac{x(a^{5}f-a^{4}be+a^{3}b^{2}d-a^{2}b^{3}c)}{2ab^{6}+2b^{7}x^{2}}+\frac{\sqrt{-\frac{a^{3}}{b^{13}}}\cdot(11a^{3}f-9a^{2}be+7ab^{2}d-5b^{3}c)\log\left(-\frac{b^{6}\sqrt{-\frac{a^{3}}{b^{13}}}\cdot(11a^{3}f-9a^{2}be+7ab^{2}d-5b^{3}c)}{11a^{4}f-9a^{3}be+7a^{2}b^{2}d-5ab^{3}c}+x\right)}{4}-\frac{\sqrt{-\frac{a^{3}}{b^{13}}}\cdot(11a^{3}f-9a^{2}be+7ab^{2}d-5b^{3}c)\log\left(\frac{b^{6}\sqrt{-\frac{a^{3}}{b^{13}}}\cdot(11a^{3}f-9a^{2}be+7ab^{2}d-5b^{3}c)}{11a^{4}f-9a^{3}be+7a^{2}b^{2}d-5ab^{3}c}+x\right)}{4}+\frac{fx^{9}}{9b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x**7*(-2*a*f/(7*b**3) + e/(7*b**2)) + x**5*(3*a**2*f/(5*b**4) - 2*a*e/(5*b*
*3) + d/(5*b**2)) + x**3*(-4*a**3*f/(3*b**5) + a**2*e/b**4 - 2*a*d/(3*b**3)
 + c/(3*b**2)) + x*(5*a**4*f/b**6 - 4*a**3*e/b**5 + 3*a**2*d/b**4 - 2*a*c/b
**3) + x*(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(2*a*b**6 + 2*b**7
*x**2) + sqrt(-a**3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)
*log(-b**6*sqrt(-a**3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*
c)/(11*a**4*f - 9*a**3*b*e + 7*a**2*b**2*d - 5*a*b**3*c) + x)/4 - sqrt(-a**
3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)*log(b**6*sqrt(-a*
*3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)/(11*a**4*f - 9*a
**3*b*e + 7*a**2*b**2*d - 5*a*b**3*c) + x)/4 + f*x**9/(9*b**2)
```

Giac [A]
time = 1.67, size = 252, normalized size = 1.05

$$\frac{(5\,a^{2}b^{3}c-7\,a^{3}b^{2}d-11\,a^{5}f+9\,a^{4}be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{6}}-\frac{a^{2}b^{3}cx-a^{3}b^{2}dx-a^{5}fx+a^{4}bxe}{2\,(bx^{2}+a)b^{6}}+\frac{35\,b^{16}fx^{9}-90\,ab^{15}fx^{7}+45\,b^{16}x^{7}e+63\,b^{16}dx^{5}+189\,a^{2}b^{14}fx^{5}-126\,ab^{15}x^{5}e+105\,b^{16}cx^{3}-210\,ab^{15}dx^{3}-420\,a^{3}b^{13}fx^{3}+315\,a^{2}b^{14}x^{3}e-630\,ab^{15}cx+945\,a^{2}b^{14}dx+1575\,a^{4}b^{12}fx-1260\,a^{3}b^{13}xe}{315\,b^{18}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*a^2*b^3*c - 7*a^3*b^2*d - 11*a^5*f + 9*a^4*b*e)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^6) - 1/2*(a^2*b^3*c*x - a^3*b^2*d*x - a^5*f*x + a^4*b*x*e)/(
```

```
(b*x^2 + a)*b^6) + 1/315*(35*b^16*f*x^9 - 90*a*b^15*f*x^7 + 45*b^16*x^7*e +
 63*b^16*d*x^5 + 189*a^2*b^14*f*x^5 - 126*a*b^15*x^5*e + 105*b^16*c*x^3 - 2
10*a*b^15*d*x^3 - 420*a^3*b^13*f*x^3 + 315*a^2*b^14*x^3*e - 630*a*b^15*c*x
+ 945*a^2*b^14*d*x + 1575*a^4*b^12*f*x - 1260*a^3*b^13*x*e)/b^18
```

Mupad [B]
time = 0.10, size = 413, normalized size = 1.72

$$x^7\left(\frac{e}{7\,b^2}-\frac{2\,a\,f}{7\,b^3}\right)-x\left(\frac{2\,a\left(\frac{c}{b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)}{b}-\frac{a^2\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b^2}\right)-x^5\left(\frac{a^2\,f}{5\,b^4}-\frac{d}{5\,b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{5\,b}\right)+x^3\left(\frac{c}{3\,b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{3\,b}\right)+\frac{f\,x^9}{9\,b^2}+\frac{x\left(\frac{f\,a^5}{2}-\frac{e\,a^4\,b}{2}+\frac{d\,a^3\,b^2}{2}-\frac{c\,a^2\,b^3}{2}\right)}{b^7\,x^2+a\,b^6}-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,\left(-11\,f\,a^3+9\,e\,a^2\,b-7\,d\,a\,b^2+5\,c\,b^3\right)}{11\,f\,a^5-9\,e\,a^4\,b+7\,d\,a^3\,b^2-5\,c\,a^2\,b^3}\right)\,\left(-11\,f\,a^3+9\,e\,a^2\,b-7\,d\,a\,b^2+5\,c\,b^3\right)}{2\,b^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x)
```

```
[Out] x^7*(e/(7*b^2) - (2*a*f)/(7*b^3)) - x*((2*a*(c/b^2 - (a^2*(e/b^2 - (2*a*f)/
b^3))/b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b))
/b - (a^2*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b^2) - x^5
*((a^2*f)/(5*b^4) - d/(5*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(5*b)) + x^3*(c
/(3*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))/(3*b^2) + (2*a*((a^2*f)/b^4 - d/b^2
+ (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(3*b)) + (f*x^9)/(9*b^2) + (x*((a^5*f)/2
- (a^2*b^3*c)/2 + (a^3*b^2*d)/2 - (a^4*b*e)/2))/(a*b^6 + b^7*x^2) - (a^(3/2
)*atan((a^(3/2)*b^(1/2)*x*(5*b^3*c - 11*a^3*f - 7*a*b^2*d + 9*a^2*b*e))/(11
*a^5*f - 5*a^2*b^3*c + 7*a^3*b^2*d - 9*a^4*b*e))*(5*b^3*c - 11*a^3*f - 7*a*
b^2*d + 9*a^2*b*e))/(2*b^(13/2))
```

3.125 $\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=202

$$\frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x}{2b^5}-\frac{\left(3b^3c-5ab^2d+7a^2be-9a^3f\right)x^3}{6ab^4}+\frac{(be-2af)x^5}{5b^3}+\frac{fx^7}{7b^2}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2\right.}{b^3}\right.}{2a\left(a+bx^2\right)}$$

```
[Out] 1/2*(-9*a^3*f+7*a^2*b*e-5*a*b^2*d+3*b^3*c)*x/b^5-1/6*(-9*a^3*f+7*a^2*b*e-5*
a*b^2*d+3*b^3*c)*x^3/a/b^4+1/5*(-2*a*f+b*e)*x^5/b^3+1/7*f*x^7/b^2+1/2*(c-a*
(a^2*f-a*b*e+b^2*d)/b^3)*x^5/a/(b*x^2+a)-1/2*(-9*a^3*f+7*a^2*b*e-5*a*b^2*d+
3*b^3*c)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(11/2)
```

Rubi [A]
time = 0.16, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1818, 1599, 1275, 211}

$$\frac{x^5\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-9a^3f+7a^2be-5ab^2d+3b^3c\right)}{2b^{11/2}}+\frac{x\left(-9a^3f+7a^2be-5ab^2d+3b^3c\right)}{2b^5}-\frac{x^3\left(-9a^3f+7a^2be-5ab^2d+3b^3c\right)}{6ab^4}+\frac{x^5(be-2af)}{5b^3}+\frac{fx^7}{7b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((3*b^3*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*x)/(2*b^5) - ((3*b^3*c - 5*a*b
^2*d + 7*a^2*b*e - 9*a^3*f)*x^3)/(6*a*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) +
(f*x^7)/(7*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^5)/(2*a*(a + b*x
^2)) - (Sqrt[a]*(3*b^3*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*ArcTan[(Sqrt[b]
*x)/Sqrt[a]])/(2*b^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
```

```
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{2a\,(a+bx^2)} - \frac{\int \frac{x^3\left(\left(3bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}\right)x-2a\left(e-\frac{af}{b}\right)x^3-2afx^5\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{2a\,(a+bx^2)} - \frac{\int \frac{x^4\left(3bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}-2a\left(e-\frac{af}{b}\right)x^2-2afx^4\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{2a\,(a+bx^2)} - \frac{\int \Bigg(-\frac{a(3b^3c-5ab^2d+7a^2be-9a^3f)}{b^4} + \frac{(3b^3c-5ab^2d+7a^2be-}{b^3}}{}$$

$$= \frac{(3b^3c-5ab^2d+7a^2be-9a^3f)\,x}{2b^5} - \frac{(3b^3c-5ab^2d+7a^2be-9a^3f)\,x^3}{6ab^4} +$$

$$= \frac{(3b^3c-5ab^2d+7a^2be-9a^3f)\,x}{2b^5} - \frac{(3b^3c-5ab^2d+7a^2be-9a^3f)\,x^3}{6ab^4} +$$

Mathematica [A]
time = 0.07, size = 187, normalized size = 0.93

$$\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x}{b^5} + \frac{(b^2d-2abe+3a^2f)\,x^3}{3b^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^7}{7b^2} + \frac{(ab^3c-a^2b^2d+a^3be-a^4f)\,x}{2b^5\,(a+bx^2)} + \frac{\sqrt{a}\,(-3b^3c+5ab^2d-7a^2be+9a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 + ((b^2*d - 2*a*b*e + 3*a
^2*f)*x^3)/(3*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) + (f*x^7)/(7*b^2) + ((a*b^
```

```
3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/(2*b^5*(a + b*x^2)) + (Sqrt[a]*(-3*b^
3*c + 5*a*b^2*d - 7*a^2*b*e + 9*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(1
1/2))
```

Maple [A]
time = 0.15, size = 182, normalized size = 0.90

method	result
default	$-\frac{-\frac{1}{7}f\,x^7b^3+\frac{2}{5}a\,b^2f\,x^5-\frac{1}{5}b^3e\,x^5-a^2bf\,x^3+\frac{2}{3}a\,b^2e\,x^3-\frac{1}{3}b^3d\,x^3+4a^3fx-3a^2bex+2a\,b^2dx-b^3cx}{b^5}+\frac{a\left(\frac{\left(-\frac{1}{2}a^3f+\frac{1}{2}a^2be-\frac{1}{2}a\,b^2d+\frac{1}{2}b^3}{b\,x^2+a}\right.}{}$
risch	$\frac{f\,x^7}{7b^2}-\frac{2af\,x^5}{5b^3}+\frac{e\,x^5}{5b^2}+\frac{a^2f\,x^3}{b^4}-\frac{2ae\,x^3}{3b^3}+\frac{d\,x^3}{3b^2}-\frac{4a^3fx}{b^5}+\frac{3a^2ex}{b^4}-\frac{2adx}{b^3}+\frac{cx}{b^2}+\frac{\left(-\frac{1}{2}a^4f+\frac{1}{2}a^3be-\frac{1}{2}a^2b^2d+\frac{1}{2}a\,b^3c\right)}{b^5(b\,x^2+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^5*(-1/7*f*x^7*b^3+2/5*a*b^2*f*x^5-1/5*b^3*e*x^5-a^2*b*f*x^3+2/3*a*b^2*
e*x^3-1/3*b^3*d*x^3+4*a^3*f*x-3*a^2*b*e*x+2*a*b^2*d*x-b^3*c*x)+a/b^5*((-1/2
*a^3*f+1/2*a^2*b*e-1/2*a*b^2*d+1/2*b^3*c)*x/(b*x^2+a)+1/2*(9*a^3*f-7*a^2*b*
e+5*a*b^2*d-3*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.52, size = 189, normalized size = 0.94

$$\frac{(ab^3c-a^2b^2d-a^4f+a^3be)x}{2\,(b^6x^2+ab^5)}-\frac{(3\,ab^3c-5\,a^2b^2d-9\,a^4f+7\,a^3be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^5}+\frac{15\,b^3fx^7-21\,(2\,ab^2f-b^3e)x^5+35\,(b^3d+3\,a^2bf-2\,ab^2e)x^3+105\,(b^3c-2\,ab^2d-4\,a^3f+3\,a^2be)x}{105\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*x/(b^6*x^2 + a*b^5) - 1/2*(3*a*
b^3*c - 5*a^2*b^2*d - 9*a^4*f + 7*a^3*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^5) + 1/105*(15*b^3*f*x^7 - 21*(2*a*b^2*f - b^3*e)*x^5 + 35*(b^3*d + 3*a^
2*b*f - 2*a*b^2*e)*x^3 + 105*(b^3*c - 2*a*b^2*d - 4*a^3*f + 3*a^2*b*e)*x)/b
^5
```

Fricas [A]
time = 4.25, size = 502, normalized size = 2.49

$$\left[\frac{60\,b^4fx^9-108\,ab^3fx^7+28\,(5\,b^4d+9\,a^2b^2f)x^5+140\,(3\,b^4c-5\,ab^3d-9\,a^3bf)x^3+105\,(3\,ab^3c-5\,a^2b^2d-9\,a^4f+(3\,b^4c-5\,ab^3d-9\,a^3bf)x^2+7\,(a^2b^2x^2+a^3b)e)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+210\,(3\,ab^3c-5\,a^2b^2d-9\,a^4f)x+14\,(6\,b^4x^7-14\,ab^3x^5+70\,a^2b^2x^3+105\,a^3bx)e}{420\,(b^6x^2+ab^5)},\ \frac{30\,b^4fx^9-54\,ab^3fx^7+14\,(5\,b^4d+9\,a^2b^2f)x^5+70\,(3\,b^4c-5\,ab^3d-9\,a^3bf)x^3-105\,(3\,ab^3c-5\,a^2b^2d-9\,a^4f+(3\,b^4c-5\,ab^3d-9\,a^3bf)x^2+7\,(a^2b^2x^2+a^3b)e)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+105\,(3\,ab^3c-5\,a^2b^2d-9\,a^4f)x+7\,(6\,b^4x^7-14\,ab^3x^5+70\,a^2b^2x^3+105\,a^3bx)e}{210\,(b^6x^2+ab^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(60*b^4*f*x^9 - 108*a*b^3*f*x^7 + 28*(5*b^4*d + 9*a^2*b^2*f)*x^5 + 1
40*(3*b^4*c - 5*a*b^3*d - 9*a^3*b*f)*x^3 + 105*(3*a*b^3*c - 5*a^2*b^2*d - 9
*a^4*f + (3*b^4*c - 5*a*b^3*d - 9*a^3*b*f)*x^2 + 7*(a^2*b^2*x^2 + a^3*b)*e)
*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210*(3*a*b^3*
c - 5*a^2*b^2*d - 9*a^4*f)*x + 14*(6*b^4*x^7 - 14*a*b^3*x^5 + 70*a^2*b^2*x^
3 + 105*a^3*b*x)*e)/(b^6*x^2 + a*b^5), 1/210*(30*b^4*f*x^9 - 54*a*b^3*f*x^7
 + 14*(5*b^4*d + 9*a^2*b^2*f)*x^5 + 70*(3*b^4*c - 5*a*b^3*d - 9*a^3*b*f)*x^
3 - 105*(3*a*b^3*c - 5*a^2*b^2*d - 9*a^4*f + (3*b^4*c - 5*a*b^3*d - 9*a^3*b
*f)*x^2 + 7*(a^2*b^2*x^2 + a^3*b)*e)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 10
5*(3*a*b^3*c - 5*a^2*b^2*d - 9*a^4*f)*x + 7*(6*b^4*x^7 - 14*a*b^3*x^5 + 70*
a^2*b^2*x^3 + 105*a^3*b*x)*e)/(b^6*x^2 + a*b^5)]
```

Sympy [A]

time = 1.14, size = 257, normalized size = 1.27

$$x^{5}\left(-\frac{2af}{5b^{3}}+\frac{e}{5b^{2}}\right)+x^{3}\left(\frac{a^{2}f}{b^{4}}-\frac{2ae}{3b^{3}}+\frac{d}{3b^{2}}\right)+x\left(-\frac{4a^{3}f}{b^{5}}+\frac{3a^{2}e}{b^{4}}-\frac{2ad}{b^{3}}+\frac{c}{b^{2}}\right)+\frac{x(-a^{4}f+a^{3}be-a^{2}b^{2}d+ab^{3}c)}{2ab^{5}+2b^{6}x^{2}}-\frac{\sqrt{-\frac{a}{b^{11}}}\cdot(9a^{3}f-7a^{2}be+5ab^{2}d-3b^{3}c)\log\left(-b^{5}\sqrt{-\frac{a}{b^{11}}}+x\right)}{4}+\frac{\sqrt{-\frac{a}{b^{11}}}\cdot(9a^{3}f-7a^{2}be+5ab^{2}d-3b^{3}c)\log\left(b^{5}\sqrt{-\frac{a}{b^{11}}}+x\right)}{4}+\frac{fx^{7}}{7b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x**5*(-2*a*f/(5*b**3) + e/(5*b**2)) + x**3*(a**2*f/b**4 - 2*a*e/(3*b**3) +
d/(3*b**2)) + x*(-4*a**3*f/b**5 + 3*a**2*e/b**4 - 2*a*d/b**3 + c/b**2) + x*
(-a**4*f + a**3*b*e - a**2*b**2*d + a*b**3*c)/(2*a*b**5 + 2*b**6*x**2) - sq
rt(-a/b**11)*(9*a**3*f - 7*a**2*b*e + 5*a*b**2*d - 3*b**3*c)*log(-b**5*sqrt
(-a/b**11) + x)/4 + sqrt(-a/b**11)*(9*a**3*f - 7*a**2*b*e + 5*a*b**2*d - 3*
b**3*c)*log(b**5*sqrt(-a/b**11) + x)/4 + f*x**7/(7*b**2)
```

Giac [A]

time = 3.47, size = 201, normalized size = 1.00

$$-\frac{(3\,ab^{3}c-5\,a^{2}b^{2}d-9\,a^{4}f+7\,a^{3}be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{5}}+\frac{ab^{3}cx-a^{2}b^{2}dx-a^{4}fx+a^{3}bxe}{2\,(bx^{2}+a)b^{5}}+\frac{15\,b^{12}fx^{7}-42\,ab^{11}fx^{5}+21\,b^{12}x^{5}e+35\,b^{12}dx^{3}+105\,a^{2}b^{10}fx^{3}-70\,ab^{11}x^{3}e+105\,b^{12}cx-210\,ab^{11}dx-420\,a^{3}b^{9}fx+315\,a^{2}b^{10}xe}{105\,b^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*a*b^3*c - 5*a^2*b^2*d - 9*a^4*f + 7*a^3*b*e)*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*b^5) + 1/2*(a*b^3*c*x - a^2*b^2*d*x - a^4*f*x + a^3*b*x*e)/((b*x
^2 + a)*b^5) + 1/105*(15*b^12*f*x^7 - 42*a*b^11*f*x^5 + 21*b^12*x^5*e + 35*
b^12*d*x^3 + 105*a^2*b^10*f*x^3 - 70*a*b^11*x^3*e + 105*b^12*c*x - 210*a*b^
11*d*x - 420*a^3*b^9*f*x + 315*a^2*b^10*x*e)/b^14
```

Mupad [B]

time = 0.97, size = 288, normalized size = 1.43

$$x^{5}\,\left(\frac{e}{5\,b^{2}}-\frac{2\,a\,f}{5\,b^{3}}\right)+x\,\left(\frac{c}{b^{2}}-\frac{a^{2}\,\left(\frac{e}{b^{2}}-\frac{2\,a\,f}{b^{3}}\right)}{b^{2}}+\frac{2\,a\,\left(\frac{a^{2}\,f}{b^{4}}-\frac{d}{b^{2}}+\frac{2\,a\,\left(\frac{e}{b^{2}}-\frac{2\,a\,f}{b^{3}}\right)}{b}\right)}{b}\right)-x^{3}\,\left(\frac{a^{2}\,f}{3\,b^{4}}-\frac{d}{3\,b^{2}}+\frac{2\,a\,\left(\frac{e}{b^{2}}-\frac{2\,a\,f}{b^{3}}\right)}{3\,b}\right)-\frac{x\,\left(\frac{f\,a^{4}}{2}-\frac{e\,a^{3}\,b}{2}+\frac{d\,a^{2}\,b^{2}}{2}-\frac{c\,a\,b^{3}}{2}\right)}{b^{6}\,x^{2}+a\,b^{5}}+\frac{f\,x^{7}}{7\,b^{2}}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,\left(-9\,f\,a^{3}+7\,e\,a^{2}\,b-5\,d\,a\,b^{2}+3\,c\,b^{3}\right)}{9\,f\,a^{4}-7\,e\,a^{3}\,b+5\,d\,a^{2}\,b^{2}-3\,c\,a\,b^{3}}\right)\,\left(-9\,f\,a^{3}+7\,e\,a^{2}\,b-5\,d\,a\,b^{2}+3\,c\,b^{3}\right)}{2\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x)
```

```
[Out] x^5*(e/(5*b^2) - (2*a*f)/(5*b^3)) + x*(c/b^2 - (a^2*(e/b^2 - (2*a*f)/b^3))/
b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b) - x^3*
((a^2*f)/(3*b^4) - d/(3*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(3*b)) - (x*((a^
4*f)/2 + (a^2*b^2*d)/2 - (a*b^3*c)/2 - (a^3*b*e)/2))/(a*b^5 + b^6*x^2) + (f
*x^7)/(7*b^2) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(3*b^3*c - 9*a^3*f - 5*a*b
^2*d + 7*a^2*b*e))/(9*a^4*f + 5*a^2*b^2*d - 3*a*b^3*c - 7*a^3*b*e))*(3*b^3*
c - 9*a^3*f - 5*a*b^2*d + 7*a^2*b*e))/(2*b^(11/2))
```

3.126 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$

Optimal. Leaf size=163

$$-\frac{(b^3c-3ab^2d+5a^2be-7a^3f)\,x}{2ab^4}+\frac{(be-2af)x^3}{3b^3}+\frac{fx^5}{5b^2}+\frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{2a\,(a+bx^2)}+\frac{(b^3c-3ab^2d+5a^2be-7}{2\sqrt{a}\,b^{9}}$$

```
[Out] -1/2*(-7*a^3*f+5*a^2*b*e-3*a*b^2*d+b^3*c)*x/a/b^4+1/3*(-2*a*f+b*e)*x^3/b^3+
1/5*f*x^5/b^2+1/2*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x^3/a/(b*x^2+a)+1/2*(-7*a^3
*f+5*a^2*b*e-3*a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(9/2)/a^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1818, 1599, 1275, 211}

$$\frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a\,(a+bx^2)}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(-7a^3f+5a^2be-3ab^2d+b^3c)}{2\sqrt{a}\,b^{9/2}}-\frac{x(-7a^3f+5a^2be-3ab^2d+b^3c)}{2ab^4}+\frac{x^3(be-2af)}{3b^3}+\frac{fx^5}{5b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] -1/2*((b^3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*x)/(a*b^4) + ((b*e - 2*a*f)
*x^3)/(3*b^3) + (f*x^5)/(5*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^
3)/(2*a*(a + b*x^2)) + ((b^3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
```

```
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{2a\,(a+bx^2)} - \frac{\int \frac{x\left(\left(bc-3ad+\frac{3a^2e}{b}-\frac{3a^3f}{b^2}\right)x-2a\left(e-\frac{af}{b}\right)x^3-2afx^5\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{2a\,(a+bx^2)} - \frac{\int \frac{x^2\left(bc-3ad+\frac{3a^2e}{b}-\frac{3a^3f}{b^2}-2a\left(e-\frac{af}{b}\right)x^2-2afx^4\right)}{a+bx^2}\,dx}{2ab}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{2a\,(a+bx^2)} - \frac{\int \Bigg(c-\frac{a(3b^2d-5abe+7a^2f)}{b^3}-\frac{2a(be-2af)x^2}{b^2}-\frac{2afx^4}{b}+}{2ab}$$

$$= -\frac{(b^3c-3ab^2d+5a^2be-7a^3f)\,x}{2ab^4}+\frac{(be-2af)x^3}{3b^3}+\frac{fx^5}{5b^2}+\frac{\Bigg(c-\frac{a(b^2d-abe+}{b^3}}{2a\,(a+b}$$

$$= -\frac{(b^3c-3ab^2d+5a^2be-7a^3f)\,x}{2ab^4}+\frac{(be-2af)x^3}{3b^3}+\frac{fx^5}{5b^2}+\frac{\Bigg(c-\frac{a(b^2d-abe+}{b^3}}{2a\,(a+b}$$

Mathematica [A]
time = 0.06, size = 148, normalized size = 0.91

$$\frac{(b^2d-2abe+3a^2f)\,x}{b^4}+\frac{(be-2af)x^3}{3b^3}+\frac{fx^5}{5b^2}-\frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{2b^4\,(a+bx^2)}-\frac{(-b^3c+3ab^2d-5a^2be+7a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\sqrt{a}\,b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

```
[Out] ((b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + ((b*e - 2*a*f)*x^3)/(3*b^3) + (f*x^5)
/(5*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*b^4*(a + b*x^2)) - ((
```

```
-(b^3*c) + 3*a*b^2*d - 5*a^2*b*e + 7*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2
*Sqrt[a]*b^(9/2))
```

Maple [A]
time = 0.14, size = 139, normalized size = 0.85

method	result
default	$\frac{\frac{1}{5} f\,x^{5} b^{2}-\frac{2}{3} a b f\,x^{3}+\frac{1}{3} b^{2} e\,x^{3}+3 a^{2} f x -2 a b e x +b^{2} d x}{b^{4}}-\frac{\frac{\left(-\frac{1}{2} a^{3} f +\frac{1}{2} a^{2} b e -\frac{1}{2} a\,b^{2} d +\frac{1}{2} b^{3} c \right) x}{b\,x^{2}+a}+\frac{\left(7 a^{3} f -5 a^{2} b e +3 a\,b^{2} d -b^{3} c \right) \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}}{b^{4}}$
risch	$\frac{f\,x^{5}}{5 b^{2}}-\frac{2 a f\,x^{3}}{3 b^{3}}+\frac{e\,x^{3}}{3 b^{2}}+\frac{3 a^{2} f x}{b^{4}}-\frac{2 a e x}{b^{3}}+\frac{d x}{b^{2}}+\frac{\left(\frac{1}{2} a^{3} f -\frac{1}{2} a^{2} b e +\frac{1}{2} a\,b^{2} d -\frac{1}{2} b^{3} c \right) x}{b^{4} \left(b\,x^{2}+a \right)}-\frac{7 \ln \left(b x -\sqrt{-a b}\right) a^{3} f}{4 b^{4} \sqrt{-a b}}+\frac{5 \ln (b x -}{4 b^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^4*(1/5*f*x^5*b^2-2/3*a*b*f*x^3+1/3*b^2*e*x^3+3*a^2*f*x-2*a*b*e*x+b^2*d*
x)-1/b^4*((-1/2*a^3*f+1/2*a^2*b*e-1/2*a*b^2*d+1/2*b^3*c)*x/(b*x^2+a)+1/2*(7
*a^3*f-5*a^2*b*e+3*a*b^2*d-b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.52, size = 145, normalized size = 0.89

$$-\frac{(b^3c - ab^2d - a^3f + a^2be)x}{2\,(b^5x^2 + ab^4)} + \frac{(b^3c - 3\,ab^2d - 7\,a^3f + 5\,a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^4} + \frac{3\,b^2fx^5 - 5\,(2\,abf - b^2e)x^3 + 15\,(b^2d + 3\,a^2f - 2\,abe)x}{15\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*x/(b^5*x^2 + a*b^4) + 1/2*(b^3*c -
 3*a*b^2*d - 7*a^3*f + 5*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) + 1
/15*(3*b^2*f*x^5 - 5*(2*a*b*f - b^2*e)*x^3 + 15*(b^2*d + 3*a^2*f - 2*a*b*e)
*x)/b^4
```

Fricas [A]
time = 3.54, size = 438, normalized size = 2.69

$$\left[\frac{12\,ab^4fx^7 - 28\,a^2b^3fx^5 + 20\,(3\,ab^4d + 7\,a^3b^2f)x^3 - 15\,(ab^3c - 3\,a^2b^2d - 7\,a^4f + (b^4c - 3\,ab^3d - 7\,a^3bf)x^2 + 5\,(a^2b^2x^2 + a^3b)e)\sqrt{-ab}\log\left(\frac{bx^2 - 2\,\sqrt{-ab}\,x - a}{bx^2 + a}\right) - 30\,(ab^4c - 3\,a^2b^3d - 7\,a^4bf)x + 10\,(2\,ab^4x^5 - 10\,a^2b^3x^3 - 15\,a^3b^2x)e}{60\,(ab^6x^2 + a^2b^5)}, \frac{6\,ab^4fx^7 - 14\,a^2b^3fx^5 + 10\,(3\,ab^4d + 7\,a^3b^2f)x^3 + 15\,(ab^3c - 3\,a^2b^2d - 7\,a^4f + (b^4c - 3\,ab^3d - 7\,a^3bf)x^2 + 5\,(a^2b^2x^2 + a^3b)e)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) - 15\,(ab^4c - 3\,a^2b^3d - 7\,a^4bf)x + 5\,(2\,ab^4x^5 - 10\,a^2b^3x^3 - 15\,a^3b^2x)e}{30\,(ab^6x^2 + a^2b^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*a*b^4*f*x^7 - 28*a^2*b^3*f*x^5 + 20*(3*a*b^4*d + 7*a^3*b^2*f)*x^3
 - 15*(a*b^3*c - 3*a^2*b^2*d - 7*a^4*f + (b^4*c - 3*a*b^3*d - 7*a^3*b*f)*x^
```

```
2 + 5*(a^2*b^2*x^2 + a^3*b)*e)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/
(b*x^2 + a)) - 30*(a*b^4*c - 3*a^2*b^3*d - 7*a^4*b*f)*x + 10*(2*a*b^4*x^5 -
 10*a^2*b^3*x^3 - 15*a^3*b^2*x)*e)/(a*b^6*x^2 + a^2*b^5), 1/30*(6*a*b^4*f*x
^7 - 14*a^2*b^3*f*x^5 + 10*(3*a*b^4*d + 7*a^3*b^2*f)*x^3 + 15*(a*b^3*c - 3*
a^2*b^2*d - 7*a^4*f + (b^4*c - 3*a*b^3*d - 7*a^3*b*f)*x^2 + 5*(a^2*b^2*x^2
+ a^3*b)*e)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 15*(a*b^4*c - 3*a^2*b^3*d - 7
*a^4*b*f)*x + 5*(2*a*b^4*x^5 - 10*a^2*b^3*x^3 - 15*a^3*b^2*x)*e)/(a*b^6*x^2
 + a^2*b^5)]
```

Sympy [A]
time = 1.05, size = 221, normalized size = 1.36

$$x^{3}\left(-\frac{2af}{3b^{3}}+\frac{e}{3b^{2}}\right)+x\left(\frac{3a^{2}f}{b^{4}}-\frac{2ae}{b^{3}}+\frac{d}{b^{2}}\right)+\frac{x(a^{3}f-a^{2}be+ab^{2}d-b^{3}c)}{2ab^{4}+2b^{5}x^{2}}+\frac{\sqrt{-\frac{1}{ab^{9}}}\cdot(7a^{3}f-5a^{2}be+3ab^{2}d-b^{3}c)\log\left(-ab^{4}\sqrt{-\frac{1}{ab^{9}}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{ab^{9}}}\cdot(7a^{3}f-5a^{2}be+3ab^{2}d-b^{3}c)\log\left(ab^{4}\sqrt{-\frac{1}{ab^{9}}}+x\right)}{4}+\frac{fx^{5}}{5b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x**3*(-2*a*f/(3*b**3) + e/(3*b**2)) + x*(3*a**2*f/b**4 - 2*a*e/b**3 + d/b**
2) + x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(2*a*b**4 + 2*b**5*x**2) + s
qrt(-1/(a*b**9))*(7*a**3*f - 5*a**2*b*e + 3*a*b**2*d - b**3*c)*log(-a*b**4*
sqrt(-1/(a*b**9)) + x)/4 - sqrt(-1/(a*b**9))*(7*a**3*f - 5*a**2*b*e + 3*a*b
**2*d - b**3*c)*log(a*b**4*sqrt(-1/(a*b**9)) + x)/4 + f*x**5/(5*b**2)
```

Giac [A]
time = 1.63, size = 152, normalized size = 0.93

$$\frac{(b^{3}c-3\,ab^{2}d-7\,a^{3}f+5\,a^{2}be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,b^{4}}-\frac{b^{3}cx-ab^{2}dx-a^{3}fx+a^{2}bxe}{2\,(bx^{2}+a)b^{4}}+\frac{3\,b^{8}fx^{5}-10\,ab^{7}fx^{3}+5\,b^{8}x^{3}e+15\,b^{8}dx+45\,a^{2}b^{6}fx-30\,ab^{7}xe}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c - 3*a*b^2*d - 7*a^3*f + 5*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*b^4) - 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^2 + a)*b^4
) + 1/15*(3*b^8*f*x^5 - 10*a*b^7*f*x^3 + 5*b^8*x^3*e + 15*b^8*d*x + 45*a^2*
b^6*f*x - 30*a*b^7*x*e)/b^10
```

Mupad [B]
time = 1.00, size = 153, normalized size = 0.94

$$x^{3}\left(\frac{e}{3\,b^{2}}-\frac{2\,a\,f}{3\,b^{3}}\right)-x\left(\frac{a^{2}\,f}{b^{4}}-\frac{d}{b^{2}}+\frac{2\,a\,\left(\frac{e}{b^{2}}-\frac{2\,a\,f}{b^{3}}\right)}{b}\right)-\frac{x\,\left(-\frac{f\,a^{3}}{2}+\frac{e\,a^{2}\,b}{2}-\frac{d\,a\,b^{2}}{2}+\frac{c\,b^{3}}{2}\right)}{b^{5}\,x^{2}+a\,b^{4}}+\frac{f\,x^{5}}{5\,b^{2}}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-7\,f\,a^{3}+5\,e\,a^{2}\,b-3\,d\,a\,b^{2}+c\,b^{3}\right)}{2\,\sqrt{a}\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x)
```

```
[Out] x^3*(e/(3*b^2) - (2*a*f)/(3*b^3)) - x*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 -
(2*a*f)/b^3))/b) - (x*((b^3*c)/2 - (a^3*f)/2 - (a*b^2*d)/2 + (a^2*b*e)/2))/
(a*b^4 + b^5*x^2) + (f*x^5)/(5*b^2) + (atan((b^(1/2)*x)/a^(1/2))*(b^3*c - 7
*a^3*f - 3*a*b^2*d + 5*a^2*b*e))/(2*a^(1/2)*b^(9/2))
```

3.127 $\int \frac{c+dx^2+ex^4+fx^6}{\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=118

$$\frac{(be-2af)x}{b^3}+\frac{fx^3}{3b^2}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(b^3c+ab^2d-3a^2be+5a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

```
[Out] (-2*a*f+b*e)*x/b^3+1/3*f*x^3/b^2+1/2*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x/a/(b*x
^2+a)+1/2*(5*a^3*f-3*a^2*b*e+a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/
2)/b^(7/2)
```

Rubi [A]

time = 0.08, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1828, 1167, 211}

$$\frac{x\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(5a^3f-3a^2be+ab^2d+b^3c\right)}{2a^{3/2}b^{7/2}}+\frac{x(be-2af)}{b^3}+\frac{fx^3}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^3)/(3*b^2) + ((c - (a*(b^2*d - a*b*e + a^2*f))
/b^3)*x)/(2*a*(a + b*x^2)) + ((b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
```

```
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{\left(a+bx^2\right)^2}\,dx &= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}-\frac{\int \frac{-\frac{b^3c+ab^2d-a^2be+a^3f}{b^3}-\frac{2a(be-af)x^2}{b^2}-\frac{2afx^4}{b}}{a+bx^2}\,dx}{2a}\\
&= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}-\frac{\int \left(-\frac{2a(be-2af)}{b^3}-\frac{2afx^2}{b^2}+\frac{-b^3c-ab^2d+3a^2be-5a^3f}{b^3(a+bx^2)}\right)\,dx}{2a}\\
&= \frac{(be-2af)x}{b^3}+\frac{fx^3}{3b^2}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(b^3c+ab^2d-3a^2be+5a^3f\right)\int}{2ab^3}\\
&= \frac{(be-2af)x}{b^3}+\frac{fx^3}{3b^2}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(b^3c+ab^2d-3a^2be+5a^3f\right)\mathrm{ta}}{2a^{3/2}b^{7/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 122, normalized size = 1.03

$$\frac{(be-2af)x}{b^3}+\frac{fx^3}{3b^2}-\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{2ab^3\left(a+bx^2\right)}+\frac{\left(b^3c+ab^2d-3a^2be+5a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x]
```

```
[Out] ((b*e - 2*a*f)*x)/b^3 + (f*x^3)/(3*b^2) - ((-(b^3*c) + a*b^2*d - a^2*b*e +
a^3*f)*x)/(2*a*b^3*(a + b*x^2)) + ((b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

Maple [A]
time = 0.16, size = 114, normalized size = 0.97

method	result
default	$-\frac{-\frac{1}{3}f\,x^3b+2afx-bex}{b^3}+\frac{-\frac{\left(a^3f-a^2be+a\,b^2d-b^3c\right)x}{2a\left(b\,x^2+a\right)}+\frac{\left(5a^3f-3a^2be+a\,b^2d+b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2a\sqrt{ab}}}{b^3}$

risch	$\frac{f\,x^{3}}{3b^{2}}-\frac{2afx}{b^{3}}+\frac{ex}{b^{2}}-\frac{(a^{3}f-a^{2}be+a\,b^{2}d-b^{3}c)\,x}{2a\,b^{3}(b\,x^{2}+a)}-\frac{5a^{2}\ln\left(bx+\sqrt{-ab}\right)f}{4b^{3}\sqrt{-ab}}+\frac{3a\ln\left(bx+\sqrt{-ab}\right)e}{4b^{2}\sqrt{-ab}}-\frac{\ln\left(bx+\sqrt{-ab}\right)}{4b\sqrt{-ab}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^3*(-1/3*f*x^3*b+2*a*f*x-b*e*x)+1/b^3*(-1/2*(a^3*f-a^2*b*e+a*b^2*d-b^3*
c)/a*x/(b*x^2+a)+1/2*(5*a^3*f-3*a^2*b*e+a*b^2*d+b^3*c)/a/(a*b)^(1/2)*arctan
(b*x/(a*b)^(1/2)))
```

Maxima [**A**]
time = 0.52, size = 121, normalized size = 1.03

$$\frac{(b^{3}c-ab^{2}d-a^{3}f+a^{2}be)x}{2\,(ab^{4}x^{2}+a^{2}b^{3})}+\frac{bfx^{3}-3\,(2\,af-be)x}{3\,b^{3}}+\frac{(b^{3}c+ab^{2}d+5\,a^{3}f-3\,a^{2}be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^3*c - a*b^2*d - a^3*f + a^2*b*e)*x/(a*b^4*x^2 + a^2*b^3) + 1/3*(b*f*
x^3 - 3*(2*a*f - b*e)*x)/b^3 + 1/2*(b^3*c + a*b^2*d + 5*a^3*f - 3*a^2*b*e)*
arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^3)
```

Fricas [**A**]
time = 1.77, size = 380, normalized size = 3.22

$$\left[\frac{4\,a^{2}b^{3}fx^{5}-20\,a^{3}b^{2}fx^{3}+3\,(ab^{3}c+a^{2}b^{2}d+5\,a^{4}f+(b^{4}c+ab^{3}d+5\,a^{3}bf)x^{2}-3\,(a^{2}b^{2}x^{2}+a^{3}b)e)\sqrt{-ab}\,\log\left(\frac{bx^{2}+2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)+6\,(ab^{4}c-a^{2}b^{3}d-5\,a^{4}bf)x+6\,(2\,a^{2}b^{3}x^{3}+3\,a^{3}b^{2}x)e}{12\,(a^{2}b^{5}x^{2}+a^{3}b^{4})},\ \frac{2\,a^{2}b^{3}fx^{5}-10\,a^{3}b^{2}fx^{3}+3\,(ab^{3}c+a^{2}b^{2}d+5\,a^{4}f+(b^{4}c+ab^{3}d+5\,a^{3}bf)x^{2}-3\,(a^{2}b^{2}x^{2}+a^{3}b)e)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+3\,(ab^{4}c-a^{2}b^{3}d-5\,a^{4}bf)x+3\,(2\,a^{2}b^{3}x^{3}+3\,a^{3}b^{2}x)e}{6\,(a^{2}b^{5}x^{2}+a^{3}b^{4})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*a^2*b^3*f*x^5 - 20*a^3*b^2*f*x^3 + 3*(a*b^3*c + a^2*b^2*d + 5*a^4*
f + (b^4*c + a*b^3*d + 5*a^3*b*f)*x^2 - 3*(a^2*b^2*x^2 + a^3*b)*e)*sqrt(-a*
b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(a*b^4*c - a^2*b^3*d -
 5*a^4*b*f)*x + 6*(2*a^2*b^3*x^3 + 3*a^3*b^2*x)*e)/(a^2*b^5*x^2 + a^3*b^4),
 1/6*(2*a^2*b^3*f*x^5 - 10*a^3*b^2*f*x^3 + 3*(a*b^3*c + a^2*b^2*d + 5*a^4*f
 + (b^4*c + a*b^3*d + 5*a^3*b*f)*x^2 - 3*(a^2*b^2*x^2 + a^3*b)*e)*sqrt(a*b)
*arctan(sqrt(a*b)*x/a) + 3*(a*b^4*c - a^2*b^3*d - 5*a^4*b*f)*x + 3*(2*a^2*b
^3*x^3 + 3*a^3*b^2*x)*e)/(a^2*b^5*x^2 + a^3*b^4)]
```

Sympy [**A**]
time = 0.85, size = 201, normalized size = 1.70

$$x\left(-\frac{2af}{b^{3}}+\frac{e}{b^{2}}\right)+\frac{x(-a^{3}f+a^{2}be-ab^{2}d+b^{3}c)}{2a^{2}b^{3}+2ab^{4}x^{2}}-\frac{\sqrt{-\frac{1}{a^{3}b^{7}}}\cdot(5a^{3}f-3a^{2}be+ab^{2}d+b^{3}c)\log\left(-a^{2}b^{3}\sqrt{-\frac{1}{a^{3}b^{7}}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^{3}b^{7}}}\cdot(5a^{3}f-3a^{2}be+ab^{2}d+b^{3}c)\log\left(a^{2}b^{3}\sqrt{-\frac{1}{a^{3}b^{7}}}+x\right)}{4}+\frac{fx^{3}}{3b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

```
[Out] x*(-2*a*f/b**3 + e/b**2) + x*(-a**3*f + a**2*b*e - a*b**2*d + b**3*c)/(2*a*
*2*b**3 + 2*a*b**4*x**2) - sqrt(-1/(a**3*b**7))*(5*a**3*f - 3*a**2*b*e + a*
b**2*d + b**3*c)*log(-a**2*b**3*sqrt(-1/(a**3*b**7)) + x)/4 + sqrt(-1/(a**3
*b**7))*(5*a**3*f - 3*a**2*b*e + a*b**2*d + b**3*c)*log(a**2*b**3*sqrt(-1/(
a**3*b**7)) + x)/4 + f*x**3/(3*b**2)
```

Giac [A]
time = 2.60, size = 126, normalized size = 1.07

$$\frac{(b^3c + ab^2d + 5\,a^3f - 3\,a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^3} + \frac{b^3cx - ab^2dx - a^3fx + a^2bxe}{2\,(bx^2+a)ab^3} + \frac{b^4fx^3 - 6\,ab^3fx + 3\,b^4xe}{3\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c + a*b^2*d + 5*a^3*f - 3*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a*b^3) + 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^2 + a)*a*b
^3) + 1/3*(b^4*f*x^3 - 6*a*b^3*f*x + 3*b^4*x*e)/b^6
```

Mupad [B]
time = 0.10, size = 113, normalized size = 0.96

$$x\left(\frac{e}{b^2} - \frac{2\,a\,f}{b^3}\right) + \frac{f\,x^3}{3\,b^2} + \frac{x\,(-f\,a^3 + e\,a^2\,b - d\,a\,b^2 + c\,b^3)}{2\,a\,(b^4\,x^2 + a\,b^3)} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(5\,f\,a^3 - 3\,e\,a^2\,b + d\,a\,b^2 + c\,b^3)}{2\,a^{3/2}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x)
```

```
[Out] x*(e/b^2 - (2*a*f)/b^3) + (f*x^3)/(3*b^2) + (x*(b^3*c - a^3*f - a*b^2*d + a
^2*b*e))/(2*a*(a*b^3 + b^4*x^2)) + (atan((b^(1/2)*x)/a^(1/2))*(b^3*c + 5*a^
3*f + a*b^2*d - 3*a^2*b*e))/(2*a^(3/2)*b^(7/2))
```

3.128 $\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2} \, dx$

Optimal. Leaf size=112

$$-\frac{c}{a^2x}+\frac{fx}{b^2}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)}-\frac{\left(3b^3c-ab^2d-a^2be+3a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}$$

```
[Out] -c/a^2/x+f*x/b^2-1/2*(b*c/a-d+a*e/b-a^2*f/b^2)*x/a/(b*x^2+a)-1/2*(3*a^3*f-a
^2*b*e-a*b^2*d+3*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(5/2)
```

Rubi [A]

time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1819, 1275, 211}

$$-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{2a\left(a+bx^2\right)}-\frac{c}{a^2x}-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(3a^3f-a^2be-ab^2d+3b^3c\right)}{2a^{5/2}b^{5/2}}+\frac{fx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) + (f*x)/b^2 - (((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)*x)/(2*a*(
a + b*x^2)) - ((3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(2*a^(5/2)*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
```

```
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2}\,dx &= -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)}-\frac{\int\frac{-2c+\left(\frac{bc}{a}-d-\frac{ae}{b}+\frac{a^2f}{b^2}\right)x^2-\frac{2afx^4}{b}}{x^2(a+bx^2)}\,dx}{2a}\\
&= -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)}-\frac{\int\left(-\frac{2af}{b^2}-\frac{2c}{ax^2}+\frac{3b^3c-ab^2d-a^2be+3a^3f}{ab^2(a+bx^2)}\right)dx}{2a}\\
&= -\frac{c}{a^2x}+\frac{fx}{b^2}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)}-\frac{\left(3b^3c-ab^2d-a^2be+3a^3f\right)\int\frac{1}{a+bx^2}\,dx}{2a^2b^2}\\
&= -\frac{c}{a^2x}+\frac{fx}{b^2}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a\left(a+bx^2\right)}-\frac{\left(3b^3c-ab^2d-a^2be+3a^3f\right)\tan^{-1}\Bigg(\frac{\sqrt{\ }}{\sqrt{\ }}}{2a^{5/2}b^{5/2}}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 115, normalized size = 1.03

$$-\frac{c}{a^2x}+\frac{fx}{b^2}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{2a^2b^2\left(a+bx^2\right)}-\frac{\left(3b^3c-ab^2d-a^2be+3a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x]
```

```
[Out] -(c/(a^2*x)) + (f*x)/b^2 + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^
2*b^2*(a + b*x^2)) - ((3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[
b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(5/2))
```

Maple [A]
time = 0.14, size = 107, normalized size = 0.96

method	result
default	$\frac{fx}{b^2}-\frac{\frac{\left(-\frac{1}{2}a^3f+\frac{1}{2}a^2be-\frac{1}{2}a\,b^2d+\frac{1}{2}b^3c\right)x}{b\,x^2+a}+\frac{\left(3a^3f-a^2be-a\,b^2d+3b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^2b^2}-\frac{c}{a^2x}$

risch	$\frac{fx}{b^2}+\frac{\frac{\left(a^3f-a^2be+a\,b^2d-3b^3c\right)x^2}{2a^2}-\frac{b^2c}{a}}{b^2x(b\,x^2+a)}-\frac{3a\ln\left(-\sqrt{-ab}\,x-a\right)f}{4b^2\sqrt{-ab}}+\frac{\ln\left(-\sqrt{-ab}\,x-a\right)e}{4b\sqrt{-ab}}+\frac{\ln\left(-\sqrt{-ab}\,x-a\right)d}{4\sqrt{-ab}\,a}-$ 3b

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] f*x/b^2-1/a^2/b^2*((-1/2*a^3*f+1/2*a^2*b*e-1/2*a*b^2*d+1/2*b^3*c)*x/(b*x^2+
a)+1/2*(3*a^3*f-a^2*b*e-a*b^2*d+3*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)
))-c/a^2/x
```

Maxima [A]
time = 0.49, size = 119, normalized size = 1.06

$$-\frac{2\,ab^2c+(3\,b^3c-ab^2d-a^3f+a^2be)x^2}{2\,(a^2b^3x^3+a^3b^2x)}+\frac{fx}{b^2}-\frac{(3\,b^3c-ab^2d+3\,a^3f-a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*b^2*c + (3*b^3*c - a*b^2*d - a^3*f + a^2*b*e)*x^2)/(a^2*b^3*x^3 +
 a^3*b^2*x) + f*x/b^2 - 1/2*(3*b^3*c - a*b^2*d + 3*a^3*f - a^2*b*e)*arctan(
b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2)
```

Fricas [A]
time = 1.51, size = 373, normalized size = 3.33

$$\left[\frac{4\,a^3b^2fx^4-2\,a^3b^2x^2e-4\,a^2b^3c-2\,(3\,ab^4c-a^2b^3d-3\,a^4bf)x^2+((3\,b^4c-ab^3d+3\,a^3bf)x^3+(3\,ab^3c-a^2b^2d+3\,a^4f)x-(a^2b^2x^3+a^3bx)e)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{4\,(a^3b^4x^3+a^4b^3x)},\ \frac{2\,a^3b^2fx^4-a^3b^2x^2e-2\,a^2b^3c-(3\,ab^4c-a^2b^3d-3\,a^4bf)x^2-((3\,b^4c-ab^3d+3\,a^3bf)x^3+(3\,ab^3c-a^2b^2d+3\,a^4f)x-(a^2b^2x^3+a^3bx)e)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{2\,(a^3b^4x^3+a^4b^3x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^3*b^2*f*x^4 - 2*a^3*b^2*x^2*e - 4*a^2*b^3*c - 2*(3*a*b^4*c - a^2*
b^3*d - 3*a^4*b*f)*x^2 + ((3*b^4*c - a*b^3*d + 3*a^3*b*f)*x^3 + (3*a*b^3*c
- a^2*b^2*d + 3*a^4*f)*x - (a^2*b^2*x^3 + a^3*b*x)*e)*sqrt(-a*b)*log((b*x^2
 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^4*x^3 + a^4*b^3*x), 1/2*(2*a^3*
b^2*f*x^4 - a^3*b^2*x^2*e - 2*a^2*b^3*c - (3*a*b^4*c - a^2*b^3*d - 3*a^4*b*
f)*x^2 - ((3*b^4*c - a*b^3*d + 3*a^3*b*f)*x^3 + (3*a*b^3*c - a^2*b^2*d + 3*
a^4*f)*x - (a^2*b^2*x^3 + a^3*b*x)*e)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3
*b^4*x^3 + a^4*b^3*x)]
```

Sympy [A]
time = 2.25, size = 197, normalized size = 1.76

$$\frac{\sqrt{-\frac{1}{a^5b^5}}\cdot(3a^3f-a^2be-ab^2d+3b^3c)\log\left(-a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{4}-\frac{\sqrt{-\frac{1}{a^5b^5}}\cdot(3a^3f-a^2be-ab^2d+3b^3c)\log\left(a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{4}+\frac{-2ab^2c+x^2(a^3f-a^2be+ab^2d-3b^3c)}{2a^3b^2x+2a^2b^3x^3}+\frac{fx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**2,x)
```

```
[Out] sqrt(-1/(a**5*b**5))*(3*a**3*f - a**2*b*e - a*b**2*d + 3*b**3*c)*log(-a**3*
b**2*sqrt(-1/(a**5*b**5)) + x)/4 - sqrt(-1/(a**5*b**5))*(3*a**3*f - a**2*b*
e - a*b**2*d + 3*b**3*c)*log(a**3*b**2*sqrt(-1/(a**5*b**5)) + x)/4 + (-2*a*
b**2*c + x**2*(a**3*f - a**2*b*e + a*b**2*d - 3*b**3*c))/(2*a**3*b**2*x + 2
*a**2*b**3*x**3) + f*x/b**2
```

Giac [A]
time = 1.78, size = 122, normalized size = 1.09

$$\frac{fx}{b^2} - \frac{(3\,b^3c - ab^2d + 3\,a^3f - a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^2b^2} - \frac{3\,b^3cx^2 - ab^2dx^2 - a^3fx^2 + a^2bx^2e + 2\,ab^2c}{2\,(bx^3 + ax)a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] f*x/b^2 - 1/2*(3*b^3*c - a*b^2*d + 3*a^3*f - a^2*b*e)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*a^2*b^2) - 1/2*(3*b^3*c*x^2 - a*b^2*d*x^2 - a^3*f*x^2 + a^2*b*x
^2*e + 2*a*b^2*c)/((b*x^3 + a*x)*a^2*b^2)
```

Mupad [B]
time = 1.00, size = 112, normalized size = 1.00

$$\frac{f\,x}{b^2} - \frac{\frac{x^2\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+3\,c\,b^3\right)}{2\,a^2}+\frac{b^2\,c}{a}}{b^3\,x^3+a\,b^2\,x} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(3\,f\,a^3-e\,a^2\,b-d\,a\,b^2+3\,c\,b^3\right)}{2\,a^{5/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x)
```

```
[Out] (f*x)/b^2 - ((x^2*(3*b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(2*a^2) + (b^2*c)/
a)/(b^3*x^3 + a*b^2*x) - (atan((b^(1/2)*x)/a^(1/2))*(3*b^3*c + 3*a^3*f - a*
b^2*d - a^2*b*e))/(2*a^(5/2)*b^(5/2))
```

3.129 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^2} \, dx$

Optimal. Leaf size=121

$$-\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\,(a+bx^2)}+\frac{(5b^3c-3ab^2d+a^2be+a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}$$

```
[Out] -1/3*c/a^2/x^3+(-a*d+2*b*c)/a^3/x+1/2*(b^2*c/a^2-b*d/a+e-a*f/b)*x/a/(b*x^2+
a)+1/2*(a^3*f+a^2*b*e-3*a*b^2*d+5*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/
b^(3/2)
```

Rubi [A]
time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1819, 1275, 211}

$$\frac{2bc-ad}{a^3x}+\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{2a\,(a+bx^2)}-\frac{c}{3a^2x^3}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(a^3f+a^2be-3ab^2d+5b^3c)}{2a^{7/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*c/(a^2*x^3) + (2*b*c - a*d)/(a^3*x) + (((b^2*c)/a^2 - (b*d)/a + e - (a
*f)/b)*x)/(2*a*(a + b*x^2)) + ((5*b^3*c - 3*a*b^2*d + a^2*b*e + a^3*f)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
```

```
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^2}\,dx &= \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)}-\frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2+\left(-\frac{b^2c}{a^2}+\frac{bd}{a}-e-\frac{af}{b}\right)x^4}{x^4(a+bx^2)}\,dx}{2a}\\
&= \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)}-\frac{\int \left(-\frac{2c}{ax^4}-\frac{2(-2bc+ad)}{a^2x^2}+\frac{-5b^3c+3ab^2d-a^2be-a^3f}{a^2b(a+bx^2)}\right)\,dx}{2a}\\
&= -\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(5b^3c-3ab^2d+a^2be+a^3f\right)\int}{2a^3b}\\
&= -\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(5b^3c-3ab^2d+a^2be+a^3f\right)\mathrm{ta}}{2a^{7/2}b^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 125, normalized size = 1.03

$$
-\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}-\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{2a^3b\left(a+bx^2\right)}+\frac{\left(5b^3c-3ab^2d+a^2be+a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x]
```

```
[Out] -1/3*c/(a^2*x^3) + (2*b*c - a*d)/(a^3*x) - ((-(b^3*c) + a*b^2*d - a^2*b*e +
 a^3*f)*x)/(2*a^3*b*(a + b*x^2)) + ((5*b^3*c - 3*a*b^2*d + a^2*b*e + a^3*f)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

Maple [A]
time = 0.13, size = 116, normalized size = 0.96

method	result
default	$\frac{-\frac{\left(a^3f-a^2be+a\,b^2d-b^3c\right)x}{2b\left(b\,x^2+a\right)}+\frac{\left(a^3f+a^2be-3a\,b^2d+5b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2b\sqrt{ab}}}{a^3}-\frac{c}{3a^2x^3}-\frac{ad-2bc}{a^3x}$

risch	$\frac{-\frac{\left(a^3f-a^2be+3a\,b^2d-5b^3c\right)x^4}{2a^3b}-\frac{(3ad-5bc)x^2}{3a^2}-\frac{c}{3a}}{x^3(b\,x^2+a)}+\frac{\Bigg(\sum_{\_R=\text{RootOf}\left(a^7\_Z^2b^3+a^6f^2+2a^5bef-6a^4b^2df+a^4b^2e^2+10a^3b^3cf-6a^3b^3de+10a^2\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(-1/2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/b*x/(b*x^2+a)+1/2*(a^3*f+a^2*b*e-
3*a*b^2*d+5*b^3*c)/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/3*c/a^2/x^3-(a*
d-2*b*c)/a^3/x
```

Maxima [A]
time = 0.51, size = 132, normalized size = 1.09

$$\frac{3\,(5\,b^3c-3\,ab^2d-a^3f+a^2be)x^4-2\,a^2bc+2\,(5\,ab^2c-3\,a^2bd)x^2}{6\,(a^3b^2x^5+a^4bx^3)}+\frac{(5\,b^3c-3\,ab^2d+a^3f+a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(5*b^3*c - 3*a*b^2*d - a^3*f + a^2*b*e)*x^4 - 2*a^2*b*c + 2*(5*a*b^2
*c - 3*a^2*b*d)*x^2)/(a^3*b^2*x^5 + a^4*b*x^3) + 1/2*(5*b^3*c - 3*a*b^2*d +
 a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b)
```

Fricas [A]
time = 1.45, size = 404, normalized size = 3.34

$$\left[\frac{6\,a^3b^2x^4e-4\,a^3b^2c+6\,(5\,ab^4c-3\,a^2b^3d-a^4bf)x^4+4\,(5\,a^2b^3c-3\,a^3b^2d)x^2-3\,((5\,b^4c-3\,ab^3d+a^3bf)x^5+(5\,ab^3c-3\,a^2b^2d+a^4f)x^3+(a^2b^2x^5+a^3bx^3)e)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{12\,(a^4b^3x^5+a^5b^2x^3)},\frac{3\,a^3b^2x^4e-2\,a^3b^2c+3\,(5\,ab^4c-3\,a^2b^3d-a^4bf)x^4+2\,(5\,a^2b^3c-3\,a^3b^2d)x^2+3\,((5\,b^4c-3\,ab^3d+a^3bf)x^5+(5\,ab^3c-3\,a^2b^2d+a^4f)x^3+(a^2b^2x^5+a^3bx^3)e)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{6\,(a^4b^3x^5+a^5b^2x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(6*a^3*b^2*x^4*e - 4*a^3*b^2*c + 6*(5*a*b^4*c - 3*a^2*b^3*d - a^4*b*f
)*x^4 + 4*(5*a^2*b^3*c - 3*a^3*b^2*d)*x^2 - 3*((5*b^4*c - 3*a*b^3*d + a^3*b
*f)*x^5 + (5*a*b^3*c - 3*a^2*b^2*d + a^4*f)*x^3 + (a^2*b^2*x^5 + a^3*b*x^3)
*e)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^3*x^5
+ a^5*b^2*x^3), 1/6*(3*a^3*b^2*x^4*e - 2*a^3*b^2*c + 3*(5*a*b^4*c - 3*a^2*b
^3*d - a^4*b*f)*x^4 + 2*(5*a^2*b^3*c - 3*a^3*b^2*d)*x^2 + 3*((5*b^4*c - 3*a
*b^3*d + a^3*b*f)*x^5 + (5*a*b^3*c - 3*a^2*b^2*d + a^4*f)*x^3 + (a^2*b^2*x^
5 + a^3*b*x^3)*e)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^3*x^5 + a^5*b^2*x
^3)]
```

Sympy [A]
time = 5.87, size = 212, normalized size = 1.75

$$-\frac{\sqrt{-\frac{1}{a^7b^3}}\,(a^3f+a^2be-3ab^2d+5b^3c)\log\left(-a^4b\sqrt{-\frac{1}{a^7b^3}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^7b^3}}\,(a^3f+a^2be-3ab^2d+5b^3c)\log\left(a^4b\sqrt{-\frac{1}{a^7b^3}}+x\right)}{4}+\frac{-2a^2bc+x^4(-3a^3f+3a^2be-9ab^2d+15b^3c)+x^2(-6a^2bd+10ab^2c)}{6a^4bx^3+6a^3b^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**7*b**3))*(a**3*f + a**2*b*e - 3*a*b**2*d + 5*b**3*c)*log(-a**4
*b*sqrt(-1/(a**7*b**3)) + x)/4 + sqrt(-1/(a**7*b**3))*(a**3*f + a**2*b*e -
3*a*b**2*d + 5*b**3*c)*log(a**4*b*sqrt(-1/(a**7*b**3)) + x)/4 + (-2*a**2*b*
c + x**4*(-3*a**3*f + 3*a**2*b*e - 9*a*b**2*d + 15*b**3*c) + x**2*(-6*a**2*
b*d + 10*a*b**2*c))/(6*a**4*b*x**3 + 6*a**3*b**2*x**5)
```

Giac [A]
time = 3.14, size = 123, normalized size = 1.02

$$\frac{(5\,b^3c-3\,ab^2d+a^3f+a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^3b}+\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{2\,(bx^2+a)a^3b}+\frac{6\,bcx^2-3\,adx^2-ac}{3\,a^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b^3*c - 3*a*b^2*d + a^3*f + a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^3*b) + 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^2 + a)*a^3
*b) + 1/3*(6*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^3*x^3)
```

Mupad [B]
time = 0.13, size = 119, normalized size = 0.98

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(f\,a^3+e\,a^2\,b-3\,d\,a\,b^2+5\,c\,b^3\right)}{2\,a^{7/2}\,b^{3/2}}-\frac{\frac{c}{3\,a}+\frac{x^2\,(3\,a\,d-5\,b\,c)}{3\,a^2}-\frac{x^4\,\left(-f\,a^3+e\,a^2\,b-3\,d\,a\,b^2+5\,c\,b^3\right)}{2\,a^3\,b}}{b\,x^5+a\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(5*b^3*c + a^3*f - 3*a*b^2*d + a^2*b*e))/(2*a^(7
/2)*b^(3/2)) - (c/(3*a) + (x^2*(3*a*d - 5*b*c))/(3*a^2) - (x^4*(5*b^3*c - a
^3*f - 3*a*b^2*d + a^2*b*e))/(2*a^3*b))/(a*x^3 + b*x^5)
```

3.130 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=152

$$-\frac{c}{5a^2x^5}+\frac{2bc-ad}{3a^3x^3}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)}-\frac{\left(7b^3c-5ab^2d+3a^2be-a^3f\right)\tan^{-1}}{2a^{9/2}\sqrt{b}}$$

```
[Out] -1/5*c/a^2/x^5+1/3*(-a*d+2*b*c)/a^3/x^3+(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x-1/2*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^4/(b*x^2+a)-1/2*(-a^3*f+3*a^2*b*e-5*a*b^
2*d+7*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)/b^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1819, 1816, 211}

$$\frac{2bc-ad}{3a^3x^3}-\frac{c}{5a^2x^5}-\frac{a^2e-2abd+3b^2c}{a^4x}-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3(-f)+3a^2be-5ab^2d+7b^3c\right)}{2a^{9/2}\sqrt{b}}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^4\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -1/5*c/(a^2*x^5) + (2*b*c - a*d)/(3*a^3*x^3) - (3*b^2*c - 2*a*b*d + a^2*e)/
(a^4*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^4*(a + b*x^2)) - ((7
*b^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(
9/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
```

```
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx &= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)}-\frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2-\frac{2\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{\left(b^3c-ab^2d+a^2be-a^3}{a^3}}{x^6(a+bx^2)}}{2a}\\
&= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)}-\frac{\int \left(-\frac{2c}{ax^6}-\frac{2(-2bc+ad)}{a^2x^4}-\frac{2\left(3b^2c-2abd+a^2e\right)}{a^3x^2}+\frac{7b^3c-}{}\right.}{2a}\\
&= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{3a^3x^3}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)}-\frac{(7b}{}\\
&= -\frac{c}{5a^2x^5}+\frac{2bc-ad}{3a^3x^3}-\frac{3b^2c-2abd+a^2e}{a^4x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)}-\frac{(7b}{}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 151, normalized size = 0.99

$$-\frac{c}{5a^2x^5}+\frac{2bc-ad}{3a^3x^3}+\frac{-3b^2c+2abd-a^2e}{a^4x}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{2a^4\left(a+bx^2\right)}+\frac{\left(-7b^3c+5ab^2d-3a^2be+a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{9/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x]
```

```
[Out] -1/5*c/(a^2*x^5) + (2*b*c - a*d)/(3*a^3*x^3) + (-3*b^2*c + 2*a*b*d - a^2*e)
/(a^4*x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^4*(a + b*x^2)) +
 ((-7*b^3*c + 5*a*b^2*d - 3*a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
2*a^(9/2)*Sqrt[b])
```

Maple [A]
time = 0.12, size = 137, normalized size = 0.90

method	result
default	$\frac{\frac{\left(\frac{1}{2}a^3f-\frac{1}{2}a^2be+\frac{1}{2}a\,b^2d-\frac{1}{2}b^3c\right)x}{b\,x^2+a}+\frac{\left(a^3f-3a^2be+5a\,b^2d-7b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^4}-\frac{c}{5a^2x^5}-\frac{ad-2bc}{3a^3x^3}-\frac{a^2e-2abd+3b^2c}{a^4x}$

risch	$\frac{\frac{\left(a^3 f-3a^2 be+5a b^2 d-7b^3 c\right)x^6}{2a^4}-\frac{\left(3a^2 e-5abd+7b^2 c\right)x^4}{3a^3}-\frac{(5ad-7bc)x^2}{15a^2}-\frac{c}{5a}}{x^5(b x^2+a)}+\Bigg(\sum_{\_R=\mathrm{RootOf}\left(a^9\_Z^2 b+a^6 f^2-6a^5 bef+10a^4 b^2 df+9a^4 b^2 e^2-\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*((1/2*a^3*f-1/2*a^2*b*e+1/2*a*b^2*d-1/2*b^3*c)*x/(b*x^2+a)+1/2*(a^3*f
-3*a^2*b*e+5*a*b^2*d-7*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/5*c/a^
2/x^5-1/3*(a*d-2*b*c)/a^3/x^3-(a^2*e-2*a*b*d+3*b^2*c)/a^4/x
```

Maxima [A]
time = 0.49, size = 154, normalized size = 1.01

$$-\frac{15\,(7\,b^3c-5\,ab^2d-a^3f+3\,a^2be)x^6+10\,(7\,ab^2c-5\,a^2bd+3\,a^3e)x^4+6\,a^3c-2\,(7\,a^2bc-5\,a^3d)x^2}{30\,(a^4bx^7+a^5x^5)}-\frac{(7\,b^3c-5\,ab^2d-a^3f+3\,a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*(7*b^3*c - 5*a*b^2*d - a^3*f + 3*a^2*b*e)*x^6 + 10*(7*a*b^2*c - 5
*a^2*b*d + 3*a^3*e)*x^4 + 6*a^3*c - 2*(7*a^2*b*c - 5*a^3*d)*x^2)/(a^4*b*x^7
 + a^5*x^5) - 1/2*(7*b^3*c - 5*a*b^2*d - a^3*f + 3*a^2*b*e)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a^4)
```

Fricas [A]
time = 2.66, size = 470, normalized size = 3.09

$$\left[-\frac{30\,(7\,ab^4c-5\,a^2b^3d-a^4bf)x^6+12\,a^4bc+20\,(7\,a^2b^3c-5\,a^3b^2d)x^4-4\,(7\,a^3b^2c-5\,a^4bd)x^2+15\,((7\,b^4c-5\,ab^3d-a^3bf)x^7+(7\,ab^3c-5\,a^2b^2d-a^4f)x^5+3\,(a^2b^2x^7+a^3bx^5)e)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+30\,(3\,a^3b^2x^6+2\,a^4bx^4)e}{60\,(a^5b^2x^7+a^6bx^5)},\ -\frac{15\,(7\,ab^4c-5\,a^2b^3d-a^4bf)x^6+6\,a^4bc+10\,(7\,a^2b^3c-5\,a^3b^2d)x^4-2\,(7\,a^3b^2c-5\,a^4bd)x^2+15\,((7\,b^4c-5\,ab^3d-a^3bf)x^7+(7\,ab^3c-5\,a^2b^2d-a^4f)x^5+3\,(a^2b^2x^7+a^3bx^5)e)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+15\,(3\,a^3b^2x^6+2\,a^4bx^4)e}{30\,(a^5b^2x^7+a^6bx^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(30*(7*a*b^4*c - 5*a^2*b^3*d - a^4*b*f)*x^6 + 12*a^4*b*c + 20*(7*a^2
*b^3*c - 5*a^3*b^2*d)*x^4 - 4*(7*a^3*b^2*c - 5*a^4*b*d)*x^2 + 15*((7*b^4*c
- 5*a*b^3*d - a^3*b*f)*x^7 + (7*a*b^3*c - 5*a^2*b^2*d - a^4*f)*x^5 + 3*(a^2
*b^2*x^7 + a^3*b*x^5)*e)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2
 + a)) + 30*(3*a^3*b^2*x^6 + 2*a^4*b*x^4)*e)/(a^5*b^2*x^7 + a^6*b*x^5), -1/
30*(15*(7*a*b^4*c - 5*a^2*b^3*d - a^4*b*f)*x^6 + 6*a^4*b*c + 10*(7*a^2*b^3*
c - 5*a^3*b^2*d)*x^4 - 2*(7*a^3*b^2*c - 5*a^4*b*d)*x^2 + 15*((7*b^4*c - 5*a
*b^3*d - a^3*b*f)*x^7 + (7*a*b^3*c - 5*a^2*b^2*d - a^4*f)*x^5 + 3*(a^2*b^2*
x^7 + a^3*b*x^5)*e)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 15*(3*a^3*b^2*x^6 + 2
*a^4*b*x^4)*e)/(a^5*b^2*x^7 + a^6*b*x^5)]
```

Sympy [A]
time = 20.56, size = 226, normalized size = 1.49

$$-\frac{\sqrt{-\frac{1}{a^9b}}\,(a^3f-3a^2be+5ab^2d-7b^3c)\log\left(-a^5\sqrt{-\frac{1}{a^9b}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{a^9b}}\,(a^3f-3a^2be+5ab^2d-7b^3c)\log\left(a^5\sqrt{-\frac{1}{a^9b}}+x\right)}{4}+\frac{-6a^3c+x^6\cdot(15a^3f-45a^2be+75ab^2d-105b^3c)+x^4(-30a^3e+50a^2bd-70ab^2c)+x^2(-10a^3d+14a^2bc)}{30a^5x^5+30a^4bx^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(a**9*b))*(a**3*f - 3*a**2*b*e + 5*a*b**2*d - 7*b**3*c)*log(-a**5*
sqrt(-1/(a**9*b)) + x)/4 + sqrt(-1/(a**9*b))*(a**3*f - 3*a**2*b*e + 5*a*b**
2*d - 7*b**3*c)*log(a**5*sqrt(-1/(a**9*b)) + x)/4 + (-6*a**3*c + x**6*(15*a
**3*f - 45*a**2*b*e + 75*a*b**2*d - 105*b**3*c) + x**4*(-30*a**3*e + 50*a**
2*b*d - 70*a*b**2*c) + x**2*(-10*a**3*d + 14*a**2*b*c))/(30*a**5*x**5 + 30*
a**4*b*x**7)
```

Giac [A]
time = 1.94, size = 151, normalized size = 0.99

$$-\frac{(7\,b^3c-5\,ab^2d-a^3f+3\,a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^4}-\frac{b^3cx-ab^2dx-a^3fx+a^2bxe}{2\,(bx^2+a)a^4}-\frac{45\,b^2cx^4-30\,abdx^4+15\,a^2x^4e-10\,abcx^2+5\,a^2dx^2+3\,a^2c}{15\,a^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*b^3*c - 5*a*b^2*d - a^3*f + 3*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*a^4) - 1/2*(b^3*c*x - a*b^2*d*x - a^3*f*x + a^2*b*x*e)/((b*x^2 + a)*a^
4) - 1/15*(45*b^2*c*x^4 - 30*a*b*d*x^4 + 15*a^2*x^4*e - 10*a*b*c*x^2 + 5*a^
2*d*x^2 + 3*a^2*c)/(a^4*x^5)
```

Mupad [B]
time = 1.00, size = 145, normalized size = 0.95

$$-\frac{\frac{c}{5\,a}+\frac{x^6\,\left(-f\,a^3+3\,e\,a^2\,b-5\,d\,a\,b^2+7\,c\,b^3\right)}{2\,a^4}+\frac{x^2\,\left(5\,a\,d-7\,b\,c\right)}{15\,a^2}+\frac{x^4\,\left(3\,e\,a^2-5\,d\,a\,b+7\,c\,b^2\right)}{3\,a^3}}{b\,x^7+a\,x^5}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+3\,e\,a^2\,b-5\,d\,a\,b^2+7\,c\,b^3\right)}{2\,a^{9/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x)
```

```
[Out] - (c/(5*a) + (x^6*(7*b^3*c - a^3*f - 5*a*b^2*d + 3*a^2*b*e))/(2*a^4) + (x^2
*(5*a*d - 7*b*c))/(15*a^2) + (x^4*(7*b^2*c + 3*a^2*e - 5*a*b*d))/(3*a^3))/(
a*x^5 + b*x^7) - (atan((b^(1/2)*x)/a^(1/2))*(7*b^3*c - a^3*f - 5*a*b^2*d +
3*a^2*b*e))/(2*a^(9/2)*b^(1/2))
```

3.131 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{c}{7a^2x^7}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{3a^4x^3}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{2a^5\,(a+bx^2)}+\sqrt{b}\,(\;$$

```
[Out] -1/7*c/a^2/x^7+1/5*(-a*d+2*b*c)/a^3/x^5+1/3*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x^
3+(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x+1/2*b*(-a^3*f+a^2*b*e-a*b^2*d+
b^3*c)*x/a^5/(b*x^2+a)+1/2*(-3*a^3*f+5*a^2*b*e-7*a*b^2*d+9*b^3*c)*arctan(x*
b^(1/2)/a^(1/2))*b^(1/2)/a^(11/2)
```

Rubi [A]
time = 0.20, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1819, 1816, 211}

$$\frac{2bc-ad}{5a^3x^5}-\frac{c}{7a^2x^7}-\frac{a^2e-2abd+3b^2c}{3a^4x^3}+\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(-3a^3f+5a^2be-7ab^2d+9b^3c)}{2a^{11/2}}+\frac{bx(a^3(-f)+a^2be-ab^2d+b^3c)}{2a^5\,(a+bx^2)}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{a^5x}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -1/7*c/(a^2*x^7) + (2*b*c - a*d)/(5*a^3*x^5) - (3*b^2*c - 2*a*b*d + a^2*e)/
(3*a^4*x^3) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) + (b*(b^3*c
 - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^5*(a + b*x^2)) + (Sqrt[b]*(9*b^3*c -
7*a*b^2*d + 5*a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
```

```
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^2}\,dx = \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^5\left(a+bx^2\right)} - \frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2-\frac{2\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{2\left(b^3c-ab^2d+a^2be-a^3}{a^3}}{x^8(a+bx^2)}}{2a}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^5\left(a+bx^2\right)} - \frac{\int \left(-\frac{2c}{ax^8}-\frac{2(-2bc+ad)}{a^2x^6}-\frac{2\left(3b^2c-2abd+a^2e\right)}{a^3x^4}-\frac{2\left(-4b^3\right.}{}\right.}{2}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{3a^4x^3}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3\right.}{}$$

$$= -\frac{c}{7a^2x^7}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{3a^4x^3}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b\left(b^3\right.}{}$$

Mathematica [A]
time = 0.07, size = 190, normalized size = 1.01

$$-\frac{c}{7a^2x^7}+\frac{2bc-ad}{5a^3x^5}+\frac{-3b^2c+2abd-a^2e}{3a^4x^3}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}-\frac{b\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{2a^5\left(a+bx^2\right)}-\frac{\sqrt{b}\left(-9b^3c+7ab^2d-5a^2be+3a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x]
```

```
[Out] -1/7*c/(a^2*x^7) + (2*b*c - a*d)/(5*a^3*x^5) + (-3*b^2*c + 2*a*b*d - a^2*e)
/(3*a^4*x^3) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) - (b*(-(b^
3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^5*(a + b*x^2)) - (Sqrt[b]*(-9*b^3
*c + 7*a*b^2*d - 5*a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(11
/2))
```

Maple [A]
time = 0.13, size = 174, normalized size = 0.92

method	result
default	$-\frac{b\left(\frac{\left(\frac{1}{2}a^3f-\frac{1}{2}a^2be+\frac{1}{2}a\,b^2d-\frac{1}{2}b^3c\right)x}{b\,x^2+a}+\frac{\left(3a^3f-5a^2be+7a\,b^2d-9b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^5}-\frac{c}{7a^2x^7}-\frac{ad-2bc}{5a^3x^5}-\frac{a^2e-2abd+3b^2c}{3a^4x^3}-$

risch	$\frac{-\frac{b\left(3a^{3}f-5a^{2}be+7a\,b^{2}d-9b^{3}c\right)x^{8}}{2a^{5}}-\frac{\left(3a^{3}f-5a^{2}be+7a\,b^{2}d-9b^{3}c\right)x^{6}}{3a^{4}}-\frac{\left(5a^{2}e-7abd+9b^{2}c\right)x^{4}}{15a^{3}}-\frac{(7ad-9bc)x^{2}}{35a^{2}}-\frac{c}{7a}}{x^{7}(b\,x^{2}+a)}+\frac{\left(\_R=\mathrm{RootOf}\left(a^{11}\_Z^{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^5*b*((1/2*a^3*f-1/2*a^2*b*e+1/2*a*b^2*d-1/2*b^3*c)*x/(b*x^2+a)+1/2*(3*
a^3*f-5*a^2*b*e+7*a*b^2*d-9*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/7
*c/a^2/x^7-1/5*(a*d-2*b*c)/a^3/x^5-1/3*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^3-(a^3
*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5/x
```

Maxima [A]
time = 0.53, size = 198, normalized size = 1.05

$$\frac{105\,(9\,b^{4}c-7\,ab^{3}d-3\,a^{3}bf+5\,a^{2}b^{2}e)x^{8}+70\,(9\,ab^{3}c-7\,a^{2}b^{2}d-3\,a^{4}f+5\,a^{3}be)x^{6}-30\,a^{4}c-14\,(9\,a^{2}b^{2}c-7\,a^{3}bd+5\,a^{4}e)x^{4}+6\,(9\,a^{3}bc-7\,a^{4}d)x^{2}}{210\,(a^{5}bx^{9}+a^{6}x^{7})}+\frac{(9\,b^{4}c-7\,ab^{3}d-3\,a^{3}bf+5\,a^{2}b^{2}e)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/210*(105*(9*b^4*c - 7*a*b^3*d - 3*a^3*b*f + 5*a^2*b^2*e)*x^8 + 70*(9*a*b^
3*c - 7*a^2*b^2*d - 3*a^4*f + 5*a^3*b*e)*x^6 - 30*a^4*c - 14*(9*a^2*b^2*c -
 7*a^3*b*d + 5*a^4*e)*x^4 + 6*(9*a^3*b*c - 7*a^4*d)*x^2)/(a^5*b*x^9 + a^6*x
^7) + 1/2*(9*b^4*c - 7*a*b^3*d - 3*a^3*b*f + 5*a^2*b^2*e)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^5)
```

Fricas [A]
time = 2.62, size = 524, normalized size = 2.77

$$\left[\frac{210\,(9\,b^{4}c-7\,ab^{3}d-3\,a^{3}bf)x^{8}+140\,(9\,ab^{3}c-7\,a^{2}b^{2}d-3\,a^{4}f)x^{6}-60\,a^{4}c-28\,(9\,a^{2}b^{2}c-7\,a^{3}bd)x^{4}+12\,(9\,a^{3}bc-7\,a^{4}d)x^{2}+105\,((9\,b^{4}c-7\,ab^{3}d-3\,a^{3}bf)x^{9}+(9\,ab^{3}c-7\,a^{2}b^{2}d-3\,a^{4}f)x^{7}+5\,(a^{2}b^{2}x^{9}+a^{3}bx^{7})e)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)+70\,(15\,a^{2}b^{2}x^{8}+10\,a^{3}bx^{6}-2\,a^{4}x^{4})e}{420\,(a^{5}bx^{9}+a^{6}x^{7})},\ \frac{105\,(9\,b^{4}c-7\,ab^{3}d-3\,a^{3}bf)x^{8}+70\,(9\,ab^{3}c-7\,a^{2}b^{2}d-3\,a^{4}f)x^{6}-30\,a^{4}c-14\,(9\,a^{2}b^{2}c-7\,a^{3}bd)x^{4}+6\,(9\,a^{3}bc-7\,a^{4}d)x^{2}+105\,((9\,b^{4}c-7\,ab^{3}d-3\,a^{3}bf)x^{9}+(9\,ab^{3}c-7\,a^{2}b^{2}d-3\,a^{4}f)x^{7}+5\,(a^{2}b^{2}x^{9}+a^{3}bx^{7})e)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+35\,(15\,a^{2}b^{2}x^{8}+10\,a^{3}bx^{6}-2\,a^{4}x^{4})e}{210\,(a^{5}bx^{9}+a^{6}x^{7})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/420*(210*(9*b^4*c - 7*a*b^3*d - 3*a^3*b*f)*x^8 + 140*(9*a*b^3*c - 7*a^2*
b^2*d - 3*a^4*f)*x^6 - 60*a^4*c - 28*(9*a^2*b^2*c - 7*a^3*b*d)*x^4 + 12*(9*
a^3*b*c - 7*a^4*d)*x^2 + 105*((9*b^4*c - 7*a*b^3*d - 3*a^3*b*f)*x^9 + (9*a*
b^3*c - 7*a^2*b^2*d - 3*a^4*f)*x^7 + 5*(a^2*b^2*x^9 + a^3*b*x^7)*e)*sqrt(-b
/a)*log((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 70*(15*a^2*b^2*x^8 +
10*a^3*b*x^6 - 2*a^4*x^4)*e)/(a^5*b*x^9 + a^6*x^7), 1/210*(105*(9*b^4*c - 7
*a*b^3*d - 3*a^3*b*f)*x^8 + 70*(9*a*b^3*c - 7*a^2*b^2*d - 3*a^4*f)*x^6 - 30
*a^4*c - 14*(9*a^2*b^2*c - 7*a^3*b*d)*x^4 + 6*(9*a^3*b*c - 7*a^4*d)*x^2 + 1
05*((9*b^4*c - 7*a*b^3*d - 3*a^3*b*f)*x^9 + (9*a*b^3*c - 7*a^2*b^2*d - 3*a^
4*f)*x^7 + 5*(a^2*b^2*x^9 + a^3*b*x^7)*e)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 3
5*(15*a^2*b^2*x^8 + 10*a^3*b*x^6 - 2*a^4*x^4)*e)/(a^5*b*x^9 + a^6*x^7)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.41, size = 201, normalized size = 1.06

$$\frac{\left(9\,b^4c-7\,ab^3d-3\,a^3bf+5\,a^2b^2e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^5}+\frac{b^4cx-ab^3dx-a^3bfx+a^2b^2xe}{2\,(bx^2+a)a^5}+\frac{420\,b^3cx^6-315\,ab^2dx^6-105\,a^3fx^6+210\,a^2bx^6e-105\,ab^2cx^4+70\,a^2bdx^4-35\,a^3x^4e+42\,a^2bcx^2-21\,a^3dx^2-15\,a^3c}{105\,a^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*b^4*c - 7*a*b^3*d - 3*a^3*b*f + 5*a^2*b^2*e)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*a^5) + 1/2*(b^4*c*x - a*b^3*d*x - a^3*b*f*x + a^2*b^2*x*e)/((b*x^
2 + a)*a^5) + 1/105*(420*b^3*c*x^6 - 315*a*b^2*d*x^6 - 105*a^3*f*x^6 + 210*
a^2*b*x^6*e - 105*a*b^2*c*x^4 + 70*a^2*b*d*x^4 - 35*a^3*x^4*e + 42*a^2*b*c*
x^2 - 21*a^3*d*x^2 - 15*a^3*c)/(a^5*x^7)
```

Mupad [**B**]
time = 0.99, size = 181, normalized size = 0.96

$$\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-3\,f\,a^3+5\,e\,a^2\,b-7\,d\,a\,b^2+9\,c\,b^3\right)}{2\,a^{11/2}}-\frac{\frac{c}{7\,a}-\frac{x^6\,\left(-3\,f\,a^3+5\,e\,a^2\,b-7\,d\,a\,b^2+9\,c\,b^3\right)}{3\,a^4}+\frac{x^2\,\left(7\,a\,d-9\,b\,c\right)}{35\,a^2}+\frac{x^4\,\left(5\,e\,a^2-7\,d\,a\,b+9\,c\,b^2\right)}{15\,a^3}-\frac{b\,x^8\,\left(-3\,f\,a^3+5\,e\,a^2\,b-7\,d\,a\,b^2+9\,c\,b^3\right)}{2\,a^5}}{b\,x^9+a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x)
```

```
[Out] (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(9*b^3*c - 3*a^3*f - 7*a*b^2*d + 5*a^2*b
*e))/(2*a^(11/2)) - (c/(7*a) - (x^6*(9*b^3*c - 3*a^3*f - 7*a*b^2*d + 5*a^2*
b*e))/(3*a^4) + (x^2*(7*a*d - 9*b*c))/(35*a^2) + (x^4*(9*b^2*c + 5*a^2*e -
7*a*b*d))/(15*a^3) - (b*x^8*(9*b^3*c - 3*a^3*f - 7*a*b^2*d + 5*a^2*b*e))/(2
*a^5))/(a*x^7 + b*x^9)
```

3.132 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx$

Optimal. Leaf size=230

$$-\frac{c}{9a^2x^9}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3}-\frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)}{a^6x}-\frac{b^2}{}$$

```
[Out] -1/9*c/a^2/x^9+1/7*(-a*d+2*b*c)/a^3/x^7+1/5*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x^
5+1/3*(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x^3-b*(-2*a^3*f+3*a^2*b*e-4*
a*b^2*d+5*b^3*c)/a^6/x-1/2*b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^6/(b*x^2+
a)-1/2*b^(3/2)*(-5*a^3*f+7*a^2*b*e-9*a*b^2*d+11*b^3*c)*arctan(x*b^(1/2)/a^(
1/2))/a^(13/2)
```

Rubi [A]
time = 0.25, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1819, 1816, 211}

$$\frac{2bc-ad}{7a^3x^7}-\frac{c}{9a^2x^9}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}-\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-5a^3f+7a^2be-9ab^2d+11b^3c\right)}{2a^{13/2}}-\frac{b^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{2a^6\left(a+bx^2\right)}-\frac{b(-2a^3f+3a^2be-4ab^2d+5b^3c)}{a^6x}+\frac{a^3(-f)+2a^2be-3ab^2d+4b^3c}{3a^5x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x]
```

```
[Out] -1/9*c/(a^2*x^9) + (2*b*c - a*d)/(7*a^3*x^7) - (3*b^2*c - 2*a*b*d + a^2*e)/
(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) - (b*(5
*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(a^6*x) - (b^2*(b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*x)/(2*a^6*(a + b*x^2)) - (b^(3/2)*(11*b^3*c - 9*a*b^2*d
+ 7*a^2*b*e - 5*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(13/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
```

```
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx = -\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{2a^6\left(a+bx^2\right)} - \frac{\int \frac{-2c+2\left(\frac{bc}{a}-d\right)x^2-\frac{2\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{2\left(b^3c-ab^2d+a^2be}{a^3}}{x}}{}$$

$$= -\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{2a^6\left(a+bx^2\right)} - \frac{\int \left(-\frac{2c}{ax^{10}}-\frac{2(-2bc+ad)}{a^2x^8}-\frac{2\left(3b^2c-2abd+a^2e\right)}{a^3x^6}-2(\right.}{}$$

$$= -\frac{c}{9a^2x^9}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3}-\frac{b(5b}{}$$

$$= -\frac{c}{9a^2x^9}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3}-\frac{b(5b}{}$$

Mathematica [A]
time = 0.08, size = 230, normalized size = 1.00

$$-\frac{c}{9a^2x^9}+\frac{2bc-ad}{7a^3x^7}+\frac{-3b^2c+2abd-a^2e}{5a^4x^5}+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3}+\frac{b(-5b^3c+4ab^2d-3a^2be+2a^3f)}{a^6x}+\frac{b^2(-b^3c+ab^2d-a^2be+a^3f)\,x}{2a^6\left(a+bx^2\right)}+\frac{b^{3/2}(-11b^3c+9ab^2d-7a^2be+5a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2a^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x]
```

```
[Out] -1/9*c/(a^2*x^9) + (2*b*c - a*d)/(7*a^3*x^7) + (-3*b^2*c + 2*a*b*d - a^2*e)
/(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) + (b*(
-5*b^3*c + 4*a*b^2*d - 3*a^2*b*e + 2*a^3*f))/(a^6*x) + (b^2*(-(b^3*c) + a*b
^2*d - a^2*b*e + a^3*f)*x)/(2*a^6*(a + b*x^2)) + (b^(3/2)*(-11*b^3*c + 9*a*
b^2*d - 7*a^2*b*e + 5*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(13/2))
```

Maple [A]
time = 0.16, size = 210, normalized size = 0.91

method	result

default	$\frac{b^2\left(\frac{\left(\frac{1}{2}a^3f-\frac{1}{2}a^2be+\frac{1}{2}a\,b^2d-\frac{1}{2}b^3c\right)x}{b\,x^2+a}+\frac{\left(5a^3f-7a^2be+9a\,b^2d-11b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{a^6}-\frac{c}{9a^2x^9}-\frac{ad-2bc}{7a^3x^7}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}-$
risch	$\frac{\frac{b^2\left(5a^3f-7a^2be+9a\,b^2d-11b^3c\right)x^{10}}{2a^6}+\frac{b\left(5a^3f-7a^2be+9a\,b^2d-11b^3c\right)x^8}{3a^5}-\frac{\left(5a^3f-7a^2be+9a\,b^2d-11b^3c\right)x^6}{15a^4}-\frac{\left(7a^2e-9abd+11b^2c\right)x^4}{35a^3}-\frac{(9ad-11}{63a}}{x^9(b\,x^2+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/a^6*((1/2*a^3*f-1/2*a^2*b*e+1/2*a*b^2*d-1/2*b^3*c)*x/(b*x^2+a)+1/2*(5*a
^3*f-7*a^2*b*e+9*a*b^2*d-11*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/9
*c/a^2/x^9-1/7*(a*d-2*b*c)/a^3/x^7-1/5*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^5-1/3*
(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5/x^3+b*(2*a^3*f-3*a^2*b*e+4*a*b^2*d-
5*b^3*c)/a^6/x
```

Maxima [A]
time = 0.52, size = 243, normalized size = 1.06

$$-\frac{315\,(11\,b^5c-9\,ab^4d-5\,a^3b^2f+7\,a^2b^3e)x^{10}+210\,(11\,ab^4c-9\,a^2b^3d-5\,a^4bf+7\,a^3b^2e)x^8-42\,(11\,a^2b^3c-9\,a^3b^2d-5\,a^5f+7\,a^4be)x^6+70\,a^5c+18\,(11\,a^3b^2c-9\,a^4bd+7\,a^5e)x^4-10\,(11\,a^4bc-9\,a^5d)x^2}{630\,(a^6bx^{11}+a^7x^9)}-\frac{(11\,b^5c-9\,ab^4d-5\,a^3b^2f+7\,a^2b^3e)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/630*(315*(11*b^5*c - 9*a*b^4*d - 5*a^3*b^2*f + 7*a^2*b^3*e)*x^10 + 210*(
11*a*b^4*c - 9*a^2*b^3*d - 5*a^4*b*f + 7*a^3*b^2*e)*x^8 - 42*(11*a^2*b^3*c
- 9*a^3*b^2*d - 5*a^5*f + 7*a^4*b*e)*x^6 + 70*a^5*c + 18*(11*a^3*b^2*c - 9*
a^4*b*d + 7*a^5*e)*x^4 - 10*(11*a^4*b*c - 9*a^5*d)*x^2)/(a^6*b*x^11 + a^7*x
^9) - 1/2*(11*b^5*c - 9*a*b^4*d - 5*a^3*b^2*f + 7*a^2*b^3*e)*arctan(b*x/sqr
t(a*b))/(sqrt(a*b)*a^6)
```

Fricas [A]
time = 3.86, size = 622, normalized size = 2.70

$$\left[-\frac{630\,(11\,b^5c-9\,ab^4d-5\,a^3b^2f)x^{10}+420\,(11\,ab^4c-9\,a^2b^3d-5\,a^4bf)x^8-84\,(11\,a^2b^3c-9\,a^3b^2d-5\,a^5f)x^6+140\,a^5c+36\,(11\,a^3b^2c-9\,a^4bd)x^4-20\,(11\,a^4bc-9\,a^5d)x^2-315\,((11\,b^5c-9\,ab^4d-5\,a^3b^2f)x^{11}+(11\,ab^4c-9\,a^2b^3d-5\,a^4bf)x^9+7\,(a^2b^3x^{11}+a^3b^2x^9)e)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+42\,(105\,a^2b^3x^{10}+70\,a^3b^2x^8-14\,a^4bx^6+6\,a^5x^4)e}{1260\,(a^6bx^{11}+a^7x^9)},-\frac{315\,(11\,b^5c-9\,ab^4d-5\,a^3b^2f)x^{10}+210\,(11\,ab^4c-9\,a^2b^3d-5\,a^4bf)x^8-42\,(11\,a^2b^3c-9\,a^3b^2d-5\,a^5f)x^6+70\,a^5c+18\,(11\,a^3b^2c-9\,a^4bd)x^4-10\,(11\,a^4bc-9\,a^5d)x^2+315\,((11\,b^5c-9\,ab^4d-5\,a^3b^2f)x^{11}+(11\,ab^4c-9\,a^2b^3d-5\,a^4bf)x^9+7\,(a^2b^3x^{11}+a^3b^2x^9)e)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+21\,(105\,a^2b^3x^{10}+70\,a^3b^2x^8-14\,a^4bx^6+6\,a^5x^4)e}{630\,(a^6bx^{11}+a^7x^9)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/1260*(630*(11*b^5*c - 9*a*b^4*d - 5*a^3*b^2*f)*x^10 + 420*(11*a*b^4*c -
 9*a^2*b^3*d - 5*a^4*b*f)*x^8 - 84*(11*a^2*b^3*c - 9*a^3*b^2*d - 5*a^5*f)*x
^6 + 140*a^5*c + 36*(11*a^3*b^2*c - 9*a^4*b*d)*x^4 - 20*(11*a^4*b*c - 9*a^5
*d)*x^2 - 315*((11*b^5*c - 9*a*b^4*d - 5*a^3*b^2*f)*x^11 + (11*a*b^4*c - 9*
```

```
a^2*b^3*d - 5*a^4*b*f)*x^9 + 7*(a^2*b^3*x^11 + a^3*b^2*x^9)*e)*sqrt(-b/a)*l
og((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 42*(105*a^2*b^3*x^10 + 70*
a^3*b^2*x^8 - 14*a^4*b*x^6 + 6*a^5*x^4)*e)/(a^6*b*x^11 + a^7*x^9), -1/630*(
315*(11*b^5*c - 9*a*b^4*d - 5*a^3*b^2*f)*x^10 + 210*(11*a*b^4*c - 9*a^2*b^3
*d - 5*a^4*b*f)*x^8 - 42*(11*a^2*b^3*c - 9*a^3*b^2*d - 5*a^5*f)*x^6 + 70*a^
5*c + 18*(11*a^3*b^2*c - 9*a^4*b*d)*x^4 - 10*(11*a^4*b*c - 9*a^5*d)*x^2 + 3
15*((11*b^5*c - 9*a*b^4*d - 5*a^3*b^2*f)*x^11 + (11*a*b^4*c - 9*a^2*b^3*d -
 5*a^4*b*f)*x^9 + 7*(a^2*b^3*x^11 + a^3*b^2*x^9)*e)*sqrt(b/a)*arctan(x*sqrt
(b/a)) + 21*(105*a^2*b^3*x^10 + 70*a^3*b^2*x^8 - 14*a^4*b*x^6 + 6*a^5*x^4)*
e)/(a^6*b*x^11 + a^7*x^9)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.26, size = 252, normalized size = 1.10

$$-\frac{\left(11\,b^{5}c - 9\,ab^{4}d - 5\,a^{3}b^{2}f + 7\,a^{2}b^{3}e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a^{6}} - \frac{b^{5}cx - ab^{4}dx - a^{3}b^{2}fx + a^{2}b^{3}xe}{2\,(bx^{2} + a)a^{6}} - \frac{1575\,b^{4}cx^{8} - 1260\,ab^{3}dx^{8} - 630\,a^{3}bfx^{8} + 945\,a^{2}b^{2}x^{8}e - 420\,ab^{3}cx^{6} + 315\,a^{2}b^{2}dx^{6} + 105\,a^{4}fx^{6} - 210\,a^{3}bx^{6}e + 189\,a^{2}b^{2}cx^{4} - 126\,a^{3}bdx^{4} + 63\,a^{4}x^{4}e - 90\,a^{3}bcx^{2} + 45\,a^{4}dx^{2} + 35\,a^{4}c}{315\,a^{6}x^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(11*b^5*c - 9*a*b^4*d - 5*a^3*b^2*f + 7*a^2*b^3*e)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*a^6) - 1/2*(b^5*c*x - a*b^4*d*x - a^3*b^2*f*x + a^2*b^3*x*e)/
((b*x^2 + a)*a^6) - 1/315*(1575*b^4*c*x^8 - 1260*a*b^3*d*x^8 - 630*a^3*b*f*
x^8 + 945*a^2*b^2*x^8*e - 420*a*b^3*c*x^6 + 315*a^2*b^2*d*x^6 + 105*a^4*f*x
^6 - 210*a^3*b*x^6*e + 189*a^2*b^2*c*x^4 - 126*a^3*b*d*x^4 + 63*a^4*x^4*e -
 90*a^3*b*c*x^2 + 45*a^4*d*x^2 + 35*a^4*c)/(a^6*x^9)
```

Mupad [**B**]
time = 1.01, size = 219, normalized size = 0.95

$$-\frac{\frac{c}{9\,a} - \frac{x^{6}\,\left(-5\,f\,a^{3}+7\,e\,a^{2}\,b-9\,d\,a\,b^{2}+11\,c\,b^{3}\right)}{15\,a^{4}} + \frac{x^{2}\,\left(9\,a\,d-11\,b\,c\right)}{63\,a^{2}} + \frac{x^{4}\,\left(7\,e\,a^{2}-9\,d\,a\,b+11\,c\,b^{2}\right)}{35\,a^{3}} + \frac{b\,x^{8}\,\left(-5\,f\,a^{3}+7\,e\,a^{2}\,b-9\,d\,a\,b^{2}+11\,c\,b^{3}\right)}{3\,a^{5}} + \frac{b^{2}\,x^{10}\,\left(-5\,f\,a^{3}+7\,e\,a^{2}\,b-9\,d\,a\,b^{2}+11\,c\,b^{3}\right)}{2\,a^{6}}}{b\,x^{11} + a\,x^{9}} - \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-5\,f\,a^{3} + 7\,e\,a^{2}\,b - 9\,d\,a\,b^{2} + 11\,c\,b^{3}\right)}{2\,a^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x)
```

```
[Out] - (c/(9*a) - (x^6*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*a^2*b*e))/(15*a^4) +
(x^2*(9*a*d - 11*b*c))/(63*a^2) + (x^4*(11*b^2*c + 7*a^2*e - 9*a*b*d))/(35*
```

```
a^3) + (b*x^8*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*a^2*b*e))/(3*a^5) + (b^2*
x^10*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*a^2*b*e))/(2*a^6))/(a*x^9 + b*x^11
) - (b^(3/2)*atan((b^(1/2)*x)/a^(1/2))*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*
a^2*b*e))/(2*a^(13/2))
```

3.133 $\int \frac{x^8\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=287

$$-\frac{a\left(15b^3c-27ab^2d+43a^2be-63a^3f\right)x}{4b^7}+\frac{\left(5b^3c-9ab^2d+15a^2be-23a^3f\right)x^3}{6b^6}-\frac{\left(5b^3c-9ab^2d+17a^2be-2\right.}{20ab^5}$$

```
[Out] -1/4*a*(-63*a^3*f+43*a^2*b*e-27*a*b^2*d+15*b^3*c)*x/b^7+1/6*(-23*a^3*f+15*a
^2*b*e-9*a*b^2*d+5*b^3*c)*x^3/b^6-1/20*(-29*a^3*f+17*a^2*b*e-9*a*b^2*d+5*b^
3*c)*x^5/a/b^5+1/7*(-3*a*f+b*e)*x^7/b^4+1/9*f*x^9/b^3+1/4*(c-a*(a^2*f-a*b*e
+b^2*d)/b^3)*x^9/a/(b*x^2+a)^2-1/8*a^2*(-17*a^3*f+13*a^2*b*e-9*a*b^2*d+5*b^
3*c)*x/b^7/(b*x^2+a)+1/8*a^(3/2)*(-143*a^3*f+99*a^2*b*e-63*a*b^2*d+35*b^3*c
)*arctan(x*b^(1/2)/a^(1/2))/b^(15/2)
```

Rubi [A]

time = 0.33, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1818, 1599, 1271, 1824, 211}

$$\frac{x^9\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{a^2x\left(-17a^3f+13a^2be-9ab^2d+5b^3c\right)}{8b^7\left(a+bx^2\right)}-\frac{ax\left(-63a^3f+43a^2be-27ab^2d+15b^3c\right)}{4b^7}+\frac{x^3\left(-23a^3f+15a^2be-9ab^2d+5b^3c\right)}{6b^6}-\frac{x^5\left(-29a^3f+17a^2be-9ab^2d+5b^3c\right)}{20ab^5}+\frac{a^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-143a^3f+99a^2be-63ab^2d+35b^3c\right)}{8b^{15/2}}+\frac{x^7(be-3af)}{7b^4}+\frac{fx^9}{9b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] -1/4*(a*(15*b^3*c - 27*a*b^2*d + 43*a^2*b*e - 63*a^3*f)*x)/b^7 + ((5*b^3*c
 - 9*a*b^2*d + 15*a^2*b*e - 23*a^3*f)*x^3)/(6*b^6) - ((5*b^3*c - 9*a*b^2*d +
 17*a^2*b*e - 29*a^3*f)*x^5)/(20*a*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*
x^9)/(9*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^9)/(4*a*(a + b*x^2)
^2) - (a^2*(5*b^3*c - 9*a*b^2*d + 13*a^2*b*e - 17*a^3*f)*x)/(8*b^7*(a + b*x
^2)) + (a^(3/2)*(35*b^3*c - 63*a*b^2*d + 99*a^2*b*e - 143*a^3*f)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(8*b^(15/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
```

```
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx &= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^9}{4a\,(a+bx^2)^2} - \frac{\int \frac{x^7\left(\left(5bc-9ad+\frac{9a^2e}{b}-\frac{9a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{(a+bx^2)^2}\,dx}{4ab} \\
&= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^9}{4a\,(a+bx^2)^2} - \frac{\int \frac{x^8\left(5bc-9ad+\frac{9a^2e}{b}-\frac{9a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{(a+bx^2)^2}\,dx}{4ab} \\
&= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^9}{4a\,(a+bx^2)^2} - \frac{a^2(5b^3c-9ab^2d+13a^2be-17a^3f)\,x}{8b^7\,(a+bx^2)} + \frac{\int \frac{a^3(5b^3c}{}}{} \\
&= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^9}{4a\,(a+bx^2)^2} - \frac{a^2(5b^3c-9ab^2d+13a^2be-17a^3f)\,x}{8b^7\,(a+bx^2)} + \frac{\int \Big(-2a^2}{} \\
&= -\frac{a(15b^3c-27ab^2d+43a^2be-63a^3f)\,x}{4b^7} + \frac{(5b^3c-9ab^2d+15a^2be-23a^3}{6b^6} \\
&= -\frac{a(15b^3c-27ab^2d+43a^2be-63a^3f)\,x}{4b^7} + \frac{(5b^3c-9ab^2d+15a^2be-23a^3}{6b^6}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 272, normalized size = 0.95

$$\frac{a(-3b^3c+6ab^2d-10a^2be+15a^3f)\,x}{b^7}+\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^3}{3b^6}+\frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5}+\frac{(be-3af)x^7}{7b^4}+\frac{fx^9}{9b^3}+\frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{4b^7\,(a+bx^2)^2}+\frac{a^2(-13b^3c+17ab^2d-21a^2be+25a^3f)\,x}{8b^7\,(a+bx^2)}-\frac{a^{3/2}(-35b^3c+63ab^2d-99a^2be+143a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x)/b^7 + ((b^3*c - 3*a*b^
2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/(3*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x^5
)/(5*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*x^9)/(9*b^3) + (a^3*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*x)/(4*b^7*(a + b*x^2)^2) + (a^2*(-13*b^3*c + 17*a
*b^2*d - 21*a^2*b*e + 25*a^3*f)*x)/(8*b^7*(a + b*x^2)) - (a^(3/2)*(-35*b^3*
c + 63*a*b^2*d - 99*a^2*b*e + 143*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*b^
(15/2))
```

Maple [A]
time = 0.16, size = 268, normalized size = 0.93

method	result

default	$\frac{\frac{1}{9}f\,x^9b^4-\frac{3}{7}a\,b^3f\,x^7+\frac{1}{7}b^4e\,x^7+\frac{6}{5}a^2b^2f\,x^5-\frac{3}{5}a\,b^3e\,x^5+\frac{1}{5}b^4d\,x^5-\frac{10}{3}a^3bf\,x^3+2a^2b^2e\,x^3-a\,b^3d\,x^3+\frac{1}{3}b^4c\,x^3+15a^4fx-10a^3bex+6a^2b}{b^7}$
risch	$\frac{f\,x^9}{9b^3}-\frac{3af\,x^7}{7b^4}+\frac{e\,x^7}{7b^3}+\frac{6a^2f\,x^5}{5b^5}-\frac{3ae\,x^5}{5b^4}+\frac{d\,x^5}{5b^3}-\frac{10a^3f\,x^3}{3b^6}+\frac{2a^2e\,x^3}{b^5}-\frac{ad\,x^3}{b^4}+\frac{c\,x^3}{3b^3}+\frac{15a^4fx}{b^7}-\frac{10a^3ex}{b^6}+6$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^7*(1/9*f*x^9*b^4-3/7*a*b^3*f*x^7+1/7*b^4*e*x^7+6/5*a^2*b^2*f*x^5-3/5*a*
b^3*e*x^5+1/5*b^4*d*x^5-10/3*a^3*b*f*x^3+2*a^2*b^2*e*x^3-a*b^3*d*x^3+1/3*b^
4*c*x^3+15*a^4*f*x-10*a^3*b*e*x+6*a^2*b^2*d*x-3*a*b^3*c*x)-a^2/b^7*(((-25/8
*a^3*b*f+21/8*a^2*e*b^2-17/8*a*d*b^3+13/8*c*b^4)*x^3-1/8*a*(23*a^3*f-19*a^2
*b*e+15*a*b^2*d-11*b^3*c)*x)/(b*x^2+a)^2+1/8*(143*a^3*f-99*a^2*b*e+63*a*b^2
*d-35*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]
time = 0.53, size = 289, normalized size = 1.01

$$-\frac{(13\,a^2b^4c-17\,a^3b^3d-25\,a^5bf+21\,a^4b^2e)x^3+(11\,a^3b^3c-15\,a^4b^2d-23\,a^6f+19\,a^5be)x}{8\,(b^9x^4+2\,ab^8x^2+a^2b^7)}+\frac{(35\,a^2b^3c-63\,a^3b^2d-143\,a^5f+99\,a^4be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^7}+\frac{35\,b^4fx^9-45\,(3\,ab^3f-b^4e)x^7+63\,(b^4d+6\,a^2b^2f-3\,ab^3e)x^5+105\,(b^4c-3\,ab^3d-10\,a^3bf+6\,a^2b^2e)x^3-315\,(3\,ab^3c-6\,a^2b^2d-15\,a^4f+10\,a^3be)x}{315\,b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((13*a^2*b^4*c - 17*a^3*b^3*d - 25*a^5*b*f + 21*a^4*b^2*e)*x^3 + (11*a
^3*b^3*c - 15*a^4*b^2*d - 23*a^6*f + 19*a^5*b*e)*x)/(b^9*x^4 + 2*a*b^8*x^2
+ a^2*b^7) + 1/8*(35*a^2*b^3*c - 63*a^3*b^2*d - 143*a^5*f + 99*a^4*b*e)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*b^7) + 1/315*(35*b^4*f*x^9 - 45*(3*a*b^3*f -
b^4*e)*x^7 + 63*(b^4*d + 6*a^2*b^2*f - 3*a*b^3*e)*x^5 + 105*(b^4*c - 3*a*b^
3*d - 10*a^3*b*f + 6*a^2*b^2*e)*x^3 - 315*(3*a*b^3*c - 6*a^2*b^2*d - 15*a^4
*f + 10*a^3*b*e)*x)/b^7
```

Fricas [A]
time = 4.23, size = 798, normalized size = 2.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/5040*(560*b^6*f*x^13 - 1040*a*b^5*f*x^11 + 16*(63*b^6*d + 143*a^2*b^4*f)
*x^9 + 48*(35*b^6*c - 63*a*b^5*d - 143*a^3*b^3*f)*x^7 - 336*(35*a*b^5*c - 6
```

```
3*a^2*b^4*d - 143*a^4*b^2*f)*x^5 - 1050*(35*a^2*b^4*c - 63*a^3*b^3*d - 143*
a^5*b*f)*x^3 + 315*(35*a^3*b^3*c - 63*a^4*b^2*d - 143*a^6*f + (35*a*b^5*c -
 63*a^2*b^4*d - 143*a^4*b^2*f)*x^4 + 2*(35*a^2*b^4*c - 63*a^3*b^3*d - 143*a
^5*b*f)*x^2 + 99*(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)*e)*sqrt(-a/b)*log((b
*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 630*(35*a^3*b^3*c - 63*a^4*b^2*
d - 143*a^6*f)*x + 18*(40*b^6*x^11 - 88*a*b^5*x^9 + 264*a^2*b^4*x^7 - 1848*
a^3*b^3*x^5 - 5775*a^4*b^2*x^3 - 3465*a^5*b*x)*e)/(b^9*x^4 + 2*a*b^8*x^2 +
a^2*b^7), 1/2520*(280*b^6*f*x^13 - 520*a*b^5*f*x^11 + 8*(63*b^6*d + 143*a^2
*b^4*f)*x^9 + 24*(35*b^6*c - 63*a*b^5*d - 143*a^3*b^3*f)*x^7 - 168*(35*a*b^
5*c - 63*a^2*b^4*d - 143*a^4*b^2*f)*x^5 - 525*(35*a^2*b^4*c - 63*a^3*b^3*d
- 143*a^5*b*f)*x^3 + 315*(35*a^3*b^3*c - 63*a^4*b^2*d - 143*a^6*f + (35*a*b
^5*c - 63*a^2*b^4*d - 143*a^4*b^2*f)*x^4 + 2*(35*a^2*b^4*c - 63*a^3*b^3*d -
 143*a^5*b*f)*x^2 + 99*(a^3*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)*e)*sqrt(a/b)*a
rctan(b*x*sqrt(a/b)/a) - 315*(35*a^3*b^3*c - 63*a^4*b^2*d - 143*a^6*f)*x +
9*(40*b^6*x^11 - 88*a*b^5*x^9 + 264*a^2*b^4*x^7 - 1848*a^3*b^3*x^5 - 5775*a
^4*b^2*x^3 - 3465*a^5*b*x)*e)/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7)]
```

Sympy [A]
time = 19.71, size = 503, normalized size = 1.75

$$x^{7}\left(-\frac{3af}{7b^{4}}+\frac{e}{7b^{3}}\right)+x^{5}\cdot\left(\frac{6a^{2}f}{5b^{5}}-\frac{3ae}{5b^{4}}+\frac{d}{5b^{3}}\right)+x^{3}\left(-\frac{10a^{3}f}{3b^{6}}+\frac{2a^{2}e}{b^{5}}-\frac{ad}{b^{4}}+\frac{c}{3b^{3}}\right)+x\left(\frac{15a^{4}f}{b^{7}}-\frac{10a^{3}e}{b^{6}}+\frac{6a^{2}d}{b^{5}}-\frac{3ac}{b^{4}}\right)+\frac{\sqrt{-\frac{a^{3}}{b^{15}}}\cdot(143a^{3}f-99a^{2}be+63ab^{2}d-35b^{3}c)\log\left(-\frac{b^{7}\sqrt{-\frac{a^{3}}{b^{15}}}\cdot(143a^{3}f-99a^{2}be+63ab^{2}d-35b^{3}c)}{143a^{4}f-99a^{3}be+63a^{2}b^{2}d-35ab^{3}c}+x\right)}{16}-\frac{\sqrt{-\frac{a^{3}}{b^{15}}}\cdot(143a^{3}f-99a^{2}be+63ab^{2}d-35b^{3}c)\log\left(\frac{b^{7}\sqrt{-\frac{a^{3}}{b^{15}}}\cdot(143a^{3}f-99a^{2}be+63ab^{2}d-35b^{3}c)}{143a^{4}f-99a^{3}be+63a^{2}b^{2}d-35ab^{3}c}+x\right)}{16}+\frac{x^{3}\cdot(25a^{5}bf-21a^{4}b^{2}e+17a^{3}b^{3}d-13a^{2}b^{4}c)+x(23a^{6}f-19a^{5}be+15a^{4}b^{2}d-11a^{3}b^{3}c)}{8a^{2}b^{7}+16ab^{8}x^{2}+8b^{9}x^{4}}+\frac{fx^{9}}{9b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] x**7*(-3*a*f/(7*b**4) + e/(7*b**3)) + x**5*(6*a**2*f/(5*b**5) - 3*a*e/(5*b*
*4) + d/(5*b**3)) + x**3*(-10*a**3*f/(3*b**6) + 2*a**2*e/b**5 - a*d/b**4 +
c/(3*b**3)) + x*(15*a**4*f/b**7 - 10*a**3*e/b**6 + 6*a**2*d/b**5 - 3*a*c/b*
*4) + sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3*c
)*log(-b**7*sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*
b**3*c)/(143*a**4*f - 99*a**3*b*e + 63*a**2*b**2*d - 35*a*b**3*c) + x)/16 -
 sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3*c)*log
(b**7*sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3*c
)/(143*a**4*f - 99*a**3*b*e + 63*a**2*b**2*d - 35*a*b**3*c) + x)/16 + (x**3
*(25*a**5*b*f - 21*a**4*b**2*e + 17*a**3*b**3*d - 13*a**2*b**4*c) + x*(23*a
**6*f - 19*a**5*b*e + 15*a**4*b**2*d - 11*a**3*b**3*c))/(8*a**2*b**7 + 16*a
*b**8*x**2 + 8*b**9*x**4) + f*x**9/(9*b**3)
```

Giac [A]
time = 2.18, size = 301, normalized size = 1.05

$$\frac{(35\,a^{2}b^{3}c-63\,a^{3}b^{2}d-143\,a^{5}f+99\,a^{4}be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{7}}-\frac{13\,a^{2}b^{4}cx^{3}-17\,a^{3}b^{3}dx^{3}-25\,a^{5}bfx^{3}+21\,a^{4}b^{2}x^{3}e+11\,a^{3}b^{3}cx-15\,a^{4}b^{2}dx-23\,a^{6}fx+19\,a^{5}bxe}{8\,(bx^{2}+a)^{2}b^{7}}+\frac{35\,b^{24}fx^{9}-135\,ab^{23}fx^{7}+45\,b^{24}x^{7}e+63\,b^{24}dx^{5}+378\,a^{2}b^{22}fx^{5}-189\,ab^{23}x^{5}e+105\,b^{24}cx^{3}-315\,ab^{23}dx^{3}-1050\,a^{3}b^{21}fx^{3}+630\,a^{2}b^{22}x^{3}e-945\,ab^{23}cx+1890\,a^{2}b^{22}dx+4725\,a^{4}b^{20}fx-3150\,a^{3}b^{21}xe}{315\,b^{27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*a^2*b^3*c - 63*a^3*b^2*d - 143*a^5*f + 99*a^4*b*e)*arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*b^7) - 1/8*(13*a^2*b^4*c*x^3 - 17*a^3*b^3*d*x^3 - 25*a^5*b
*f*x^3 + 21*a^4*b^2*x^3*e + 11*a^3*b^3*c*x - 15*a^4*b^2*d*x - 23*a^6*f*x +
19*a^5*b*x*e)/((b*x^2 + a)^2*b^7) + 1/315*(35*b^24*f*x^9 - 135*a*b^23*f*x^7
 + 45*b^24*x^7*e + 63*b^24*d*x^5 + 378*a^2*b^22*f*x^5 - 189*a*b^23*x^5*e +
105*b^24*c*x^3 - 315*a*b^23*d*x^3 - 1050*a^3*b^21*f*x^3 + 630*a^2*b^22*x^3*
e - 945*a*b^23*c*x + 1890*a^2*b^22*d*x + 4725*a^4*b^20*f*x - 3150*a^3*b^21*
x*e)/b^27
```

Mupad [B]
time = 1.00, size = 506, normalized size = 1.76

$$x^7\left(\frac{e}{7\,b^3}-\frac{3\,a\,f}{7\,b^4}\right)+x^3\left(\frac{c}{3\,b^3}-\frac{a^3\,f}{3\,b^6}-\frac{a^2\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{a\,\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)+\frac{x\,\left(\frac{23\,f\,a^6}{8}-\frac{19\,e\,a^5\,b}{8}+\frac{15\,d\,a^4\,b^2}{8}-\frac{11\,c\,a^3\,b^3}{8}\right)-x^3\,\left(-\frac{25\,f\,a^5\,b}{8}+\frac{21\,e\,a^4\,b^2}{8}-\frac{17\,d\,a^3\,b^3}{8}+\frac{13\,c\,a^2\,b^4}{8}\right)}{a^2\,b^7+2\,a\,b^8\,x^2+b^9\,x^4}-x\left(\frac{3\,a\,\left(\frac{c}{b^3}-\frac{a^3\,f}{b^6}-\frac{3\,a^2\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{3\,a\,\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)}{b}-\frac{3\,a^2\,\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b^2}+\frac{a^3\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^3}\right)-x^5\left(\frac{3\,a^2\,f}{5\,b^5}-\frac{d}{5\,b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{5\,b}\right)+\frac{f\,x^9}{9\,b^3}-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,\left(-143\,f\,a^3+99\,e\,a^2\,b-63\,d\,a\,b^2+35\,c\,b^3\right)}{143\,f\,a^5-99\,e\,a^4\,b+63\,d\,a^3\,b^2-35\,c\,a^2\,b^3}\right)\,\left(-143\,f\,a^3+99\,e\,a^2\,b-63\,d\,a\,b^2+35\,c\,b^3\right)}{8\,b^{15/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

```
[Out] x^7*(e/(7*b^3) - (3*a*f)/(7*b^4)) + x^3*(c/(3*b^3) - (a^3*f)/(3*b^6) - (a^2
*(e/b^3 - (3*a*f)/b^4))/b^2 + (a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*
a*f)/b^4))/b))/b) + (x*((23*a^6*f)/8 - (11*a^3*b^3*c)/8 + (15*a^4*b^2*d)/8
- (19*a^5*b*e)/8) - x^3*((13*a^2*b^4*c)/8 - (17*a^3*b^3*d)/8 + (21*a^4*b^2*
e)/8 - (25*a^5*b*f)/8))/(a^2*b^7 + b^9*x^4 + 2*a*b^8*x^2) - x*((3*a*(c/b^3
- (a^3*f)/b^6 - (3*a^2*(e/b^3 - (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b^5 - d
/b^3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b))/b - (3*a^2*((3*a^2*f)/b^5 - d/b^
3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b^2 + (a^3*(e/b^3 - (3*a*f)/b^4))/b^3)
- x^5*((3*a^2*f)/(5*b^5) - d/(5*b^3) + (3*a*(e/b^3 - (3*a*f)/b^4))/(5*b)) +
 (f*x^9)/(9*b^3) - (a^(3/2)*atan((a^(3/2)*b^(1/2)*x*(35*b^3*c - 143*a^3*f -
 63*a*b^2*d + 99*a^2*b*e))/(143*a^5*f - 35*a^2*b^3*c + 63*a^3*b^2*d - 99*a^
4*b*e))*(35*b^3*c - 143*a^3*f - 63*a*b^2*d + 99*a^2*b*e))/(8*b^(15/2))
```

3.134 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=247

$$\frac{(3b^3c-7ab^2d+13a^2be-21a^3f)\,x}{2b^6}-\frac{(3b^3c-7ab^2d+15a^2be-27a^3f)\,x^3}{12ab^5}+\frac{(be-3af)x^5}{5b^4}+\frac{fx^7}{7b^3}+\frac{\left(c-\frac{a(b^2d-a}{b}\right.}{4a\,(a+}$$

```
[Out] 1/2*(-21*a^3*f+13*a^2*b*e-7*a*b^2*d+3*b^3*c)*x/b^6-1/12*(-27*a^3*f+15*a^2*b
*e-7*a*b^2*d+3*b^3*c)*x^3/a/b^5+1/5*(-3*a*f+b*e)*x^5/b^4+1/7*f*x^7/b^3+1/4*
(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x^7/a/(b*x^2+a)^2+1/8*a*(-15*a^3*f+11*a^2*b*e
-7*a*b^2*d+3*b^3*c)*x/b^6/(b*x^2+a)-1/8*(-99*a^3*f+63*a^2*b*e-35*a*b^2*d+15
*b^3*c)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(13/2)
```

Rubi [A]
time = 0.27, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1818, 1599, 1271, 1824, 211}

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\,(a+bx^2)^2}-\frac{\sqrt{a}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(-99a^3f+63a^2be-35ab^2d+15b^3c)}{8b^{13/2}}+\frac{ax(-15a^3f+11a^2be-7ab^2d+3b^3c)}{8b^6\,(a+bx^2)}+\frac{x(-21a^3f+13a^2be-7ab^2d+3b^3c)}{2b^6}-\frac{x^3(-27a^3f+15a^2be-7ab^2d+3b^3c)}{12ab^5}+\frac{x^5(be-3af)}{5b^4}+\frac{fx^7}{7b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] ((3*b^3*c - 7*a*b^2*d + 13*a^2*b*e - 21*a^3*f)*x)/(2*b^6) - ((3*b^3*c - 7*a
*b^2*d + 15*a^2*b*e - 27*a^3*f)*x^3)/(12*a*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^
4) + (f*x^7)/(7*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^7)/(4*a*(a
+ b*x^2)^2) + (a*(3*b^3*c - 7*a*b^2*d + 11*a^2*b*e - 15*a^3*f)*x)/(8*b^6*(a
 + b*x^2)) - (Sqrt[a]*(15*b^3*c - 35*a*b^2*d + 63*a^2*b*e - 99*a^3*f)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/(8*b^(13/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
```

```
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^5\left(\left(3bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{(a+bx^2)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{x^6\left(3bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{(a+bx^2)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{4a\left(a+bx^2\right)^2} + \frac{a(3b^3c-7ab^2d+11a^2be-15a^3f)\,x}{8b^6\left(a+bx^2\right)} + \frac{\int \frac{-a^2\left(3b^3c\right.}{}}{}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{4a\left(a+bx^2\right)^2} + \frac{a(3b^3c-7ab^2d+11a^2be-15a^3f)\,x}{8b^6\left(a+bx^2\right)} + \frac{\int \left(4a(3b\right.}{}$$

$$= \frac{(3b^3c-7ab^2d+13a^2be-21a^3f)\,x}{2b^6} - \frac{(3b^3c-7ab^2d+15a^2be-27a^3f)\,x^3}{12ab^5}$$

$$= \frac{(3b^3c-7ab^2d+13a^2be-21a^3f)\,x}{2b^6} - \frac{(3b^3c-7ab^2d+15a^2be-27a^3f)\,x^3}{12ab^5}$$

Mathematica [A]
time = 0.10, size = 232, normalized size = 0.94

$$\frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x}{b^6} + \frac{(b^2d-3abe+6a^2f)\,x^3}{3b^5} + \frac{(be-3af)x^5}{5b^4} + \frac{fx^7}{7b^3} + \frac{a^2(-b^3c+ab^2d-a^2be+a^3f)\,x}{4b^6\left(a+bx^2\right)^2} + \frac{a(9b^3c-13ab^2d+17a^2be-21a^3f)\,x}{8b^6\left(a+bx^2\right)} + \frac{\sqrt{a}\,(-15b^3c+35ab^2d-63a^2be+99a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] ((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x)/b^6 + ((b^2*d - 3*a*b*e + 6*
a^2*f)*x^3)/(3*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^4) + (f*x^7)/(7*b^3) + (a^2*
(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*b^6*(a + b*x^2)^2) + (a*(9*b^3
*c - 13*a*b^2*d + 17*a^2*b*e - 21*a^3*f)*x)/(8*b^6*(a + b*x^2)) + (Sqrt[a]*
(-15*b^3*c + 35*a*b^2*d - 63*a^2*b*e + 99*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]
])/(8*b^(13/2))
```

Maple [A]
time = 0.16, size = 219, normalized size = 0.89

method	result

default	$-\frac{-\frac{1}{7}f\,x^7b^3+\frac{3}{5}a\,b^2f\,x^5-\frac{1}{5}b^3e\,x^5-2a^2bf\,x^3+a\,b^2e\,x^3-\frac{1}{3}b^3d\,x^3+10a^3fx-6a^2bex+3a\,b^2dx-b^3cx}{b^6}+\frac{a\left(\frac{\left(-\frac{21}{8}a^3bf+\frac{17}{8}a^2e\,b^2-\frac{13}{8}ad$
risch	$\frac{f\,x^7}{7b^3}-\frac{3af\,x^5}{5b^4}+\frac{e\,x^5}{5b^3}+\frac{2a^2f\,x^3}{b^5}-\frac{ae\,x^3}{b^4}+\frac{d\,x^3}{3b^3}-\frac{10a^3fx}{b^6}+\frac{6a^2ex}{b^5}-\frac{3adx}{b^4}+\frac{cx}{b^3}+\frac{\left(-\frac{21}{8}a^4bf+\frac{17}{8}a^3b^2e-\frac{13}{8}a^2b^3d+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^6*(-1/7*f*x^7*b^3+3/5*a*b^2*f*x^5-1/5*b^3*e*x^5-2*a^2*b*f*x^3+a*b^2*e*
x^3-1/3*b^3*d*x^3+10*a^3*f*x-6*a^2*b*e*x+3*a*b^2*d*x-b^3*c*x)+a/b^6*(((-21/
8*a^3*b*f+17/8*a^2*e*b^2-13/8*a*d*b^3+9/8*c*b^4)*x^3-1/8*a*(19*a^3*f-15*a^2
*b*e+11*a*b^2*d-7*b^3*c)*x)/(b*x^2+a)^2+1/8*(99*a^3*f-63*a^2*b*e+35*a*b^2*d
-15*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]

time = 0.52, size = 244, normalized size = 0.99

$$\frac{(9\,ab^4c-13\,a^2b^3d-21\,a^4bf+17\,a^3b^2e)x^3+(7\,a^2b^3c-11\,a^3b^2d-19\,a^5f+15\,a^4be)x}{8\,(b^8x^4+2\,ab^7x^2+a^2b^6)}-\frac{(15\,ab^3c-35\,a^2b^2d-99\,a^4f+63\,a^3be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^6}+\frac{15\,b^3fx^7-21\,(3\,ab^2f-b^3e)x^5+35\,(b^3d+6\,a^2bf-3\,ab^2e)x^3+105\,(b^3c-3\,ab^2d-10\,a^3f+6\,a^2be)x}{105\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((9*a*b^4*c - 13*a^2*b^3*d - 21*a^4*b*f + 17*a^3*b^2*e)*x^3 + (7*a^2*b^
3*c - 11*a^3*b^2*d - 19*a^5*f + 15*a^4*b*e)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2
*b^6) - 1/8*(15*a*b^3*c - 35*a^2*b^2*d - 99*a^4*f + 63*a^3*b*e)*arctan(b*x/
sqrt(a*b))/(sqrt(a*b)*b^6) + 1/105*(15*b^3*f*x^7 - 21*(3*a*b^2*f - b^3*e)*x
^5 + 35*(b^3*d + 6*a^2*b*f - 3*a*b^2*e)*x^3 + 105*(b^3*c - 3*a*b^2*d - 10*a
^3*f + 6*a^2*b*e)*x)/b^6
```

Fricas [A]

time = 1.72, size = 700, normalized size = 2.83

$$\left[\frac{240\,b^5fx^{11}-528\,ab^4fx^9+16\,(35\,b^5d+99\,a^2b^3f)x^7+112\,(15\,b^5c-35\,ab^4d-99\,a^3b^2f)x^5+350\,(15\,ab^4c-35\,a^2b^3d-99\,a^4bf)x^3+105\,(15\,a^2b^3c-35\,a^3b^2d-99\,a^5f+(15\,b^5c-35\,ab^4d-99\,a^3b^2f)x^4+2\,(15\,ab^4c-35\,a^2b^3d-99\,a^4bf)x^2+63\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)e)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+210\,(15\,a^2b^3c-35\,a^3b^2d-99\,a^5f)x+42\,(8\,b^5x^9-24\,ab^4x^7+168\,a^2b^3x^5+525\,a^3b^2x^3+315\,a^4bx)e}{1680\,(b^8x^4+2\,ab^7x^2+a^2b^6)},\frac{120\,b^5fx^{11}-264\,ab^4fx^9+8\,(35\,b^5d+99\,a^2b^3f)x^7+56\,(15\,b^5c-35\,ab^4d-99\,a^3b^2f)x^5+175\,(15\,ab^4c-35\,a^2b^3d-99\,a^4bf)x^3-105\,(15\,a^2b^3c-35\,a^3b^2d-99\,a^5f+(15\,b^5c-35\,ab^4d-99\,a^3b^2f)x^4+2\,(15\,ab^4c-35\,a^2b^3d-99\,a^4bf)x^2+63\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)e)\sqrt{\frac{a}{b}}\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+105\,(15\,a^2b^3c-35\,a^3b^2d-99\,a^5f)x+21\,(8\,b^5x^9-24\,ab^4x^7+168\,a^2b^3x^5+525\,a^3b^2x^3+315\,a^4bx)e}{840\,(b^8x^4+2\,ab^7x^2+a^2b^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/1680*(240*b^5*f*x^11 - 528*a*b^4*f*x^9 + 16*(35*b^5*d + 99*a^2*b^3*f)*x^
7 + 112*(15*b^5*c - 35*a*b^4*d - 99*a^3*b^2*f)*x^5 + 350*(15*a*b^4*c - 35*a
^2*b^3*d - 99*a^4*b*f)*x^3 + 105*(15*a^2*b^3*c - 35*a^3*b^2*d - 99*a^5*f +
(15*b^5*c - 35*a*b^4*d - 99*a^3*b^2*f)*x^4 + 2*(15*a*b^4*c - 35*a^2*b^3*d -
```

```
 99*a^4*b*f)*x^2 + 63*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*e)*sqrt(-a/b)*l
og((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210*(15*a^2*b^3*c - 35*a^3
*b^2*d - 99*a^5*f)*x + 42*(8*b^5*x^9 - 24*a*b^4*x^7 + 168*a^2*b^3*x^5 + 525
*a^3*b^2*x^3 + 315*a^4*b*x)*e)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6), 1/840*(12
0*b^5*f*x^11 - 264*a*b^4*f*x^9 + 8*(35*b^5*d + 99*a^2*b^3*f)*x^7 + 56*(15*b
^5*c - 35*a*b^4*d - 99*a^3*b^2*f)*x^5 + 175*(15*a*b^4*c - 35*a^2*b^3*d - 99
*a^4*b*f)*x^3 - 105*(15*a^2*b^3*c - 35*a^3*b^2*d - 99*a^5*f + (15*b^5*c - 3
5*a*b^4*d - 99*a^3*b^2*f)*x^4 + 2*(15*a*b^4*c - 35*a^2*b^3*d - 99*a^4*b*f)*
x^2 + 63*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*e)*sqrt(a/b)*arctan(b*x*sqrt
(a/b)/a) + 105*(15*a^2*b^3*c - 35*a^3*b^2*d - 99*a^5*f)*x + 21*(8*b^5*x^9 -
 24*a*b^4*x^7 + 168*a^2*b^3*x^5 + 525*a^3*b^2*x^3 + 315*a^4*b*x)*e)/(b^8*x^
4 + 2*a*b^7*x^2 + a^2*b^6)]
```

Sympy [**A**]
time = 18.40, size = 316, normalized size = 1.28

$$x^{5}\left(-\frac{3af}{5b^{4}}+\frac{e}{5b^{3}}\right)+x^{3}\cdot\left(\frac{2a^{2}f}{b^{5}}-\frac{ae}{b^{4}}+\frac{d}{3b^{3}}\right)+x\left(-\frac{10a^{3}f}{b^{6}}+\frac{6a^{2}e}{b^{5}}-\frac{3ad}{b^{4}}+\frac{c}{b^{3}}\right)-\frac{\sqrt{-\frac{a}{b^{13}}}\cdot(99a^{3}f-63a^{2}be+35ab^{2}d-15b^{3}c)\log\left(-b^{6}\sqrt{-\frac{a}{b^{13}}}+x\right)}{16}+\frac{\sqrt{-\frac{a}{b^{13}}}\cdot(99a^{3}f-63a^{2}be+35ab^{2}d-15b^{3}c)\log\left(b^{6}\sqrt{-\frac{a}{b^{13}}}+x\right)}{16}+\frac{x^{3}(-21a^{4}bf+17a^{3}b^{2}e-13a^{2}b^{3}d+9ab^{4}c)+x(-19a^{5}f+15a^{4}be-11a^{3}b^{2}d+7a^{2}b^{3}c)}{8a^{2}b^{6}+16ab^{7}x^{2}+8b^{8}x^{4}}+\frac{fx^{7}}{7b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] x**5*(-3*a*f/(5*b**4) + e/(5*b**3)) + x**3*(2*a**2*f/b**5 - a*e/b**4 + d/(3
*b**3)) + x*(-10*a**3*f/b**6 + 6*a**2*e/b**5 - 3*a*d/b**4 + c/b**3) - sqrt(
-a/b**13)*(99*a**3*f - 63*a**2*b*e + 35*a*b**2*d - 15*b**3*c)*log(-b**6*sqr
t(-a/b**13) + x)/16 + sqrt(-a/b**13)*(99*a**3*f - 63*a**2*b*e + 35*a*b**2*d
 - 15*b**3*c)*log(b**6*sqrt(-a/b**13) + x)/16 + (x**3*(-21*a**4*b*f + 17*a*
*3*b**2*e - 13*a**2*b**3*d + 9*a*b**4*c) + x*(-19*a**5*f + 15*a**4*b*e - 11
*a**3*b**2*d + 7*a**2*b**3*c))/(8*a**2*b**6 + 16*a*b**7*x**2 + 8*b**8*x**4)
 + f*x**7/(7*b**3)
```

Giac [**A**]
time = 1.99, size = 250, normalized size = 1.01

$$-\frac{(15\,ab^{3}c-35\,a^{2}b^{2}d-99\,a^{4}f+63\,a^{3}be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{6}}+\frac{9\,ab^{4}cx^{3}-13\,a^{2}b^{3}dx^{3}-21\,a^{4}bfx^{3}+17\,a^{3}b^{2}x^{3}e+7\,a^{2}b^{3}cx-11\,a^{3}b^{2}dx-19\,a^{5}fx+15\,a^{4}bxe}{8\,(bx^{2}+a)^{2}b^{6}}+\frac{15\,b^{18}fx^{7}-63\,ab^{17}fx^{5}+21\,b^{18}x^{5}e+35\,b^{18}dx^{3}+210\,a^{2}b^{16}fx^{3}-105\,ab^{17}x^{3}e+105\,b^{18}cx-315\,ab^{17}dx-1050\,a^{3}b^{15}fx+630\,a^{2}b^{16}xe}{105\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*a*b^3*c - 35*a^2*b^2*d - 99*a^4*f + 63*a^3*b*e)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*b^6) + 1/8*(9*a*b^4*c*x^3 - 13*a^2*b^3*d*x^3 - 21*a^4*b*f*x^
3 + 17*a^3*b^2*x^3*e + 7*a^2*b^3*c*x - 11*a^3*b^2*d*x - 19*a^5*f*x + 15*a^4
*b*x*e)/((b*x^2 + a)^2*b^6) + 1/105*(15*b^18*f*x^7 - 63*a*b^17*f*x^5 + 21*b
^18*x^5*e + 35*b^18*d*x^3 + 210*a^2*b^16*f*x^3 - 105*a*b^17*x^3*e + 105*b^1
8*c*x - 315*a*b^17*d*x - 1050*a^3*b^15*f*x + 630*a^2*b^16*x*e)/b^21
```

Mupad [B]
time = 0.11, size = 348, normalized size = 1.41

$$x^5\left(\frac{e}{5\,b^3}-\frac{3\,a\,f}{5\,b^4}\right)+x\left(\frac{c}{b^3}-\frac{a^3\,f}{b^6}-\frac{3\,a^2\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{3\,a\,\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)-x^3\left(\frac{a^2\,f}{b^5}-\frac{d}{3\,b^3}+\frac{a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)-\frac{\left(\frac{21\,f\,a^4\,b}{8}-\frac{17\,e\,a^3\,b^2}{8}+\frac{13\,d\,a^2\,b^3}{8}-\frac{9\,c\,a\,b^4}{8}\right)\,x^3+\left(\frac{19\,f\,a^5}{8}-\frac{15\,e\,a^4\,b}{8}+\frac{11\,d\,a^3\,b^2}{8}-\frac{7\,c\,a^2\,b^3}{8}\right)\,x}{a^2\,b^6+2\,a\,b^7\,x^2+b^8\,x^4}+\frac{f\,x^7}{7\,b^3}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,\left(-99\,f\,a^3+63\,e\,a^2\,b-35\,d\,a\,b^2+15\,c\,b^3\right)}{99\,f\,a^4-63\,e\,a^3\,b+35\,d\,a^2\,b^2-15\,c\,a\,b^3}\right)\,\left(-99\,f\,a^3+63\,e\,a^2\,b-35\,d\,a\,b^2+15\,c\,b^3\right)}{8\,b^{13/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

```
[Out] x^5*(e/(5*b^3) - (3*a*f)/(5*b^4)) + x*(c/b^3 - (a^3*f)/b^6 - (3*a^2*(e/b^3
- (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b
^4))/b))/b) - x^3*((a^2*f)/b^5 - d/(3*b^3) + (a*(e/b^3 - (3*a*f)/b^4))/b) -
 (x*((19*a^5*f)/8 - (7*a^2*b^3*c)/8 + (11*a^3*b^2*d)/8 - (15*a^4*b*e)/8) +
x^3*((13*a^2*b^3*d)/8 - (17*a^3*b^2*e)/8 - (9*a*b^4*c)/8 + (21*a^4*b*f)/8))
/(a^2*b^6 + b^8*x^4 + 2*a*b^7*x^2) + (f*x^7)/(7*b^3) + (a^(1/2)*atan((a^(1/
2)*b^(1/2)*x*(15*b^3*c - 99*a^3*f - 35*a*b^2*d + 63*a^2*b*e))/(99*a^4*f + 3
5*a^2*b^2*d - 15*a*b^3*c - 63*a^3*b*e))*(15*b^3*c - 99*a^3*f - 35*a*b^2*d +
 63*a^2*b*e))/(8*b^(13/2))
```

3.135 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

Optimal. Leaf size=207

$$-\frac{(b^3c-5ab^2d+13a^2be-25a^3f)\,x}{4ab^5}+\frac{(be-3af)x^3}{3b^4}+\frac{fx^5}{5b^3}+\frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{4a\,(a+bx^2)^2}-\frac{(b^3c-5ab^2d+9a^2be-1}{8b^5\,(a+bx^2)}$$

```
[Out] -1/4*(-25*a^3*f+13*a^2*b*e-5*a*b^2*d+b^3*c)*x/a/b^5+1/3*(-3*a*f+b*e)*x^3/b^
4+1/5*f*x^5/b^3+1/4*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x^5/a/(b*x^2+a)^2-1/8*(-1
3*a^3*f+9*a^2*b*e-5*a*b^2*d+b^3*c)*x/b^5/(b*x^2+a)+1/8*(-63*a^3*f+35*a^2*b*
e-15*a*b^2*d+3*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)/a^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1818, 1599, 1271, 1824, 211}

$$\frac{x^5\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\,(a+bx^2)^2}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(-63a^3f+35a^2be-15ab^2d+3b^3c)}{8\sqrt{a}\,b^{11/2}}-\frac{x(-13a^3f+9a^2be-5ab^2d+b^3c)}{8b^5\,(a+bx^2)}-\frac{x(-25a^3f+13a^2be-5ab^2d+b^3c)}{4ab^5}+\frac{x^3(be-3af)}{3b^4}+\frac{fx^5}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] -1/4*((b^3*c - 5*a*b^2*d + 13*a^2*b*e - 25*a^3*f)*x)/(a*b^5) + ((b*e - 3*a*
f)*x^3)/(3*b^4) + (f*x^5)/(5*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*
x^5)/(4*a*(a + b*x^2)^2) - ((b^3*c - 5*a*b^2*d + 9*a^2*b*e - 13*a^3*f)*x)/(
8*b^5*(a + b*x^2)) + ((3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx &= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{x^3\left(\left(bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}\\
&= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{x^4\left(bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{\left(a+bx^2\right)^2}\,dx}{4ab}\\
&= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2}-\frac{\left(b^3c-5ab^2d+9a^2be-13a^3f\right)x}{8b^5\left(a+bx^2\right)}+\frac{\int \frac{a\left(b^3c-5ab^2d\right.}{}}{}\\
&= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^5}{4a\left(a+bx^2\right)^2}-\frac{\left(b^3c-5ab^2d+9a^2be-13a^3f\right)x}{8b^5\left(a+bx^2\right)}+\frac{\int \left(-2(b^3c\right.}{}\\
&= -\frac{\left(b^3c-5ab^2d+13a^2be-25a^3f\right)x}{4ab^5}+\frac{(be-3af)x^3}{3b^4}+\frac{fx^5}{5b^3}+\frac{\left(c-\frac{a\left(b^2d-\right.}{}\right.}{4a\left(a-\right.}\\
&= -\frac{\left(b^3c-5ab^2d+13a^2be-25a^3f\right)x}{4ab^5}+\frac{(be-3af)x^3}{3b^4}+\frac{fx^5}{5b^3}+\frac{\left(c-\frac{a\left(b^2d-\right.}{}\right.}{4a\left(a-\right.}
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 176, normalized size = 0.85

$$\frac{x(945a^4f-525a^3b(e-3fx^2)+a^2b^2(225d-875ex^2+504fx^4)-ab^3(45c-375dx^2+280ex^4+72fx^6)+b^4x^2(-75c+8(15dx^2+5ex^4+3fx^6)))}{120b^5\left(a+bx^2\right)^2}+\frac{(3b^3c-15ab^2d+35a^2be-63a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\sqrt{a}\,b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (x*(945*a^4*f - 525*a^3*b*(e - 3*f*x^2) + a^2*b^2*(225*d - 875*e*x^2 + 504*
f*x^4) - a*b^3*(45*c - 375*d*x^2 + 280*e*x^4 + 72*f*x^6) + b^4*x^2*(-75*c +
 8*(15*d*x^2 + 5*e*x^4 + 3*f*x^6))))/(120*b^5*(a + b*x^2)^2) + ((3*b^3*c -
15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]
*b^(11/2))
```

Maple [A]

time = 0.14, size = 177, normalized size = 0.86

method	result
default	$\frac{\frac{1}{5}f\,x^5b^2-abf\,x^3+\frac{1}{3}b^2e\,x^3+6a^2fx-3abex+b^2dx}{b^5}-\frac{\frac{\left(-\frac{17}{8}a^3bf+\frac{13}{8}a^2e\,b^2-\frac{9}{8}ad\,b^3+\frac{5}{8}c\,b^4\right)x^3-\frac{a\left(15a^3f-11a^2be+7a\,b^2d-3b^3c\right)x}{8}}{\left(b\,x^2+a\right)^2}+ \left(63a^3f-}{b^5}$
risch	$\frac{f\,x^5}{5b^3}-\frac{af\,x^3}{b^4}+\frac{e\,x^3}{3b^3}+\frac{6a^2fx}{b^5}-\frac{3aex}{b^4}+\frac{dx}{b^3}+\frac{(\frac{17}{8}a^3bf-\frac{13}{8}a^2e\,b^2+\frac{9}{8}ad\,b^3-\frac{5}{8}c\,b^4)x^3+\frac{a\left(15a^3f-11a^2be+7a\,b^2d-3b^3c\right)x}{8}}{b^5(b\,x^2+a)^2}-63$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b^5*(1/5*f*x^5*b^2-a*b*f*x^3+1/3*b^2*e*x^3+6*a^2*f*x-3*a*b*e*x+b^2*d*x)-1
/b^5*(((-17/8*a^3*b*f+13/8*a^2*e*b^2-9/8*a*d*b^3+5/8*c*b^4)*x^3-1/8*a*(15*a
^3*f-11*a^2*b*e+7*a*b^2*d-3*b^3*c)*x)/(b*x^2+a)^2+1/8*(63*a^3*f-35*a^2*b*e+
15*a*b^2*d-3*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]

time = 0.50, size = 199, normalized size = 0.96

$$-\frac{(5\,b^4c-9\,ab^3d-17\,a^3bf+13\,a^2b^2e)x^3+(3\,ab^3c-7\,a^2b^2d-15\,a^4f+11\,a^3be)x}{8\,(b^7x^4+2\,ab^6x^2+a^2b^5)}+\frac{(3\,b^3c-15\,ab^2d-63\,a^3f+35\,a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^5}+\frac{3\,b^2fx^5-5\,(3\,abf-b^2e)x^3+15\,(b^2d+6\,a^2f-3\,abe)x}{15\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*b^4*c - 9*a*b^3*d - 17*a^3*b*f + 13*a^2*b^2*e)*x^3 + (3*a*b^3*c -
7*a^2*b^2*d - 15*a^4*f + 11*a^3*b*e)*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5) +
 1/8*(3*b^3*c - 15*a*b^2*d - 63*a^3*f + 35*a^2*b*e)*arctan(b*x/sqrt(a*b))/(
```

```
sqrt(a*b)*b^5) + 1/15*(3*b^2*f*x^5 - 5*(3*a*b*f - b^2*e)*x^3 + 15*(b^2*d +
6*a^2*f - 3*a*b*e)*x)/b^5
```

Fricas [A]
time = 2.85, size = 636, normalized size = 3.07

$$\left[\frac{48\,ab^{5}fx^{9} - 144\,a^{2}b^{4}fx^{7} + 48\,(5\,ab^{5}d + 21\,a^{3}b^{3}f)x^{5} - 150\,(ab^{5}c - 5\,a^{2}b^{4}d - 21\,a^{4}b^{2}f)x^{3} - 15\,(3\,a^{2}b^{3}c - 15\,a^{3}b^{2}d - 63\,a^{5}f + 3\,(b^{5}c - 5\,ab^{4}d - 21\,a^{3}b^{2}f)x^{4} + 6\,(ab^{4}c - 5\,a^{2}b^{3}d - 21\,a^{4}bf)x^{2} + 35\,(a^{2}b^{3}x^{4} + 2\,a^{3}b^{2}x^{2} + a^{4}b)e)\sqrt{-ab}\,\log\left(\frac{bx^{2} - 2\,\sqrt{-ab}\,x - a}{bx^{2} + a}\right) - 90\,(a^{2}b^{4}c - 5\,a^{3}b^{3}d - 21\,a^{5}bf)x + 10\,(8\,ab^{5}x^{7} - 56\,a^{2}b^{4}x^{5} - 175\,a^{3}b^{3}x^{3} - 105\,a^{4}b^{2}x)e}{240\,(ab^{8}x^{4} + 2\,a^{2}b^{7}x^{2} + a^{3}b^{6})}, \frac{24\,ab^{5}fx^{9} - 72\,a^{2}b^{4}fx^{7} + 24\,(5\,ab^{5}d + 21\,a^{3}b^{3}f)x^{5} - 75\,(ab^{5}c - 5\,a^{2}b^{4}d - 21\,a^{4}b^{2}f)x^{3} + 15\,(3\,a^{2}b^{3}c - 15\,a^{3}b^{2}d - 63\,a^{5}f + 3\,(b^{5}c - 5\,ab^{4}d - 21\,a^{3}b^{2}f)x^{4} + 6\,(ab^{4}c - 5\,a^{2}b^{3}d - 21\,a^{4}bf)x^{2} + 35\,(a^{2}b^{3}x^{4} + 2\,a^{3}b^{2}x^{2} + a^{4}b)e)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right) - 45\,(a^{2}b^{4}c - 5\,a^{3}b^{3}d - 21\,a^{5}bf)x + 5\,(8\,ab^{5}x^{7} - 56\,a^{2}b^{4}x^{5} - 175\,a^{3}b^{3}x^{3} - 105\,a^{4}b^{2}x)e}{120\,(ab^{8}x^{4} + 2\,a^{2}b^{7}x^{2} + a^{3}b^{6})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(48*a*b^5*f*x^9 - 144*a^2*b^4*f*x^7 + 48*(5*a*b^5*d + 21*a^3*b^3*f)*
x^5 - 150*(a*b^5*c - 5*a^2*b^4*d - 21*a^4*b^2*f)*x^3 - 15*(3*a^2*b^3*c - 15
*a^3*b^2*d - 63*a^5*f + 3*(b^5*c - 5*a*b^4*d - 21*a^3*b^2*f)*x^4 + 6*(a*b^4
*c - 5*a^2*b^3*d - 21*a^4*b*f)*x^2 + 35*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*
b)*e)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 90*(a^2*b^
4*c - 5*a^3*b^3*d - 21*a^5*b*f)*x + 10*(8*a*b^5*x^7 - 56*a^2*b^4*x^5 - 175*
a^3*b^3*x^3 - 105*a^4*b^2*x)*e)/(a*b^8*x^4 + 2*a^2*b^7*x^2 + a^3*b^6), 1/12
0*(24*a*b^5*f*x^9 - 72*a^2*b^4*f*x^7 + 24*(5*a*b^5*d + 21*a^3*b^3*f)*x^5 -
75*(a*b^5*c - 5*a^2*b^4*d - 21*a^4*b^2*f)*x^3 + 15*(3*a^2*b^3*c - 15*a^3*b^
2*d - 63*a^5*f + 3*(b^5*c - 5*a*b^4*d - 21*a^3*b^2*f)*x^4 + 6*(a*b^4*c - 5*
a^2*b^3*d - 21*a^4*b*f)*x^2 + 35*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*e)*s
qrt(a*b)*arctan(sqrt(a*b)*x/a) - 45*(a^2*b^4*c - 5*a^3*b^3*d - 21*a^5*b*f)*
x + 5*(8*a*b^5*x^7 - 56*a^2*b^4*x^5 - 175*a^3*b^3*x^3 - 105*a^4*b^2*x)*e)/(
a*b^8*x^4 + 2*a^2*b^7*x^2 + a^3*b^6)]
```

Sympy [A]
time = 16.44, size = 280, normalized size = 1.35

$$x^{3}\left(-\frac{af}{b^{4}} + \frac{e}{3b^{3}}\right) + x\left(\frac{6a^{2}f}{b^{5}} - \frac{3ae}{b^{4}} + \frac{d}{b^{3}}\right) + \frac{\sqrt{-\frac{1}{ab^{11}}}\cdot(63a^{3}f - 35a^{2}be + 15ab^{2}d - 3b^{3}c)\log\left(-ab^{5}\sqrt{-\frac{1}{ab^{11}}} + x\right)}{16} - \frac{\sqrt{-\frac{1}{ab^{11}}}\cdot(63a^{3}f - 35a^{2}be + 15ab^{2}d - 3b^{3}c)\log\left(ab^{5}\sqrt{-\frac{1}{ab^{11}}} + x\right)}{16} + \frac{x^{3}\cdot(17a^{3}bf - 13a^{2}b^{2}e + 9ab^{3}d - 5b^{4}c) + x(15a^{4}f - 11a^{3}be + 7a^{2}b^{2}d - 3ab^{3}c)}{8a^{2}b^{5} + 16ab^{6}x^{2} + 8b^{7}x^{4}} + \frac{fx^{5}}{5b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] x**3*(-a*f/b**4 + e/(3*b**3)) + x*(6*a**2*f/b**5 - 3*a*e/b**4 + d/b**3) + s
qrt(-1/(a*b**11))*(63*a**3*f - 35*a**2*b*e + 15*a*b**2*d - 3*b**3*c)*log(-a
*b**5*sqrt(-1/(a*b**11)) + x)/16 - sqrt(-1/(a*b**11))*(63*a**3*f - 35*a**2*
b*e + 15*a*b**2*d - 3*b**3*c)*log(a*b**5*sqrt(-1/(a*b**11)) + x)/16 + (x**3
*(17*a**3*b*f - 13*a**2*b**2*e + 9*a*b**3*d - 5*b**4*c) + x*(15*a**4*f - 11
*a**3*b*e + 7*a**2*b**2*d - 3*a*b**3*c))/(8*a**2*b**5 + 16*a*b**6*x**2 + 8*
b**7*x**4) + f*x**5/(5*b**3)
```

Giac [A]
time = 1.53, size = 200, normalized size = 0.97

$$\frac{(3\,b^{3}c - 15\,ab^{2}d - 63\,a^{3}f + 35\,a^{2}be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,b^{5}} - \frac{5\,b^{4}cx^{3} - 9\,ab^{3}dx^{3} - 17\,a^{3}bfx^{3} + 13\,a^{2}b^{2}x^{3}e + 3\,ab^{3}cx - 7\,a^{2}b^{2}dx - 15\,a^{4}fx + 11\,a^{3}bxe}{8\,(bx^{2} + a)^{2}b^{5}} + \frac{3\,b^{12}fx^{5} - 15\,ab^{11}fx^{3} + 5\,b^{12}x^{3}e + 15\,b^{12}dx + 90\,a^{2}b^{10}fx - 45\,ab^{11}xe}{15\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b^3*c - 15*a*b^2*d - 63*a^3*f + 35*a^2*b*e)*arctan(b*x/sqrt(a*b))/(s
qrt(a*b)*b^5) - 1/8*(5*b^4*c*x^3 - 9*a*b^3*d*x^3 - 17*a^3*b*f*x^3 + 13*a^2*
b^2*x^3*e + 3*a*b^3*c*x - 7*a^2*b^2*d*x - 15*a^4*f*x + 11*a^3*b*x*e)/((b*x^
2 + a)^2*b^5) + 1/15*(3*b^12*f*x^5 - 15*a*b^11*f*x^3 + 5*b^12*x^3*e + 15*b^
12*d*x + 90*a^2*b^10*f*x - 45*a*b^11*x*e)/b^15
```

Mupad [**B**]
time = 0.95, size = 206, normalized size = 1.00

$$x^3\left(\frac{e}{3\,b^3}-\frac{a\,f}{b^4}\right)-x\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)-\frac{x^3\,\left(-\frac{17\,f\,a^3\,b}{8}+\frac{13\,e\,a^2\,b^2}{8}-\frac{9\,d\,a\,b^3}{8}+\frac{5\,c\,b^4}{8}\right)-x\,\left(\frac{15\,f\,a^4}{8}-\frac{11\,e\,a^3\,b}{8}+\frac{7\,d\,a^2\,b^2}{8}-\frac{3\,c\,a\,b^3}{8}\right)}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4}+\frac{f\,x^5}{5\,b^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-63\,f\,a^3+35\,e\,a^2\,b-15\,d\,a\,b^2+3\,c\,b^3\right)}{8\,\sqrt{a}\,b^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

```
[Out] x^3*(e/(3*b^3) - (a*f)/b^4) - x*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a
*f)/b^4))/b) - (x^3*((5*b^4*c)/8 + (13*a^2*b^2*e)/8 - (9*a*b^3*d)/8 - (17*a
^3*b*f)/8) - x*((15*a^4*f)/8 + (7*a^2*b^2*d)/8 - (3*a*b^3*c)/8 - (11*a^3*b*
e)/8))/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) + (f*x^5)/(5*b^3) + (atan((b^(1/2)
*x)/a^(1/2))*(3*b^3*c - 63*a^3*f - 15*a*b^2*d + 35*a^2*b*e))/(8*a^(1/2)*b^(
11/2))
```

3.136 $\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=167

$$\frac{(be-3af)x}{b^4}+\frac{fx^3}{3b^3}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x^3}{4a\left(a+bx^2\right)^2}-\frac{\left(b^3c+3ab^2d-7a^2be+11a^3f\right)x}{8ab^4\left(a+bx^2\right)}+\frac{\left(b^3c+3ab^2d-15a^2be+35\right.}{8a^{3/2}b^{9/}}$$

```
[Out] (-3*a*f+b*e)*x/b^4+1/3*f*x^3/b^3+1/4*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x^3/a/(b
*x^2+a)^2-1/8*(11*a^3*f-7*a^2*b*e+3*a*b^2*d+b^3*c)*x/a/b^4/(b*x^2+a)+1/8*(3
5*a^3*f-15*a^2*b*e+3*a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(9/
2)
```

Rubi [A]
time = 0.17, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = $\{1818, 1599, 1271, 1167, 211\}$

$$\frac{x^3\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{x\left(11a^3f-7a^2be+3ab^2d+b^3c\right)}{8ab^4\left(a+bx^2\right)}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(35a^3f-15a^2be+3ab^2d+b^3c\right)}{8a^{3/2}b^{9/2}}+\frac{x(be-3af)}{b^4}+\frac{fx^3}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] ((b*e - 3*a*f)*x)/b^4 + (f*x^3)/(3*b^3) + ((c - (a*(b^2*d - a*b*e + a^2*f))
/b^3)*x^3)/(4*a*(a + b*x^2)^2) - ((b^3*c + 3*a*b^2*d - 7*a^2*b*e + 11*a^3*f
)*x)/(8*a*b^4*(a + b*x^2)) + ((b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*a^3*f)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
```

```
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{4a\,(a+bx^2)^2} - \frac{\int \frac{x\left(-\left(bc+3ad-\frac{3a^2e}{b}+\frac{3a^3f}{b^2}\right)x-4a\left(e-\frac{af}{b}\right)x^3-4afx^5\right)}{(a+bx^2)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{4a\,(a+bx^2)^2} - \frac{\int \frac{x^2\left(-bc-3ad+\frac{3a^2e}{b}-\frac{3a^3f}{b^2}-4a\left(e-\frac{af}{b}\right)x^2-4afx^4\right)}{(a+bx^2)^2}\,dx}{4ab}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{4a\,(a+bx^2)^2} - \frac{(b^3c+3ab^2d-7a^2be+11a^3f)\,x}{8ab^4\,(a+bx^2)} + \frac{\int \frac{b^3c+3ab^2d-}{}}{}$$

$$= \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{4a\,(a+bx^2)^2} - \frac{(b^3c+3ab^2d-7a^2be+11a^3f)\,x}{8ab^4\,(a+bx^2)} + \frac{\int \Big(8a(be-}{}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^3}{3b^3} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{4a\,(a+bx^2)^2} - \frac{(b^3c+3ab^2d-7a^2be+1}{8ab^4\,(a+bx^2)}$$

$$= \frac{(be-3af)x}{b^4} + \frac{fx^3}{3b^3} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{4a\,(a+bx^2)^2} - \frac{(b^3c+3ab^2d-7a^2be+1}{8ab^4\,(a+bx^2)}$$

Mathematica [A]

time = 0.09, size = 156, normalized size = 0.93

$$\frac{x(-105a^4f+3b^4cx^2+5a^3b(9e-35fx^2)+a^2b^2(-9d+75ex^2-56fx^4)+ab^3(-3c-15dx^2+24ex^4+8fx^6))}{24ab^4\,(a+bx^2)^2} + \frac{(b^3c+3ab^2d-15a^2be+35a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

```
[Out] (x*(-105*a^4*f + 3*b^4*c*x^2 + 5*a^3*b*(9*e - 35*f*x^2) + a^2*b^2*(-9*d + 7
5*e*x^2 - 56*f*x^4) + a*b^3*(-3*c - 15*d*x^2 + 24*e*x^4 + 8*f*x^6)))/(24*a*
b^4*(a + b*x^2)^2) + ((b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*a^3*f)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(9/2))
```

Maple [A]

time = 0.15, size = 151, normalized size = 0.90

method	result
default	$-\frac{-\frac{1}{3}f\,x^3b+3afx-bex}{b^4}+\frac{\frac{-\frac{b\left(13a^3f-9a^2be+5a\,b^2d-b^3c\right)x^3}{8a}+\left(-\frac{11}{8}a^3f+\frac{7}{8}a^2be-\frac{3}{8}a\,b^2d-\frac{1}{8}b^3c\right)x}{\left(b\,x^2+a\right)^2}+\frac{\left(35a^3f-15a^2be+3a\,b^2d+b^3c\right)\arctan\Big(}{8a\sqrt{ab}}}{b^4}$

risch	$\frac{f\,x^{3}}{3b^{3}}-\frac{3afx}{b^{4}}+\frac{ex}{b^{3}}+\frac{-\frac{b\left(13a^{3}f-9a^{2}be+5a\,b^{2}d-b^{3}c\right)x^{3}}{8a}+(-\frac{11}{8}a^{3}f+\frac{7}{8}a^{2}be-\frac{3}{8}a\,b^{2}d-\frac{1}{8}b^{3}c)x}{b^{4}(b\,x^{2}+a)^{2}}-\frac{35a^{2}\ln\left(bx+\sqrt{-ab}\right)f}{16b^{4}\sqrt{-ab}}+\frac{15a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^4*(-1/3*f*x^3*b+3*a*f*x-b*e*x)+1/b^4*((-1/8*b*(13*a^3*f-9*a^2*b*e+5*a*
b^2*d-b^3*c)/a*x^3+(-11/8*a^3*f+7/8*a^2*b*e-3/8*a*b^2*d-1/8*b^3*c)*x)/(b*x^
2+a)^2+1/8*(35*a^3*f-15*a^2*b*e+3*a*b^2*d+b^3*c)/a/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2)))
```

Maxima [A]

time = 0.52, size = 174, normalized size = 1.04

$$\frac{(b^4c-5\,ab^3d-13\,a^3bf+9\,a^2b^2e)x^3-(ab^3c+3\,a^2b^2d+11\,a^4f-7\,a^3be)x}{8\,(ab^6x^4+2\,a^2b^5x^2+a^3b^4)}+\frac{bfx^3-3\,(3\,af-be)x}{3\,b^4}+\frac{(b^3c+3\,ab^2d+35\,a^3f-15\,a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((b^4*c - 5*a*b^3*d - 13*a^3*b*f + 9*a^2*b^2*e)*x^3 - (a*b^3*c + 3*a^2*
b^2*d + 11*a^4*f - 7*a^3*b*e)*x)/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4) + 1/
3*(b*f*x^3 - 3*(3*a*f - b*e)*x)/b^4 + 1/8*(b^3*c + 3*a*b^2*d + 35*a^3*f - 1
5*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4)
```

Fricas [A]

time = 2.11, size = 579, normalized size = 3.47

$$\left[\frac{16\,a^2b^4fx^7-112\,a^3b^3fx^5+2\,(3\,ab^5c-15\,a^2b^4d-175\,a^4b^2f)x^3+3\,(a^2b^3c+3\,a^3b^2d+35\,a^5f+(b^5c+3\,ab^4d+35\,a^3b^2f)x^4+2\,(ab^4c+3\,a^2b^3d+35\,a^4bf)x^2-15\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)e)\sqrt{-ab}\,\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-6\,(a^2b^4c+3\,a^3b^3d+35\,a^5bf)x+6\,(8\,a^2b^4x^5+25\,a^3b^3x^3+15\,a^4b^2x)e}{48\,(a^2b^7x^4+2\,a^3b^6x^2+a^4b^5)},\ \frac{8\,a^2b^4fx^7-56\,a^3b^3fx^5+(3\,ab^5c-15\,a^2b^4d-175\,a^4b^2f)x^3+3\,(a^2b^3c+3\,a^3b^2d+35\,a^5f+(b^5c+3\,ab^4d+35\,a^3b^2f)x^4+2\,(ab^4c+3\,a^2b^3d+35\,a^4bf)x^2-15\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)e)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-3\,(a^2b^4c+3\,a^3b^3d+35\,a^5bf)x+3\,(8\,a^2b^4x^5+25\,a^3b^3x^3+15\,a^4b^2x)e}{24\,(a^2b^7x^4+2\,a^3b^6x^2+a^4b^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*a^2*b^4*f*x^7 - 112*a^3*b^3*f*x^5 + 2*(3*a*b^5*c - 15*a^2*b^4*d -
 175*a^4*b^2*f)*x^3 + 3*(a^2*b^3*c + 3*a^3*b^2*d + 35*a^5*f + (b^5*c + 3*a*
b^4*d + 35*a^3*b^2*f)*x^4 + 2*(a*b^4*c + 3*a^2*b^3*d + 35*a^4*b*f)*x^2 - 15
*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*e)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a
*b)*x - a)/(b*x^2 + a)) - 6*(a^2*b^4*c + 3*a^3*b^3*d + 35*a^5*b*f)*x + 6*(8
*a^2*b^4*x^5 + 25*a^3*b^3*x^3 + 15*a^4*b^2*x)*e)/(a^2*b^7*x^4 + 2*a^3*b^6*x
^2 + a^4*b^5), 1/24*(8*a^2*b^4*f*x^7 - 56*a^3*b^3*f*x^5 + (3*a*b^5*c - 15*a
^2*b^4*d - 175*a^4*b^2*f)*x^3 + 3*(a^2*b^3*c + 3*a^3*b^2*d + 35*a^5*f + (b^
5*c + 3*a*b^4*d + 35*a^3*b^2*f)*x^4 + 2*(a*b^4*c + 3*a^2*b^3*d + 35*a^4*b*f
)*x^2 - 15*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*e)*sqrt(a*b)*arctan(sqrt(a
*b)*x/a) - 3*(a^2*b^4*c + 3*a^3*b^3*d + 35*a^5*b*f)*x + 3*(8*a^2*b^4*x^5 +
25*a^3*b^3*x^3 + 15*a^4*b^2*x)*e)/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*b^5)]
```

Sympy [A]

time = 6.29, size = 260, normalized size = 1.56

$$x\left(-\frac{3af}{b^4}+\frac{e}{b^3}\right)-\frac{\sqrt{-\frac{1}{a^3b^9}}\cdot(35a^3f-15a^2be+3ab^2d+b^3c)\log\left(-a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^3b^9}}\cdot(35a^3f-15a^2be+3ab^2d+b^3c)\log\left(a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{16}+\frac{x^3(-13a^3bf+9a^2b^2e-5ab^3d+b^4c)+x(-11a^4f+7a^3be-3a^2b^2d-ab^3c)}{8a^3b^4+16a^2b^5x^2+8ab^6x^4}+\frac{fx^3}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] x*(-3*a*f/b**4 + e/b**3) - sqrt(-1/(a**3*b**9))*(35*a**3*f - 15*a**2*b*e +
3*a*b**2*d + b**3*c)*log(-a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/16 + sqrt(-1/
(a**3*b**9))*(35*a**3*f - 15*a**2*b*e + 3*a*b**2*d + b**3*c)*log(a**2*b**4*
sqrt(-1/(a**3*b**9)) + x)/16 + (x**3*(-13*a**3*b*f + 9*a**2*b**2*e - 5*a*b*
*3*d + b**4*c) + x*(-11*a**4*f + 7*a**3*b*e - 3*a**2*b**2*d - a*b**3*c))/(8
*a**3*b**4 + 16*a**2*b**5*x**2 + 8*a*b**6*x**4) + f*x**3/(3*b**3)
```

Giac [A]

time = 2.07, size = 173, normalized size = 1.04

$$\frac{(b^3c+3\,ab^2d+35\,a^3f-15\,a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,ab^4}+\frac{b^4cx^3-5\,ab^3dx^3-13\,a^3bfx^3+9\,a^2b^2x^3e-ab^3cx-3\,a^2b^2dx-11\,a^4fx+7\,a^3bxe}{8\,(bx^2+a)^2ab^4}+\frac{b^6fx^3-9\,ab^5fx+3\,b^6xe}{3\,b^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(b^3*c + 3*a*b^2*d + 35*a^3*f - 15*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a*b^4) + 1/8*(b^4*c*x^3 - 5*a*b^3*d*x^3 - 13*a^3*b*f*x^3 + 9*a^2*b^2*
x^3*e - a*b^3*c*x - 3*a^2*b^2*d*x - 11*a^4*f*x + 7*a^3*b*x*e)/((b*x^2 + a)^
2*a*b^4) + 1/3*(b^6*f*x^3 - 9*a*b^5*f*x + 3*b^6*x*e)/b^9
```

Mupad [B]

time = 1.02, size = 163, normalized size = 0.98

$$x\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)-\frac{x\left(\frac{11\,f\,a^3}{8}-\frac{7\,e\,a^2\,b}{8}+\frac{3\,d\,a\,b^2}{8}+\frac{c\,b^3}{8}\right)-\frac{x^3\,\left(-13\,f\,a^3\,b+9\,e\,a^2\,b^2-5\,d\,a\,b^3+c\,b^4\right)}{8\,a}}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4}+\frac{f\,x^3}{3\,b^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(35\,f\,a^3-15\,e\,a^2\,b+3\,d\,a\,b^2+c\,b^3\right)}{8\,a^{3/2}\,b^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

```
[Out] x*(e/b^3 - (3*a*f)/b^4) - (x*((b^3*c)/8 + (11*a^3*f)/8 + (3*a*b^2*d)/8 - (7
*a^2*b*e)/8) - (x^3*(b^4*c + 9*a^2*b^2*e - 5*a*b^3*d - 13*a^3*b*f))/(8*a))/
(a^2*b^4 + b^6*x^4 + 2*a*b^5*x^2) + (f*x^3)/(3*b^3) + (atan((b^(1/2)*x)/a^(
1/2))*(b^3*c + 35*a^3*f + 3*a*b^2*d - 15*a^2*b*e))/(8*a^(3/2)*b^(9/2))
```

3.137 $\int \frac{c+dx^2+ex^4+fx^6}{\left(a+bx^2\right)^3} dx$

Optimal. Leaf size=147

$$\frac{fx}{b^3}+\frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(3b^3c+ab^2d-5a^2be+9a^3f\right)x}{8a^2b^3\left(a+bx^2\right)}+\frac{\left(3b^3c+ab^2d+3a^2be-15a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}$$

```
[Out] f*x/b^3+1/4*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x/a/(b*x^2+a)^2+1/8*(9*a^3*f-5*a^
2*b*e+a*b^2*d+3*b^3*c)*x/a^2/b^3/(b*x^2+a)+1/8*(-15*a^3*f+3*a^2*b*e+a*b^2*d
+3*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(7/2)
```

Rubi [A]
time = 0.10, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$,
Rules used = {1828, 1171, 396, 211}

$$\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}+\frac{x(9a^3f-5a^2be+ab^2d+3b^3c)}{8a^2b^3\left(a+bx^2\right)}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-15a^3f+3a^2be+ab^2d+3b^3c\right)}{8a^{5/2}b^{7/2}}+\frac{fx}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x]
```

```
[Out] (f*x)/b^3 + ((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x)/(4*a*(a + b*x^2)^2) +
 ((3*b^3*c + a*b^2*d - 5*a^2*b*e + 9*a^3*f)*x)/(8*a^2*b^3*(a + b*x^2)) + ((
3*b^3*c + a*b^2*d + 3*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a
^(5/2)*b^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
```

```
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{\left(a+bx^2\right)^3}\,dx &= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{-\frac{3b^3c+ab^2d-a^2be+a^3f}{b^3}-\frac{4a(be-af)x^2}{b^2}-\frac{4afx^4}{b}}{\left(a+bx^2\right)^2}\,dx}{4a} \\
&= \frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(3b^3c+ab^2d-5a^2be+9a^3f\right)x}{8a^2b^3\left(a+bx^2\right)}+\frac{\int \frac{\frac{3b^3c+ab^2d+3a^2be-7a^3}{b^3}}{a+bx^2}}{8a^2} \\
&= \frac{fx}{b^3}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(3b^3c+ab^2d-5a^2be+9a^3f\right)x}{8a^2b^3\left(a+bx^2\right)}+\frac{(3b^3c+ab^2}{} \\
&= \frac{fx}{b^3}+\frac{\left(c-\frac{a\left(b^2d-abe+a^2f\right)}{b^3}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(3b^3c+ab^2d-5a^2be+9a^3f\right)x}{8a^2b^3\left(a+bx^2\right)}+\frac{(3b^3c+ab^2}{}
\end{aligned}
$$

Mathematica [A]
time = 0.08, size = 141, normalized size = 0.96

$$\frac{x\left(15a^4f+3b^4cx^2+ab^3\left(5c+dx^2\right)+a^3b\left(-3e+25fx^2\right)-a^2b^2\left(d+5ex^2-8fx^4\right)\right)}{8a^2b^3\left(a+bx^2\right)^2}+\frac{\left(3b^3c+ab^2d+3a^2be-15a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x]
```

```
[Out] (x*(15*a^4*f + 3*b^4*c*x^2 + a*b^3*(5*c + d*x^2) + a^3*b*(-3*e + 25*f*x^2)
- a^2*b^2*(d + 5*e*x^2 - 8*f*x^4)))/(8*a^2*b^3*(a + b*x^2)^2) + ((3*b^3*c +
 a*b^2*d + 3*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^
(7/2))
```

Maple [A]

time = 0.14, size = 139, normalized size = 0.95

method	result
default	$\frac{fx}{b^{3}}-\frac{\frac{-\frac{b\left(9a^{3}f-5a^{2}be+a\,b^{2}d+3b^{3}c\right)x^{3}}{8a^{2}}-\frac{\left(7a^{3}f-3a^{2}be-a\,b^{2}d+5b^{3}c\right)x}{8a}}{\left(b\,x^{2}+a\right)^{2}}+\frac{\left(15a^{3}f-3a^{2}be-a\,b^{2}d-3b^{3}c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^{2}\sqrt{ab}}}{b^{3}}$
risch	$\frac{fx}{b^{3}}+\frac{\frac{b\left(9a^{3}f-5a^{2}be+a\,b^{2}d+3b^{3}c\right)x^{3}}{8a^{2}}+\frac{\left(7a^{3}f-3a^{2}be-a\,b^{2}d+5b^{3}c\right)x}{8a}}{b^{3}\left(b\,x^{2}+a\right)^{2}}-\frac{15a\ln\left(bx-\sqrt{-ab}\right)f}{16b^{3}\sqrt{-ab}}+\frac{3\ln\left(bx-\sqrt{-ab}\right)e}{16b^{2}\sqrt{-ab}}+\frac{\ln\left(bx-\right.}{16b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] f*x/b^3-1/b^3*((-1/8*b*(9*a^3*f-5*a^2*b*e+a*b^2*d+3*b^3*c)/a^2*x^3-1/8*(7*a
^3*f-3*a^2*b*e-a*b^2*d+5*b^3*c)/a*x)/(b*x^2+a)^2+1/8*(15*a^3*f-3*a^2*b*e-a*
b^2*d-3*b^3*c)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

Maxima [A]

time = 0.52, size = 157, normalized size = 1.07

$$\frac{\left(3\,b^{4}c+ab^{3}d+9\,a^{3}bf-5\,a^{2}b^{2}e\right)x^{3}+\left(5\,ab^{3}c-a^{2}b^{2}d+7\,a^{4}f-3\,a^{3}be\right)x}{8\left(a^{2}b^{5}x^{4}+2\,a^{3}b^{4}x^{2}+a^{4}b^{3}\right)}+\frac{fx}{b^{3}}+\frac{\left(3\,b^{3}c+ab^{2}d-15\,a^{3}f+3\,a^{2}be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{2}b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((3*b^4*c + a*b^3*d + 9*a^3*b*f - 5*a^2*b^2*e)*x^3 + (5*a*b^3*c - a^2*b
^2*d + 7*a^4*f - 3*a^3*b*e)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3) + f*
x/b^3 + 1/8*(3*b^3*c + a*b^2*d - 15*a^3*f + 3*a^2*b*e)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*a^2*b^3)
```

Fricas [A]

time = 0.81, size = 531, normalized size = 3.61

$$\left[\frac{16\,a^{3}b^{3}fx^{5}+2\left(3\,ab^{5}c+a^{2}b^{4}d+25\,a^{4}b^{2}f\right)x^{3}-\left(3\,a^{2}b^{3}c+a^{3}b^{2}d-15\,a^{5}f+\left(3\,b^{5}c+ab^{4}d-15\,a^{3}b^{2}f\right)x^{4}+2\left(3\,ab^{4}c+a^{2}b^{3}d-15\,a^{4}bf\right)x^{2}+3\left(a^{2}b^{3}x^{4}+2\,a^{3}b^{2}x^{2}+a^{4}b\right)e\right)\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)+2\left(5\,a^{2}b^{4}c-a^{3}b^{3}d+15\,a^{5}bf\right)x-2\left(5\,a^{3}b^{3}x^{3}+3\,a^{4}b^{2}x\right)e}{16\left(a^{3}b^{6}x^{4}+2\,a^{4}b^{5}x^{2}+a^{5}b^{4}\right)},\ \frac{8\,a^{3}b^{3}fx^{5}+\left(3\,ab^{5}c+a^{2}b^{4}d+25\,a^{4}b^{2}f\right)x^{3}+\left(3\,a^{2}b^{3}c+a^{3}b^{2}d-15\,a^{5}f+\left(3\,b^{5}c+ab^{4}d-15\,a^{3}b^{2}f\right)x^{4}+2\left(3\,ab^{4}c+a^{2}b^{3}d-15\,a^{4}bf\right)x^{2}+3\left(a^{2}b^{3}x^{4}+2\,a^{3}b^{2}x^{2}+a^{4}b\right)e\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(5\,a^{2}b^{4}c-a^{3}b^{3}d+15\,a^{5}bf\right)x-\left(5\,a^{3}b^{3}x^{3}+3\,a^{4}b^{2}x\right)e}{8\left(a^{3}b^{6}x^{4}+2\,a^{4}b^{5}x^{2}+a^{5}b^{4}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*a^3*b^3*f*x^5 + 2*(3*a*b^5*c + a^2*b^4*d + 25*a^4*b^2*f)*x^3 - (3
*a^2*b^3*c + a^3*b^2*d - 15*a^5*f + (3*b^5*c + a*b^4*d - 15*a^3*b^2*f)*x^4
+ 2*(3*a*b^4*c + a^2*b^3*d - 15*a^4*b*f)*x^2 + 3*(a^2*b^3*x^4 + 2*a^3*b^2*x
^2 + a^4*b)*e)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2
*(5*a^2*b^4*c - a^3*b^3*d + 15*a^5*b*f)*x - 2*(5*a^3*b^3*x^3 + 3*a^4*b^2*x)
```

```
*e)/(a^3*b^6*x^4 + 2*a^4*b^5*x^2 + a^5*b^4), 1/8*(8*a^3*b^3*f*x^5 + (3*a*b^
5*c + a^2*b^4*d + 25*a^4*b^2*f)*x^3 + (3*a^2*b^3*c + a^3*b^2*d - 15*a^5*f +
 (3*b^5*c + a*b^4*d - 15*a^3*b^2*f)*x^4 + 2*(3*a*b^4*c + a^2*b^3*d - 15*a^4
*b*f)*x^2 + 3*(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b)*e)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) + (5*a^2*b^4*c - a^3*b^3*d + 15*a^5*b*f)*x - (5*a^3*b^3*x^3 + 3
*a^4*b^2*x)*e)/(a^3*b^6*x^4 + 2*a^4*b^5*x^2 + a^5*b^4)]
```

Sympy [A]
time = 3.83, size = 243, normalized size = 1.65

$$\frac{\sqrt{-\frac{1}{a^5b^7}}\cdot(15a^3f-3a^2be-ab^2d-3b^3c)\log\left(-a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{16}-\frac{\sqrt{-\frac{1}{a^5b^7}}\cdot(15a^3f-3a^2be-ab^2d-3b^3c)\log\left(a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{16}+\frac{x^3\cdot(9a^3bf-5a^2b^2e+ab^3d+3b^4c)+x(7a^4f-3a^3be-a^2b^2d+5ab^3c)}{8a^4b^3+16a^3b^4x^2+8a^2b^5x^4}+\frac{fx}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

```
[Out] sqrt(-1/(a**5*b**7))*(15*a**3*f - 3*a**2*b*e - a*b**2*d - 3*b**3*c)*log(-a*
*3*b**3*sqrt(-1/(a**5*b**7)) + x)/16 - sqrt(-1/(a**5*b**7))*(15*a**3*f - 3*
a**2*b*e - a*b**2*d - 3*b**3*c)*log(a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/16
+ (x**3*(9*a**3*b*f - 5*a**2*b**2*e + a*b**3*d + 3*b**4*c) + x*(7*a**4*f -
3*a**3*b*e - a**2*b**2*d + 5*a*b**3*c))/(8*a**4*b**3 + 16*a**3*b**4*x**2 +
8*a**2*b**5*x**4) + f*x/b**3
```

Giac [A]
time = 1.49, size = 149, normalized size = 1.01

$$\frac{fx}{b^3}+\frac{(3\,b^3c+ab^2d-15\,a^3f+3\,a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b^3}+\frac{3\,b^4cx^3+ab^3dx^3+9\,a^3bfx^3-5\,a^2b^2x^3e+5\,ab^3cx-a^2b^2dx+7\,a^4fx-3\,a^3bxe}{8\,(bx^2+a)^2a^2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] f*x/b^3 + 1/8*(3*b^3*c + a*b^2*d - 15*a^3*f + 3*a^2*b*e)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*a^2*b^3) + 1/8*(3*b^4*c*x^3 + a*b^3*d*x^3 + 9*a^3*b*f*x^3 -
5*a^2*b^2*x^3*e + 5*a*b^3*c*x - a^2*b^2*d*x + 7*a^4*f*x - 3*a^3*b*x*e)/((b*
x^2 + a)^2*a^2*b^3)
```

Mupad [B]
time = 1.05, size = 148, normalized size = 1.01

$$\frac{\frac{x\,(7\,f\,a^3-3\,e\,a^2\,b-d\,a\,b^2+5\,c\,b^3)}{8\,a}+\frac{x^3\,(9\,f\,a^3\,b-5\,e\,a^2\,b^2+d\,a\,b^3+3\,c\,b^4)}{8\,a^2}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4}+\frac{f\,x}{b^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(-15\,f\,a^3+3\,e\,a^2\,b+d\,a\,b^2+3\,c\,b^3)}{8\,a^{5/2}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x)
```

```
[Out] ((x*(5*b^3*c + 7*a^3*f - a*b^2*d - 3*a^2*b*e))/(8*a) + (x^3*(3*b^4*c - 5*a^
2*b^2*e + a*b^3*d + 9*a^3*b*f))/(8*a^2))/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2)
+ (f*x)/b^3 + (atan((b^(1/2)*x)/a^(1/2))*(3*b^3*c - 15*a^3*f + a*b^2*d + 3*
a^2*b*e))/(8*a^(5/2)*b^(7/2))
```

3.138 $\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=153

$$-\frac{c}{a^3x}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\left(7b^3c-3ab^2d-a^2be+5a^3f\right)x}{8a^3b^2\left(a+bx^2\right)}-\frac{\left(15b^3c-3ab^2d-a^2be-3a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}}{\sqrt{a}}\right.}{8a^{7/2}b^{5/2}}$$

```
[Out] -c/a^3/x-1/4*(b*c/a-d+a*e/b-a^2*f/b^2)*x/a/(b*x^2+a)^2-1/8*(5*a^3*f-a^2*b*e
-3*a*b^2*d+7*b^3*c)*x/a^3/b^2/(b*x^2+a)-1/8*(-3*a^3*f-a^2*b*e-3*a*b^2*d+15*
b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(5/2)
```

Rubi [A]

time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1819, 1273, 464, 211}

$$-\frac{c}{a^3x}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{4a\left(a+bx^2\right)^2}-\frac{x\left(5a^3f-a^2be-3ab^2d+7b^3c\right)}{8a^3b^2\left(a+bx^2\right)}-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-3a^3f-a^2be-3ab^2d+15b^3c\right)}{8a^{7/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(c/(a^3*x)) - (((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)*x)/(4*a*(a + b*x^2)^2
) - ((7*b^3*c - 3*a*b^2*d - a^2*b*e + 5*a^3*f)*x)/(8*a^3*b^2*(a + b*x^2)) -
 ((15*b^3*c - 3*a*b^2*d - a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*a^(7/2)*b^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
```

```
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^3}\,dx = -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{-4c+\left(\frac{3bc}{a}-3d-\frac{ae}{b}+\frac{a^2f}{b^2}\right)x^2-\frac{4afx^4}{b}}{x^2(a+bx^2)^2}\,dx}{4a}$$

$$= -\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\left(7b^3c-3ab^2d-a^2be+5a^3f\right)x}{8a^3b^2\left(a+bx^2\right)}+ \int \frac{8ab^2c-(7b^3c-3ab}{x^2(a} \quad 8a$$

$$= -\frac{c}{a^3x}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\left(7b^3c-3ab^2d-a^2be+5a^3f\right)x}{8a^3b^2\left(a+bx^2\right)}-(15b^3c-$$

$$= -\frac{c}{a^3x}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{4a\left(a+bx^2\right)^2}-\frac{\left(7b^3c-3ab^2d-a^2be+5a^3f\right)x}{8a^3b^2\left(a+bx^2\right)}-(15b^3c-$$

Mathematica [A]

time = 0.09, size = 155, normalized size = 1.01

$$-\frac{c}{a^3x}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{4a^2b^2\left(a+bx^2\right)^2}-\frac{\left(7b^3c-3ab^2d-a^2be+5a^3f\right)x}{8a^3b^2\left(a+bx^2\right)}+\frac{\left(-15b^3c+3ab^2d+a^2be+3a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x]
```

```
[Out] -(c/(a^3*x)) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^2*b^2*(a + b
*x^2)^2) - ((7*b^3*c - 3*a*b^2*d - a^2*b*e + 5*a^3*f)*x)/(8*a^3*b^2*(a + b*
x^2)) + ((-15*b^3*c + 3*a*b^2*d + a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/(8*a^(7/2)*b^(5/2))
```

Maple [A]
time = 0.15, size = 140, normalized size = 0.92

method	result
default	$\frac{\frac{-\frac{\left(5a^3f-a^2be-3a\,b^2d+7b^3c\right)x^3}{8b}-\frac{a\left(3a^3f+a^2be-5a\,b^2d+9b^3c\right)x}{8b^2}}{\left(b\,x^2+a\right)^2}+\frac{\left(3a^3f+a^2be+3a\,b^2d-15b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8b^2\sqrt{ab}}}{a^3}-\frac{c}{a^3x}$
risch	$\frac{-\frac{\left(5a^3f-a^2be-3a\,b^2d+15b^3c\right)x^4}{8a^3b}-\frac{\left(3a^3f+a^2be-5a\,b^2d+25b^3c\right)x^2}{8a^2b^2}-\frac{c}{a}}{x(b\,x^2+a)^2}-\frac{3\ln\left(-\sqrt{-ab}\,x+a\right)f}{16\sqrt{-ab}\,b^2}-\frac{\ln\left(-\sqrt{-ab}\,x+a\right)e}{16\sqrt{-ab}\,ba}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*((-1/8*(5*a^3*f-a^2*b*e-3*a*b^2*d+7*b^3*c)/b*x^3-1/8*a*(3*a^3*f+a^2*b
*e-5*a*b^2*d+9*b^3*c)/b^2*x)/(b*x^2+a)^2+1/8*(3*a^3*f+a^2*b*e+3*a*b^2*d-15*
b^3*c)/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-c/a^3/x
```

Maxima [A]
time = 0.52, size = 164, normalized size = 1.07

$$-\frac{8\,a^2b^2c+(15\,b^4c-3\,ab^3d+5\,a^3bf-a^2b^2e)x^4+(25\,ab^3c-5\,a^2b^2d+3\,a^4f+a^3be)x^2}{8\,(a^3b^4x^5+2\,a^4b^3x^3+a^5b^2x)}-\frac{(15\,b^3c-3\,ab^2d-3\,a^3f-a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(8*a^2*b^2*c + (15*b^4*c - 3*a*b^3*d + 5*a^3*b*f - a^2*b^2*e)*x^4 + (2
5*a*b^3*c - 5*a^2*b^2*d + 3*a^4*f + a^3*b*e)*x^2)/(a^3*b^4*x^5 + 2*a^4*b^3*
x^3 + a^5*b^2*x) - 1/8*(15*b^3*c - 3*a*b^2*d - 3*a^3*f - a^2*b*e)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2)
```

Fricas [A]
time = 1.62, size = 553, normalized size = 3.61

$$\left[-\frac{16\,a^3b^3c+2\,(15\,ab^5c-3\,a^2b^4d+5\,a^4b^2f)x^4+2\,(25\,a^2b^4c-5\,a^3b^3d+3\,a^5bf)x^2-(3\,(5\,b^5c-ab^4d-a^3b^2f)x^5+6\,(5\,ab^4c-a^2b^3d-a^4bf)x^3+3\,(5\,a^2b^3c-a^3b^2d-a^5f)x-(a^2b^3x^5+2\,a^3b^2x^3+a^4bx)e)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\,(a^3b^3x^4-a^4b^2x^2)e}{16\,(a^4b^5x^5+2\,a^5b^4x^3+a^6b^3x)},-\frac{8\,a^3b^3c+(15\,ab^5c-3\,a^2b^4d+5\,a^4b^2f)x^4+(25\,a^2b^4c-5\,a^3b^3d+3\,a^5bf)x^2+(3\,(5\,b^5c-ab^4d-a^3b^2f)x^5+6\,(5\,ab^4c-a^2b^3d-a^4bf)x^3+3\,(5\,a^2b^3c-a^3b^2d-a^5f)x-(a^2b^3x^5+2\,a^3b^2x^3+a^4bx)e)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-(a^3b^3x^4-a^4b^2x^2)e}{8\,(a^4b^5x^5+2\,a^5b^4x^3+a^6b^3x)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(16*a^3*b^3*c + 2*(15*a*b^5*c - 3*a^2*b^4*d + 5*a^4*b^2*f)*x^4 + 2*(
25*a^2*b^4*c - 5*a^3*b^3*d + 3*a^5*b*f)*x^2 - (3*(5*b^5*c - a*b^4*d - a^3*b
^2*f)*x^5 + 6*(5*a*b^4*c - a^2*b^3*d - a^4*b*f)*x^3 + 3*(5*a^2*b^3*c - a^3*
b^2*d - a^5*f)*x - (a^2*b^3*x^5 + 2*a^3*b^2*x^3 + a^4*b*x)*e)*sqrt(-a*b)*lo
g((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(a^3*b^3*x^4 - a^4*b^2*x^2)
*e)/(a^4*b^5*x^5 + 2*a^5*b^4*x^3 + a^6*b^3*x), -1/8*(8*a^3*b^3*c + (15*a*b^
5*c - 3*a^2*b^4*d + 5*a^4*b^2*f)*x^4 + (25*a^2*b^4*c - 5*a^3*b^3*d + 3*a^5*
b*f)*x^2 + (3*(5*b^5*c - a*b^4*d - a^3*b^2*f)*x^5 + 6*(5*a*b^4*c - a^2*b^3*
d - a^4*b*f)*x^3 + 3*(5*a^2*b^3*c - a^3*b^2*d - a^5*f)*x - (a^2*b^3*x^5 + 2
*a^3*b^2*x^3 + a^4*b*x)*e)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (a^3*b^3*x^4 -
 a^4*b^2*x^2)*e)/(a^4*b^5*x^5 + 2*a^5*b^4*x^3 + a^6*b^3*x)]
```

Sympy [A]
time = 12.30, size = 250, normalized size = 1.63

$$-\frac{\sqrt{-\frac{1}{a^7b^5}}\cdot(3a^3f+a^2be+3ab^2d-15b^3c)\log\left(-a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{a^7b^5}}\cdot(3a^3f+a^2be+3ab^2d-15b^3c)\log\left(a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{16}+\frac{-8a^2b^2c+x^4(-5a^3bf+a^2b^2e+3ab^3d-15b^4c)+x^2(-3a^4f-a^3be+5a^2b^2d-25ab^3c)}{8a^5b^2x+16a^4b^3x^3+8a^3b^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(a**7*b**5))*(3*a**3*f + a**2*b*e + 3*a*b**2*d - 15*b**3*c)*log(-a
**4*b**2*sqrt(-1/(a**7*b**5)) + x)/16 + sqrt(-1/(a**7*b**5))*(3*a**3*f + a*
*2*b*e + 3*a*b**2*d - 15*b**3*c)*log(a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/16
 + (-8*a**2*b**2*c + x**4*(-5*a**3*b*f + a**2*b**2*e + 3*a*b**3*d - 15*b**4
*c) + x**2*(-3*a**4*f - a**3*b*e + 5*a**2*b**2*d - 25*a*b**3*c))/(8*a**5*b*
*2*x + 16*a**4*b**3*x**3 + 8*a**3*b**4*x**5)
```

Giac [A]
time = 1.13, size = 153, normalized size = 1.00

$$-\frac{c}{a^3x}-\frac{(15\,b^3c-3\,ab^2d-3\,a^3f-a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3b^2}-\frac{7\,b^4cx^3-3\,ab^3dx^3+5\,a^3bfx^3-a^2b^2x^3e+9\,ab^3cx-5\,a^2b^2dx+3\,a^4fx+a^3bxe}{8\,(bx^2+a)^2a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -c/(a^3*x) - 1/8*(15*b^3*c - 3*a*b^2*d - 3*a^3*f - a^2*b*e)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a^3*b^2) - 1/8*(7*b^4*c*x^3 - 3*a*b^3*d*x^3 + 5*a^3*b*f*x
^3 - a^2*b^2*x^3*e + 9*a*b^3*c*x - 5*a^2*b^2*d*x + 3*a^4*f*x + a^3*b*x*e)/(
(b*x^2 + a)^2*a^3*b^2)
```

Mupad [B]
time = 1.09, size = 149, normalized size = 0.97

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(3\,f\,a^3+e\,a^2\,b+3\,d\,a\,b^2-15\,c\,b^3\right)}{8\,a^{7/2}\,b^{5/2}}-\frac{\frac{c}{a}+\frac{x^4\,\left(5\,f\,a^3-e\,a^2\,b-3\,d\,a\,b^2+15\,c\,b^3\right)}{8\,a^3\,b}+\frac{x^2\,\left(3\,f\,a^3+e\,a^2\,b-5\,d\,a\,b^2+25\,c\,b^3\right)}{8\,a^2\,b^2}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(3*a^3*f - 15*b^3*c + 3*a*b^2*d + a^2*b*e))/(8*a
^(7/2)*b^(5/2)) - (c/a + (x^4*(15*b^3*c + 5*a^3*f - 3*a*b^2*d - a^2*b*e))/(
8*a^3*b) + (x^2*(25*b^3*c + 3*a^3*f - 5*a*b^2*d + a^2*b*e))/(8*a^2*b^2))/(a
^2*x + b^2*x^5 + 2*a*b*x^3)
```

3.139 $\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=168

$$-\frac{c}{3a^3x^3}+\frac{3bc-ad}{a^4x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)x}{8a^4b\left(a+bx^2\right)}+\frac{(35b^3c-15ab^2d+3a^2be+}{8a^{9/2}b^{3/2}}$$

```
[Out] -1/3*c/a^3/x^3+(-a*d+3*b*c)/a^4/x+1/4*(b^2*c/a^2-b*d/a+e-a*f/b)*x/a/(b*x^2+
a)^2+1/8*(a^3*f+3*a^2*b*e-7*a*b^2*d+11*b^3*c)*x/a^4/b/(b*x^2+a)+1/8*(a^3*f+
3*a^2*b*e-15*a*b^2*d+35*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)/b^(3/2)
```

Rubi [A]

time = 0.16, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1819, 1273, 1275, 211}

$$\frac{3bc-ad}{a^4x}-\frac{c}{3a^3x^3}+\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{4a\left(a+bx^2\right)^2}+\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(a^3f+3a^2be-15ab^2d+35b^3c\right)}{8a^{9/2}b^{3/2}}+\frac{x\left(a^3f+3a^2be-7ab^2d+11b^3c\right)}{8a^4b\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] -1/3*c/(a^3*x^3) + (3*b*c - a*d)/(a^4*x) + (((b^2*c)/a^2 - (b*d)/a + e - (a
*f)/b)*x)/(4*a*(a + b*x^2)^2) + ((11*b^3*c - 7*a*b^2*d + 3*a^2*b*e + a^3*f)
*x)/(8*a^4*b*(a + b*x^2)) + ((35*b^3*c - 15*a*b^2*d + 3*a^2*b*e + a^3*f)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*b^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx = \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2} - \frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2+\left(-\frac{3b^2c}{a^2}+\frac{3bd}{a}-3e-\frac{af}{b}\right)x^4}{x^4\left(a+bx^2\right)^2}\,dx}{4a}$$

$$= \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)x}{8a^4b\left(a+bx^2\right)} - \int \frac{-8a^2b^2c+8ab^2(2bc-}{}$$

$$= \frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)x}{8a^4b\left(a+bx^2\right)} - \int \left(-\frac{8ab^2c}{x^4}+\frac{8b^2(3b}{x}\right.$$

$$= -\frac{c}{3a^3x^3}+\frac{3bc-ad}{a^4x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)}{8a^4b\left(a+bx^2\right)}$$

$$= -\frac{c}{3a^3x^3}+\frac{3bc-ad}{a^4x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2}+\frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)}{8a^4b\left(a+bx^2\right)}$$

Mathematica [A]
time = 0.10, size = 169, normalized size = 1.01

$$\frac{-3a^4fx^4+105b^4cx^6+5ab^3x^4(35c-9dx^2)+a^2b^2x^2(56c-75dx^2+9ex^4)+a^3b(-8c+3x^2(-8d+5ex^2+fx^4))}{24a^4bx^3\left(a+bx^2\right)^2}+\frac{\left(35b^3c-15ab^2d+3a^2be+a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{9/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x]
```

```
[Out] (-3*a^4*f*x^4 + 105*b^4*c*x^6 + 5*a*b^3*x^4*(35*c - 9*d*x^2) + a^2*b^2*x^2*
(56*c - 75*d*x^2 + 9*e*x^4) + a^3*b*(-8*c + 3*x^2*(-8*d + 5*e*x^2 + f*x^4))
)/(24*a^4*b*x^3*(a + b*x^2)^2) + ((35*b^3*c - 15*a*b^2*d + 3*a^2*b*e + a^3*
f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*b^(3/2))
```

Maple [A]
time = 0.16, size = 152, normalized size = 0.90

method	result
default	$\frac{\frac{\left(\frac{1}{8}a^3f+\frac{3}{8}a^2be-\frac{7}{8}a\,b^2d+\frac{11}{8}b^3c\right)x^3-\frac{a\left(a^3f-5a^2be+9a\,b^2d-13b^3c\right)x}{8b}}{\left(b\,x^2+a\right)^2}+\frac{\left(a^3f+3a^2be-15a\,b^2d+35b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8b\sqrt{ab}}}{a^4}-\frac{c}{3a^3x^3}-\frac{ad-3}{a^4x}$
risch	$\frac{\frac{\left(a^3f+3a^2be-15a\,b^2d+35b^3c\right)x^6}{8a^4}-\frac{\left(3a^3f-15a^2be+75a\,b^2d-175b^3c\right)x^4}{24a^3b}-\frac{(3ad-7bc)x^2}{3a^2}-\frac{c}{3a}}{x^3(b\,x^2+a)^2}-\frac{\ln\left(-\sqrt{-ab}\,x+a\right)f}{16\sqrt{-ab}\,ba}-\frac{3\ln\left(-\sqrt{-ab}\right.}{16\sqrt{-ab}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(((1/8*a^3*f+3/8*a^2*b*e-7/8*a*b^2*d+11/8*b^3*c)*x^3-1/8*a*(a^3*f-5*a
^2*b*e+9*a*b^2*d-13*b^3*c)/b*x)/(b*x^2+a)^2+1/8*(a^3*f+3*a^2*b*e-15*a*b^2*d
+35*b^3*c)/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/3*c/a^3/x^3-(a*d-3*b*c)
/a^4/x
```

Maxima [A]
time = 0.50, size = 184, normalized size = 1.10

$$\frac{3\,(35\,b^4c-15\,ab^3d+a^3bf+3\,a^2b^2e)x^6-8\,a^3bc+(175\,ab^3c-75\,a^2b^2d-3\,a^4f+15\,a^3be)x^4+8\,(7\,a^2b^2c-3\,a^3bd)x^2}{24\,(a^4b^3x^7+2\,a^5b^2x^5+a^6bx^3)}+\frac{(35\,b^3c-15\,ab^2d+a^3f+3\,a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(3*(35*b^4*c - 15*a*b^3*d + a^3*b*f + 3*a^2*b^2*e)*x^6 - 8*a^3*b*c + (
175*a*b^3*c - 75*a^2*b^2*d - 3*a^4*f + 15*a^3*b*e)*x^4 + 8*(7*a^2*b^2*c - 3
*a^3*b*d)*x^2)/(a^4*b^3*x^7 + 2*a^5*b^2*x^5 + a^6*b*x^3) + 1/8*(35*b^3*c -
15*a*b^2*d + a^3*f + 3*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b)
```

Fricas [A]
time = 3.83, size = 606, normalized size = 3.61

$$\left[-\frac{16\,a^4b^2c-6\,(35\,ab^5c-15\,a^2b^4d+a^4b^2f)x^6-2\,(175\,a^2b^4c-75\,a^3b^3d-3\,a^5bf)x^4-16\,(7\,a^3b^3c-3\,a^4b^2d)x^2+3\,((35\,b^5c-15\,ab^4d+a^3b^2f)x^7+2\,(35\,ab^4c-15\,a^2b^3d+a^4bf)x^5+(35\,a^2b^3c-15\,a^3b^2d+a^5f)x^3+3\,(a^2b^3x^7+2\,a^3b^2x^5+a^4bx^3)e)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-6\,(3\,a^3b^3x^6+5\,a^4b^2x^4)e}{48\,(a^5b^4x^7+2\,a^6b^3x^5+a^7b^2x^3)},-\frac{8\,a^4b^2c-3\,(35\,ab^5c-15\,a^2b^4d+a^4b^2f)x^6-(175\,a^2b^4c-75\,a^3b^3d-3\,a^5bf)x^4-8\,(7\,a^3b^3c-3\,a^4b^2d)x^2-3\,((35\,b^5c-15\,ab^4d+a^3b^2f)x^7+2\,(35\,ab^4c-15\,a^2b^3d+a^4bf)x^5+(35\,a^2b^3c-15\,a^3b^2d+a^5f)x^3+3\,(a^2b^3x^7+2\,a^3b^2x^5+a^4bx^3)e)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-3\,(3\,a^3b^3x^6+5\,a^4b^2x^4)e}{24\,(a^5b^4x^7+2\,a^6b^3x^5+a^7b^2x^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(16*a^4*b^2*c - 6*(35*a*b^5*c - 15*a^2*b^4*d + a^4*b^2*f)*x^6 - 2*(1
75*a^2*b^4*c - 75*a^3*b^3*d - 3*a^5*b*f)*x^4 - 16*(7*a^3*b^3*c - 3*a^4*b^2*
d)*x^2 + 3*((35*b^5*c - 15*a*b^4*d + a^3*b^2*f)*x^7 + 2*(35*a*b^4*c - 15*a^
2*b^3*d + a^4*b*f)*x^5 + (35*a^2*b^3*c - 15*a^3*b^2*d + a^5*f)*x^3 + 3*(a^2
*b^3*x^7 + 2*a^3*b^2*x^5 + a^4*b*x^3)*e)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*
b)*x - a)/(b*x^2 + a)) - 6*(3*a^3*b^3*x^6 + 5*a^4*b^2*x^4)*e)/(a^5*b^4*x^7
+ 2*a^6*b^3*x^5 + a^7*b^2*x^3), -1/24*(8*a^4*b^2*c - 3*(35*a*b^5*c - 15*a^2
*b^4*d + a^4*b^2*f)*x^6 - (175*a^2*b^4*c - 75*a^3*b^3*d - 3*a^5*b*f)*x^4 -
8*(7*a^3*b^3*c - 3*a^4*b^2*d)*x^2 - 3*((35*b^5*c - 15*a*b^4*d + a^3*b^2*f)*
x^7 + 2*(35*a*b^4*c - 15*a^2*b^3*d + a^4*b*f)*x^5 + (35*a^2*b^3*c - 15*a^3*
b^2*d + a^5*f)*x^3 + 3*(a^2*b^3*x^7 + 2*a^3*b^2*x^5 + a^4*b*x^3)*e)*sqrt(a*
b)*arctan(sqrt(a*b)*x/a) - 3*(3*a^3*b^3*x^6 + 5*a^4*b^2*x^4)*e)/(a^5*b^4*x^
7 + 2*a^6*b^3*x^5 + a^7*b^2*x^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.68, size = 170, normalized size = 1.01

$$\frac{(35\,b^3c-15\,ab^2d+a^3f+3\,a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^4b}+\frac{11\,b^4cx^3-7\,ab^3dx^3+a^3bfx^3+3\,a^2b^2x^3e+13\,ab^3cx-9\,a^2b^2dx-a^4fx+5\,a^3bxe}{8\,(bx^2+a)^2a^4b}+\frac{9\,bcx^2-3\,adx^2-ac}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*b^3*c - 15*a*b^2*d + a^3*f + 3*a^2*b*e)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a^4*b) + 1/8*(11*b^4*c*x^3 - 7*a*b^3*d*x^3 + a^3*b*f*x^3 + 3*a^2*b^2*
x^3*e + 13*a*b^3*c*x - 9*a^2*b^2*d*x - a^4*f*x + 5*a^3*b*x*e)/((b*x^2 + a)^
2*a^4*b) + 1/3*(9*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^4*x^3)
```

Mupad [**B**]
time = 1.03, size = 166, normalized size = 0.99

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(f\,a^3+3\,e\,a^2\,b-15\,d\,a\,b^2+35\,c\,b^3\right)}{8\,a^{9/2}\,b^{3/2}}-\frac{\frac{c}{3\,a}-\frac{x^6\,\left(f\,a^3+3\,e\,a^2\,b-15\,d\,a\,b^2+35\,c\,b^3\right)}{8\,a^4}+\frac{x^2\,\left(3\,a\,d-7\,b\,c\right)}{3\,a^2}-\frac{x^4\,\left(-3\,f\,a^3+15\,e\,a^2\,b-75\,d\,a\,b^2+175\,c\,b^3\right)}{24\,a^3\,b}}{a^2\,x^3+2\,a\,b\,x^5+b^2\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x)
```

```
[Out] (atan((b^(1/2)*x)/a^(1/2))*(35*b^3*c + a^3*f - 15*a*b^2*d + 3*a^2*b*e))/(8*
a^(9/2)*b^(3/2)) - (c/(3*a) - (x^6*(35*b^3*c + a^3*f - 15*a*b^2*d + 3*a^2*b
*e))/(8*a^4) + (x^2*(3*a*d - 7*b*c))/(3*a^2) - (x^4*(175*b^3*c - 3*a^3*f -
75*a*b^2*d + 15*a^2*b*e))/(24*a^3*b))/(a^2*x^3 + b^2*x^7 + 2*a*b*x^5)
```

3.140 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=196

$$-\frac{c}{5a^3x^5}+\frac{3bc-ad}{3a^4x^3}-\frac{6b^2c-3abd+a^2e}{a^5x}-\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2}-\frac{\left(15b^3c-11ab^2d+7a^2be-3a^3f\right)x}{8a^5\left(a+bx^2\right)}-$$

```
[Out] -1/5*c/a^3/x^5+1/3*(-a*d+3*b*c)/a^4/x^3+(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x-1/4*
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^4/(b*x^2+a)^2-1/8*(-3*a^3*f+7*a^2*b*e-11
*a*b^2*d+15*b^3*c)*x/a^5/(b*x^2+a)-1/8*(-3*a^3*f+15*a^2*b*e-35*a*b^2*d+63*b
^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(11/2)/b^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1819, 1816, 211}

$$\frac{3bc-ad}{3a^4x^3}-\frac{c}{5a^3x^5}-\frac{a^2e-3abd+6b^2c}{a^5x}-\frac{\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-3a^3f+15a^2be-35ab^2d+63b^3c\right)}{8a^{11/2}\sqrt{b}}-\frac{x\left(-3a^3f+7a^2be-11ab^2d+15b^3c\right)}{8a^5\left(a+bx^2\right)}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^4\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -1/5*c/(a^3*x^5) + (3*b*c - a*d)/(3*a^4*x^3) - (6*b^2*c - 3*a*b*d + a^2*e)/
(a^5*x) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^4*(a + b*x^2)^2) - (
(15*b^3*c - 11*a*b^2*d + 7*a^2*b*e - 3*a^3*f)*x)/(8*a^5*(a + b*x^2)) - ((63
*b^3*c - 35*a*b^2*d + 15*a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8
*a^(11/2)*Sqrt[b])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
```

```
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx = -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2} - \frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2-\frac{4\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{3\left(b^3c-ab^2d+a^2be-\right.}{a^3}}{x^6\left(a+bx^2\right)^2}}{4a}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2} - \frac{\left(15b^3c-11ab^2d+7a^2be-3a^3f\right)x}{8a^5\left(a+bx^2\right)} + \frac{\int \frac{8c-8}{}}{}$$

$$= -\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2} - \frac{\left(15b^3c-11ab^2d+7a^2be-3a^3f\right)x}{8a^5\left(a+bx^2\right)} + \frac{\int \Big(\frac{8c}{ax^6}}{}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{3a^4x^3} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2} - \frac{(1}{}$$

$$= -\frac{c}{5a^3x^5} + \frac{3bc-ad}{3a^4x^3} - \frac{6b^2c-3abd+a^2e}{a^5x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^4\left(a+bx^2\right)^2} - \frac{(1}{}$$

Mathematica [A]
time = 0.08, size = 196, normalized size = 1.00

$$-\frac{c}{5a^3x^5}+\frac{3bc-ad}{3a^4x^3}+\frac{-6b^2c+3abd-a^2e}{a^5x}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{4a^4\left(a+bx^2\right)^2}+\frac{\left(-15b^3c+11ab^2d-7a^2be+3a^3f\right)x}{8a^5\left(a+bx^2\right)}+\frac{\left(-63b^3c+35ab^2d-15a^2be+3a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{11/2}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x]
```

```
[Out] -1/5*c/(a^3*x^5) + (3*b*c - a*d)/(3*a^4*x^3) + (-6*b^2*c + 3*a*b*d - a^2*e)
/(a^5*x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^4*(a + b*x^2)^2)
 + ((-15*b^3*c + 11*a*b^2*d - 7*a^2*b*e + 3*a^3*f)*x)/(8*a^5*(a + b*x^2)) +
 ((-63*b^3*c + 35*a*b^2*d - 15*a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/(8*a^(11/2)*Sqrt[b])
```

Maple [A]
time = 0.16, size = 176, normalized size = 0.90

---

method	result
default	$\frac{\frac{\left(\frac{3}{8}a^3bf-\frac{7}{8}a^2eb^2+\frac{11}{8}adb^3-\frac{15}{8}cb^4\right)x^3+\frac{a\left(5a^3f-9a^2be+13ab^2d-17b^3c\right)x}{8}}{\left(bx^2+a\right)^2}+\frac{\left(3a^3f-15a^2be+35ab^2d-63b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{a^5}-\frac{c}{5a^3x^5}-$
risch	$\frac{\frac{b\left(3a^3f-15a^2be+35ab^2d-63b^3c\right)x^8}{8a^5}+\frac{5\left(3a^3f-15a^2be+35ab^2d-63b^3c\right)x^6}{24a^4}-\frac{\left(15a^2e-35abd+63b^2c\right)x^4}{15a^3}-\frac{(5ad-9bc)x^2}{15a^2}-\frac{c}{5a}}{x^5(bx^2+a)^2}-\frac{3\ln\left(-\sqrt{-ab}\right.}{16\sqrt{-ab}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(((3/8*a^3*b*f-7/8*a^2*e*b^2+11/8*a*d*b^3-15/8*c*b^4)*x^3+1/8*a*(5*a^
3*f-9*a^2*b*e+13*a*b^2*d-17*b^3*c)*x)/(b*x^2+a)^2+1/8*(3*a^3*f-15*a^2*b*e+3
5*a*b^2*d-63*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/5*c/a^3/x^5-1/3*
(a*d-3*b*c)/a^4/x^3-(a^2*e-3*a*b*d+6*b^2*c)/a^5/x
```

Maxima [A]
time = 0.55, size = 206, normalized size = 1.05

$$-\frac{15\,(63\,b^4c-35\,ab^3d-3\,a^3bf+15\,a^2b^2e)x^8+25\,(63\,ab^3c-35\,a^2b^2d-3\,a^4f+15\,a^3be)x^6+24\,a^4c+8\,(63\,a^2b^2c-35\,a^3bd+15\,a^4e)x^4-8\,(9\,a^3bc-5\,a^4d)x^2}{120\,(a^5b^2x^9+2\,a^6bx^7+a^7x^5)}-\frac{(63\,b^3c-35\,ab^2d-3\,a^3f+15\,a^2be)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/120*(15*(63*b^4*c - 35*a*b^3*d - 3*a^3*b*f + 15*a^2*b^2*e)*x^8 + 25*(63*
a*b^3*c - 35*a^2*b^2*d - 3*a^4*f + 15*a^3*b*e)*x^6 + 24*a^4*c + 8*(63*a^2*b
^2*c - 35*a^3*b*d + 15*a^4*e)*x^4 - 8*(9*a^3*b*c - 5*a^4*d)*x^2)/(a^5*b^2*x
^9 + 2*a^6*b*x^7 + a^7*x^5) - 1/8*(63*b^3*c - 35*a*b^2*d - 3*a^3*f + 15*a^2
*b*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

Fricas [A]
time = 5.35, size = 668, normalized size = 3.41

$$\left[-\frac{30\,(63\,ab^5c-35\,a^2b^4d-3\,a^4b^2f)x^8+48\,a^5bc+50\,(63\,a^2b^4c-35\,a^3b^3d-3\,a^5bf)x^6+112\,(9\,a^3b^3c-5\,a^4b^2d)x^4-16\,(9\,a^4b^2c-5\,a^5bd)x^2+15\,((63\,b^5c-35\,ab^4d-3\,a^3b^2f)x^9+2\,(63\,ab^4c-35\,a^2b^3d-3\,a^4bf)x^7+(63\,a^2b^3c-35\,a^3b^2d-3\,a^5f)x^5+15\,(a^2b^3x^9+2\,a^3b^2x^7+a^4bx^5)e)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)+30\,(15\,a^3b^3x^8+25\,a^4b^2x^6+8\,a^5bx^4)e}{240\,(a^6b^3x^9+2\,a^7b^2x^7+a^8bx^5)},-\frac{15\,(63\,ab^5c-35\,a^2b^4d-3\,a^4b^2f)x^8+24\,a^5bc+25\,(63\,a^2b^4c-35\,a^3b^3d-3\,a^5bf)x^6+56\,(9\,a^3b^3c-5\,a^4b^2d)x^4-8\,(9\,a^4b^2c-5\,a^5bd)x^2+15\,((63\,b^5c-35\,ab^4d-3\,a^3b^2f)x^9+2\,(63\,ab^4c-35\,a^2b^3d-3\,a^4bf)x^7+(63\,a^2b^3c-35\,a^3b^2d-3\,a^5f)x^5+15\,(a^2b^3x^9+2\,a^3b^2x^7+a^4bx^5)e)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+15\,(15\,a^3b^3x^8+25\,a^4b^2x^6+8\,a^5bx^4)e}{120\,(a^6b^3x^9+2\,a^7b^2x^7+a^8bx^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(30*(63*a*b^5*c - 35*a^2*b^4*d - 3*a^4*b^2*f)*x^8 + 48*a^5*b*c + 50
*(63*a^2*b^4*c - 35*a^3*b^3*d - 3*a^5*b*f)*x^6 + 112*(9*a^3*b^3*c - 5*a^4*b
^2*d)*x^4 - 16*(9*a^4*b^2*c - 5*a^5*b*d)*x^2 + 15*((63*b^5*c - 35*a*b^4*d -
 3*a^3*b^2*f)*x^9 + 2*(63*a*b^4*c - 35*a^2*b^3*d - 3*a^4*b*f)*x^7 + (63*a^2
*b^3*c - 35*a^3*b^2*d - 3*a^5*f)*x^5 + 15*(a^2*b^3*x^9 + 2*a^3*b^2*x^7 + a^
```

```
4*b*x^5)*e)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 30*(
15*a^3*b^3*x^8 + 25*a^4*b^2*x^6 + 8*a^5*b*x^4)*e)/(a^6*b^3*x^9 + 2*a^7*b^2*
x^7 + a^8*b*x^5), -1/120*(15*(63*a*b^5*c - 35*a^2*b^4*d - 3*a^4*b^2*f)*x^8
+ 24*a^5*b*c + 25*(63*a^2*b^4*c - 35*a^3*b^3*d - 3*a^5*b*f)*x^6 + 56*(9*a^3
*b^3*c - 5*a^4*b^2*d)*x^4 - 8*(9*a^4*b^2*c - 5*a^5*b*d)*x^2 + 15*((63*b^5*c
 - 35*a*b^4*d - 3*a^3*b^2*f)*x^9 + 2*(63*a*b^4*c - 35*a^2*b^3*d - 3*a^4*b*f
)*x^7 + (63*a^2*b^3*c - 35*a^3*b^2*d - 3*a^5*f)*x^5 + 15*(a^2*b^3*x^9 + 2*a
^3*b^2*x^7 + a^4*b*x^5)*e)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 15*(15*a^3*b^3
*x^8 + 25*a^4*b^2*x^6 + 8*a^5*b*x^4)*e)/(a^6*b^3*x^9 + 2*a^7*b^2*x^7 + a^8*
b*x^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.41, size = 198, normalized size = 1.01

$$-\frac{\left(63\,b^{3}c-35\,ab^{2}d-3\,a^{3}f+15\,a^{2}be\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^{5}}-\frac{15\,b^{4}cx^{3}-11\,ab^{3}dx^{3}-3\,a^{3}bfx^{3}+7\,a^{2}b^{2}x^{3}e+17\,ab^{3}cx-13\,a^{2}b^{2}dx-5\,a^{4}fx+9\,a^{3}bxe}{8\,\left(bx^{2}+a\right)^{2}a^{5}}-\frac{90\,b^{2}cx^{4}-45\,abdx^{4}+15\,a^{2}x^{4}e-15\,abcx^{2}+5\,a^{2}dx^{2}+3\,a^{2}c}{15\,a^{5}x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(63*b^3*c - 35*a*b^2*d - 3*a^3*f + 15*a^2*b*e)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*a^5) - 1/8*(15*b^4*c*x^3 - 11*a*b^3*d*x^3 - 3*a^3*b*f*x^3 + 7*a^2
*b^2*x^3*e + 17*a*b^3*c*x - 13*a^2*b^2*d*x - 5*a^4*f*x + 9*a^3*b*x*e)/((b*x
^2 + a)^2*a^5) - 1/15*(90*b^2*c*x^4 - 45*a*b*d*x^4 + 15*a^2*x^4*e - 15*a*b*
c*x^2 + 5*a^2*d*x^2 + 3*a^2*c)/(a^5*x^5)
```

Mupad [**B**]
time = 1.04, size = 192, normalized size = 0.98

$$-\frac{\frac{c}{5\,a}+\frac{5\,x^6\,\left(-3\,f\,a^3+15\,e\,a^2\,b-35\,d\,a\,b^2+63\,c\,b^3\right)}{24\,a^4}+\frac{x^2\,\left(5\,a\,d-9\,b\,c\right)}{15\,a^2}+\frac{x^4\,\left(15\,e\,a^2-35\,d\,a\,b+63\,c\,b^2\right)}{15\,a^3}+\frac{b\,x^8\,\left(-3\,f\,a^3+15\,e\,a^2\,b-35\,d\,a\,b^2+63\,c\,b^3\right)}{8\,a^5}}{a^2\,x^5+2\,a\,b\,x^7+b^2\,x^9}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-3\,f\,a^3+15\,e\,a^2\,b-35\,d\,a\,b^2+63\,c\,b^3\right)}{8\,a^{11/2}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x)
```

```
[Out] - (c/(5*a) + (5*x^6*(63*b^3*c - 3*a^3*f - 35*a*b^2*d + 15*a^2*b*e))/(24*a^4
) + (x^2*(5*a*d - 9*b*c))/(15*a^2) + (x^4*(63*b^2*c + 15*a^2*e - 35*a*b*d))
/(15*a^3) + (b*x^8*(63*b^3*c - 3*a^3*f - 35*a*b^2*d + 15*a^2*b*e))/(8*a^5))
/(a^2*x^5 + b^2*x^9 + 2*a*b*x^7) - (atan((b^(1/2)*x)/a^(1/2))*(63*b^3*c - 3
*a^3*f - 35*a*b^2*d + 15*a^2*b*e))/(8*a^(11/2)*b^(1/2))
```

3.141 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=234

$$-\frac{c}{7a^3x^7}+\frac{3bc-ad}{5a^4x^5}-\frac{6b^2c-3abd+a^2e}{3a^5x^3}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x}+\frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^5\left(a+bx^2\right)^2}+b(19b$$

```
[Out] -1/7*c/a^3/x^7+1/5*(-a*d+3*b*c)/a^4/x^5+1/3*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x^
3+(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x+1/4*b*(-a^3*f+a^2*b*e-a*b^2*d
+b^3*c)*x/a^5/(b*x^2+a)^2+1/8*b*(-7*a^3*f+11*a^2*b*e-15*a*b^2*d+19*b^3*c)*x
/a^6/(b*x^2+a)+1/8*(-15*a^3*f+35*a^2*b*e-63*a*b^2*d+99*b^3*c)*arctan(x*b^(1
/2)/a^(1/2))*b^(1/2)/a^(13/2)
```

Rubi [A]
time = 0.33, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1819, 1816, 211}

$$\frac{3bc-ad}{5a^4x^5}-\frac{c}{7a^3x^7}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}+\frac{\sqrt{b}\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-15a^3f+35a^2be-63ab^2d+99b^3c\right)}{8a^{13/2}}+\frac{bx\left(-7a^3f+11a^2be-15ab^2d+19b^3c\right)}{8a^6\left(a+bx^2\right)}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{a^6x}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^5\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x]
```

```
[Out] -1/7*c/(a^3*x^7) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/
(3*a^5*x^3) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^5*(a + b*x^2)^2) + (b*(19*b^3*c - 15
*a*b^2*d + 11*a^2*b*e - 7*a^3*f)*x)/(8*a^6*(a + b*x^2)) + (Sqrt[b]*(99*b^3*
c - 63*a*b^2*d + 35*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(
13/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
```

```
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx = \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^5\left(a+bx^2\right)^2} - \frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2-\frac{4\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{4\left(b^3c-ab^2d+a^2be-\right.}{a^3}}{x^8\left(a+bx^2\right)^2}}{4a}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^5\left(a+bx^2\right)^2} + \frac{b\left(19b^3c-15ab^2d+11a^2be-7a^3f\right)x}{8a^6\left(a+bx^2\right)} + \int \frac{8c-}{}$$

$$= \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^5\left(a+bx^2\right)^2} + \frac{b\left(19b^3c-15ab^2d+11a^2be-7a^3f\right)x}{8a^6\left(a+bx^2\right)} + \int \left(\frac{8}{ax}\right.$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{3a^5x^3} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + b$$

$$= -\frac{c}{7a^3x^7} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{3a^5x^3} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + b$$

Mathematica [A]
time = 0.10, size = 234, normalized size = 1.00

$$-\frac{c}{7a^3x^7} + \frac{3bc-ad}{5a^4x^5} - \frac{6b^2c-3abd+a^2e}{3a^5x^3} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x} + \frac{b\left(b^3c-ab^2d+a^2be-a^3f\right)x}{4a^5\left(a+bx^2\right)^2} + \frac{b\left(19b^3c-15ab^2d+11a^2be-7a^3f\right)x}{8a^6\left(a+bx^2\right)} + \frac{\sqrt{b}\left(99b^3c-63ab^2d+35a^2be-15a^3f\right)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{13/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x]
```

```
[Out] -1/7*c/(a^3*x^7) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)/
(3*a^5*x^3) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^3*
c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^5*(a + b*x^2)^2) + (b*(19*b^3*c - 15
*a*b^2*d + 11*a^2*b*e - 7*a^3*f)*x)/(8*a^6*(a + b*x^2)) + (Sqrt[b]*(99*b^3*
c - 63*a*b^2*d + 35*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(
13/2))
```

Maple [A]
time = 0.14, size = 212, normalized size = 0.91

method	result
default	$-\frac{b\left(\frac{\left(\frac{7}{8}a^3bf-\frac{11}{8}a^2e\,b^2+\frac{15}{8}ad\,b^3-\frac{19}{8}c\,b^4\right)x^3+\frac{a\left(9a^3f-13a^2be+17a\,b^2d-21b^3c\right)x}{8}}{\left(b\,x^2+a\right)^2}+\frac{\left(15a^3f-35a^2be+63a\,b^2d-99b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^6}-$
risch	$\frac{-\frac{b^2\left(15a^3f-35a^2be+63a\,b^2d-99b^3c\right)x^{10}}{8a^6}-\frac{5b\left(15a^3f-35a^2be+63a\,b^2d-99b^3c\right)x^8}{24a^5}-\frac{\left(15a^3f-35a^2be+63a\,b^2d-99b^3c\right)x^6}{15a^4}-\frac{\left(35a^2e-63abd+99b^2c\right)}{105a^3}}{x^7\left(b\,x^2+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -b/a^6*(((7/8*a^3*b*f-11/8*a^2*e*b^2+15/8*a*d*b^3-19/8*c*b^4)*x^3+1/8*a*(9*
a^3*f-13*a^2*b*e+17*a*b^2*d-21*b^3*c)*x)/(b*x^2+a)^2+1/8*(15*a^3*f-35*a^2*b
*e+63*a*b^2*d-99*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/7*c/a^3/x^7-
1/5*(a*d-3*b*c)/a^4/x^5-1/3*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x^3-(a^3*f-3*a^2*b*
e+6*a*b^2*d-10*b^3*c)/a^6/x
```

Maxima [A]
time = 0.53, size = 252, normalized size = 1.08

$$\frac{105\,(99\,b^5c-63\,ab^4d-15\,a^3b^2f+35\,a^2b^3e)x^{10}+175\,(99\,ab^4c-63\,a^2b^3d-15\,a^4bf+35\,a^3b^2e)x^8+56\,(99\,a^2b^3c-63\,a^3b^2d-15\,a^5f+35\,a^4be)x^6-120\,a^5c-8\,(99\,a^3b^2c-63\,a^4bd+35\,a^5e)x^4+24\,(11\,a^4bc-7\,a^5d)x^2}{840\,(a^6b^2x^{11}+2\,a^7bx^9+a^8x^7)}+\frac{(99\,b^4c-63\,ab^3d-15\,a^3bf+35\,a^2b^2e)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/840*(105*(99*b^5*c - 63*a*b^4*d - 15*a^3*b^2*f + 35*a^2*b^3*e)*x^10 + 175
*(99*a*b^4*c - 63*a^2*b^3*d - 15*a^4*b*f + 35*a^3*b^2*e)*x^8 + 56*(99*a^2*b
^3*c - 63*a^3*b^2*d - 15*a^5*f + 35*a^4*b*e)*x^6 - 120*a^5*c - 8*(99*a^3*b^
2*c - 63*a^4*b*d + 35*a^5*e)*x^4 + 24*(11*a^4*b*c - 7*a^5*d)*x^2)/(a^6*b^2*
x^11 + 2*a^7*b*x^9 + a^8*x^7) + 1/8*(99*b^4*c - 63*a*b^3*d - 15*a^3*b*f + 3
5*a^2*b^2*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6)
```

Fricas [A]
time = 4.15, size = 726, normalized size = 3.10

$$\left[\frac{630\,(33\,b^5c-21\,ab^4d-5\,a^3b^2f)x^{10}+1050\,(33\,ab^4c-21\,a^2b^3d-5\,a^4bf)x^8+336\,(33\,a^2b^3c-21\,a^3b^2d-5\,a^5f)x^6-240\,a^5c-144\,(11\,a^3b^2c-7\,a^4bd)x^4+48\,(11\,a^4bc-7\,a^5d)x^2+105\,(3\,(33\,b^5c-21\,ab^4d-5\,a^3b^2f)x^{11}+6\,(33\,ab^4c-21\,a^2b^3d-5\,a^4bf)x^9+3\,(33\,a^2b^3c-21\,a^3b^2d-5\,a^5f)x^7+35\,(a^2b^3x^{11}+2\,a^3b^2x^9+a^4bx^7)e)\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+70\,(105\,a^2b^3x^{10}+175\,a^3b^2x^8+56\,a^4bx^6-8\,a^5x^4)e}{1680\,(a^6b^2x^{11}+2\,a^7bx^9+a^8x^7)},\ \frac{315\,(33\,b^5c-21\,ab^4d-5\,a^3b^2f)x^{10}+525\,(33\,ab^4c-21\,a^2b^3d-5\,a^4bf)x^8+168\,(33\,a^2b^3c-21\,a^3b^2d-5\,a^5f)x^6-120\,a^5c-72\,(11\,a^3b^2c-7\,a^4bd)x^4+24\,(11\,a^4bc-7\,a^5d)x^2+105\,(3\,(33\,b^5c-21\,ab^4d-5\,a^3b^2f)x^{11}+6\,(33\,ab^4c-21\,a^2b^3d-5\,a^4bf)x^9+3\,(33\,a^2b^3c-21\,a^3b^2d-5\,a^5f)x^7+35\,(a^2b^3x^{11}+2\,a^3b^2x^9+a^4bx^7)e)\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+35\,(105\,a^2b^3x^{10}+175\,a^3b^2x^8+56\,a^4bx^6-8\,a^5x^4)e}{840\,(a^6b^2x^{11}+2\,a^7bx^9+a^8x^7)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/1680*(630*(33*b^5*c - 21*a*b^4*d - 5*a^3*b^2*f)*x^10 + 1050*(33*a*b^4*c
- 21*a^2*b^3*d - 5*a^4*b*f)*x^8 + 336*(33*a^2*b^3*c - 21*a^3*b^2*d - 5*a^5*
```

```
f)*x^6 - 240*a^5*c - 144*(11*a^3*b^2*c - 7*a^4*b*d)*x^4 + 48*(11*a^4*b*c -
7*a^5*d)*x^2 + 105*(3*(33*b^5*c - 21*a*b^4*d - 5*a^3*b^2*f)*x^11 + 6*(33*a*
b^4*c - 21*a^2*b^3*d - 5*a^4*b*f)*x^9 + 3*(33*a^2*b^3*c - 21*a^3*b^2*d - 5*
a^5*f)*x^7 + 35*(a^2*b^3*x^11 + 2*a^3*b^2*x^9 + a^4*b*x^7)*e)*sqrt(-b/a)*lo
g((b*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 70*(105*a^2*b^3*x^10 + 175*
a^3*b^2*x^8 + 56*a^4*b*x^6 - 8*a^5*x^4)*e)/(a^6*b^2*x^11 + 2*a^7*b*x^9 + a^
8*x^7), 1/840*(315*(33*b^5*c - 21*a*b^4*d - 5*a^3*b^2*f)*x^10 + 525*(33*a*b
^4*c - 21*a^2*b^3*d - 5*a^4*b*f)*x^8 + 168*(33*a^2*b^3*c - 21*a^3*b^2*d - 5
*a^5*f)*x^6 - 120*a^5*c - 72*(11*a^3*b^2*c - 7*a^4*b*d)*x^4 + 24*(11*a^4*b*
c - 7*a^5*d)*x^2 + 105*(3*(33*b^5*c - 21*a*b^4*d - 5*a^3*b^2*f)*x^11 + 6*(3
3*a*b^4*c - 21*a^2*b^3*d - 5*a^4*b*f)*x^9 + 3*(33*a^2*b^3*c - 21*a^3*b^2*d
- 5*a^5*f)*x^7 + 35*(a^2*b^3*x^11 + 2*a^3*b^2*x^9 + a^4*b*x^7)*e)*sqrt(b/a)
*arctan(x*sqrt(b/a)) + 35*(105*a^2*b^3*x^10 + 175*a^3*b^2*x^8 + 56*a^4*b*x^
6 - 8*a^5*x^4)*e)/(a^6*b^2*x^11 + 2*a^7*b*x^9 + a^8*x^7)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.80, size = 250, normalized size = 1.07

$$\frac{\left(99\,b^4c-63\,ab^3d-15\,a^3bf+35\,a^2b^2e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^6}+\frac{19\,b^5cx^3-15\,ab^4dx^3-7\,a^3b^2fx^3+11\,a^2b^3x^3e+21\,ab^4cx-17\,a^2b^3dx-9\,a^4bfx+13\,a^3b^2xe}{8\,\left(bx^2+a\right)^2a^6}+\frac{1050\,b^3cx^6-630\,ab^2dx^6-105\,a^3fx^6+315\,a^2bx^6e-210\,ab^2cx^4+105\,a^2bdx^4-35\,a^3x^4e+63\,a^2bcx^2-21\,a^3dx^2-15\,a^3c}{105\,a^6x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*(99*b^4*c - 63*a*b^3*d - 15*a^3*b*f + 35*a^2*b^2*e)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*a^6) + 1/8*(19*b^5*c*x^3 - 15*a*b^4*d*x^3 - 7*a^3*b^2*f*x^3 +
 11*a^2*b^3*x^3*e + 21*a*b^4*c*x - 17*a^2*b^3*d*x - 9*a^4*b*f*x + 13*a^3*b^
2*x*e)/((b*x^2 + a)^2*a^6) + 1/105*(1050*b^3*c*x^6 - 630*a*b^2*d*x^6 - 105*
a^3*f*x^6 + 315*a^2*b*x^6*e - 210*a*b^2*c*x^4 + 105*a^2*b*d*x^4 - 35*a^3*x^
4*e + 63*a^2*b*c*x^2 - 21*a^3*d*x^2 - 15*a^3*c)/(a^6*x^7)
```

Mupad [**B**]
time = 1.05, size = 230, normalized size = 0.98

$$\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-15\,f\,a^3+35\,e\,a^2\,b-63\,d\,a\,b^2+99\,c\,b^3\right)}{8\,a^{13/2}}-\frac{\frac{c}{7\,a}-\frac{x^6\,\left(-15\,f\,a^3+35\,e\,a^2\,b-63\,d\,a\,b^2+99\,c\,b^3\right)}{15\,a^4}+\frac{x^2\,\left(7\,a\,d-11\,b\,c\right)}{35\,a^2}+\frac{x^4\,\left(35\,e\,a^2-63\,d\,a\,b+99\,c\,b^2\right)}{105\,a^3}-\frac{5\,b\,x^8\,\left(-15\,f\,a^3+35\,e\,a^2\,b-63\,d\,a\,b^2+99\,c\,b^3\right)}{24\,a^5}-\frac{b^2\,x^{10}\,\left(-15\,f\,a^3+35\,e\,a^2\,b-63\,d\,a\,b^2+99\,c\,b^3\right)}{8\,a^6}}{a^2\,x^7+2\,a\,b\,x^9+b^2\,x^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x)
```

```
[Out] (b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(99*b^3*c - 15*a^3*f - 63*a*b^2*d + 35*a
^2*b*e))/(8*a^(13/2)) - (c/(7*a) - (x^6*(99*b^3*c - 15*a^3*f - 63*a*b^2*d +
 35*a^2*b*e))/(15*a^4) + (x^2*(7*a*d - 11*b*c))/(35*a^2) + (x^4*(99*b^2*c +
 35*a^2*e - 63*a*b*d))/(105*a^3) - (5*b*x^8*(99*b^3*c - 15*a^3*f - 63*a*b^2
*d + 35*a^2*b*e))/(24*a^5) - (b^2*x^10*(99*b^3*c - 15*a^3*f - 63*a*b^2*d +
35*a^2*b*e))/(8*a^6))/(a^2*x^7 + b^2*x^11 + 2*a*b*x^9)
```

3.142 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx$

Optimal. Leaf size=277

$$-\frac{c}{9a^3x^9}+\frac{3bc-ad}{7a^4x^7}-\frac{6b^2c-3abd+a^2e}{5a^5x^5}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3}-\frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)}{a^7x}-$$

```
[Out] -1/9*c/a^3/x^9+1/7*(-a*d+3*b*c)/a^4/x^7+1/5*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x^
5+1/3*(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x^3-b*(-3*a^3*f+6*a^2*b*e-1
0*a*b^2*d+15*b^3*c)/a^7/x-1/4*b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^6/(b*x
^2+a)^2-1/8*b^2*(-11*a^3*f+15*a^2*b*e-19*a*b^2*d+23*b^3*c)*x/a^7/(b*x^2+a)-
1/8*b^(3/2)*(-35*a^3*f+63*a^2*b*e-99*a*b^2*d+143*b^3*c)*arctan(x*b^(1/2)/a^
(1/2))/a^(15/2)
```

Rubi [A]
time = 0.41, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1819, 1816, 211}

$$\frac{3bc-ad}{7a^4x^7}-\frac{c}{9a^3x^9}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}-\frac{b^{3/2}\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-35a^3f+63a^2be-99ab^2d+143b^3c\right)}{8a^{15/2}}-\frac{b^2x(-11a^3f+15a^2be-19ab^2d+23b^3c)}{8a^7\left(a+bx^2\right)}-\frac{b(-3a^3f+6a^2be-10ab^2d+15b^3c)}{a^7x}+\frac{a^3(-f)+3a^2be-6ab^2d+10b^3c}{3a^6x^3}-\frac{b^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{4a^6\left(a+bx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x]
```

```
[Out] -1/9*c/(a^3*x^9) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e)/
(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) - (b*(
15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f))/(a^7*x) - (b^2*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f)*x)/(4*a^6*(a + b*x^2)^2) - (b^2*(23*b^3*c - 19*a*b^2*
d + 15*a^2*b*e - 11*a^3*f)*x)/(8*a^7*(a + b*x^2)) - (b^(3/2)*(143*b^3*c - 9
9*a*b^2*d + 63*a^2*b*e - 35*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(15/2)
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx = -\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^6\left(a+bx^2\right)^2} - \frac{\int \frac{-4c+4\left(\frac{bc}{a}-d\right)x^2-\frac{4\left(b^2c-abd+a^2e\right)x^4}{a^2}+\frac{4\left(b^3c-ab^2d+a^2be}{a^3}}{x}}{}$$

$$= -\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^6\left(a+bx^2\right)^2} - \frac{b^2(23b^3c-19ab^2d+15a^2be-11a^3f)\,x}{8a^7\left(a+bx^2\right)} + \frac{\int}{}$$

$$= -\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^6\left(a+bx^2\right)^2} - \frac{b^2(23b^3c-19ab^2d+15a^2be-11a^3f)\,x}{8a^7\left(a+bx^2\right)} + \frac{\int}{}$$

$$= -\frac{c}{9a^3x^9} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3} - \frac{b(}{}$$

$$= -\frac{c}{9a^3x^9} + \frac{3bc-ad}{7a^4x^7} - \frac{6b^2c-3abd+a^2e}{5a^5x^5} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3} - \frac{b(}{}$$

Mathematica [A]
time = 0.10, size = 276, normalized size = 1.00

$$-\frac{c}{9a^3x^9}+\frac{3bc-ad}{7a^4x^7}-\frac{6b^2c-3abd+a^2e}{5a^5x^5}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3}+\frac{b(-15b^3c+10ab^2d-6a^2be+3a^3f)}{a^7x}+\frac{b^2(-b^3c+ab^2d-a^2be+a^3f)\,x}{4a^6\left(a+bx^2\right)^2}+\frac{b^2(-23b^3c+19ab^2d-15a^2be+11a^3f)\,x}{8a^7\left(a+bx^2\right)}+\frac{b^{3/2}(-143b^3c+99ab^2d-63a^2be+35a^3f)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x]
```

```
[Out] -1/9*c/(a^3*x^9) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e)/
(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) + (b*(
-15*b^3*c + 10*a*b^2*d - 6*a^2*b*e + 3*a^3*f))/(a^7*x) + (b^2*(-(b^3*c) + a
*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^6*(a + b*x^2)^2) + (b^2*(-23*b^3*c + 19*a
*b^2*d - 15*a^2*b*e + 11*a^3*f)*x)/(8*a^7*(a + b*x^2)) + (b^(3/2)*(-143*b^3
*c + 99*a*b^2*d - 63*a^2*b*e + 35*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^
(15/2))
```

Maple [A]
time = 0.17, size = 248, normalized size = 0.90

method	result
default	$\frac{b^2\left(\frac{\left(\frac{11}{8}a^3bf-\frac{15}{8}a^2e\,b^2+\frac{19}{8}ad\,b^3-\frac{23}{8}c\,b^4\right)x^3+\frac{a\left(13a^3f-17a^2be+21a\,b^2d-25b^3c\right)x}{8}}{\left(b\,x^2+a\right)^2}+\frac{\left(35a^3f-63a^2be+99a\,b^2d-143b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{a^7}$
risch	$\frac{\frac{b^3\left(35a^3f-63a^2be+99a\,b^2d-143b^3c\right)x^{12}}{8a^7}+\frac{5b^2\left(35a^3f-63a^2be+99a\,b^2d-143b^3c\right)x^{10}}{24a^6}+\frac{b\left(35a^3f-63a^2be+99a\,b^2d-143b^3c\right)x^8}{15a^5}-\left(35a^3f-63a^2be\right.}{x^9\left(b\,x^2+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b^2/a^7*((((11/8*a^3*b*f-15/8*a^2*e*b^2+19/8*a*d*b^3-23/8*c*b^4)*x^3+1/8*a*(
13*a^3*f-17*a^2*b*e+21*a*b^2*d-25*b^3*c)*x)/(b*x^2+a)^2+1/8*(35*a^3*f-63*a^
2*b*e+99*a*b^2*d-143*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))-1/9*c/a^3/
x^9-1/7*(a*d-3*b*c)/a^4/x^7-1/5*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x^5-1/3*(a^3*f-
3*a^2*b*e+6*a*b^2*d-10*b^3*c)/a^6/x^3+b*(3*a^3*f-6*a^2*b*e+10*a*b^2*d-15*b^
3*c)/a^7/x
```

Maxima [A]
time = 0.50, size = 297, normalized size = 1.07

$$-\frac{315\,(143\,b^6c-99\,ab^5d-35\,a^3b^3f+63\,a^2b^4e)x^{12}+525\,(143\,ab^5c-99\,a^2b^4d-35\,a^4b^2f+63\,a^3b^3e)x^{10}+168\,(143\,a^2b^4c-99\,a^3b^3d-35\,a^5bf+63\,a^4b^2e)x^8+280\,a^6c-24\,(143\,a^3b^3c-99\,a^4b^2d-35\,a^6f+63\,a^5be)x^6+8\,(143\,a^4b^2c-99\,a^5bd+63\,a^6e)x^4-40\,(13\,a^5bc-9\,a^6d)x^2}{2520\,(a^7b^2x^{13}+2\,a^8bx^{11}+a^9x^9)}-\frac{(143\,b^5c-99\,ab^4d-35\,a^3b^2f+63\,a^2b^3e)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2520*(315*(143*b^6*c - 99*a*b^5*d - 35*a^3*b^3*f + 63*a^2*b^4*e)*x^12 +
525*(143*a*b^5*c - 99*a^2*b^4*d - 35*a^4*b^2*f + 63*a^3*b^3*e)*x^10 + 168*(
143*a^2*b^4*c - 99*a^3*b^3*d - 35*a^5*b*f + 63*a^4*b^2*e)*x^8 + 280*a^6*c -
 24*(143*a^3*b^3*c - 99*a^4*b^2*d - 35*a^6*f + 63*a^5*b*e)*x^6 + 8*(143*a^4
*b^2*c - 99*a^5*b*d + 63*a^6*e)*x^4 - 40*(13*a^5*b*c - 9*a^6*d)*x^2)/(a^7*b
^2*x^13 + 2*a^8*b*x^11 + a^9*x^9) - 1/8*(143*b^5*c - 99*a*b^4*d - 35*a^3*b^
2*f + 63*a^2*b^3*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7)
```

Fricas [A]
time = 6.08, size = 820, normalized size = 2.96

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/5040*(630*(143*b^6*c - 99*a*b^5*d - 35*a^3*b^3*f)*x^12 + 1050*(143*a*b^
5*c - 99*a^2*b^4*d - 35*a^4*b^2*f)*x^10 + 336*(143*a^2*b^4*c - 99*a^3*b^3*d
 - 35*a^5*b*f)*x^8 + 560*a^6*c - 48*(143*a^3*b^3*c - 99*a^4*b^2*d - 35*a^6*
f)*x^6 + 176*(13*a^4*b^2*c - 9*a^5*b*d)*x^4 - 80*(13*a^5*b*c - 9*a^6*d)*x^2
 - 315*((143*b^6*c - 99*a*b^5*d - 35*a^3*b^3*f)*x^13 + 2*(143*a*b^5*c - 99*
a^2*b^4*d - 35*a^4*b^2*f)*x^11 + (143*a^2*b^4*c - 99*a^3*b^3*d - 35*a^5*b*f
)*x^9 + 63*(a^2*b^4*x^13 + 2*a^3*b^3*x^11 + a^4*b^2*x^9)*e)*sqrt(-b/a)*log(
(b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 126*(315*a^2*b^4*x^12 + 525*a
^3*b^3*x^10 + 168*a^4*b^2*x^8 - 24*a^5*b*x^6 + 8*a^6*x^4)*e)/(a^7*b^2*x^13
+ 2*a^8*b*x^11 + a^9*x^9), -1/2520*(315*(143*b^6*c - 99*a*b^5*d - 35*a^3*b^
3*f)*x^12 + 525*(143*a*b^5*c - 99*a^2*b^4*d - 35*a^4*b^2*f)*x^10 + 168*(143
*a^2*b^4*c - 99*a^3*b^3*d - 35*a^5*b*f)*x^8 + 280*a^6*c - 24*(143*a^3*b^3*c
 - 99*a^4*b^2*d - 35*a^6*f)*x^6 + 88*(13*a^4*b^2*c - 9*a^5*b*d)*x^4 - 40*(1
3*a^5*b*c - 9*a^6*d)*x^2 + 315*((143*b^6*c - 99*a*b^5*d - 35*a^3*b^3*f)*x^1
3 + 2*(143*a*b^5*c - 99*a^2*b^4*d - 35*a^4*b^2*f)*x^11 + (143*a^2*b^4*c - 9
9*a^3*b^3*d - 35*a^5*b*f)*x^9 + 63*(a^2*b^4*x^13 + 2*a^3*b^3*x^11 + a^4*b^2
*x^9)*e)*sqrt(b/a)*arctan(x*sqrt(b/a)) + 63*(315*a^2*b^4*x^12 + 525*a^3*b^3
*x^10 + 168*a^4*b^2*x^8 - 24*a^5*b*x^6 + 8*a^6*x^4)*e)/(a^7*b^2*x^13 + 2*a^
8*b*x^11 + a^9*x^9)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 2.04, size = 301, normalized size = 1.09

$$-\frac{\left(143\,b^5c-99\,ab^4d-35\,a^3b^2f+63\,a^2b^3e\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^7}-\frac{23\,b^6cx^3-19\,ab^5dx^3-11\,a^3b^3fx^3+15\,a^2b^4x^3e+25\,ab^5cx-21\,a^2b^4dx-13\,a^4b^2fx+17\,a^3b^3xe}{8\,\left(bx^2+a\right)^2a^7}-\frac{4725\,b^4cx^8-3150\,ab^3dx^8-945\,a^3bfx^8+1890\,a^2b^2x^8e-1050\,ab^3cx^6+630\,a^2b^2dx^6+105\,a^4fx^6-315\,a^3bx^6e+378\,a^2b^2cx^4-189\,a^3bdx^4+63\,a^4x^4e-135\,a^3bcx^2+45\,a^4dx^2+35\,a^4c}{315\,a^7x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(143*b^5*c - 99*a*b^4*d - 35*a^3*b^2*f + 63*a^2*b^3*e)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*a^7) - 1/8*(23*b^6*c*x^3 - 19*a*b^5*d*x^3 - 11*a^3*b^3*f*
x^3 + 15*a^2*b^4*x^3*e + 25*a*b^5*c*x - 21*a^2*b^4*d*x - 13*a^4*b^2*f*x + 1
7*a^3*b^3*x*e)/((b*x^2 + a)^2*a^7) - 1/315*(4725*b^4*c*x^8 - 3150*a*b^3*d*x
^8 - 945*a^3*b*f*x^8 + 1890*a^2*b^2*x^8*e - 1050*a*b^3*c*x^6 + 630*a^2*b^2*
d*x^6 + 105*a^4*f*x^6 - 315*a^3*b*x^6*e + 378*a^2*b^2*c*x^4 - 189*a^3*b*d*x
^4 + 63*a^4*x^4*e - 135*a^3*b*c*x^2 + 45*a^4*d*x^2 + 35*a^4*c)/(a^7*x^9)
```

Mupad [B]

time = 1.07, size = 268, normalized size = 0.97

$$-\frac{\frac{c}{9\,a}-\frac{x^6\,\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\,a\,b^2+143\,c\,b^3\right)}{105\,a^4}+\frac{x^2\,\left(9\,a\,d-13\,b\,c\right)}{63\,a^2}+\frac{x^4\,\left(63\,e\,a^2-99\,d\,a\,b+143\,c\,b^2\right)}{315\,a^3}+\frac{b\,x^8\,\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\,a\,b^2+143\,c\,b^3\right)}{15\,a^5}+\frac{5\,b^2\,x^{10}\,\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\,a\,b^2+143\,c\,b^3\right)}{24\,a^6}+\frac{b^3\,x^{12}\,\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\,a\,b^2+143\,c\,b^3\right)}{8\,a^7}}{a^2\,x^9+2\,a\,b\,x^{11}+b^2\,x^{13}}-\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\,a\,b^2+143\,c\,b^3\right)}{8\,a^{15/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x)
```

```
[Out] - (c/(9*a) - (x^6*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(105*a^
4) + (x^2*(9*a*d - 13*b*c))/(63*a^2) + (x^4*(143*b^2*c + 63*a^2*e - 99*a*b*
d))/(315*a^3) + (b*x^8*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(1
5*a^5) + (5*b^2*x^10*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(24*
a^6) + (b^3*x^12*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(8*a^7))
/(a^2*x^9 + b^2*x^13 + 2*a*b*x^11) - (b^(3/2)*atan((b^(1/2)*x)/a^(1/2))*(14
3*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(8*a^(15/2))
```

3.143 $\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=214

$$\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^6}-\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\left(a+bx^2\right)^{3/2}}{3b^6}+\frac{(b^3c-3ab^2d+6a^2be}{5}$$

```
[Out] -1/3*a*(-5*a^3*f+4*a^2*b*e-3*a*b^2*d+2*b^3*c)*(b*x^2+a)^(3/2)/b^6+1/5*(-10*
a^3*f+6*a^2*b*e-3*a*b^2*d+b^3*c)*(b*x^2+a)^(5/2)/b^6+1/7*(10*a^2*f-4*a*b*e+
b^2*d)*(b*x^2+a)^(7/2)/b^6+1/9*(-5*a*f+b*e)*(b*x^2+a)^(9/2)/b^6+1/11*f*(b*x
^2+a)^(11/2)/b^6+a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a)^(1/2)/b^6
```

Rubi [A]
time = 0.18, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1813, 1634}

$$\frac{\left(a+bx^2\right)^{7/2}\left(10a^2f-4abe+b^2d\right)}{7b^6}+\frac{\left(a+bx^2\right)^{5/2}\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{5b^6}-\frac{a\left(a+bx^2\right)^{3/2}\left(-5a^3f+4a^2be-3ab^2d+2b^3c\right)}{3b^6}+\frac{a^2\sqrt{a+bx^2}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^6}+\frac{\left(a+bx^2\right)^{9/2}(be-5af)}{9b^6}+\frac{f\left(a+bx^2\right)^{11/2}}{11b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (a*(2*b^3*c
 - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/(3*b^6) + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)^(5/2))/(5*b^6) + ((b^2*d - 4*
a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2))/(7*b^6) + ((b*e - 5*a*f)*(a + b*x^2)^(
9/2))/(9*b^6) + (f*(a + b*x^2)^(11/2))/(11*b^6)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(c+dx+ex^2+fx^3)}{\sqrt{a+bx}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)}{b^5\sqrt{a+bx}}+\frac{a(-2b^3c+3ab^2d-4a^2b}{b^5}\right.\right.\\ &= \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^6}-\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f}{3b^6}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 158, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\,(-1280a^5f+128a^4b(11e+5fx^2)-16a^3b^2(99d+44ex^2+30fx^4)+8a^2b^3(231c+99dx^2+66ex^4+50fx^6)-2ab^4x^2(462c+297dx^2+220ex^4+175fx^6)+b^5x^4(693c+5(99dx^2+77ex^4+63fx^6)))}{3465b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-1280*a^5*f + 128*a^4*b*(11*e + 5*f*x^2) - 16*a^3*b^2*(99
*d + 44*e*x^2 + 30*f*x^4) + 8*a^2*b^3*(231*c + 99*d*x^2 + 66*e*x^4 + 50*f*x
^6) - 2*a*b^4*x^2*(462*c + 297*d*x^2 + 220*e*x^4 + 175*f*x^6) + b^5*x^4*(69
3*c + 5*(99*d*x^2 + 77*e*x^4 + 63*f*x^6))))/(3465*b^6)
```

Maple [A]
time = 0.12, size = 382, normalized size = 1.79

method	result
gosper	$-\frac{\sqrt{b\,x^2+a}\,(-315f\,x^{10}b^5+350a\,b^4f\,x^8-385b^5e\,x^8-400a^2b^3f\,x^6+440a\,b^4e\,x^6-495b^5d\,x^6+480a^3b^2f\,x^4-528a^2b^3e\,x^4+594a\,b}{3465b^6}$
trager	$-\frac{\sqrt{b\,x^2+a}\,(-315f\,x^{10}b^5+350a\,b^4f\,x^8-385b^5e\,x^8-400a^2b^3f\,x^6+440a\,b^4e\,x^6-495b^5d\,x^6+480a^3b^2f\,x^4-528a^2b^3e\,x^4+594a\,b}{3465b^6}$
risch	$-\frac{\sqrt{b\,x^2+a}\,(-315f\,x^{10}b^5+350a\,b^4f\,x^8-385b^5e\,x^8-400a^2b^3f\,x^6+440a\,b^4e\,x^6-495b^5d\,x^6+480a^3b^2f\,x^4-528a^2b^3e\,x^4+594a\,b}{3465b^6}$

default	$f\left(\frac{x^{10}\sqrt{b\,x^{2}+a}}{11b}-\frac{10a\left(\frac{x^{8}\sqrt{b\,x^{2}+a}}{9b}-\frac{8a\left(\frac{x^{6}\sqrt{b\,x^{2}+a}}{7b}-\frac{6a\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5b}-\frac{4a\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}}}{3b^{2}}\right.}{5b}\right.}{7b}\right.}{9b}\right.}{11b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/11*x^10/b*(b*x^2+a)^(1/2)-10/11*a/b*(1/9*x^8/b*(b*x^2+a)^(1/2)-8/9*a/b
*(1/7*x^6/b*(b*x^2+a)^(1/2)-6/7*a/b*(1/5*x^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/3
*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))))))+e*(1/9*x^8/b*(b*x^2+a
)^(1/2)-8/9*a/b*(1/7*x^6/b*(b*x^2+a)^(1/2)-6/7*a/b*(1/5*x^4/b*(b*x^2+a)^(1/
2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2)))))+d*(1/7*
x^6/b*(b*x^2+a)^(1/2)-6/7*a/b*(1/5*x^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/3*x^2/b
*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))))+c*(1/5*x^4/b*(b*x^2+a)^(1/2)-
4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2)))
```

Maxima [A]

time = 0.29, size = 352, normalized size = 1.64

$$\frac{\sqrt{bx^2+a}\,fx^{10}}{11\,b}-\frac{10\,\sqrt{bx^2+a}\,afx^8}{99\,b^2}+\frac{\sqrt{bx^2+a}\,x^8e}{9\,b}+\frac{\sqrt{bx^2+a}\,dx^6}{7\,b}+\frac{80\,\sqrt{bx^2+a}\,a^2fx^6}{693\,b^3}-\frac{8\,\sqrt{bx^2+a}\,ax^6e}{63\,b^2}+\frac{\sqrt{bx^2+a}\,cx^4}{5\,b}-\frac{6\,\sqrt{bx^2+a}\,adx^4}{35\,b^2}-\frac{32\,\sqrt{bx^2+a}\,a^3fx^4}{231\,b^4}+\frac{16\,\sqrt{bx^2+a}\,a^2x^4e}{105\,b^3}-\frac{4\,\sqrt{bx^2+a}\,acx^2}{15\,b^2}+\frac{8\,\sqrt{bx^2+a}\,a^2dx^2}{35\,b^3}+\frac{128\,\sqrt{bx^2+a}\,a^4fx^2}{693\,b^5}-\frac{64\,\sqrt{bx^2+a}\,a^3x^2e}{315\,b^4}+\frac{8\,\sqrt{bx^2+a}\,a^2c}{15\,b^3}-\frac{16\,\sqrt{bx^2+a}\,a^3d}{35\,b^4}-\frac{256\,\sqrt{bx^2+a}\,a^5f}{693\,b^6}+\frac{128\,\sqrt{bx^2+a}\,a^4e}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/11*sqrt(b*x^2 + a)*f*x^10/b - 10/99*sqrt(b*x^2 + a)*a*f*x^8/b^2 + 1/9*sqr
t(b*x^2 + a)*x^8*e/b + 1/7*sqrt(b*x^2 + a)*d*x^6/b + 80/693*sqrt(b*x^2 + a)
*a^2*f*x^6/b^3 - 8/63*sqrt(b*x^2 + a)*a*x^6*e/b^2 + 1/5*sqrt(b*x^2 + a)*c*x
^4/b - 6/35*sqrt(b*x^2 + a)*a*d*x^4/b^2 - 32/231*sqrt(b*x^2 + a)*a^3*f*x^4/
b^4 + 16/105*sqrt(b*x^2 + a)*a^2*x^4*e/b^3 - 4/15*sqrt(b*x^2 + a)*a*c*x^2/b
^2 + 8/35*sqrt(b*x^2 + a)*a^2*d*x^2/b^3 + 128/693*sqrt(b*x^2 + a)*a^4*f*x^2
/b^5 - 64/315*sqrt(b*x^2 + a)*a^3*x^2*e/b^4 + 8/15*sqrt(b*x^2 + a)*a^2*c/b^
3 - 16/35*sqrt(b*x^2 + a)*a^3*d/b^4 - 256/693*sqrt(b*x^2 + a)*a^5*f/b^6 + 1
28/315*sqrt(b*x^2 + a)*a^4*e/b^5
```

Fricas [**A**]
time = 2.98, size = 186, normalized size = 0.87

$$\frac{(315\,b^5fx^{10}-350\,ab^4fx^8+5\,(99\,b^5d+80\,a^2b^3f)x^6+1848\,a^2b^3c-1584\,a^3b^2d-1280\,a^5f+3\,(231\,b^5c-198\,ab^4d-160\,a^3b^2f)x^4-4\,(231\,ab^4c-198\,a^2b^3d-160\,a^4bf)x^2+11\,(35\,b^5x^8-40\,ab^4x^6+48\,a^2b^3x^4-64\,a^3b^2x^2+128\,a^4b)e)\sqrt{bx^2+a}}{3465\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^5*f*x^10 - 350*a*b^4*f*x^8 + 5*(99*b^5*d + 80*a^2*b^3*f)*x^6
+ 1848*a^2*b^3*c - 1584*a^3*b^2*d - 1280*a^5*f + 3*(231*b^5*c - 198*a*b^4*d
 - 160*a^3*b^2*f)*x^4 - 4*(231*a*b^4*c - 198*a^2*b^3*d - 160*a^4*b*f)*x^2 +
 11*(35*b^5*x^8 - 40*a*b^4*x^6 + 48*a^2*b^3*x^4 - 64*a^3*b^2*x^2 + 128*a^4*
b)*e)*sqrt(b*x^2 + a)/b^6
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(214) = 428$.
time = 0.56, size = 442, normalized size = 2.07

$$\begin{cases} - \frac{256a^5f\sqrt{a+bx^2}}{693b^6} + \frac{128a^4e\sqrt{a+bx^2}}{315b^5} + \frac{128a^4fx^2\sqrt{a+bx^2}}{693b^5} - \frac{16a^3d\sqrt{a+bx^2}}{35b^4} - \frac{64a^3ex^2\sqrt{a+bx^2}}{315b^4} - \frac{32a^3fx^4\sqrt{a+bx^2}}{231b^4} + \frac{8a^2c\sqrt{a+bx^2}}{15b^3} + \frac{8a^2dx^2\sqrt{a+bx^2}}{35b^3} + \frac{16a^2ex^4\sqrt{a+bx^2}}{105b^3} + \frac{80a^2fx^6\sqrt{a+bx^2}}{693b^3} - \frac{4acx^2\sqrt{a+bx^2}}{15b^2} - \frac{6adx^4\sqrt{a+bx^2}}{35b^2} - \frac{8aex^6\sqrt{a+bx^2}}{63b^2} - \frac{10afx^8\sqrt{a+bx^2}}{99b^2} + \frac{cx^4\sqrt{a+bx^2}}{5b} + \frac{dx^6\sqrt{a+bx^2}}{7b} + \frac{ex^8\sqrt{a+bx^2}}{9b} + \frac{fx^{10}\sqrt{a+bx^2}}{11b} & \text{for}\: b \neq 0 \\ \frac{\frac{cx^6}{6} + \frac{dx^8}{8} + \frac{ex^{10}}{10} + \frac{fx^{12}}{12}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-256*a**5*f*sqrt(a + b*x**2)/(693*b**6) + 128*a**4*e*sqrt(a + b*
x**2)/(315*b**5) + 128*a**4*f*x**2*sqrt(a + b*x**2)/(693*b**5) - 16*a**3*d*
sqrt(a + b*x**2)/(35*b**4) - 64*a**3*e*x**2*sqrt(a + b*x**2)/(315*b**4) - 3
2*a**3*f*x**4*sqrt(a + b*x**2)/(231*b**4) + 8*a**2*c*sqrt(a + b*x**2)/(15*b
**3) + 8*a**2*d*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*e*x**4*sqrt(a + b
*x**2)/(105*b**3) + 80*a**2*f*x**6*sqrt(a + b*x**2)/(693*b**3) - 4*a*c*x**2
*sqrt(a + b*x**2)/(15*b**2) - 6*a*d*x**4*sqrt(a + b*x**2)/(35*b**2) - 8*a*e
*x**6*sqrt(a + b*x**2)/(63*b**2) - 10*a*f*x**8*sqrt(a + b*x**2)/(99*b**2) +
 c*x**4*sqrt(a + b*x**2)/(5*b) + d*x**6*sqrt(a + b*x**2)/(7*b) + e*x**8*sqr
```

```
t(a + b*x**2)/(9*b) + f*x**10*sqrt(a + b*x**2)/(11*b), Ne(b, 0)), ((c*x**6/
6 + d*x**8/8 + e*x**10/10 + f*x**12/12)/sqrt(a), True))
```

Giac [A]
time = 1.60, size = 264, normalized size = 1.23

$$\frac{\left(a^{2} b^{3} c - a^{3} b^{2} d - a^{5} f + a^{4} b e\right) \sqrt{b x^{2} + a}}{b^{6}} + \frac{693 \, {\left(b x^{2} + a\right)}^{\frac{5}{2}} b^{3} c - 2310 \, {\left(b x^{2} + a\right)}^{\frac{3}{2}} a b^{3} c + 495 \, {\left(b x^{2} + a\right)}^{\frac{7}{2}} b^{2} d - 2079 \, {\left(b x^{2} + a\right)}^{\frac{5}{2}} a b^{2} d + 3465 \, {\left(b x^{2} + a\right)}^{\frac{3}{2}} a^{2} b^{2} d + 315 \, {\left(b x^{2} + a\right)}^{\frac{11}{2}} f - 1925 \, {\left(b x^{2} + a\right)}^{\frac{9}{2}} a f + 4950 \, {\left(b x^{2} + a\right)}^{\frac{7}{2}} a^{2} f - 6930 \, {\left(b x^{2} + a\right)}^{\frac{5}{2}} a^{3} f + 5775 \, {\left(b x^{2} + a\right)}^{\frac{3}{2}} a^{4} f + 385 \, {\left(b x^{2} + a\right)}^{\frac{9}{2}} b e - 1980 \, {\left(b x^{2} + a\right)}^{\frac{7}{2}} a b e + 4158 \, {\left(b x^{2} + a\right)}^{\frac{5}{2}} a^{2} b e - 4620 \, {\left(b x^{2} + a\right)}^{\frac{3}{2}} a^{3} b e}{3465 \, b^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*sqrt(b*x^2 + a)/b^6 + 1/3465*(693
*(b*x^2 + a)^(5/2)*b^3*c - 2310*(b*x^2 + a)^(3/2)*a*b^3*c + 495*(b*x^2 + a)
^(7/2)*b^2*d - 2079*(b*x^2 + a)^(5/2)*a*b^2*d + 3465*(b*x^2 + a)^(3/2)*a^2*
b^2*d + 315*(b*x^2 + a)^(11/2)*f - 1925*(b*x^2 + a)^(9/2)*a*f + 4950*(b*x^2
 + a)^(7/2)*a^2*f - 6930*(b*x^2 + a)^(5/2)*a^3*f + 5775*(b*x^2 + a)^(3/2)*a
^4*f + 385*(b*x^2 + a)^(9/2)*b*e - 1980*(b*x^2 + a)^(7/2)*a*b*e + 4158*(b*x
^2 + a)^(5/2)*a^2*b*e - 4620*(b*x^2 + a)^(3/2)*a^3*b*e)/b^6
```

Mupad [B]
time = 1.19, size = 186, normalized size = 0.87

$$\sqrt{b\,x^2+a}\,\left(\frac{x^6\,\left(400\,f\,a^2\,b^3-440\,e\,a\,b^4+495\,d\,b^5\right)}{3465\,b^6}-\frac{1280\,f\,a^5-1408\,e\,a^4\,b+1584\,d\,a^3\,b^2-1848\,c\,a^2\,b^3}{3465\,b^6}+\frac{x^4\,\left(-480\,f\,a^3\,b^2+528\,e\,a^2\,b^3-594\,d\,a\,b^4+693\,c\,b^5\right)}{3465\,b^6}+\frac{f\,x^{10}}{11\,b}+\frac{x^8\,\left(385\,b^5\,e-350\,a\,b^4\,f\right)}{3465\,b^6}-\frac{4\,a\,x^2\,\left(-160\,f\,a^3+176\,e\,a^2\,b-198\,d\,a\,b^2+231\,c\,b^3\right)}{3465\,b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((x^6*(495*b^5*d + 400*a^2*b^3*f - 440*a*b^4*e))/(3465*b^
6) - (1280*a^5*f - 1848*a^2*b^3*c + 1584*a^3*b^2*d - 1408*a^4*b*e)/(3465*b^
6) + (x^4*(693*b^5*c + 528*a^2*b^3*e - 480*a^3*b^2*f - 594*a*b^4*d))/(3465*
b^6) + (f*x^10)/(11*b) + (x^8*(385*b^5*e - 350*a*b^4*f))/(3465*b^6) - (4*a*
x^2*(231*b^3*c - 160*a^3*f - 198*a*b^2*d + 176*a^2*b*e))/(3465*b^5))
```

3.144 $\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=167

$$-\frac{a(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^5}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,(a+bx^2)^{3/2}}{3b^5}+\frac{(b^2d-3abe+6a^2f)}{5b^5}$$

```
[Out] 1/3*(-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^3*c)*(b*x^2+a)^(3/2)/b^5+1/5*(6*a^2*f-3
*a*b*e+b^2*d)*(b*x^2+a)^(5/2)/b^5+1/7*(-4*a*f+b*e)*(b*x^2+a)^(7/2)/b^5+1/9*
f*(b*x^2+a)^(9/2)/b^5-a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a)^(1/2)/b^5
```

Rubi [A]

time = 0.13, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1813, 1634}

$$\frac{(a+bx^2)^{5/2}(6a^2f-3abe+b^2d)}{5b^5}+\frac{(a+bx^2)^{3/2}(-4a^3f+3a^2be-2ab^2d+b^3c)}{3b^5}-\frac{a\sqrt{a+bx^2}\,(a^3(-f)+a^2be-ab^2d+b^3c)}{b^5}+\frac{(a+bx^2)^{7/2}(be-4af)}{7b^5}+\frac{f(a+bx^2)^{9/2}}{9b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5) + ((b^3*c -
2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*b^5) + ((b^2*d - 3*a
*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) + ((b*e - 4*a*f)*(a + b*x^2)^(7/
2))/(7*b^5) + (f*(a + b*x^2)^(9/2))/(9*b^5)
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(c+dx+ex^2+fx^3)}{\sqrt{a+bx}}\,dx,x,x^2\right) \\
&= \frac{1}{2}\text{Subst}\Bigg(\int \Bigg(\frac{a(-b^3c+ab^2d-a^2be+a^3f)}{b^4\sqrt{a+bx}}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f}{b^4} \\
&= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^5}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,(}{3b^5}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 122, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\,(128a^4f-16a^3b(9e+4fx^2)+24a^2b^2(7d+3ex^2+2fx^4)-2ab^3(105c+42dx^2+27ex^4+20fx^6)+b^4x^2(105c+63dx^2+45ex^4+35fx^6))}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(128*a^4*f - 16*a^3*b*(9*e + 4*f*x^2) + 24*a^2*b^2*(7*d +
3*e*x^2 + 2*f*x^4) - 2*a*b^3*(105*c + 42*d*x^2 + 27*e*x^4 + 20*f*x^6) + b^4
*x^2*(105*c + 63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5)
```

Maple [A]
time = 0.11, size = 286, normalized size = 1.71

method	result
gosper	$\frac{\sqrt{b\,x^2+a}\,(35f\,x^8b^4-40a\,b^3f\,x^6+45b^4e\,x^6+48a^2b^2f\,x^4-54a\,b^3e\,x^4+63b^4d\,x^4-64a^3bf\,x^2+72a^2b^2e\,x^2-84a\,b^3d\,x^2+105b^4c\,x^2+1}{315b^5}$
trager	$\frac{\sqrt{b\,x^2+a}\,(35f\,x^8b^4-40a\,b^3f\,x^6+45b^4e\,x^6+48a^2b^2f\,x^4-54a\,b^3e\,x^4+63b^4d\,x^4-64a^3bf\,x^2+72a^2b^2e\,x^2-84a\,b^3d\,x^2+105b^4c\,x^2+1}{315b^5}$
risch	$\frac{\sqrt{b\,x^2+a}\,(35f\,x^8b^4-40a\,b^3f\,x^6+45b^4e\,x^6+48a^2b^2f\,x^4-54a\,b^3e\,x^4+63b^4d\,x^4-64a^3bf\,x^2+72a^2b^2e\,x^2-84a\,b^3d\,x^2+105b^4c\,x^2+1}{315b^5}$

default	$f\left(\frac{x^{8}\sqrt{b\,x^{2}+a}}{9b}-\frac{8a\left(\frac{x^{6}\sqrt{b\,x^{2}+a}}{7b}-\frac{6a\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5b}-\frac{4a\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)}{5b}\right)}{7b}\right)}{9b}\right)+e$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/9*x^8/b*(b*x^2+a)^(1/2)-8/9*a/b*(1/7*x^6/b*(b*x^2+a)^(1/2)-6/7*a/b*(1/
5*x^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2
+a)^(1/2)))))+e*(1/7*x^6/b*(b*x^2+a)^(1/2)-6/7*a/b*(1/5*x^4/b*(b*x^2+a)^(1/
2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))))+d*(1/5*x
^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)
^(1/2)))+c*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.27, size = 267, normalized size = 1.60

$$\frac{\sqrt{bx^2+a}\,fx^8}{9\,b}-\frac{8\,\sqrt{bx^2+a}\,afx^6}{63\,b^2}+\frac{\sqrt{bx^2+a}\,x^6e}{7\,b}+\frac{\sqrt{bx^2+a}\,dx^4}{5\,b}+\frac{16\,\sqrt{bx^2+a}\,a^2fx^4}{105\,b^3}-\frac{6\,\sqrt{bx^2+a}\,ax^4e}{35\,b^2}+\frac{\sqrt{bx^2+a}\,cx^2}{3\,b}-\frac{4\,\sqrt{bx^2+a}\,adx^2}{15\,b^2}-\frac{64\,\sqrt{bx^2+a}\,a^3fx^2}{315\,b^4}+\frac{8\,\sqrt{bx^2+a}\,a^2x^2e}{35\,b^3}-\frac{2\,\sqrt{bx^2+a}\,ac}{3\,b^2}+\frac{8\,\sqrt{bx^2+a}\,a^2d}{15\,b^3}+\frac{128\,\sqrt{bx^2+a}\,a^4f}{315\,b^5}-\frac{16\,\sqrt{bx^2+a}\,a^3e}{35\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(b*x^2 + a)*f*x^8/b - 8/63*sqrt(b*x^2 + a)*a*f*x^6/b^2 + 1/7*sqrt(b
*x^2 + a)*x^6*e/b + 1/5*sqrt(b*x^2 + a)*d*x^4/b + 16/105*sqrt(b*x^2 + a)*a^
2*f*x^4/b^3 - 6/35*sqrt(b*x^2 + a)*a*x^4*e/b^2 + 1/3*sqrt(b*x^2 + a)*c*x^2/
b - 4/15*sqrt(b*x^2 + a)*a*d*x^2/b^2 - 64/315*sqrt(b*x^2 + a)*a^3*f*x^2/b^4
 + 8/35*sqrt(b*x^2 + a)*a^2*x^2*e/b^3 - 2/3*sqrt(b*x^2 + a)*a*c/b^2 + 8/15*
sqrt(b*x^2 + a)*a^2*d/b^3 + 128/315*sqrt(b*x^2 + a)*a^4*f/b^5 - 16/35*sqrt(
b*x^2 + a)*a^3*e/b^4
```

Fricas [A]
time = 5.72, size = 141, normalized size = 0.84

$$\frac{(35\,b^4fx^8-40\,ab^3fx^6-210\,ab^3c+168\,a^2b^2d+128\,a^4f+3\,(21\,b^4d+16\,a^2b^2f)x^4+(105\,b^4c-84\,ab^3d-64\,a^3bf)x^2+9\,(5\,b^4x^6-6\,ab^3x^4+8\,a^2b^2x^2-16\,a^3b)e)\sqrt{bx^2+a}}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*f*x^8 - 40*a*b^3*f*x^6 - 210*a*b^3*c + 168*a^2*b^2*d + 128*a^
4*f + 3*(21*b^4*d + 16*a^2*b^2*f)*x^4 + (105*b^4*c - 84*a*b^3*d - 64*a^3*b*
f)*x^2 + 9*(5*b^4*x^6 - 6*a*b^3*x^4 + 8*a^2*b^2*x^2 - 16*a^3*b)*e)*sqrt(b*x
^2 + a)/b^5
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(163) = 326$.
time = 0.45, size = 340, normalized size = 2.04

$$\begin{cases} \frac{128a^4f\sqrt{a+bx^2}}{315b^5} - \frac{16a^3e\sqrt{a+bx^2}}{35b^4} - \frac{64a^3fx^2\sqrt{a+bx^2}}{315b^4} + \frac{8a^2d\sqrt{a+bx^2}}{15b^3} + \frac{8a^2ex^2\sqrt{a+bx^2}}{35b^3} + \frac{16a^2fx^4\sqrt{a+bx^2}}{105b^3} - \frac{2ac\sqrt{a+bx^2}}{3b^2} - \frac{4adx^2\sqrt{a+bx^2}}{15b^2} - \frac{6aex^4\sqrt{a+bx^2}}{35b^2} - \frac{8afx^6\sqrt{a+bx^2}}{63b^2} + \frac{cx^2\sqrt{a+bx^2}}{3b} + \frac{dx^4\sqrt{a+bx^2}}{5b} + \frac{ex^6\sqrt{a+bx^2}}{7b} + \frac{fx^8\sqrt{a+bx^2}}{9b} & \text{for}\ b \neq 0 \\ \frac{\frac{cx^4}{4} + \frac{dx^6}{6} + \frac{ex^8}{8} + \frac{fx^{10}}{10}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((128*a**4*f*sqrt(a + b*x**2)/(315*b**5) - 16*a**3*e*sqrt(a + b*x*
*2)/(35*b**4) - 64*a**3*f*x**2*sqrt(a + b*x**2)/(315*b**4) + 8*a**2*d*sqrt(
a + b*x**2)/(15*b**3) + 8*a**2*e*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*
f*x**4*sqrt(a + b*x**2)/(105*b**3) - 2*a*c*sqrt(a + b*x**2)/(3*b**2) - 4*a*
d*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*e*x**4*sqrt(a + b*x**2)/(35*b**2) -
 8*a*f*x**6*sqrt(a + b*x**2)/(63*b**2) + c*x**2*sqrt(a + b*x**2)/(3*b) + d*
x**4*sqrt(a + b*x**2)/(5*b) + e*x**6*sqrt(a + b*x**2)/(7*b) + f*x**8*sqrt(a
 + b*x**2)/(9*b), Ne(b, 0)), ((c*x**4/4 + d*x**6/6 + e*x**8/8 + f*x**10/10)
/sqrt(a), True))
```

Giac [**A**]
time = 1.76, size = 197, normalized size = 1.18

$$-\frac{\left(ab^3c - a^2b^2d - a^4f + a^3be\right)\sqrt{bx^2+a}}{b^5} + \frac{105\,(bx^2+a)^{\frac{3}{2}}b^3c + 63\,(bx^2+a)^{\frac{5}{2}}b^2d - 210\,(bx^2+a)^{\frac{3}{2}}ab^2d + 35\,(bx^2+a)^{\frac{9}{2}}f - 180\,(bx^2+a)^{\frac{7}{2}}af + 378\,(bx^2+a)^{\frac{5}{2}}a^2f - 420\,(bx^2+a)^{\frac{3}{2}}a^3f + 45\,(bx^2+a)^{\frac{7}{2}}be - 189\,(bx^2+a)^{\frac{5}{2}}abe + 315\,(bx^2+a)^{\frac{3}{2}}a^2be}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*sqrt(b*x^2 + a)/b^5 + 1/315*(105*(
b*x^2 + a)^(3/2)*b^3*c + 63*(b*x^2 + a)^(5/2)*b^2*d - 210*(b*x^2 + a)^(3/2)
*a*b^2*d + 35*(b*x^2 + a)^(9/2)*f - 180*(b*x^2 + a)^(7/2)*a*f + 378*(b*x^2
+ a)^(5/2)*a^2*f - 420*(b*x^2 + a)^(3/2)*a^3*f + 45*(b*x^2 + a)^(7/2)*b*e -
 189*(b*x^2 + a)^(5/2)*a*b*e + 315*(b*x^2 + a)^(3/2)*a^2*b*e)/b^5
```

Mupad [**B**]
time = 1.11, size = 146, normalized size = 0.87

$$\sqrt{b\,x^2+a}\,\left(\frac{128\,f\,a^4-144\,e\,a^3\,b+168\,d\,a^2\,b^2-210\,c\,a\,b^3}{315\,b^5}+\frac{x^4\,\left(48\,f\,a^2\,b^2-54\,e\,a\,b^3+63\,d\,b^4\right)}{315\,b^5}+\frac{f\,x^8}{9\,b}+\frac{x^6\,\left(45\,b^4\,e-40\,a\,b^3\,f\right)}{315\,b^5}+\frac{x^2\,\left(-64\,f\,a^3\,b+72\,e\,a^2\,b^2-84\,d\,a\,b^3+105\,c\,b^4\right)}{315\,b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((128*a^4*f + 168*a^2*b^2*d - 210*a*b^3*c - 144*a^3*b*e)/
(315*b^5) + (x^4*(63*b^4*d + 48*a^2*b^2*f - 54*a*b^3*e))/(315*b^5) + (f*x^8
)/(9*b) + (x^6*(45*b^4*e - 40*a*b^3*f))/(315*b^5) + (x^2*(105*b^4*c + 72*a^
2*b^2*e - 84*a*b^3*d - 64*a^3*b*f))/(315*b^5))
```

3.145 $\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=121

$$\frac{(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^4}+\frac{(b^2d-2abe+3a^2f)(a+bx^2)^{3/2}}{3b^4}+\frac{(be-3af)(a+bx^2)^{5/2}}{5b^4}+\frac{f(a+bx}{7b^4}$$

```
[Out] 1/3*(3*a^2*f-2*a*b*e+b^2*d)*(b*x^2+a)^(3/2)/b^4+1/5*(-3*a*f+b*e)*(b*x^2+a)^
(5/2)/b^4+1/7*f*(b*x^2+a)^(7/2)/b^4+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a
)^(1/2)/b^4
```

Rubi [A]

time = 0.09, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1813, 1864}

$$\frac{(a+bx^2)^{3/2}(3a^2f-2abe+b^2d)}{3b^4}+\frac{\sqrt{a+bx^2}\,(a^3(-f)+a^2be-ab^2d+b^3c)}{b^4}+\frac{(a+bx^2)^{5/2}(be-3af)}{5b^4}+\frac{f(a+bx^2)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + ((b^2*d - 2*a*b
*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + ((b*e - 3*a*f)*(a + b*x^2)^(5/2)
)/(5*b^4) + (f*(a + b*x^2)^(7/2))/(7*b^4)
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\sqrt{a+bx}} + \frac{(b^2d-2abe+3a^2f)\sqrt{a+bx}}{b^3} +$$

$$= \frac{(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^4} + \frac{(b^2d-2abe+3a^2f)(a+bx^2)^{3/2}}{3b^4} -$$

Mathematica [A]
time = 0.07, size = 89, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\,(-48a^3f+8a^2b(7e+3fx^2)-2ab^2(35d+14ex^2+9fx^4)+b^3(105c+35dx^2+21ex^4+15fx^6))}{105b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x^2) - 2*a*b^2*(35*d + 14*
e*x^2 + 9*f*x^4) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^4)
```

Maple [A]
time = 0.11, size = 193, normalized size = 1.60

method	result
gosper	$-\frac{\sqrt{b\,x^2+a}\,(-15f\,x^6b^3+18a\,b^2f\,x^4-21b^3e\,x^4-24a^2bf\,x^2+28a\,b^2e\,x^2-35b^3d\,x^2+48a^3f-56a^2be+70a\,b^2d-105b^3c)}{105b^4}$
trager	$-\frac{\sqrt{b\,x^2+a}\,(-15f\,x^6b^3+18a\,b^2f\,x^4-21b^3e\,x^4-24a^2bf\,x^2+28a\,b^2e\,x^2-35b^3d\,x^2+48a^3f-56a^2be+70a\,b^2d-105b^3c)}{105b^4}$
risch	$-\frac{\sqrt{b\,x^2+a}\,(-15f\,x^6b^3+18a\,b^2f\,x^4-21b^3e\,x^4-24a^2bf\,x^2+28a\,b^2e\,x^2-35b^3d\,x^2+48a^3f-56a^2be+70a\,b^2d-105b^3c)}{105b^4}$
default	$f\left(\frac{x^6\sqrt{b\,x^2+a}}{7b}-\frac{6a\left(\frac{x^4\sqrt{b\,x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)}{5b}\right)}{7b}\right)+e\left(\frac{x^4\sqrt{b\,x^2+a}}{5b}-\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/7*x^6/b*(b*x^2+a)^(1/2)-6/7*a/b*(1/5*x^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/
3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))))+e*(1/5*x^4/b*(b*x^2+a)
```

```
^(1/2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2)))+d*(1/
3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))+c*(b*x^2+a)^(1/2)/b
```

Maxima [A]
time = 0.30, size = 183, normalized size = 1.51

$$\frac{\sqrt{bx^2+a}\,fx^6}{7\,b}-\frac{6\,\sqrt{bx^2+a}\,afx^4}{35\,b^2}+\frac{\sqrt{bx^2+a}\,x^4e}{5\,b}+\frac{\sqrt{bx^2+a}\,dx^2}{3\,b}+\frac{8\,\sqrt{bx^2+a}\,a^2fx^2}{35\,b^3}-\frac{4\,\sqrt{bx^2+a}\,ax^2e}{15\,b^2}+\frac{\sqrt{bx^2+a}\,c}{b}-\frac{2\,\sqrt{bx^2+a}\,ad}{3\,b^2}-\frac{16\,\sqrt{bx^2+a}\,a^3f}{35\,b^4}+\frac{8\,\sqrt{bx^2+a}\,a^2e}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(b*x^2 + a)*f*x^6/b - 6/35*sqrt(b*x^2 + a)*a*f*x^4/b^2 + 1/5*sqrt(b
*x^2 + a)*x^4*e/b + 1/3*sqrt(b*x^2 + a)*d*x^2/b + 8/35*sqrt(b*x^2 + a)*a^2*
f*x^2/b^3 - 4/15*sqrt(b*x^2 + a)*a*x^2*e/b^2 + sqrt(b*x^2 + a)*c/b - 2/3*sq
rt(b*x^2 + a)*a*d/b^2 - 16/35*sqrt(b*x^2 + a)*a^3*f/b^4 + 8/15*sqrt(b*x^2 +
 a)*a^2*e/b^3
```

Fricas [A]
time = 4.03, size = 99, normalized size = 0.82

$$\frac{\left(15\,b^3fx^6-18\,ab^2fx^4+105\,b^3c-70\,ab^2d-48\,a^3f+\left(35\,b^3d+24\,a^2bf\right)x^2+7\left(3\,b^3x^4-4\,ab^2x^2+8\,a^2b\right)e\right)\sqrt{bx^2+a}}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^3*f*x^6 - 18*a*b^2*f*x^4 + 105*b^3*c - 70*a*b^2*d - 48*a^3*f +
(35*b^3*d + 24*a^2*b*f)*x^2 + 7*(3*b^3*x^4 - 4*a*b^2*x^2 + 8*a^2*b)*e)*sqrt
(b*x^2 + a)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(112) = 224.
time = 0.36, size = 238, normalized size = 1.97

$$\begin{cases} -\frac{16a^3f\sqrt{a+bx^2}}{35b^4}+\frac{8a^2e\sqrt{a+bx^2}}{15b^3}+\frac{8a^2fx^2\sqrt{a+bx^2}}{35b^3}-\frac{2ad\sqrt{a+bx^2}}{3b^2}-\frac{4aex^2\sqrt{a+bx^2}}{15b^2}-\frac{6afx^4\sqrt{a+bx^2}}{35b^2}+\frac{c\sqrt{a+bx^2}}{b}+\frac{dx^2\sqrt{a+bx^2}}{3b}+\frac{ex^4\sqrt{a+bx^2}}{5b}+\frac{fx^6\sqrt{a+bx^2}}{7b} & \text{for}\: b \neq 0 \\ \frac{\frac{cx^2}{2}+\frac{dx^4}{4}+\frac{ex^6}{6}+\frac{fx^8}{8}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-16*a**3*f*sqrt(a + b*x**2)/(35*b**4) + 8*a**2*e*sqrt(a + b*x**2
)/(15*b**3) + 8*a**2*f*x**2*sqrt(a + b*x**2)/(35*b**3) - 2*a*d*sqrt(a + b*x
**2)/(3*b**2) - 4*a*e*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*f*x**4*sqrt(a +
 b*x**2)/(35*b**2) + c*sqrt(a + b*x**2)/b + d*x**2*sqrt(a + b*x**2)/(3*b) +
 e*x**4*sqrt(a + b*x**2)/(5*b) + f*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0)),
((c*x**2/2 + d*x**4/4 + e*x**6/6 + f*x**8/8)/sqrt(a), True))
```

Giac [A]
time = 1.52, size = 130, normalized size = 1.07

$$\frac{(b^3c - ab^2d - a^3f + a^2be)\sqrt{bx^2 + a}}{b^4} + \frac{35\,(bx^2 + a)^{\frac{3}{2}}b^2d + 15\,(bx^2 + a)^{\frac{7}{2}}f - 63\,(bx^2 + a)^{\frac{5}{2}}af + 105\,(bx^2 + a)^{\frac{3}{2}}a^2f + 21\,(bx^2 + a)^{\frac{5}{2}}be - 70\,(bx^2 + a)^{\frac{3}{2}}abe}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*sqrt(b*x^2 + a)/b^4 + 1/105*(35*(b*x^2
+ a)^(3/2)*b^2*d + 15*(b*x^2 + a)^(7/2)*f - 63*(b*x^2 + a)^(5/2)*a*f + 105*
(b*x^2 + a)^(3/2)*a^2*f + 21*(b*x^2 + a)^(5/2)*b*e - 70*(b*x^2 + a)^(3/2)*a
*b*e)/b^4
```

Mupad [B]
time = 1.06, size = 103, normalized size = 0.85

$$\sqrt{b\,x^2 + a}\,\left(\frac{-48\,f\,a^3 + 56\,e\,a^2\,b - 70\,d\,a\,b^2 + 105\,c\,b^3}{105\,b^4} + \frac{f\,x^6}{7\,b} + \frac{x^2\,(24\,f\,a^2\,b - 28\,e\,a\,b^2 + 35\,d\,b^3)}{105\,b^4} + \frac{x^4\,(21\,b^3\,e - 18\,a\,b^2\,f)}{105\,b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((105*b^3*c - 48*a^3*f - 70*a*b^2*d + 56*a^2*b*e)/(105*b^
4) + (f*x^6)/(7*b) + (x^2*(35*b^3*d - 28*a*b^2*e + 24*a^2*b*f))/(105*b^4) +
 (x^4*(21*b^3*e - 18*a*b^2*f))/(105*b^4))
```

3.146 $\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=103

$$\frac{(b^2d-abe+a^2f)\sqrt{a+bx^2}}{b^3}+\frac{(be-2af)\,(a+bx^2)^{3/2}}{3b^3}+\frac{f(a+bx^2)^{5/2}}{5b^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] 1/3*(-2*a*f+b*e)*(b*x^2+a)^(3/2)/b^3+1/5*f*(b*x^2+a)^(5/2)/b^3-c*arctanh((b
*x^2+a)^(1/2)/a^(1/2))/a^(1/2)+(a^2*f-a*b*e+b^2*d)*(b*x^2+a)^(1/2)/b^3
```

Rubi [A]
time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1813, 1634, 65, 214}

$$\frac{\sqrt{a+bx^2}\,(a^2f-abe+b^2d)}{b^3}+\frac{(a+bx^2)^{3/2}\,(be-2af)}{3b^3}+\frac{f(a+bx^2)^{5/2}}{5b^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] ((b^2*d - a*b*e + a^2*f)*Sqrt[a + b*x^2])/b^3 + ((b*e - 2*a*f)*(a + b*x^2)^
(3/2))/(3*b^3) + (f*(a + b*x^2)^(5/2))/(5*b^3) - (c*ArcTanh[Sqrt[a + b*x^2]
/Sqrt[a]])/Sqrt[a]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1634

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
```

```
xpon[Px, x], 2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^2d-abe+a^2f}{b^2\sqrt{a+bx}}+\frac{c}{x\sqrt{a+bx}}+\frac{(be-2af)\sqrt{a+bx}}{b^2}+\frac{f(a+}{b}\right.\right.\\ &= \frac{(b^2d-abe+a^2f)\sqrt{a+bx^2}}{b^3}+\frac{(be-2af)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{1}{2}c\\ &= \frac{(b^2d-abe+a^2f)\sqrt{a+bx^2}}{b^3}+\frac{(be-2af)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}+\frac{cS}{}\\ &= \frac{(b^2d-abe+a^2f)\sqrt{a+bx^2}}{b^3}+\frac{(be-2af)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{c\,t}{}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 86, normalized size = 0.83

$$\frac{\sqrt{a+bx^2}\,(8a^2f-2ab(5e+2fx^2)+b^2(15d+5ex^2+3fx^4))}{15b^3}-\frac{c\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(8*a^2*f - 2*a*b*(5*e + 2*f*x^2) + b^2*(15*d + 5*e*x^2 + 3
*f*x^4)))/(15*b^3) - (c*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

Maple [A]
time = 0.11, size = 139, normalized size = 1.35

method	result

default	$f\left(\frac{x^4\sqrt{b\,x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)}{5b}\right)+e\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+\frac{d\sqrt{b}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/5*x^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(
b*x^2+a)^(1/2)))+e*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))+d*
(b*x^2+a)^(1/2)/b-c/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [A]
time = 0.30, size = 124, normalized size = 1.20

$$\frac{\sqrt{bx^2+a}\,fx^4}{5\,b}-\frac{4\,\sqrt{bx^2+a}\,afx^2}{15\,b^2}+\frac{\sqrt{bx^2+a}\,x^2e}{3\,b}-\frac{c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}+\frac{\sqrt{bx^2+a}\,d}{b}+\frac{8\,\sqrt{bx^2+a}\,a^2f}{15\,b^3}-\frac{2\,\sqrt{bx^2+a}\,ae}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(b*x^2 + a)*f*x^4/b - 4/15*sqrt(b*x^2 + a)*a*f*x^2/b^2 + 1/3*sqrt(b
*x^2 + a)*x^2*e/b - c*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^2 +
a)*d/b + 8/15*sqrt(b*x^2 + a)*a^2*f/b^3 - 2/3*sqrt(b*x^2 + a)*a*e/b^2
```

Fricas [A]
time = 2.82, size = 211, normalized size = 2.05

$$\left[\frac{15\,\sqrt{a}\,b^3c\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,ab^2fx^4-4\,a^2bfx^2+15\,ab^2d+8\,a^3f+5\,(ab^2x^2-2\,a^2b)e)\sqrt{bx^2+a}}{30\,ab^3},\ \frac{15\,\sqrt{-a}\,b^3c\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(3\,ab^2fx^4-4\,a^2bfx^2+15\,ab^2d+8\,a^3f+5\,(ab^2x^2-2\,a^2b)e)\sqrt{bx^2+a}}{15\,ab^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*sqrt(a)*b^3*c*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
 + 2*(3*a*b^2*f*x^4 - 4*a^2*b*f*x^2 + 15*a*b^2*d + 8*a^3*f + 5*(a*b^2*x^2 -
 2*a^2*b)*e)*sqrt(b*x^2 + a))/(a*b^3), 1/15*(15*sqrt(-a)*b^3*c*arctan(sqrt(
-a)/sqrt(b*x^2 + a)) + (3*a*b^2*f*x^4 - 4*a^2*b*f*x^2 + 15*a*b^2*d + 8*a^3*
f + 5*(a*b^2*x^2 - 2*a^2*b)*e)*sqrt(b*x^2 + a))/(a*b^3)]
```

Sympy [A]
time = 15.20, size = 102, normalized size = 0.99

$$\frac{f(a+bx^2)^{\frac{5}{2}}}{5b^3}-\frac{(a+bx^2)^{\frac{3}{2}}\cdot(2af-be)}{3b^3}+\frac{\sqrt{a+bx^2}\,(a^2f-abe+b^2d)}{b^3}+\frac{c\,\mathrm{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\,\sqrt{a+bx^2}}\right)}{a\sqrt{-\frac{1}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x/(b*x**2+a)**(1/2),x)
```

```
[Out] f*(a + b*x**2)**(5/2)/(5*b**3) - (a + b*x**2)**(3/2)*(2*a*f - b*e)/(3*b**3)
 + sqrt(a + b*x**2)*(a**2*f - a*b*e + b**2*d)/b**3 + c*atan(1/(sqrt(-1/a)*s
qrt(a + b*x**2)))/(a*sqrt(-1/a))
```

Giac [A]
time = 1.74, size = 127, normalized size = 1.23

$$\frac{c\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{15\sqrt{bx^2+a}\,b^{14}d+3\,(bx^2+a)^{\frac{5}{2}}b^{12}f-10\,(bx^2+a)^{\frac{3}{2}}ab^{12}f+15\sqrt{bx^2+a}\,a^2b^{12}f+5\,(bx^2+a)^{\frac{3}{2}}b^{13}e-15\sqrt{bx^2+a}\,ab^{13}e}{15\,b^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] c*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/15*(15*sqrt(b*x^2 + a)*b^14
*d + 3*(b*x^2 + a)^(5/2)*b^12*f - 10*(b*x^2 + a)^(3/2)*a*b^12*f + 15*sqrt(b
*x^2 + a)*a^2*b^12*f + 5*(b*x^2 + a)^(3/2)*b^13*e - 15*sqrt(b*x^2 + a)*a*b^
13*e)/b^15
```

Mupad [B]
time = 1.81, size = 99, normalized size = 0.96

$$\sqrt{b\,x^2+a}\,\left(\frac{8\,a^2\,f}{15\,b^3}+\frac{f\,x^4}{5\,b}-\frac{4\,a\,f\,x^2}{15\,b^2}\right)+\frac{d\,\sqrt{b\,x^2+a}}{b}-\frac{c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{e\,\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x*(a + b*x^2)^(1/2)),x)
```

```
[Out] (a + b*x^2)^(1/2)*((8*a^2*f)/(15*b^3) + (f*x^4)/(5*b) - (4*a*f*x^2)/(15*b^2
)) + (d*(a + b*x^2)^(1/2))/b - (c*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2)
 - (e*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

3.147 $\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=100

$$\frac{(be-af)\sqrt{a+bx^2}}{b^2}-\frac{c\sqrt{a+bx^2}}{2ax^2}+\frac{f(a+bx^2)^{3/2}}{3b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

```
[Out] 1/3*f*(b*x^2+a)^(3/2)/b^2+1/2*(-2*a*d+b*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))
/a^(3/2)+(-a*f+b*e)*(b*x^2+a)^(1/2)/b^2-1/2*c*(b*x^2+a)^(1/2)/a/x^2
```

Rubi [A]
time = 0.14, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {1813, 1635, 911, 1167, 214}

$$\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{a+bx^2}\,(be-af)}{b^2}+\frac{f(a+bx^2)^{3/2}}{3b^2}-\frac{c\sqrt{a+bx^2}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] ((b*e - a*f)*Sqrt[a + b*x^2])/b^2 - (c*Sqrt[a + b*x^2])/(2*a*x^2) + (f*(a +
 b*x^2)^(3/2))/(3*b^2) + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(
2*a^(3/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1635

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)\\
&= -\frac{c\sqrt{a+bx^2}}{2ax^2}-\frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc-2ad)-aex-afx^2}{x\sqrt{a+bx}}\,dx,x,x^2\right)}{2a}\\
&= -\frac{c\sqrt{a+bx^2}}{2ax^2}-\frac{\text{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(bc-2ad)+a^2be-a^3f}{b^2}-\frac{\left(abe-2a^2f\right)x^2}{b^2}-\frac{afx^4}{b^2}}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx^2}}{ab}\\
&= -\frac{c\sqrt{a+bx^2}}{2ax^2}-\frac{\text{Subst}\left(\int \left(-a\left(e-\frac{af}{b}\right)-\frac{afx^2}{b}+\frac{bc-2ad}{2\left(-\frac{a}{b}+\frac{x^2}{b}\right)}\right)\,dx,x,\sqrt{a+bx^2}}{ab}\\
&= \frac{(be-af)\sqrt{a+bx^2}}{b^2}-\frac{c\sqrt{a+bx^2}}{2ax^2}+\frac{f\left(a+bx^2\right)^{3/2}}{3b^2}-\frac{(bc-2ad)\text{Subst}\left(\int\right.}{2}\\
&= \frac{(be-af)\sqrt{a+bx^2}}{b^2}-\frac{c\sqrt{a+bx^2}}{2ax^2}+\frac{f\left(a+bx^2\right)^{3/2}}{3b^2}+\frac{(bc-2ad)\tanh^{-1}\left(\sqrt{}\right.}{2a^{3/2}}
\end{aligned}
$$

Mathematica [A]

time = 0.20, size = 92, normalized size = 0.92

$$\frac{\sqrt{a+bx^2}\,(-3b^2c+6abex^2-4a^2fx^2+2abfx^4)}{6ab^2x^2}+\frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^3*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-3*b^2*c + 6*a*b*e*x^2 - 4*a^2*f*x^2 + 2*a*b*f*x^4))/(6*a
*b^2*x^2) + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2))
```

Maple [A]
time = 0.13, size = 129, normalized size = 1.29

method	result
risch	$-\frac{c\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{f\,x^2\sqrt{b\,x^2+a}}{3b}-\frac{2af\sqrt{b\,x^2+a}}{3b^2}+\frac{e\sqrt{b\,x^2+a}}{b}-\frac{d\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}+\ln($
default	$f\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+\frac{e\sqrt{b\,x^2+a}}{b}+c\left(-\frac{\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))+e*(b*x^2+a)^(1/2)/b
+c*(-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1
/2))/x))-d/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [A]
time = 0.31, size = 105, normalized size = 1.05

$$\frac{\sqrt{bx^2+a}\,fx^2}{3\,b}+\frac{bc\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{d\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}-\frac{2\,\sqrt{bx^2+a}\,af}{3\,b^2}+\frac{\sqrt{bx^2+a}\,e}{b}-\frac{\sqrt{bx^2+a}\,c}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(b*x^2 + a)*f*x^2/b + 1/2*b*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2)
 - d*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) - 2/3*sqrt(b*x^2 + a)*a*f/b^2 +
sqrt(b*x^2 + a)*e/b - 1/2*sqrt(b*x^2 + a)*c/(a*x^2)
```

Fricas [A]
time = 2.48, size = 212, normalized size = 2.12

$$\left[-\frac{3\,(b^3c-2\,ab^2d)\sqrt{a}\,x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,(2\,a^2bfx^4-4\,a^3fx^2+6\,a^2bx^2e-3\,ab^2c)\sqrt{bx^2+a}}{12\,a^2b^2x^2},-\frac{3\,(b^3c-2\,ab^2d)\sqrt{-a}\,x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(2\,a^2bfx^4-4\,a^3fx^2+6\,a^2bx^2e-3\,ab^2c)\sqrt{bx^2+a}}{6\,a^2b^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(b^3*c - 2*a*b^2*d)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*s
qrt(a) + 2*a)/x^2) - 2*(2*a^2*b*f*x^4 - 4*a^3*f*x^2 + 6*a^2*b*x^2*e - 3*a*b
^2*c)*sqrt(b*x^2 + a))/(a^2*b^2*x^2), -1/6*(3*(b^3*c - 2*a*b^2*d)*sqrt(-a)*
x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (2*a^2*b*f*x^4 - 4*a^3*f*x^2 + 6*a^2
*b*x^2*e - 3*a*b^2*c)*sqrt(b*x^2 + a))/(a^2*b^2*x^2)]
```

Sympy [A]
time = 29.21, size = 138, normalized size = 1.38

$$e\left(\begin{cases}\frac{x^2}{2\sqrt{a}} & \text{for}\ b=0\\ \frac{\sqrt{a+bx^2}}{b} & \text{otherwise}\end{cases}\right)+f\left(\begin{cases}-\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for}\ b\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise}\end{cases}\right)-\frac{\sqrt{b}\,c\sqrt{\frac{a}{bx^2}+1}}{2ax}-\frac{d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}}+\frac{bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**3/(b*x**2+a)**(1/2),x)
```

```
[Out] e*Piecewise((x**2/(2*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**2)/b, True)) + f*P
iecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), Ne(
b, 0)), (x**4/(4*sqrt(a)), True)) - sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(2*a*x)
- d*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + b*c*asinh(sqrt(a)/(sqrt(b)*x))/(2*
a**(3/2))
```

Giac [A]
time = 1.79, size = 114, normalized size = 1.14

$$-\frac{\frac{3\,(b^2c-2\,abd)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a}+\frac{3\,\sqrt{bx^2+a}\,bc}{ax^2}-\frac{2\left((bx^2+a)^{\frac{3}{2}}b^2f-3\,\sqrt{bx^2+a}\,ab^2f+3\,\sqrt{bx^2+a}\,b^3e\right)}{b^3}}{6\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(3*(b^2*c - 2*a*b*d)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) + 3
*sqrt(b*x^2 + a)*b*c/(a*x^2) - 2*((b*x^2 + a)^(3/2)*b^2*f - 3*sqrt(b*x^2 +
a)*a*b^2*f + 3*sqrt(b*x^2 + a)*b^3*e)/b^3)/b
```

Mupad [B]
time = 1.95, size = 99, normalized size = 0.99

$$\frac{e\,\sqrt{b\,x^2+a}}{b}-\frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{c\,\sqrt{b\,x^2+a}}{2\,a\,x^2}+\frac{b\,c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{f\,\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^3*(a + b*x^2)^(1/2)),x)
```

```
[Out] (e*(a + b*x^2)^(1/2))/b - (d*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (c
*(a + b*x^2)^(1/2))/(2*a*x^2) + (b*c*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a
^(3/2)) - (f*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

3.148 $\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=114

$$\frac{f\sqrt{a+bx^2}}{b}-\frac{c\sqrt{a+bx^2}}{4ax^4}+\frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2}-\frac{(3b^2c-4abd+8a^2e)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

```
[Out] -1/8*(8*a^2*e-4*a*b*d+3*b^2*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+f*(
b*x^2+a)^(1/2)/b-1/4*c*(b*x^2+a)^(1/2)/a/x^4+1/8*(-4*a*d+3*b*c)*(b*x^2+a)^(
1/2)/a^2/x^2
```

Rubi [A]
time = 0.17, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1813, 1635, 911, 1171, 396, 214}

$$\frac{\sqrt{a+bx^2}\,(3bc-4ad)}{8a^2x^2}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(8a^2e-4abd+3b^2c)}{8a^{5/2}}-\frac{c\sqrt{a+bx^2}}{4ax^4}+\frac{f\sqrt{a+bx^2}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (f*Sqrt[a + b*x^2])/b - (c*Sqrt[a + b*x^2])/(4*a*x^4) + ((3*b*c - 4*a*d)*Sq
rt[a + b*x^2])/(8*a^2*x^2) - ((3*b^2*c - 4*a*b*d + 8*a^2*e)*ArcTanh[Sqrt[a
+ b*x^2]/Sqrt[a]])/(8*a^(5/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
```

```
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1635

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^3\sqrt{a+bx}}\,dx,x,x^2\right)$$

$$= -\frac{c\sqrt{a+bx^2}}{4ax^4} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(3bc-4ad)-2aex-2afx^2}{x^2\sqrt{a+bx}}\,dx,x,x^2\right)}{4a}$$

$$= -\frac{c\sqrt{a+bx^2}}{4ax^4} - \frac{\text{Subst}\left(\int \frac{\frac{\frac{1}{2}b^2(3bc-4ad)+2a^2be-2a^3f}{b^2}-\frac{\left(2abe-4a^2f\right)x^2}{b^2}-\frac{2afx^4}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^2}\,dx,x,\sqrt{a+\right.}{2ab}$$

$$= -\frac{c\sqrt{a+bx^2}}{4ax^4} + \frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}\left(-3bc+4ad-\frac{8a^2e}{b}+\frac{8a^3f}{b^2}\right)-\frac{4a^2}{b}}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{4a^2}$$

$$= \frac{f\sqrt{a+bx^2}}{b} - \frac{c\sqrt{a+bx^2}}{4ax^4} + \frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2} + \frac{\left(3bc-4ad+\frac{8a^2e}{b}\right)\text{Su}}{}$$

$$= \frac{f\sqrt{a+bx^2}}{b} - \frac{c\sqrt{a+bx^2}}{4ax^4} + \frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2} - \frac{(3b^2c-4abd+8a^2e)\,t}{8a}$$

Mathematica [A]
time = 0.21, size = 102, normalized size = 0.89

$$\frac{\sqrt{a+bx^2}\,(-2abc+3b^2cx^2-4abdx^2+8a^2fx^4)}{8a^2bx^4} + \frac{(-3b^2c+4abd-8a^2e)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^5*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*a*b*c + 3*b^2*c*x^2 - 4*a*b*d*x^2 + 8*a^2*f*x^4))/(8*a
^2*b*x^4) + ((-3*b^2*c + 4*a*b*d - 8*a^2*e)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]
])/(8*a^(5/2))
```

Maple [A]
time = 0.13, size = 167, normalized size = 1.46

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(4ad\,x^2-3c\,x^2b+2ac)}{8a^2x^4}+\frac{f\sqrt{b\,x^2+a}}{b}-\frac{e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}+\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right.}{2a^{\frac{3}{2}}}$

default	$\frac{f\sqrt{b\,x^{2}+a}}{b}+c\left(-\frac{\sqrt{b\,x^{2}+a}}{4a\,x^{4}}-\frac{3b\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)+d\Bigg(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(b*x^2+a)^(1/2)/b+c*(-1/4/a/x^4*(b*x^2+a)^(1/2)-3/4*b/a*(-1/2*(b*x^2+a)^(
1/2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+d*(-1/2*(b
*x^2+a)^(1/2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))-e/
a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [**A**]
time = 0.29, size = 129, normalized size = 1.13

$$-\frac{3\,b^{2}c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{5}{2}}}+\frac{bd\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)e}{\sqrt{a}}+\frac{\sqrt{bx^{2}+a}\,f}{b}+\frac{3\,\sqrt{bx^{2}+a}\,bc}{8\,a^{2}x^{2}}-\frac{\sqrt{bx^{2}+a}\,d}{2\,ax^{2}}-\frac{\sqrt{bx^{2}+a}\,c}{4\,ax^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/8*b^2*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/2*b*d*arcsinh(a/(sqrt(
a*b)*abs(x)))/a^(3/2) - arcsinh(a/(sqrt(a*b)*abs(x)))*e/sqrt(a) + sqrt(b*x^
2 + a)*f/b + 3/8*sqrt(b*x^2 + a)*b*c/(a^2*x^2) - 1/2*sqrt(b*x^2 + a)*d/(a*x
^2) - 1/4*sqrt(b*x^2 + a)*c/(a*x^4)
```

Fricas [**A**]
time = 1.24, size = 233, normalized size = 2.04

$$\left[\frac{(8\,a^{2}bx^{4}e+(3\,b^{3}c-4\,ab^{2}d)x^{4})\sqrt{a}\,\log\left(-\frac{bx^{2}-2\,\sqrt{bx^{2}+a}\,\sqrt{a}+2\,a}{x^{2}}\right)+2\,(8\,a^{3}fx^{4}-2\,a^{2}bc+(3\,ab^{2}c-4\,a^{2}bd)x^{2})\sqrt{bx^{2}+a}}{16\,a^{3}bx^{4}},\ \frac{(8\,a^{2}bx^{4}e+(3\,b^{3}c-4\,ab^{2}d)x^{4})\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^{2}+a}}\right)+(8\,a^{3}fx^{4}-2\,a^{2}bc+(3\,ab^{2}c-4\,a^{2}bd)x^{2})\sqrt{bx^{2}+a}}{8\,a^{3}bx^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((8*a^2*b*x^4*e + (3*b^3*c - 4*a*b^2*d)*x^4)*sqrt(a)*log(-(b*x^2 - 2*
sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(8*a^3*f*x^4 - 2*a^2*b*c + (3*a*b^2
*c - 4*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4), 1/8*((8*a^2*b*x^4*e + (3
*b^3*c - 4*a*b^2*d)*x^4)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (8*a^3
*f*x^4 - 2*a^2*b*c + (3*a*b^2*c - 4*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x
^4)]
```

Sympy [A]

time = 60.98, size = 194, normalized size = 1.70

$$f\left(\begin{cases} \frac{x^2}{2\sqrt{a}} & \text{for}\ b = 0 \\ \frac{\sqrt{a+bx^2}}{b} & \text{otherwise} \end{cases}\right) - \frac{c}{4\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{\sqrt{b}\,c}{8ax^3\sqrt{\frac{a}{bx^2}+1}} - \frac{\sqrt{b}\,d\sqrt{\frac{a}{bx^2}+1}}{2ax} + \frac{3b^{\frac{3}{2}}c}{8a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}} + \frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{3}{2}}} - \frac{3b^2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**5/(b*x**2+a)**(1/2),x)
```

```
[Out] f*Piecewise((x**2/(2*sqrt(a)), Eq(b, 0)), (sqrt(a + b*x**2)/b, True)) - c/(
4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)*c/(8*a*x**3*sqrt(a/(b*x**2)
+ 1)) - sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(2*a*x) + 3*b**(3/2)*c/(8*a**2*x*sqr
t(a/(b*x**2) + 1)) - e*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + b*d*asinh(sqrt(
a)/(sqrt(b)*x))/(2*a**(3/2)) - 3*b**2*c*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5
/2))
```

Giac [A]

time = 2.10, size = 141, normalized size = 1.24

$$\frac{8\sqrt{bx^2+a}\,f + \frac{(3\,b^3c-4\,ab^2d+8\,a^2be)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^2} + \frac{3\,(bx^2+a)^{\frac{3}{2}}b^3c-5\sqrt{bx^2+a}\,ab^3c-4\,(bx^2+a)^{\frac{3}{2}}ab^2d+4\sqrt{bx^2+a}\,a^2b^2d}{a^2b^2x^4}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(8*sqrt(b*x^2 + a)*f + (3*b^3*c - 4*a*b^2*d + 8*a^2*b*e)*arctan(sqrt(b*
x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 + a)^(3/2)*b^3*c - 5*sqrt(b*x
^2 + a)*a*b^3*c - 4*(b*x^2 + a)^(3/2)*a*b^2*d + 4*sqrt(b*x^2 + a)*a^2*b^2*d
)/(a^2*b^2*x^4))/b
```

Mupad [B]

time = 2.19, size = 133, normalized size = 1.17

$$\frac{f\,\sqrt{b\,x^2+a}}{b} - \frac{e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{5\,c\,\sqrt{b\,x^2+a}}{8\,a\,x^4} + \frac{3\,c\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4} - \frac{d\,\sqrt{b\,x^2+a}}{2\,a\,x^2} + \frac{b\,d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}} - \frac{3\,b^2\,c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^5*(a + b*x^2)^(1/2)),x)
```

```
[Out] (f*(a + b*x^2)^(1/2))/b - (e*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (5
*c*(a + b*x^2)^(1/2))/(8*a*x^4) + (3*c*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (d*
(a + b*x^2)^(1/2))/(2*a*x^2) + (b*d*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^
(3/2)) - (3*b^2*c*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2))
```

3.149 $\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=146

$$-\frac{c\sqrt{a+bx^2}}{6ax^6}+\frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4}-\frac{(5b^2c-6abd+8a^2e)\sqrt{a+bx^2}}{16a^3x^2}+\frac{(5b^3c-6ab^2d+8a^2be-16a^3f)\,\mathrm{ta}}{16a^{7/2}}$$

```
[Out] 1/16*(-16*a^3*f+8*a^2*b*e-6*a*b^2*d+5*b^3*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2
))/a^(7/2)-1/6*c*(b*x^2+a)^(1/2)/a/x^6+1/24*(-6*a*d+5*b*c)*(b*x^2+a)^(1/2)/
a^2/x^4-1/16*(8*a^2*e-6*a*b*d+5*b^2*c)*(b*x^2+a)^(1/2)/a^3/x^2
```

Rubi [A]
time = 0.20, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$,
Rules used = {1813, 1635, 911, 1171, 393, 214}

$$\frac{\sqrt{a+bx^2}\,(5bc-6ad)}{24a^2x^4}-\frac{\sqrt{a+bx^2}\,(8a^2e-6abd+5b^2c)}{16a^3x^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(-16a^3f+8a^2be-6ab^2d+5b^3c)}{16a^{7/2}}-\frac{c\sqrt{a+bx^2}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/6*(c*Sqrt[a + b*x^2])/(a*x^6) + ((5*b*c - 6*a*d)*Sqrt[a + b*x^2])/(24*a^
2*x^4) - ((5*b^2*c - 6*a*b*d + 8*a^2*e)*Sqrt[a + b*x^2])/(16*a^3*x^2) + ((5
*b^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
)/(16*a^(7/2))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
```

```
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1635

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^4\sqrt{a+bx}}\,dx,x,x^2\right)$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5bc-6ad)-3aex-3afx^2}{x^3\sqrt{a+bx}}\,dx,x,x^2\right)}{6a}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} - \frac{\text{Subst}\Bigg(\int \frac{\frac{\frac{1}{2}b^2(5bc-6ad)+3a^2be-3a^3f}{b^2}-\frac{\left(3abe-6a^2f\right)x^2}{b^2}-\frac{3afx^4}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^3}\,dx,x,\sqrt{a+bx}}{3ab}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} + \frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4} - \frac{\text{Subst}\Bigg(\int \frac{-\frac{3}{2}\left(5bc-6ad+\frac{8a^2e}{b}-\frac{8a^3f}{b^2}\right)-\frac{12a^2f}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^2}}{12a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} + \frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4} - \frac{\left(5b^2c-6abd+8a^2e\right)\sqrt{a+bx^2}}{16a^3x^2} + $$

$$= -\frac{c\sqrt{a+bx^2}}{6ax^6} + \frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4} - \frac{\left(5b^2c-6abd+8a^2e\right)\sqrt{a+bx^2}}{16a^3x^2} + $$

Mathematica [A]

time = 0.24, size = 126, normalized size = 0.86

$$\frac{\sqrt{a+bx^2}\left(-8a^2c+10abcx^2-12a^2dx^2-15b^2cx^4+18abdx^4-24a^2ex^4\right)}{48a^3x^6}+\frac{\left(5b^3c-6ab^2d+8a^2be-16a^3f\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^7*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*a^2*c + 10*a*b*c*x^2 - 12*a^2*d*x^2 - 15*b^2*c*x^4 + 1
8*a*b*d*x^4 - 24*a^2*e*x^4))/(48*a^3*x^6) + ((5*b^3*c - 6*a*b^2*d + 8*a^2*b
*e - 16*a^3*f)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(7/2))
```

Maple [A]

time = 0.13, size = 250, normalized size = 1.71

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(24a^2e\,x^4-18abd\,x^4+15b^2c\,x^4+12a^2d\,x^2-10abc\,x^2+8a^2c\right)}{48a^3x^6}-\frac{f\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{\sqrt{a}}+$

default	$d\left(-\frac{\sqrt{b\,x^2+a}}{4a\,x^4}-\frac{3b\left(-\frac{\sqrt{b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)+c\left(-\frac{\sqrt{b\,x^2+a}}{6a\,x^6}-\right.$ $5b\left(-\frac{\sqrt{b\,x}}{4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/4/a/x^4*(b*x^2+a)^(1/2)-3/4*b/a*(-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(
3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+c*(-1/6/a/x^6*(b*x^2+a)^(1/2)-
5/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(1/2)-3/4*b/a*(-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2
*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))+e*(-1/2*(b*x^2+a)^(1/2)
/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))-f/a^(1/2)*ln((2
*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

Maxima [A]

time = 0.29, size = 195, normalized size = 1.34

$$\frac{5\,b^3c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{7}{2}}}-\frac{3\,b^2d\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{8\,a^{\frac{5}{2}}}-\frac{f\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{\sqrt{a}}+\frac{b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)e}{2\,a^{\frac{3}{2}}}-\frac{5\,\sqrt{bx^2+a}\,b^2c}{16\,a^3x^2}+\frac{3\,\sqrt{bx^2+a}\,bd}{8\,a^2x^2}-\frac{\sqrt{bx^2+a}\,e}{2\,ax^2}+\frac{5\,\sqrt{bx^2+a}\,bc}{24\,a^2x^4}-\frac{\sqrt{bx^2+a}\,d}{4\,ax^4}-\frac{\sqrt{bx^2+a}\,c}{6\,ax^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/16*b^3*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 3/8*b^2*d*arcsinh(a/(sqr
t(a*b)*abs(x)))/a^(5/2) - f*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + 1/2*b*a
rcsinh(a/(sqrt(a*b)*abs(x)))*e/a^(3/2) - 5/16*sqrt(b*x^2 + a)*b^2*c/(a^3*x^
2) + 3/8*sqrt(b*x^2 + a)*b*d/(a^2*x^2) - 1/2*sqrt(b*x^2 + a)*e/(a*x^2) + 5/
24*sqrt(b*x^2 + a)*b*c/(a^2*x^4) - 1/4*sqrt(b*x^2 + a)*d/(a*x^4) - 1/6*sqrt
(b*x^2 + a)*c/(a*x^6)
```

Fricas [A]

time = 1.15, size = 281, normalized size = 1.92

$$\left[\frac{3\,(8\,a^2bx^6e+(5\,b^3c-6\,ab^2d-16\,a^3f)x^6)\sqrt{a}\,\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)-2\,(24\,a^3x^4e+3\,(5\,ab^2c-6\,a^2bd)x^4+8\,a^3c-2\,(5\,a^2bc-6\,a^3d)x^2)\sqrt{bx^2+a}}{96\,a^4x^6},-\frac{3\,(8\,a^2bx^6e+(5\,b^3c-6\,ab^2d-16\,a^3f)x^6)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(24\,a^3x^4e+3\,(5\,ab^2c-6\,a^2bd)x^4+8\,a^3c-2\,(5\,a^2bc-6\,a^3d)x^2)\sqrt{bx^2+a}}{48\,a^4x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*a^2*b*x^6*e + (5*b^3*c - 6*a*b^2*d - 16*a^3*f)*x^6)*sqrt(a)*log
(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(24*a^3*x^4*e + 3*(5*a
*b^2*c - 6*a^2*b*d)*x^4 + 8*a^3*c - 2*(5*a^2*b*c - 6*a^3*d)*x^2)*sqrt(b*x^2
 + a))/(a^4*x^6), -1/48*(3*(8*a^2*b*x^6*e + (5*b^3*c - 6*a*b^2*d - 16*a^3*f
)*x^6)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (24*a^3*x^4*e + 3*(5*a*b
^2*c - 6*a^2*b*d)*x^4 + 8*a^3*c - 2*(5*a^2*b*c - 6*a^3*d)*x^2)*sqrt(b*x^2 +
 a))/(a^4*x^6)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(141) = 282$.
time = 85.98, size = 303, normalized size = 2.08

$$-\frac{c}{6\sqrt{b}\,x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{d}{4\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}\,c}{24ax^5\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}\,d}{8ax^3\sqrt{\frac{a}{bx^2}+1}}-\frac{\sqrt{b}\,e\sqrt{\frac{a}{bx^2}+1}}{2ax}-\frac{5b^{\frac{3}{2}}c}{48a^2x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{3b^{\frac{3}{2}}d}{8a^2x\sqrt{\frac{a}{bx^2}+1}}-\frac{5b^{\frac{5}{2}}c}{16a^3x\sqrt{\frac{a}{bx^2}+1}}-\frac{f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{\sqrt{a}}+\frac{be\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{5}{2}}}+\frac{5b^3c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**7/(b*x**2+a)**(1/2),x)
```

```
[Out] -c/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - d/(4*sqrt(b)*x**5*sqrt(a/(b*x**2
) + 1)) + sqrt(b)*c/(24*a*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)*d/(8*a*x**3*
sqrt(a/(b*x**2) + 1)) - sqrt(b)*e*sqrt(a/(b*x**2) + 1)/(2*a*x) - 5*b**(3/2)
*c/(48*a**2*x**3*sqrt(a/(b*x**2) + 1)) + 3*b**(3/2)*d/(8*a**2*x*sqrt(a/(b*x
**2) + 1)) - 5*b**(5/2)*c/(16*a**3*x*sqrt(a/(b*x**2) + 1)) - f*asinh(sqrt(a
)/(sqrt(b)*x))/sqrt(a) + b*e*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2)) - 3*b*
*2*d*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) + 5*b**3*c*asinh(sqrt(a)/(sqrt
(b)*x))/(16*a**(7/2))
```

Giac [**A**]
time = 1.08, size = 232, normalized size = 1.59

$$-\frac{\frac{3\,(5\,b^4c-6\,ab^3d-16\,a^3bf+8\,a^2b^2e)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{15\,(bx^2+a)^{\frac{5}{2}}b^4c-40\,(bx^2+a)^{\frac{3}{2}}ab^4c+33\,\sqrt{bx^2+a}\,a^2b^4c-18\,(bx^2+a)^{\frac{5}{2}}ab^3d+48\,(bx^2+a)^{\frac{3}{2}}a^2b^3d-30\,\sqrt{bx^2+a}\,a^3b^3d+24\,(bx^2+a)^{\frac{5}{2}}a^2b^2e-48\,(bx^2+a)^{\frac{3}{2}}a^3b^2e+24\,\sqrt{bx^2+a}\,a^4b^2e}{a^3b^3x^6}}{48\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/48*(3*(5*b^4*c - 6*a*b^3*d - 16*a^3*b*f + 8*a^2*b^2*e)*arctan(sqrt(b*x^2
 + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x^2 + a)^(5/2)*b^4*c - 40*(b*x^2 +
a)^(3/2)*a*b^4*c + 33*sqrt(b*x^2 + a)*a^2*b^4*c - 18*(b*x^2 + a)^(5/2)*a*b^
3*d + 48*(b*x^2 + a)^(3/2)*a^2*b^3*d - 30*sqrt(b*x^2 + a)*a^3*b^3*d + 24*(b
*x^2 + a)^(5/2)*a^2*b^2*e - 48*(b*x^2 + a)^(3/2)*a^3*b^2*e + 24*sqrt(b*x^2
+ a)*a^4*b^2*e)/(a^3*b^3*x^6))/b
```

Mupad [B]

time = 2.54, size = 199, normalized size = 1.36

$$\frac{5\,c\,{\left(b\,x^2+a\right)}^{3/2}}{6\,a^2\,x^6}-\frac{11\,c\,\sqrt{b\,x^2+a}}{16\,a\,x^6}-\frac{f\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{5\,c\,{\left(b\,x^2+a\right)}^{5/2}}{16\,a^3\,x^6}-\frac{5\,d\,\sqrt{b\,x^2+a}}{8\,a\,x^4}+\frac{3\,d\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4}-\frac{e\,\sqrt{b\,x^2+a}}{2\,a\,x^2}+\frac{b\,e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{3\,b^2\,d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}-\frac{b^3\,c\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{16\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^7*(a + b*x^2)^(1/2)),x)
```

```
[Out] (5*c*(a + b*x^2)^(3/2))/(6*a^2*x^6) - (11*c*(a + b*x^2)^(1/2))/(16*a*x^6) -
 (f*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (5*c*(a + b*x^2)^(5/2))/(16
*a^3*x^6) - (5*d*(a + b*x^2)^(1/2))/(8*a*x^4) + (3*d*(a + b*x^2)^(3/2))/(8*
a^2*x^4) - (e*(a + b*x^2)^(1/2))/(2*a*x^2) + (b*e*atanh((a + b*x^2)^(1/2)/a
^(1/2)))/(2*a^(3/2)) - (b^3*c*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*
a^(7/2)) - (3*b^2*d*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2))
```

3.150 $\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=195

$$-\frac{c\sqrt{a+bx^2}}{8ax^8}+\frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6}-\frac{(35b^2c-40abd+48a^2e)\sqrt{a+bx^2}}{192a^3x^4}+\frac{(35b^3c-40ab^2d+48a^2be-64}{128a^4x^2}$$

```
[Out] -1/128*b*(-64*a^3*f+48*a^2*b*e-40*a*b^2*d+35*b^3*c)*arctanh((b*x^2+a)^(1/2)
/a^(1/2))/a^(9/2)-1/8*c*(b*x^2+a)^(1/2)/a/x^8+1/48*(-8*a*d+7*b*c)*(b*x^2+a)
^(1/2)/a^2/x^6-1/192*(48*a^2*e-40*a*b*d+35*b^2*c)*(b*x^2+a)^(1/2)/a^3/x^4+1
/128*(-64*a^3*f+48*a^2*b*e-40*a*b^2*d+35*b^3*c)*(b*x^2+a)^(1/2)/a^4/x^2
```

Rubi [A]

time = 0.24, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {1813, 1635, 911, 1171, 393, 205, 214}

$$\frac{\sqrt{a+bx^2}\,(7bc-8ad)}{48a^2x^6}-\frac{\sqrt{a+bx^2}\,(48a^2e-40abd+35b^2c)}{192a^3x^4}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(-64a^3f+48a^2be-40ab^2d+35b^3c)}{128a^{9/2}}+\frac{\sqrt{a+bx^2}\,(-64a^3f+48a^2be-40ab^2d+35b^3c)}{128a^4x^2}-\frac{c\sqrt{a+bx^2}}{8ax^8}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^9*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/8*(c*Sqrt[a + b*x^2])/(a*x^8) + ((7*b*c - 8*a*d)*Sqrt[a + b*x^2])/(48*a^
2*x^6) - ((35*b^2*c - 40*a*b*d + 48*a^2*e)*Sqrt[a + b*x^2])/(192*a^3*x^4) +
 ((35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*f)*Sqrt[a + b*x^2])/(128*a^4
*x^2) - (b*(35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*f)*ArcTanh[Sqrt[a +
 b*x^2]/Sqrt[a]])/(128*a^(9/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1635

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{x^5\sqrt{a+bx}}\,dx, x, x^2\right)$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(7bc-8ad)-4aex-4afx^2}{x^4\sqrt{a+bx}}\,dx, x, x^2\right)}{8a}$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} - \frac{\text{Subst}\Bigg(\int \frac{\frac{\frac{1}{2}b^2(7bc-8ad)+4a^2be-4a^3f}{b^2}-\frac{\left(4abe-8a^2f\right)x^2}{b^2}-\frac{4afx^4}{b^2}}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^4}\,dx, x, \sqrt{a+bx}}{4ab}$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{\text{Subst}\Bigg(\int \frac{\frac{1}{2}\left(-35bc+40ad-\frac{48a^2e}{b}+\frac{48a^3f}{b^2}\right)-}{\left(-\frac{a}{b}+\frac{x^2}{b}\right)^3}}{24a^2}$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{\left(35b^2c-40abd+48a^2e\right)\sqrt{a+bx^2}}{192a^3x^4} -$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{\left(35b^2c-40abd+48a^2e\right)\sqrt{a+bx^2}}{192a^3x^4} -$$

$$= -\frac{c\sqrt{a+bx^2}}{8ax^8} + \frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{\left(35b^2c-40abd+48a^2e\right)\sqrt{a+bx^2}}{192a^3x^4} -$$

Mathematica [A]
time = 0.38, size = 159, normalized size = 0.82

$$\frac{\frac{\sqrt{a}\sqrt{a+bx^2}\left(105b^3cx^6-10ab^2x^4\left(7c+12dx^2\right)+8a^2bx^2\left(7c+10dx^2+18ex^4\right)-16a^3\left(3c+4dx^2+6ex^4+12fx^6\right)\right)}{x^8}-3b\left(35b^3c-40ab^2d+48a^2be-64a^3f\right)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{384a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^9*Sqrt[a + b*x^2]),x]
```

```
[Out] ((Sqrt[a]*Sqrt[a + b*x^2]*(105*b^3*c*x^6 - 10*a*b^2*x^4*(7*c + 12*d*x^2) +
8*a^2*b*x^2*(7*c + 10*d*x^2 + 18*e*x^4) - 16*a^3*(3*c + 4*d*x^2 + 6*e*x^4 +
 12*f*x^6)))/x^8 - 3*b*(35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*f)*ArcT
anh[Sqrt[a + b*x^2]/Sqrt[a]])/(384*a^(9/2))
```

Maple [A]
time = 0.14, size = 342, normalized size = 1.75

method	result

risch	$-\frac{\sqrt{b x^2+a}\,(192a^3 f x^6-144a^2be x^6+120a b^2 d x^6-105b^3 c x^6+96a^3 e x^4-80a^2bd x^4+70a b^2 c x^4+64a^3 d x^2-56a^2bc x^2+48a^3c)}{384a^4x^8}$
default	$e\left(-\frac{\sqrt{b x^2+a}}{4a x^4}-\frac{3b\left(-\frac{\sqrt{b x^2+a}}{2a x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)+d\left(-\frac{\sqrt{b x^2+a}}{6a x^6}-\frac{5b\left(-\frac{\sqrt{b x}}{4}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/4/a/x^4*(b*x^2+a)^(1/2)-3/4*b/a*(-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(
3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))+d*(-1/6/a/x^6*(b*x^2+a)^(1/2)-
5/6*b/a*(-1/4/a/x^4*(b*x^2+a)^(1/2)-3/4*b/a*(-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2
*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))+f*(-1/2*(b*x^2+a)^(1/2)
/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))+c*(-1/8/a/x^8*(
```

```
b*x^2+a)^(1/2)-7/8*b/a*(-1/6/a/x^6*(b*x^2+a)^(1/2)-5/6*b/a*(-1/4/a/x^4*(b*x
^2+a)^(1/2)-3/4*b/a*(-1/2*(b*x^2+a)^(1/2)/a/x^2+1/2*b/a^(3/2)*ln((2*a+2*a^(
1/2)*(b*x^2+a)^(1/2))/x)))))
```

Maxima [A]
time = 0.29, size = 278, normalized size = 1.43

$$-\frac{35\,b^4c\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{128\,a^{\frac{9}{2}}}+\frac{5\,b^3d\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{16\,a^{\frac{7}{2}}}+\frac{bf\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)}{2\,a^{\frac{3}{2}}}-\frac{3\,b^2\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|x|}\right)e}{8\,a^{\frac{5}{2}}}+\frac{35\,\sqrt{bx^2+a}\,b^3c}{128\,a^4x^2}-\frac{5\,\sqrt{bx^2+a}\,b^2d}{16\,a^3x^2}-\frac{\sqrt{bx^2+a}\,f}{2\,ax^2}+\frac{3\,\sqrt{bx^2+a}\,be}{8\,a^2x^2}-\frac{35\,\sqrt{bx^2+a}\,b^2c}{192\,a^3x^4}+\frac{5\,\sqrt{bx^2+a}\,bd}{24\,a^2x^4}-\frac{\sqrt{bx^2+a}\,e}{4\,ax^4}+\frac{7\,\sqrt{bx^2+a}\,bc}{48\,a^2x^6}-\frac{\sqrt{bx^2+a}\,d}{6\,ax^6}-\frac{\sqrt{bx^2+a}\,c}{8\,ax^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -35/128*b^4*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2) + 5/16*b^3*d*arcsinh(a/
(sqrt(a*b)*abs(x)))/a^(7/2) + 1/2*b*f*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2)
 - 3/8*b^2*arcsinh(a/(sqrt(a*b)*abs(x)))*e/a^(5/2) + 35/128*sqrt(b*x^2 + a)
*b^3*c/(a^4*x^2) - 5/16*sqrt(b*x^2 + a)*b^2*d/(a^3*x^2) - 1/2*sqrt(b*x^2 +
a)*f/(a*x^2) + 3/8*sqrt(b*x^2 + a)*b*e/(a^2*x^2) - 35/192*sqrt(b*x^2 + a)*b
^2*c/(a^3*x^4) + 5/24*sqrt(b*x^2 + a)*b*d/(a^2*x^4) - 1/4*sqrt(b*x^2 + a)*e
/(a*x^4) + 7/48*sqrt(b*x^2 + a)*b*c/(a^2*x^6) - 1/6*sqrt(b*x^2 + a)*d/(a*x^
6) - 1/8*sqrt(b*x^2 + a)*c/(a*x^8)
```

Fricas [A]
time = 1.08, size = 371, normalized size = 1.90

$$\left[\frac{3\,(48\,a^2b^2x^8e+(35\,b^4c-40\,ab^3d-64\,a^3bf)x^8)\sqrt{a}\,\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\,\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,(35\,ab^3c-40\,a^2b^2d-64\,a^4f)x^6-48\,a^4c-10\,(7\,a^2b^2c-8\,a^3bd)x^4+8\,(7\,a^3bc-8\,a^4d)x^2+48\,(3\,a^3bx^6-2\,a^4x^4)e)\sqrt{bx^2+a}}{768\,a^5x^8},\frac{3\,(48\,a^2b^2x^8e+(35\,b^4c-40\,ab^3d-64\,a^3bf)x^8)\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(3\,(35\,ab^3c-40\,a^2b^2d-64\,a^4f)x^6-48\,a^4c-10\,(7\,a^2b^2c-8\,a^3bd)x^4+8\,(7\,a^3bc-8\,a^4d)x^2+48\,(3\,a^3bx^6-2\,a^4x^4)e)\sqrt{bx^2+a}}{384\,a^5x^8}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(48*a^2*b^2*x^8*e + (35*b^4*c - 40*a*b^3*d - 64*a^3*b*f)*x^8)*sqr
t(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*(35*a*b^3*c
 - 40*a^2*b^2*d - 64*a^4*f)*x^6 - 48*a^4*c - 10*(7*a^2*b^2*c - 8*a^3*b*d)*x
^4 + 8*(7*a^3*b*c - 8*a^4*d)*x^2 + 48*(3*a^3*b*x^6 - 2*a^4*x^4)*e)*sqrt(b*x
^2 + a))/(a^5*x^8), 1/384*(3*(48*a^2*b^2*x^8*e + (35*b^4*c - 40*a*b^3*d - 6
4*a^3*b*f)*x^8)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(35*a*b^3*c
- 40*a^2*b^2*d - 64*a^4*f)*x^6 - 48*a^4*c - 10*(7*a^2*b^2*c - 8*a^3*b*d)*x^
4 + 8*(7*a^3*b*c - 8*a^4*d)*x^2 + 48*(3*a^3*b*x^6 - 2*a^4*x^4)*e)*sqrt(b*x^
2 + a))/(a^5*x^8)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(196) = 392.
time = 166.55, size = 444, normalized size = 2.28

$$-\frac{c}{8\sqrt{b}\,x^9\sqrt{\frac{a}{bx^2}+1}}-\frac{d}{6\sqrt{b}\,x^7\sqrt{\frac{a}{bx^2}+1}}-\frac{e}{4\sqrt{b}\,x^5\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}\,c}{48ax^7\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}\,d}{24ax^5\sqrt{\frac{a}{bx^2}+1}}+\frac{\sqrt{b}\,e}{8ax^3\sqrt{\frac{a}{bx^2}+1}}-\frac{\sqrt{b}\,f\sqrt{\frac{a}{bx^2}+1}}{2ax}-\frac{7b^{\frac{3}{2}}c}{192a^2x^5\sqrt{\frac{a}{bx^2}+1}}-\frac{5b^{\frac{3}{2}}d}{48a^2x^3\sqrt{\frac{a}{bx^2}+1}}+\frac{3b^{\frac{3}{2}}e}{8a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{35b^{\frac{5}{2}}c}{384a^3x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{5b^{\frac{5}{2}}d}{16a^3x\sqrt{\frac{a}{bx^2}+1}}+\frac{35b^{\frac{7}{2}}c}{128a^4x\sqrt{\frac{a}{bx^2}+1}}+\frac{bf\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{2a^{\frac{3}{2}}}-\frac{3b^2e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{8a^{\frac{5}{2}}}+\frac{5b^3d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{16a^{\frac{7}{2}}}-\frac{35b^4c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\,x}\right)}{128a^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**9/(b*x**2+a)**(1/2),x)
```

```
[Out] -c/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - d/(6*sqrt(b)*x**7*sqrt(a/(b*x**2
) + 1)) - e/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)*c/(48*a*x**7*sq
rt(a/(b*x**2) + 1)) + sqrt(b)*d/(24*a*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)*
e/(8*a*x**3*sqrt(a/(b*x**2) + 1)) - sqrt(b)*f*sqrt(a/(b*x**2) + 1)/(2*a*x)
- 7*b**(3/2)*c/(192*a**2*x**5*sqrt(a/(b*x**2) + 1)) - 5*b**(3/2)*d/(48*a**2
*x**3*sqrt(a/(b*x**2) + 1)) + 3*b**(3/2)*e/(8*a**2*x*sqrt(a/(b*x**2) + 1))
+ 35*b**(5/2)*c/(384*a**3*x**3*sqrt(a/(b*x**2) + 1)) - 5*b**(5/2)*d/(16*a**
3*x*sqrt(a/(b*x**2) + 1)) + 35*b**(7/2)*c/(128*a**4*x*sqrt(a/(b*x**2) + 1))
 + b*f*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2)) - 3*b**2*e*asinh(sqrt(a)/(sq
rt(b)*x))/(8*a**(5/2)) + 5*b**3*d*asinh(sqrt(a)/(sqrt(b)*x))/(16*a**(7/2))
- 35*b**4*c*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(9/2))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(174) = 348$.
time = 1.11, size = 361, normalized size = 1.85

$$\frac{\frac{3\,(35\,b^5c-40\,ab^4d-64\,a^3b^2f+48\,a^2b^3e)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^4}+\frac{105\,(bx^2+a)^{\frac{7}{2}}b^5c-385\,(bx^2+a)^{\frac{5}{2}}ab^5c+511\,(bx^2+a)^{\frac{3}{2}}a^2b^5c-279\,\sqrt{bx^2+a}\,a^3b^5c-120\,(bx^2+a)^{\frac{7}{2}}ab^4d+440\,(bx^2+a)^{\frac{5}{2}}a^2b^4d-584\,(bx^2+a)^{\frac{3}{2}}a^3b^4d+264\,\sqrt{bx^2+a}\,a^4b^4d-192\,(bx^2+a)^{\frac{7}{2}}a^3b^2f+576\,(bx^2+a)^{\frac{5}{2}}a^4b^2f-576\,(bx^2+a)^{\frac{3}{2}}a^5b^2f+192\,\sqrt{bx^2+a}\,a^6b^2f+144\,(bx^2+a)^{\frac{7}{2}}a^2b^3e-528\,(bx^2+a)^{\frac{5}{2}}a^3b^3e+624\,(bx^2+a)^{\frac{3}{2}}a^4b^3e-240\,\sqrt{bx^2+a}\,a^5b^3e}{a^4b^4x^8}}{384\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(3*(35*b^5*c - 40*a*b^4*d - 64*a^3*b^2*f + 48*a^2*b^3*e)*arctan(sqrt(
b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) + (105*(b*x^2 + a)^(7/2)*b^5*c - 385*(b
*x^2 + a)^(5/2)*a*b^5*c + 511*(b*x^2 + a)^(3/2)*a^2*b^5*c - 279*sqrt(b*x^2
+ a)*a^3*b^5*c - 120*(b*x^2 + a)^(7/2)*a*b^4*d + 440*(b*x^2 + a)^(5/2)*a^2*
b^4*d - 584*(b*x^2 + a)^(3/2)*a^3*b^4*d + 264*sqrt(b*x^2 + a)*a^4*b^4*d - 1
92*(b*x^2 + a)^(7/2)*a^3*b^2*f + 576*(b*x^2 + a)^(5/2)*a^4*b^2*f - 576*(b*x
^2 + a)^(3/2)*a^5*b^2*f + 192*sqrt(b*x^2 + a)*a^6*b^2*f + 144*(b*x^2 + a)^(
7/2)*a^2*b^3*e - 528*(b*x^2 + a)^(5/2)*a^3*b^3*e + 624*(b*x^2 + a)^(3/2)*a^
4*b^3*e - 240*sqrt(b*x^2 + a)*a^5*b^3*e)/(a^4*b^4*x^8))/b
```

Mupad [**B**]
time = 2.91, size = 277, normalized size = 1.42

$$\frac{511\,c\,{\left(b\,x^2+a\right)}^{3/2}}{384\,a^2\,x^8}-\frac{93\,c\,\sqrt{b\,x^2+a}}{128\,a\,x^8}-\frac{385\,c\,{\left(b\,x^2+a\right)}^{5/2}}{384\,a^3\,x^8}+\frac{35\,c\,{\left(b\,x^2+a\right)}^{7/2}}{128\,a^4\,x^8}-\frac{11\,d\,\sqrt{b\,x^2+a}}{16\,a\,x^6}+\frac{5\,d\,{\left(b\,x^2+a\right)}^{3/2}}{6\,a^2\,x^6}-\frac{5\,d\,{\left(b\,x^2+a\right)}^{5/2}}{16\,a^3\,x^6}-\frac{5\,e\,\sqrt{b\,x^2+a}}{8\,a\,x^4}+\frac{3\,e\,{\left(b\,x^2+a\right)}^{3/2}}{8\,a^2\,x^4}-\frac{f\,\sqrt{b\,x^2+a}}{2\,a\,x^2}+\frac{b\,f\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}-\frac{3\,b^2\,e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}+\frac{b^4\,c\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,35{}\mathrm{i}}{128\,a^{9/2}}-\frac{b^3\,d\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,5{}\mathrm{i}}{16\,a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^9*(a + b*x^2)^(1/2)),x)
```

```
[Out] (511*c*(a + b*x^2)^(3/2))/(384*a^2*x^8) - (93*c*(a + b*x^2)^(1/2))/(128*a*x
^8) - (385*c*(a + b*x^2)^(5/2))/(384*a^3*x^8) + (35*c*(a + b*x^2)^(7/2))/(1
28*a^4*x^8) - (11*d*(a + b*x^2)^(1/2))/(16*a*x^6) + (5*d*(a + b*x^2)^(3/2))
/(6*a^2*x^6) - (5*d*(a + b*x^2)^(5/2))/(16*a^3*x^6) - (5*e*(a + b*x^2)^(1/2
))/(8*a*x^4) + (3*e*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (f*(a + b*x^2)^(1/2))/
```

```
(2*a*x^2) + (b*f*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) + (b^4*c*ata
n(((a + b*x^2)^(1/2)*1i)/a^(1/2))*35i)/(128*a^(9/2)) - (b^3*d*atan(((a + b*
x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(7/2)) - (3*b^2*e*atanh((a + b*x^2)^(1/2)
/a^(1/2)))/(8*a^(5/2))
```

3.151 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=245

$$-\frac{a(96b^3c-80ab^2d+70a^2be-63a^3f)\,x\sqrt{a+bx^2}}{256b^5}+\frac{(96b^3c-80ab^2d+70a^2be-63a^3f)\,x^3\sqrt{a+bx^2}}{384b^4}+(80$$

```
[Out] 1/256*a^2*(-63*a^3*f+70*a^2*b*e-80*a*b^2*d+96*b^3*c)*arctanh(x*b^(1/2)/(b*x
^2+a)^(1/2))/b^(11/2)-1/256*a*(-63*a^3*f+70*a^2*b*e-80*a*b^2*d+96*b^3*c)*x*
(b*x^2+a)^(1/2)/b^5+1/384*(-63*a^3*f+70*a^2*b*e-80*a*b^2*d+96*b^3*c)*x^3*(b
*x^2+a)^(1/2)/b^4+1/480*(63*a^2*f-70*a*b*e+80*b^2*d)*x^5*(b*x^2+a)^(1/2)/b^
3+1/80*(-9*a*f+10*b*e)*x^7*(b*x^2+a)^(1/2)/b^2+1/10*f*x^9*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.18, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1823, 1281, 470, 327, 223, 212}

$$\frac{x^5\sqrt{a+bx^2}\,(63a^2f-70abe+80b^2d)}{480b^3}+\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)(-63a^3f+70a^2be-80ab^2d+96b^3c)}{256b^{11/2}}-\frac{ax\sqrt{a+bx^2}\,(-63a^3f+70a^2be-80ab^2d+96b^3c)}{256b^5}+\frac{x^3\sqrt{a+bx^2}\,(-63a^3f+70a^2be-80ab^2d+96b^3c)}{384b^4}+\frac{x^7\sqrt{a+bx^2}\,(10be-9af)}{80b^2}+\frac{fx^9\sqrt{a+bx^2}}{10b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] -1/256*(a*(96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*x*Sqrt[a + b*x^2]
)/b^5 + ((96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*x^3*Sqrt[a + b*x^2
])/(384*b^4) + ((80*b^2*d - 70*a*b*e + 63*a^2*f)*x^5*Sqrt[a + b*x^2])/(480*
b^3) + ((10*b*e - 9*a*f)*x^7*Sqrt[a + b*x^2])/(80*b^2) + (f*x^9*Sqrt[a + b*
x^2])/(10*b) + (a^2*(96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*b^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{fx^9\sqrt{a+bx^2}}{10b} + \frac{\int \frac{x^4\left(10bc+10bdx^2+(10be-9af)x^4\right)}{\sqrt{a+bx^2}}\,dx}{10b}$$

$$= \frac{(10be-9af)x^7\sqrt{a+bx^2}}{80b^2} + \frac{fx^9\sqrt{a+bx^2}}{10b} + \frac{\int \frac{x^4\left(80b^2c+\left(80b^2d-70abe+63a^2f\right.}{\sqrt{a+bx^2}}}{80b^2}$$

$$= \frac{\left(80b^2d-70abe+63a^2f\right)x^5\sqrt{a+bx^2}}{480b^3} + \frac{(10be-9af)x^7\sqrt{a+bx^2}}{80b^2} + fx$$

$$= \frac{\left(96b^3c-80ab^2d+70a^2be-63a^3f\right)x^3\sqrt{a+bx^2}}{384b^4} + \frac{(80b^2d-70abe+63a}{480b^3}$$

$$= -\frac{a\left(96b^3c-80ab^2d+70a^2be-63a^3f\right)x\sqrt{a+bx^2}}{256b^5} + (96b^3c-80ab^2d+$$

$$= -\frac{a\left(96b^3c-80ab^2d+70a^2be-63a^3f\right)x\sqrt{a+bx^2}}{256b^5} + (96b^3c-80ab^2d+$$

$$= -\frac{a\left(96b^3c-80ab^2d+70a^2be-63a^3f\right)x\sqrt{a+bx^2}}{256b^5} + (96b^3c-80ab^2d+$$

Mathematica [A]
time = 0.34, size = 186, normalized size = 0.76

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\left(945a^4f-210a^3b(5e+3fx^2)+4a^2b^2(300d+175ex^2+126fx^4)+32b^4x^2(30c+20dx^2+15ex^4+12fx^6)-16ab^3(90c+50dx^2+35ex^4+27fx^6)\right)+15a^2\left(-96b^3c+80ab^2d-70a^2be+63a^3f\right)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{3840b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(945*a^4*f - 210*a^3*b*(5*e + 3*f*x^2) + 4*a^2*b
^2*(300*d + 175*e*x^2 + 126*f*x^4) + 32*b^4*x^2*(30*c + 20*d*x^2 + 15*e*x^4
 + 12*f*x^6) - 16*a*b^3*(90*c + 50*d*x^2 + 35*e*x^4 + 27*f*x^6)) + 15*a^2*(
-96*b^3*c + 80*a*b^2*d - 70*a^2*b*e + 63*a^3*f)*Log[-(Sqrt[b]*x) + Sqrt[a +
 b*x^2]])/(3840*b^(11/2))
```

Maple [A]
time = 0.12, size = 402, normalized size = 1.64

method	result
risch	$\frac{x\left(384f\,x^8b^4-432a\,b^3f\,x^6+480b^4e\,x^6+504a^2b^2f\,x^4-560a\,b^3e\,x^4+640b^4d\,x^4-630a^3bf\,x^2+700a^2b^2e\,x^2-800a\,b^3d\,x^2+960b^4c\,x^2+94\right.}{3840b^5}$

default	$f\left(\frac{x^{9}\sqrt{b\,x^{2}+a}}{10b}-\frac{9a\left(\frac{x^{7}\sqrt{b\,x^{2}+a}}{8b}-\frac{7a\left(\frac{x^{5}\sqrt{b\,x^{2}+a}}{6b}-\frac{5a\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\right.}{2}\right.}{4b}\right.}{6b}\right.}{8b}\right.}{10b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/10*x^9/b*(b*x^2+a)^(1/2)-9/10*a/b*(1/8*x^7/b*(b*x^2+a)^(1/2)-7/8*a/b*(
1/6*x^5/b*(b*x^2+a)^(1/2)-5/6*a/b*(1/4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x
*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))))+e*(1/8*
x^7/b*(b*x^2+a)^(1/2)-7/8*a/b*(1/6*x^5/b*(b*x^2+a)^(1/2)-5/6*a/b*(1/4*x^3/b
*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2
)+(b*x^2+a)^(1/2))))))+d*(1/6*x^5/b*(b*x^2+a)^(1/2)-5/6*a/b*(1/4*x^3/b*(b*x
^2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*
x^2+a)^(1/2)))))+c*(1/4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2
)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))
```

Maxima [A]

time = 0.30, size = 344, normalized size = 1.40

$$\frac{\sqrt{bx^2+a}\,fx^9}{10\,b}-\frac{9\,\sqrt{bx^2+a}\,afx^7}{80\,b^2}+\frac{\sqrt{bx^2+a}\,x^7e}{8\,b}+\frac{\sqrt{bx^2+a}\,dx^5}{6\,b}+\frac{21\,\sqrt{bx^2+a}\,a^2fx^5}{160\,b^3}-\frac{7\,\sqrt{bx^2+a}\,ax^5e}{48\,b^2}+\frac{\sqrt{bx^2+a}\,cx^3}{4\,b}-\frac{5\,\sqrt{bx^2+a}\,adx^3}{24\,b^2}-\frac{21\,\sqrt{bx^2+a}\,a^3fx^3}{128\,b^4}+\frac{35\,\sqrt{bx^2+a}\,a^2x^3e}{192\,b^3}-\frac{3\,\sqrt{bx^2+a}\,acx}{8\,b^2}+\frac{5\,\sqrt{bx^2+a}\,a^2dx}{16\,b^3}+\frac{63\,\sqrt{bx^2+a}\,a^4fx}{256\,b^5}+\frac{3\,a^2c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{5\,a^3d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}}-\frac{63\,a^5f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{11}{2}}}-\frac{35\,\sqrt{bx^2+a}\,a^3xe}{128\,b^4}+\frac{35\,a^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)e}{128\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(b*x^2 + a)*f*x^9/b - 9/80*sqrt(b*x^2 + a)*a*f*x^7/b^2 + 1/8*sqrt(
b*x^2 + a)*x^7*e/b + 1/6*sqrt(b*x^2 + a)*d*x^5/b + 21/160*sqrt(b*x^2 + a)*a
^2*f*x^5/b^3 - 7/48*sqrt(b*x^2 + a)*a*x^5*e/b^2 + 1/4*sqrt(b*x^2 + a)*c*x^3
/b - 5/24*sqrt(b*x^2 + a)*a*d*x^3/b^2 - 21/128*sqrt(b*x^2 + a)*a^3*f*x^3/b^
4 + 35/192*sqrt(b*x^2 + a)*a^2*x^3*e/b^3 - 3/8*sqrt(b*x^2 + a)*a*c*x/b^2 +
5/16*sqrt(b*x^2 + a)*a^2*d*x/b^3 + 63/256*sqrt(b*x^2 + a)*a^4*f*x/b^5 + 3/8
*a^2*c*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/16*a^3*d*arcsinh(b*x/sqrt(a*b))/b
^(7/2) - 63/256*a^5*f*arcsinh(b*x/sqrt(a*b))/b^(11/2) - 35/128*sqrt(b*x^2 +
 a)*a^3*x*e/b^4 + 35/128*a^4*arcsinh(b*x/sqrt(a*b))*e/b^(9/2)
```

Fricas [A]

time = 1.20, size = 432, normalized size = 1.76

$$\left[\frac{15\,(96\,a^2b^3c-80\,a^3b^2d-63\,a^5f+70\,a^4be)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(384\,b^5fx^9-432\,ab^4fx^7+8\,(80\,b^5d+63\,a^2b^3f)x^5+10\,(96\,b^5c-80\,ab^4d-63\,a^3b^2f)x^3-15\,(96\,ab^4c-80\,a^2b^3d-63\,a^4bf)x+10\,(48\,b^5x^7-56\,ab^4x^5+70\,a^2b^3x^3-105\,a^3b^2x)e)\sqrt{bx^2+a}}{7680\,b^6},\,-\frac{15\,(96\,a^2b^3c-80\,a^3b^2d-63\,a^5f+70\,a^4be)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(384\,b^5fx^9-432\,ab^4fx^7+8\,(80\,b^5d+63\,a^2b^3f)x^5+10\,(96\,b^5c-80\,ab^4d-63\,a^3b^2f)x^3-15\,(96\,ab^4c-80\,a^2b^3d-63\,a^4bf)x+10\,(48\,b^5x^7-56\,ab^4x^5+70\,a^2b^3x^3-105\,a^3b^2x)e)\sqrt{bx^2+a}}{3840\,b^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/7680*(15*(96*a^2*b^3*c - 80*a^3*b^2*d - 63*a^5*f + 70*a^4*b*e)*sqrt(b)*l
og(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(384*b^5*f*x^9 - 432*a*b
^4*f*x^7 + 8*(80*b^5*d + 63*a^2*b^3*f)*x^5 + 10*(96*b^5*c - 80*a*b^4*d - 63
*a^3*b^2*f)*x^3 - 15*(96*a*b^4*c - 80*a^2*b^3*d - 63*a^4*b*f)*x + 10*(48*b^
5*x^7 - 56*a*b^4*x^5 + 70*a^2*b^3*x^3 - 105*a^3*b^2*x)*e)*sqrt(b*x^2 + a))/
b^6, -1/3840*(15*(96*a^2*b^3*c - 80*a^3*b^2*d - 63*a^5*f + 70*a^4*b*e)*sqrt
(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (384*b^5*f*x^9 - 432*a*b^4*f*x^7
+ 8*(80*b^5*d + 63*a^2*b^3*f)*x^5 + 10*(96*b^5*c - 80*a*b^4*d - 63*a^3*b^2*
f)*x^3 - 15*(96*a*b^4*c - 80*a^2*b^3*d - 63*a^4*b*f)*x + 10*(48*b^5*x^7 - 5
6*a*b^4*x^5 + 70*a^2*b^3*x^3 - 105*a^3*b^2*x)*e)*sqrt(b*x^2 + a))/b^6]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.87, size = 224, normalized size = 0.91

$$\frac{1}{3840}\left(2\left(4\left(6\left(\frac{8\,fx^2}{b}-\frac{9\,ab^7f-10\,b^8e}{b^9}\right)x^2+\frac{80\,b^8d+63\,a^2b^6f-70\,ab^7e}{b^9}\right)x^2+\frac{5\,(96\,b^8c-80\,ab^7d-63\,a^3b^5f+70\,a^2b^6e)}{b^9}\right)x^2-\frac{15\,(96\,ab^7c-80\,a^2b^6d-63\,a^4b^4f+70\,a^3b^5e)}{b^9}\right)\sqrt{bx^2+a}\,x-\frac{(96\,a^2b^3c-80\,a^3b^2d-63\,a^5f+70\,a^4be)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3840*(2*(4*(6*(8*f*x^2/b - (9*a*b^7*f - 10*b^8*e)/b^9)*x^2 + (80*b^8*d +
63*a^2*b^6*f - 70*a*b^7*e)/b^9)*x^2 + 5*(96*b^8*c - 80*a*b^7*d - 63*a^3*b^5
*f + 70*a^2*b^6*e)/b^9)*x^2 - 15*(96*a*b^7*c - 80*a^2*b^6*d - 63*a^4*b^4*f
+ 70*a^3*b^5*e)/b^9)*sqrt(b*x^2 + a)*x - 1/256*(96*a^2*b^3*c - 80*a^3*b^2*d
 - 63*a^5*f + 70*a^4*b*e)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(f\,x^6+e\,x^4+d\,x^2+c\right)}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2), x)
```

3.152 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=194

$$\frac{(64b^3c-48ab^2d+40a^2be-35a^3f)\,x\sqrt{a+bx^2}}{128b^4}+\frac{(48b^2d-40abe+35a^2f)\,x^3\sqrt{a+bx^2}}{192b^3}+\frac{(8be-7af)x^5\sqrt{a}}{48b^2}$$

```
[Out] -1/128*a*(-35*a^3*f+40*a^2*b*e-48*a*b^2*d+64*b^3*c)*arctanh(x*b^(1/2)/(b*x^
2+a)^(1/2))/b^(9/2)+1/128*(-35*a^3*f+40*a^2*b*e-48*a*b^2*d+64*b^3*c)*x*(b*x
^2+a)^(1/2)/b^4+1/192*(35*a^2*f-40*a*b*e+48*b^2*d)*x^3*(b*x^2+a)^(1/2)/b^3+
1/48*(-7*a*f+8*b*e)*x^5*(b*x^2+a)^(1/2)/b^2+1/8*f*x^7*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.15, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1823, 1281, 470, 327, 223, 212}

$$\frac{x^3\sqrt{a+bx^2}\,(35a^2f-40abe+48b^2d)}{192b^3}-\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)(-35a^3f+40a^2be-48ab^2d+64b^3c)}{128b^{9/2}}+\frac{x\sqrt{a+bx^2}\,(-35a^3f+40a^2be-48ab^2d+64b^3c)}{128b^4}+\frac{x^5\sqrt{a+bx^2}\,(8be-7af)}{48b^2}+\frac{fx^7\sqrt{a+bx^2}}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] ((64*b^3*c - 48*a*b^2*d + 40*a^2*b*e - 35*a^3*f)*x*Sqrt[a + b*x^2])/(128*b^
4) + ((48*b^2*d - 40*a*b*e + 35*a^2*f)*x^3*Sqrt[a + b*x^2])/(192*b^3) + ((8
*b*e - 7*a*f)*x^5*Sqrt[a + b*x^2])/(48*b^2) + (f*x^7*Sqrt[a + b*x^2])/(8*b)
 - (a*(64*b^3*c - 48*a*b^2*d + 40*a^2*b*e - 35*a^3*f)*ArcTanh[(Sqrt[b]*x)/S
qrt[a + b*x^2]])/(128*b^(9/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx &= \frac{fx^7\sqrt{a+bx^2}}{8b}+\frac{\int \frac{x^2\left(8bc+8bdx^2+(8be-7af)x^4\right)}{\sqrt{a+bx^2}}\,dx}{8b}\\ &= \frac{(8be-7af)x^5\sqrt{a+bx^2}}{48b^2}+\frac{fx^7\sqrt{a+bx^2}}{8b}+\\ &= \frac{\left(48b^2d-40abe+35a^2f\right)x^3\sqrt{a+bx^2}}{192b^3}+\frac{(8be-7af)x^5\sqrt{a+bx^2}}{48b^2}+\\ &= \frac{\left(64c-\frac{a\left(48b^2d-40abe+35a^2f\right)}{b^3}\right)x\sqrt{a+bx^2}}{128b}+\\ &= \frac{\left(64c-\frac{a\left(48b^2d-40abe+35a^2f\right)}{b^3}\right)x\sqrt{a+bx^2}}{128b}+\\ &= \frac{\left(64c-\frac{a\left(48b^2d-40abe+35a^2f\right)}{b^3}\right)x\sqrt{a+bx^2}}{128b}+\end{aligned}$$

Mathematica [A]

time = 0.28, size = 151, normalized size = 0.78

$$\frac{\sqrt{b}\,x\sqrt{a+bx^2}\,(-105a^3f+10a^2b(12e+7fx^2)-8ab^2(18d+10ex^2+7fx^4)+16b^3(12c+6dx^2+4ex^4+3fx^6))-3a(-64b^3c+48ab^2d-40a^2be+35a^3f)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{384b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[b]*x*Sqrt[a + b*x^2]*(-105*a^3*f + 10*a^2*b*(12*e + 7*f*x^2) - 8*a*b^
2*(18*d + 10*e*x^2 + 7*f*x^4) + 16*b^3*(12*c + 6*d*x^2 + 4*e*x^4 + 3*f*x^6)
) - 3*a*(-64*b^3*c + 48*a*b^2*d - 40*a^2*b*e + 35*a^3*f)*Log[-(Sqrt[b]*x) +
 Sqrt[a + b*x^2]])/(384*b^(9/2))
```

Maple [A]

time = 0.12, size = 306, normalized size = 1.58

method	result
risch	$-\frac{x\left(-48f\,x^6b^3+56a\,b^2f\,x^4-64b^3e\,x^4-70a^2bf\,x^2+80a\,b^2e\,x^2-96b^3d\,x^2+105a^3f-120a^2be+144a\,b^2d-192b^3c\right)\sqrt{b\,x^2+a}}{384b^4}+$

default	$f\left(\frac{x^{7} \sqrt{b\,x^{2}+a}}{8 b}-\frac{7 a \left(\frac{x^{5} \sqrt{b\,x^{2}+a}}{6 b}-\frac{5 a \left(\frac{x^{3} \sqrt{b\,x^{2}+a}}{4 b}-\frac{3 a \left(\frac{x \sqrt{b\,x^{2}+a}}{2 b}-\frac{a \ln \left(x \sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2 b^{\frac{3}{2}}}\right)}{4 b}\right)}{6 b}\right)}{8 b}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/8*x^7/b*(b*x^2+a)^(1/2)-7/8*a/b*(1/6*x^5/b*(b*x^2+a)^(1/2)-5/6*a/b*(1/
4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x
*b^(1/2)+(b*x^2+a)^(1/2))))))+e*(1/6*x^5/b*(b*x^2+a)^(1/2)-5/6*a/b*(1/4*x^3
/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1
/2)+(b*x^2+a)^(1/2)))))+d*(1/4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+
a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))+c*(1/2*x*(b*x^2+a)
^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))
```

Maxima [A]

time = 0.29, size = 259, normalized size = 1.34

$$\frac{\sqrt{bx^2+a}\,fx^7}{8\,b}-\frac{7\,\sqrt{bx^2+a}\,afx^5}{48\,b^2}+\frac{\sqrt{bx^2+a}\,x^5e}{6\,b}+\frac{\sqrt{bx^2+a}\,dx^3}{4\,b}+\frac{35\,\sqrt{bx^2+a}\,a^2fx^3}{192\,b^3}-\frac{5\,\sqrt{bx^2+a}\,ax^3e}{24\,b^2}+\frac{\sqrt{bx^2+a}\,cx}{2\,b}-\frac{3\,\sqrt{bx^2+a}\,adx}{8\,b^2}-\frac{35\,\sqrt{bx^2+a}\,a^3fx}{128\,b^4}-\frac{ac\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{3\,a^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}+\frac{35\,a^4f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{9}{2}}}+\frac{5\,\sqrt{bx^2+a}\,a^2xe}{16\,b^3}-\frac{5\,a^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)e}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*f*x^7/b - 7/48*sqrt(b*x^2 + a)*a*f*x^5/b^2 + 1/6*sqrt(b
*x^2 + a)*x^5*e/b + 1/4*sqrt(b*x^2 + a)*d*x^3/b + 35/192*sqrt(b*x^2 + a)*a^
2*f*x^3/b^3 - 5/24*sqrt(b*x^2 + a)*a*x^3*e/b^2 + 1/2*sqrt(b*x^2 + a)*c*x/b
- 3/8*sqrt(b*x^2 + a)*a*d*x/b^2 - 35/128*sqrt(b*x^2 + a)*a^3*f*x/b^4 - 1/2*
```

```
a*c*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/8*a^2*d*arcsinh(b*x/sqrt(a*b))/b^(5/
2) + 35/128*a^4*f*arcsinh(b*x/sqrt(a*b))/b^(9/2) + 5/16*sqrt(b*x^2 + a)*a^2
*x*e/b^3 - 5/16*a^3*arcsinh(b*x/sqrt(a*b))*e/b^(7/2)
```

Fricas [**A**]
time = 2.73, size = 343, normalized size = 1.77

$$\left[\frac{3\,(64\,ab^3c-48\,a^2b^2d-35\,a^4f+40\,a^3be)\sqrt{b}\,\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(48\,b^4fx^7-56\,ab^3fx^5+2\,(48\,b^4d+35\,a^2b^2f)x^3+3\,(64\,b^4c-48\,ab^3d-35\,a^3bf)x+8\,(8\,b^4x^5-10\,ab^3x^3+15\,a^2b^2x)e)\sqrt{bx^2+a}}{768\,b^5}, \frac{3\,(64\,ab^3c-48\,a^2b^2d-35\,a^4f+40\,a^3be)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(48\,b^4fx^7-56\,ab^3fx^5+2\,(48\,b^4d+35\,a^2b^2f)x^3+3\,(64\,b^4c-48\,ab^3d-35\,a^3bf)x+8\,(8\,b^4x^5-10\,ab^3x^3+15\,a^2b^2x)e)\sqrt{bx^2+a}}{384\,b^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(64*a*b^3*c - 48*a^2*b^2*d - 35*a^4*f + 40*a^3*b*e)*sqrt(b)*log(-
2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(48*b^4*f*x^7 - 56*a*b^3*f*x
^5 + 2*(48*b^4*d + 35*a^2*b^2*f)*x^3 + 3*(64*b^4*c - 48*a*b^3*d - 35*a^3*b*
f)*x + 8*(8*b^4*x^5 - 10*a*b^3*x^3 + 15*a^2*b^2*x)*e)*sqrt(b*x^2 + a))/b^5,
 1/384*(3*(64*a*b^3*c - 48*a^2*b^2*d - 35*a^4*f + 40*a^3*b*e)*sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)) + (48*b^4*f*x^7 - 56*a*b^3*f*x^5 + 2*(48*b^4
*d + 35*a^2*b^2*f)*x^3 + 3*(64*b^4*c - 48*a*b^3*d - 35*a^3*b*f)*x + 8*(8*b^
4*x^5 - 10*a*b^3*x^3 + 15*a^2*b^2*x)*e)*sqrt(b*x^2 + a))/b^5]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(196) = 392.
time = 47.78, size = 444, normalized size = 2.29

$$-\frac{35a^{\frac{7}{2}}fx}{128b^4\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{5}{2}}ex}{16b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{35a^{\frac{5}{2}}fx^3}{384b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{3}{2}}dx}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{3}{2}}ex^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{7a^{\frac{3}{2}}fx^5}{192b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}\,cx\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{\sqrt{a}\,dx^3}{8b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,ex^5}{24b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,fx^7}{48b\sqrt{1+\frac{bx^2}{a}}}+\frac{35a^4f\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{128b^{\frac{9}{2}}}-\frac{5a^3e\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{7}{2}}}+\frac{3a^2d\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{ac\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+\frac{dx^5}{4\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}+\frac{ex^7}{6\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}+\frac{fx^9}{8\sqrt{a}\,\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] -35*a**(7/2)*f*x/(128*b**4*sqrt(1 + b*x**2/a)) + 5*a**(5/2)*e*x/(16*b**3*sq
rt(1 + b*x**2/a)) - 35*a**(5/2)*f*x**3/(384*b**3*sqrt(1 + b*x**2/a)) - 3*a*
*(3/2)*d*x/(8*b**2*sqrt(1 + b*x**2/a)) + 5*a**(3/2)*e*x**3/(48*b**2*sqrt(1
+ b*x**2/a)) + 7*a**(3/2)*f*x**5/(192*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*c*
x*sqrt(1 + b*x**2/a)/(2*b) - sqrt(a)*d*x**3/(8*b*sqrt(1 + b*x**2/a)) - sqrt
(a)*e*x**5/(24*b*sqrt(1 + b*x**2/a)) - sqrt(a)*f*x**7/(48*b*sqrt(1 + b*x**2
/a)) + 35*a**4*f*asinh(sqrt(b)*x/sqrt(a))/(128*b**(9/2)) - 5*a**3*e*asinh(s
qrt(b)*x/sqrt(a))/(16*b**(7/2)) + 3*a**2*d*asinh(sqrt(b)*x/sqrt(a))/(8*b**(
5/2)) - a*c*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)) + d*x**5/(4*sqrt(a)*sqrt(
1 + b*x**2/a)) + e*x**7/(6*sqrt(a)*sqrt(1 + b*x**2/a)) + f*x**9/(8*sqrt(a)*
sqrt(1 + b*x**2/a))
```

Giac [**A**]
time = 1.40, size = 175, normalized size = 0.90

$$\frac{1}{384}\left(2\left(4\left(\frac{6\,fx^2}{b}-\frac{7\,ab^5f-8\,b^6e}{b^7}\right)x^2+\frac{48\,b^6d+35\,a^2b^4f-40\,ab^5e}{b^7}\right)x^2+\frac{3\,(64\,b^6c-48\,ab^5d-35\,a^3b^3f+40\,a^2b^4e)}{b^7}\right)\sqrt{bx^2+a}\,x+\frac{(64\,ab^3c-48\,a^2b^2d-35\,a^4f+40\,a^3be)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*(2*(4*(6*f*x^2/b - (7*a*b^5*f - 8*b^6*e)/b^7)*x^2 + (48*b^6*d + 35*a^
2*b^4*f - 40*a*b^5*e)/b^7)*x^2 + 3*(64*b^6*c - 48*a*b^5*d - 35*a^3*b^3*f +
40*a^2*b^4*e)/b^7)*sqrt(b*x^2 + a)*x + 1/128*(64*a*b^3*c - 48*a^2*b^2*d - 3
5*a^4*f + 40*a^3*b*e)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(f\,x^6+e\,x^4+d\,x^2+c\right)}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

```
[Out] int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2), x)
```

3.153 $\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=145

$$\frac{(8b^2d-6abe+5a^2f)\,x\sqrt{a+bx^2}}{16b^3}+\frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2}+\frac{fx^5\sqrt{a+bx^2}}{6b}+\frac{(16b^3c-8ab^2d+6a^2be-5a}{16b^7}$$

```
[Out] 1/16*(-5*a^3*f+6*a^2*b*e-8*a*b^2*d+16*b^3*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(1
/2))/b^(7/2)+1/16*(5*a^2*f-6*a*b*e+8*b^2*d)*x*(b*x^2+a)^(1/2)/b^3+1/24*(-5*
a*f+6*b*e)*x^3*(b*x^2+a)^(1/2)/b^2+1/6*f*x^5*(b*x^2+a)^(1/2)/b
```

Rubi [A]
time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$,
Rules used = {1829, 1173, 396, 223, 212}

$$\frac{x\sqrt{a+bx^2}\,(5a^2f-6abe+8b^2d)}{16b^3}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)(-5a^3f+6a^2be-8ab^2d+16b^3c)}{16b^{7/2}}+\frac{x^3\sqrt{a+bx^2}\,(6be-5af)}{24b^2}+\frac{fx^5\sqrt{a+bx^2}}{6b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/Sqrt[a + b*x^2],x]
```

```
[Out] ((8*b^2*d - 6*a*b*e + 5*a^2*f)*x*Sqrt[a + b*x^2])/(16*b^3) + ((6*b*e - 5*a*
f)*x^3*Sqrt[a + b*x^2])/(24*b^2) + (f*x^5*Sqrt[a + b*x^2])/(6*b) + ((16*b^3
*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])
/(16*b^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 1829

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx = \frac{fx^5\sqrt{a+bx^2}}{6b} + \frac{\int \frac{6bc+6bdx^2+(6be-5af)x^4}{\sqrt{a+bx^2}}\,dx}{6b}$$

$$= \frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b} + \frac{\int \frac{24b^2c+3(8b^2d-6abe+5a^2f)x^2}{\sqrt{a+bx^2}}\,dx}{24b^2}$$

$$= \frac{(8b^2d-6abe+5a^2f)\,x\sqrt{a+bx^2}}{16b^3} + \frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b}$$

$$= \frac{(8b^2d-6abe+5a^2f)\,x\sqrt{a+bx^2}}{16b^3} + \frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b}$$

$$= \frac{(8b^2d-6abe+5a^2f)\,x\sqrt{a+bx^2}}{16b^3} + \frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b}$$

Mathematica [A]
time = 0.16, size = 121, normalized size = 0.83

$$\frac{x\sqrt{a+bx^2}\,(24b^2d-18abe+15a^2f+12b^2ex^2-10abfx^2+8b^2fx^4)}{48b^3} + \frac{(-16b^3c+8ab^2d-6a^2be+5a^3f)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{16b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[a + b*x^2]*(24*b^2*d - 18*a*b*e + 15*a^2*f + 12*b^2*e*x^2 - 10*a*b*
f*x^2 + 8*b^2*f*x^4))/(48*b^3) + ((-16*b^3*c + 8*a*b^2*d - 6*a^2*b*e + 5*a^
3*f)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(16*b^(7/2))
```

Maple [A]
time = 0.12, size = 215, normalized size = 1.48

method	result
risch	$\frac{x\left(8f\,b^2x^4-10abf\,x^2+12b^2e\,x^2+15a^2f-18abe+24b^2d\right)\sqrt{b\,x^2+a}}{48b^3}-\frac{5\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)a^3f}{16b^{\frac{7}{2}}}+\frac{3\ln\left(x\sqrt{b}+\sqrt{b}\right.}{8b^{\frac{5}{2}}}$
default	$f\left(\frac{x^5\sqrt{b\,x^2+a}}{6b}-\frac{5a\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)}{6b}\right)+e\left(\frac{x^3\sqrt{b\,x^2+}}{4b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/6*x^5/b*(b*x^2+a)^(1/2)-5/6*a/b*(1/4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/
2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+e*(1/4
*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*
b^(1/2)+(b*x^2+a)^(1/2))))+d*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^
(1/2)+(b*x^2+a)^(1/2)))+c*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

Maxima [A]
time = 0.27, size = 177, normalized size = 1.22

$$\frac{\sqrt{bx^2+a}\,fx^5}{6\,b}-\frac{5\,\sqrt{bx^2+a}\,afx^3}{24\,b^2}+\frac{\sqrt{bx^2+a}\,x^3e}{4\,b}+\frac{\sqrt{bx^2+a}\,dx}{2\,b}+\frac{5\,\sqrt{bx^2+a}\,a^2fx}{16\,b^3}+\frac{c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{ad\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}-\frac{5\,a^3f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}}-\frac{3\,\sqrt{bx^2+a}\,axe}{8\,b^2}+\frac{3\,a^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)e}{8\,b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(b*x^2 + a)*f*x^5/b - 5/24*sqrt(b*x^2 + a)*a*f*x^3/b^2 + 1/4*sqrt(b
*x^2 + a)*x^3*e/b + 1/2*sqrt(b*x^2 + a)*d*x/b + 5/16*sqrt(b*x^2 + a)*a^2*f*
x/b^3 + c*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2*a*d*arcsinh(b*x/sqrt(a*b))/b
^(3/2) - 5/16*a^3*f*arcsinh(b*x/sqrt(a*b))/b^(7/2) - 3/8*sqrt(b*x^2 + a)*a*
x*e/b^2 + 3/8*a^2*arcsinh(b*x/sqrt(a*b))*e/b^(5/2)
```

Fricas [A]

time = 2.44, size = 260, normalized size = 1.79

$$\left[\frac{3\,(16\,b^3c-8\,ab^2d-5\,a^3f+6\,a^2be)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(8\,b^3fx^5-10\,ab^2fx^3+3\,(8\,b^3d+5\,a^2bf)x+6\,(2\,b^3x^3-3\,ab^2x)e)\sqrt{bx^2+a}}{96\,b^4},\, -\frac{3\,(16\,b^3c-8\,ab^2d-5\,a^3f+6\,a^2be)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)-(8\,b^3fx^5-10\,ab^2fx^3+3\,(8\,b^3d+5\,a^2bf)x+6\,(2\,b^3x^3-3\,ab^2x)e)\sqrt{bx^2+a}}{48\,b^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(16*b^3*c - 8*a*b^2*d - 5*a^3*f + 6*a^2*b*e)*sqrt(b)*log(-2*b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(8*b^3*f*x^5 - 10*a*b^2*f*x^3 + 3*(8
*b^3*d + 5*a^2*b*f)*x + 6*(2*b^3*x^3 - 3*a*b^2*x)*e)*sqrt(b*x^2 + a))/b^4,
-1/48*(3*(16*b^3*c - 8*a*b^2*d - 5*a^3*f + 6*a^2*b*e)*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - (8*b^3*f*x^5 - 10*a*b^2*f*x^3 + 3*(8*b^3*d + 5*a^2
*b*f)*x + 6*(2*b^3*x^3 - 3*a*b^2*x)*e)*sqrt(b*x^2 + a))/b^4]
```

Sympy [A]

time = 11.34, size = 362, normalized size = 2.50

$$\frac{5a^{\frac{5}{2}}fx}{16b^3\sqrt{1+\frac{bx^2}{a}}}-\frac{3a^{\frac{3}{2}}ex}{8b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{5a^{\frac{3}{2}}fx^3}{48b^2\sqrt{1+\frac{bx^2}{a}}}+\frac{\sqrt{a}\,dx\sqrt{1+\frac{bx^2}{a}}}{2b}-\frac{\sqrt{a}\,ex^3}{8b\sqrt{1+\frac{bx^2}{a}}}-\frac{\sqrt{a}\,fx^5}{24b\sqrt{1+\frac{bx^2}{a}}}-\frac{5a^3f\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{16b^{\frac{7}{2}}}+\frac{3a^2e\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}}-\frac{ad\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}}+c\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\ b>0\wedge a<0\end{cases}\right)+\frac{ex^5}{4\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}+\frac{fx^7}{6\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

```
[Out] 5*a**(5/2)*f*x/(16*b**3*sqrt(1 + b*x**2/a)) - 3*a**(3/2)*e*x/(8*b**2*sqrt(1
 + b*x**2/a)) + 5*a**(3/2)*f*x**3/(48*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*d*
x*sqrt(1 + b*x**2/a)/(2*b) - sqrt(a)*e*x**3/(8*b*sqrt(1 + b*x**2/a)) - sqrt
(a)*f*x**5/(24*b*sqrt(1 + b*x**2/a)) - 5*a**3*f*asinh(sqrt(b)*x/sqrt(a))/(1
6*b**(7/2)) + 3*a**2*e*asinh(sqrt(b)*x/sqrt(a))/(8*b**(5/2)) - a*d*asinh(sq
rt(b)*x/sqrt(a))/(2*b**(3/2)) + c*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/
sqrt(a), (a > 0) & (b < 0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0)
 & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0)))
+ e*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a)) + f*x**7/(6*sqrt(a)*sqrt(1 + b*x**2
/a))
```

Giac [A]

time = 1.25, size = 129, normalized size = 0.89

$$\frac{1}{48}\left(2\left(\frac{4\,fx^2}{b}-\frac{5\,ab^3f-6\,b^4e}{b^5}\right)x^2+\frac{3\,(8\,b^4d+5\,a^2b^2f-6\,ab^3e)}{b^5}\right)\sqrt{bx^2+a}\,x-\frac{(16\,b^3c-8\,ab^2d-5\,a^3f+6\,a^2be)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*f*x^2/b - (5*a*b^3*f - 6*b^4*e)/b^5)*x^2 + 3*(8*b^4*d + 5*a^2*b^
2*f - 6*a*b^3*e)/b^5)*sqrt(b*x^2 + a)*x - 1/16*(16*b^3*c - 8*a*b^2*d - 5*a^
3*f + 6*a^2*b*e)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^6+e\,x^4+d\,x^2+c}{\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^(1/2),x)
```

```
[Out] int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^(1/2), x)
```

3.154 $\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=117

$$-\frac{c\sqrt{a+bx^2}}{ax}+\frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2}+\frac{fx^3\sqrt{a+bx^2}}{4b}+\frac{(8b^2d-4abe+3a^2f)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

```
[Out] 1/8*(3*a^2*f-4*a*b*e+8*b^2*d)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-c*
(b*x^2+a)^(1/2)/a/x+1/8*(-3*a*f+4*b*e)*x*(b*x^2+a)^(1/2)/b^2+1/4*f*x^3*(b*x
^2+a)^(1/2)/b
```

Rubi [A]

time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1821, 1599, 1173, 396, 223, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)(3a^2f-4abe+8b^2d)}{8b^{5/2}}+\frac{x\sqrt{a+bx^2}\,(4be-3af)}{8b^2}-\frac{c\sqrt{a+bx^2}}{ax}+\frac{fx^3\sqrt{a+bx^2}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] -((c*Sqrt[a + b*x^2])/(a*x)) + ((4*b*e - 3*a*f)*x*Sqrt[a + b*x^2])/(8*b^2)
+ (f*x^3*Sqrt[a + b*x^2])/(4*b) + ((8*b^2*d - 4*a*b*e + 3*a^2*f)*ArcTanh[(S
qrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx &= -\frac{c\sqrt{a+bx^2}}{ax}-\frac{\int \frac{-adx-aex^3-afx^5}{x\sqrt{a+bx^2}}\,dx}{a}\\
&= -\frac{c\sqrt{a+bx^2}}{ax}-\frac{\int \frac{-ad-aex^2-afx^4}{\sqrt{a+bx^2}}\,dx}{a}\\
&= -\frac{c\sqrt{a+bx^2}}{ax}+\frac{fx^3\sqrt{a+bx^2}}{4b}-\frac{\int \frac{-4abd-a(4be-3af)x^2}{\sqrt{a+bx^2}}\,dx}{4ab}\\
&= -\frac{c\sqrt{a+bx^2}}{ax}+\frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2}+\frac{fx^3\sqrt{a+bx^2}}{4b}+\frac{(8ab^2d-a^2(4be}{}\\
&= -\frac{c\sqrt{a+bx^2}}{ax}+\frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2}+\frac{fx^3\sqrt{a+bx^2}}{4b}+\frac{(8ab^2d-a^2(4be}{}\\
&= -\frac{c\sqrt{a+bx^2}}{ax}+\frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2}+\frac{fx^3\sqrt{a+bx^2}}{4b}+\frac{(8b^2d-4abe+}{}
\end{aligned}$$

Mathematica [A]
time = 0.21, size = 105, normalized size = 0.90

$$\frac{\sqrt{a+bx^2}\,(-8b^2c+4abex^2-3a^2fx^2+2abfx^4)}{8ab^2x}+\frac{(-8b^2d+4abe-3a^2f)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-8*b^2*c + 4*a*b*e*x^2 - 3*a^2*f*x^2 + 2*a*b*f*x^4))/(8*a
*b^2*x) + ((-8*b^2*d + 4*a*b*e - 3*a^2*f)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2
]])/(8*b^(5/2))
```

Maple [A]
time = 0.13, size = 145, normalized size = 1.24

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\;(-2abf\,x^4+3a^2f\,x^2-4abe\,x^2+8b^2c)}{8b^2ax}+\frac{3\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)a^2f}{8b^{\frac{5}{2}}}-\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)ae}{2b^{\frac{3}{2}}}+$
default	$f\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+e\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x\right.}{2b^{\frac{3}{2}}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/4*x^3/b*(b*x^2+a)^(1/2)-3/4*a/b*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)
*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))+e*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*l
n(x*b^(1/2)+(b*x^2+a)^(1/2)))+d*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)-c*(b*
x^2+a)^(1/2)/a/x
```

Maxima [A]
time = 0.27, size = 120, normalized size = 1.03

$$\frac{\sqrt{bx^2+a}\,fx^3}{4\,b}-\frac{3\,\sqrt{bx^2+a}\,afx}{8\,b^2}+\frac{d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}+\frac{3\,a^2f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}+\frac{\sqrt{bx^2+a}\,xe}{2\,b}-\frac{a\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)e}{2\,b^{\frac{3}{2}}}-\frac{\sqrt{bx^2+a}\,c}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(b*x^2 + a)*f*x^3/b - 3/8*sqrt(b*x^2 + a)*a*f*x/b^2 + d*arcsinh(b*x
/sqrt(a*b))/sqrt(b) + 3/8*a^2*f*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 1/2*sqrt(b
```

```
*x^2 + a)*x*e/b - 1/2*a*arcsinh(b*x/sqrt(a*b))*e/b^(3/2) - sqrt(b*x^2 + a)*
c/(a*x)
```

Fricas [A]
time = 3.30, size = 229, normalized size = 1.96

$$\left[\frac{(4\,a^2bxe - (8\,ab^2d + 3\,a^3f)x)\sqrt{b}\,\log\left(-2\,bx^2 + 2\,\sqrt{bx^2 + a}\,\sqrt{b}\,x - a\right) + 2\,(2\,ab^2fx^4 - 3\,a^2bfx^2 + 4\,ab^2x^2e - 8\,b^3c)\sqrt{bx^2 + a}}{16\,ab^3x}, \frac{(4\,a^2bxe - (8\,ab^2d + 3\,a^3f)x)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2 + a}}\right) + (2\,ab^2fx^4 - 3\,a^2bfx^2 + 4\,ab^2x^2e - 8\,b^3c)\sqrt{bx^2 + a}}{8\,ab^3x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((4*a^2*b*x*e - (8*a*b^2*d + 3*a^3*f)*x)*sqrt(b)*log(-2*b*x^2 + 2*sqr
t(b*x^2 + a)*sqrt(b)*x - a) + 2*(2*a*b^2*f*x^4 - 3*a^2*b*f*x^2 + 4*a*b^2*x^
2*e - 8*b^3*c)*sqrt(b*x^2 + a))/(a*b^3*x), 1/8*((4*a^2*b*x*e - (8*a*b^2*d +
 3*a^3*f)*x)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*a*b^2*f*x^4 -
 3*a^2*b*f*x^2 + 4*a*b^2*x^2*e - 8*b^3*c)*sqrt(b*x^2 + a))/(a*b^3*x)]
```

Sympy [A]
time = 4.23, size = 250, normalized size = 2.14

$$-\frac{3a^{\frac{3}{2}}fx}{8b^2\sqrt{1 + \frac{bx^2}{a}}} + \frac{\sqrt{a}\,ex\sqrt{1 + \frac{bx^2}{a}}}{2b} - \frac{\sqrt{a}\,fx^3}{8b\sqrt{1 + \frac{bx^2}{a}}} + \frac{3a^2f\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8b^{\frac{5}{2}}} - \frac{ae\operatorname{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} + d\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a > 0 \wedge b < 0\\ \frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a > 0 \wedge b > 0\\ \frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\ b > 0 \wedge a < 0\end{cases}\right) - \frac{\sqrt{b}\,c\sqrt{\frac{a}{bx^2} + 1}}{a} + \frac{fx^5}{4\sqrt{a}\,\sqrt{1 + \frac{bx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**(3/2)*f*x/(8*b**2*sqrt(1 + b*x**2/a)) + sqrt(a)*e*x*sqrt(1 + b*x**2/a
)/(2*b) - sqrt(a)*f*x**3/(8*b*sqrt(1 + b*x**2/a)) + 3*a**2*f*asinh(sqrt(b)*
x/sqrt(a))/(8*b**(5/2)) - a*e*asinh(sqrt(b)*x/sqrt(a))/(2*b**(3/2)) + d*Pie
cewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt(a/
b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*sqrt
(-b/a))/sqrt(-a), (b > 0) & (a < 0))) - sqrt(b)*c*sqrt(a/(b*x**2) + 1)/a +
f*x**5/(4*sqrt(a)*sqrt(1 + b*x**2/a))
```

Giac [A]
time = 1.27, size = 121, normalized size = 1.03

$$\frac{1}{8}\,\sqrt{bx^2 + a}\,\left(\frac{2\,fx^2}{b} - \frac{3\,abf - 4\,b^2e}{b^3}\right)x + \frac{2\,\sqrt{b}\,c}{\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^2 - a} - \frac{\left(8\,b^{\frac{5}{2}}d + 3\,a^2\sqrt{b}\,f - 4\,ab^{\frac{3}{2}}e\right)\log\left(\left(\sqrt{b}\,x - \sqrt{bx^2 + a}\right)^2\right)}{16\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(b*x^2 + a)*(2*f*x^2/b - (3*a*b*f - 4*b^2*e)/b^3)*x + 2*sqrt(b)*c/(
(sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) - 1/16*(8*b^(5/2)*d + 3*a^2*sqrt(b)*f
- 4*a*b^(3/2)*e)*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b^3
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{f\,x^6+e\,x^4+d\,x^2+c}{x^2\,\sqrt{b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^(1/2)),x)
```

```
[Out] int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^(1/2)), x)
```

3.155 $\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=110

$$-\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x}+\frac{fx\sqrt{a+bx^2}}{2b}+\frac{(2be-af)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

```
[Out] 1/2*(-a*f+2*b*e)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-1/3*c*(b*x^2+a)
^(1/2)/a/x^3+1/3*(-3*a*d+2*b*c)*(b*x^2+a)^(1/2)/a^2/x+1/2*f*x*(b*x^2+a)^(1/
2)/b
```

Rubi [A]

time = 0.09, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1821, 1599, 1279, 396, 223, 212}

$$\frac{\sqrt{a+bx^2}\,(2bc-3ad)}{3a^2x}+\frac{(2be-af)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}-\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{fx\sqrt{a+bx^2}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/3*(c*Sqrt[a + b*x^2])/(a*x^3) + ((2*b*c - 3*a*d)*Sqrt[a + b*x^2])/(3*a^2
*x) + (f*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*e - a*f)*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]])/(2*b^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx &= -\frac{c\sqrt{a+bx^2}}{3ax^3}-\frac{\int \frac{(2bc-3ad)x-3aex^3-3afx^5}{x^3\sqrt{a+bx^2}}\,dx}{3a}\\
&= -\frac{c\sqrt{a+bx^2}}{3ax^3}-\frac{\int \frac{2bc-3ad-3aex^2-3afx^4}{x^2\sqrt{a+bx^2}}\,dx}{3a}\\
&= -\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x}+\frac{\int \frac{3a^2e+3a^2fx^2}{\sqrt{a+bx^2}}\,dx}{3a^2}\\
&= -\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x}+\frac{fx\sqrt{a+bx^2}}{2b}+\frac{(2be-af)\int \frac{1}{\sqrt{a+}}}{2b}\\
&= -\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x}+\frac{fx\sqrt{a+bx^2}}{2b}+\frac{(2be-af)\text{Subst}\Big(}{}\\
&= -\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x}+\frac{fx\sqrt{a+bx^2}}{2b}+\frac{(2be-af)\tanh^{-1}}{2b^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 95, normalized size = 0.86

$$\frac{\sqrt{a+bx^2}\,(-2abc+4b^2cx^2-6abdx^2+3a^2fx^4)}{6a^2bx^3}+\frac{(-2be+af)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(-2*a*b*c + 4*b^2*c*x^2 - 6*a*b*d*x^2 + 3*a^2*f*x^4))/(6*a
^2*b*x^3) + ((-2*b*e + a*f)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(3/2)
)
```

Maple [A]
time = 0.12, size = 119, normalized size = 1.08

method	result
risch	$\frac{\sqrt{b\,x^2+a}\,(3a^2f\,x^4-6abd\,x^2+4b^2c\,x^2-2abc)}{6b\,a^2x^3}-\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)af}{2b^{\frac{3}{2}}}+\frac{e\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$
default	$f\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)+\frac{e\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+c\Big(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x}}{3a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/2*x*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))+e*l
n(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)+c*(-1/3/a/x^3*(b*x^2+a)^(1/2)+2/3*b/a^
2*(b*x^2+a)^(1/2)/x)-d*(b*x^2+a)^(1/2)/a/x
```

Maxima [A]
time = 0.28, size = 103, normalized size = 0.94

$$\frac{\sqrt{bx^2+a}\,fx}{2\,b} - \frac{af\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)e}{\sqrt{b}} + \frac{2\,\sqrt{bx^2+a}\,bc}{3\,a^2x} - \frac{\sqrt{bx^2+a}\,d}{ax} - \frac{\sqrt{bx^2+a}\,c}{3\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*f*x/b - 1/2*a*f*arcsinh(b*x/sqrt(a*b))/b^(3/2) + arcsin
h(b*x/sqrt(a*b))*e/sqrt(b) + 2/3*sqrt(b*x^2 + a)*b*c/(a^2*x) - sqrt(b*x^2 +
 a)*d/(a*x) - 1/3*sqrt(b*x^2 + a)*c/(a*x^3)
```

Fricas [A]
time = 4.37, size = 215, normalized size = 1.95

$$\left[-\frac{3\,(a^3fx^3-2\,a^2bx^3e)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(3\,a^2bfx^4-2\,ab^2c+2\,(2\,b^3c-3\,ab^2d)x^2)\sqrt{bx^2+a}}{12\,a^2b^2x^3}, \frac{3\,(a^3fx^3-2\,a^2bx^3e)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(3\,a^2bfx^4-2\,ab^2c+2\,(2\,b^3c-3\,ab^2d)x^2)\sqrt{bx^2+a}}{6\,a^2b^2x^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*(a^3*f*x^3 - 2*a^2*b*x^3*e)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 +
 a)*sqrt(b)*x - a) - 2*(3*a^2*b*f*x^4 - 2*a*b^2*c + 2*(2*b^3*c - 3*a*b^2*d)
*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^3), 1/6*(3*(a^3*f*x^3 - 2*a^2*b*x^3*e)*sq
rt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (3*a^2*b*f*x^4 - 2*a*b^2*c + 2*
(2*b^3*c - 3*a*b^2*d)*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^3)]
```

Sympy [A]
time = 2.23, size = 197, normalized size = 1.79

$$\frac{\sqrt{a}\,fx\sqrt{1+\frac{bx^2}{a}}}{2b} - \frac{af\,\mathrm{asinh}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2b^{\frac{3}{2}}} + e\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\,\mathrm{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b<0\\ \frac{\sqrt{\frac{a}{b}}\,\mathrm{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\ a>0\wedge b>0\\ \frac{\sqrt{-\frac{a}{b}}\,\mathrm{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\ b>0\wedge a<0\end{cases}\right) - \frac{\sqrt{b}\,c\sqrt{\frac{a}{bx^2}+1}}{3ax^2} - \frac{\sqrt{b}\,d\sqrt{\frac{a}{bx^2}+1}}{a} + \frac{2b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*f*x*sqrt(1 + b*x**2/a)/(2*b) - a*f*asinh(sqrt(b)*x/sqrt(a))/(2*b**(
3/2)) + e*Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b <
0)), (sqrt(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)
*acosh(x*sqrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) - sqrt(b)*c*sqrt(a/(b*x*
*2) + 1)/(3*a*x**2) - sqrt(b)*d*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/2)*c*sqrt(
a/(b*x**2) + 1)/(3*a**2)
```

Giac [A]
time = 1.21, size = 176, normalized size = 1.60

$$\frac{\sqrt{bx^2+a}\,fx}{2\,b}+\frac{\left(a\sqrt{b}\,f-2\,b^{\frac{3}{2}}e\right)\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)}{4\,b^2}+\frac{2\left(3\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4\sqrt{b}\,d+6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}c-6\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a\sqrt{b}\,d-2\,ab^{\frac{3}{2}}c+3\,a^2\sqrt{b}\,d\right)}{3\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(b*x^2 + a)*f*x/b + 1/4*(a*sqrt(b)*f - 2*b^(3/2)*e)*log((sqrt(b)*x
- sqrt(b*x^2 + a))^2)/b^2 + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*sqrt(b)*
d + 6*(sqrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c - 6*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a*sqrt(b)*d - 2*a*b^(3/2)*c + 3*a^2*sqrt(b)*d)/((sqrt(b)*x - sqrt(
b*x^2 + a))^2 - a)^3
```

Mupad [B]
time = 2.20, size = 143, normalized size = 1.30

$$\begin{cases} -\frac{-f\,x^6-3\,e\,x^4+3\,d\,x^2+c}{3\,\sqrt{a}\,x^3} & \text{if}\ \ b=0\\ \frac{e\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{d\,\sqrt{b\,x^2+a}}{a\,x}-\frac{a\,f\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}+\frac{f\,x\,\sqrt{b\,x^2+a}}{2\,b}-\frac{c\,\sqrt{b\,x^2+a}\,\left(a-2\,b\,x^2\right)}{3\,a^2\,x^3} & \text{if}\ \ b\neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^(1/2)),x)
```

```
[Out] piecewise(b == 0, -(c + 3*d*x^2 - 3*e*x^4 - f*x^6)/(3*a^(1/2)*x^3), b ~= 0,
 (e*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (d*(a + b*x^2)^(1/2))/(a*
x) - (a*f*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)) + (f*x*(a + b
*x^2)^(1/2))/(2*b) - (c*(a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3))
```

3.156 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=118

$$-\frac{c\sqrt{a+bx^2}}{5ax^5}+\frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3}-\frac{(8b^2c-10abd+15a^2e)\sqrt{a+bx^2}}{15a^3x}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

```
[Out] f*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)-1/5*c*(b*x^2+a)^(1/2)/a/x^5+1/
15*(-5*a*d+4*b*c)*(b*x^2+a)^(1/2)/a^2/x^3-1/15*(15*a^2*e-10*a*b*d+8*b^2*c)*
(b*x^2+a)^(1/2)/a^3/x
```

Rubi [A]
time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1821, 1599, 1279, 462, 223, 212}

$$\frac{\sqrt{a+bx^2}\,(4bc-5ad)}{15a^2x^3}-\frac{\sqrt{a+bx^2}\,(15a^2e-10abd+8b^2c)}{15a^3x}-\frac{c\sqrt{a+bx^2}}{5ax^5}+\frac{f\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/5*(c*Sqrt[a + b*x^2])/(a*x^5) + ((4*b*c - 5*a*d)*Sqrt[a + b*x^2])/(15*a^
2*x^3) - ((8*b^2*c - 10*a*b*d + 15*a^2*e)*Sqrt[a + b*x^2])/(15*a^3*x) + (f*
ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
```

```
Q[m + n, -1]))
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx &= -\frac{c\sqrt{a+bx^2}}{5ax^5}-\frac{\int \frac{(4bc-5ad)x-5aex^3-5afx^5}{x^5\sqrt{a+bx^2}}\,dx}{5a} \\
&= -\frac{c\sqrt{a+bx^2}}{5ax^5}-\frac{\int \frac{4bc-5ad-5aex^2-5afx^4}{x^4\sqrt{a+bx^2}}\,dx}{5a} \\
&= -\frac{c\sqrt{a+bx^2}}{5ax^5}+\frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3}+\frac{\int \frac{8b^2c-10abd+15a^2e+15a^2fx^2}{x^2\sqrt{a+bx^2}}\,dx}{15a^2} \\
&= -\frac{c\sqrt{a+bx^2}}{5ax^5}+\frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3}-\frac{(8b^2c-10abd+15a^2e)\sqrt{a+bx^2}}{15a^3x}+ \\
&= -\frac{c\sqrt{a+bx^2}}{5ax^5}+\frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3}-\frac{(8b^2c-10abd+15a^2e)\sqrt{a+bx^2}}{15a^3x}+ \\
&= -\frac{c\sqrt{a+bx^2}}{5ax^5}+\frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3}-\frac{(8b^2c-10abd+15a^2e)\sqrt{a+bx^2}}{15a^3x}+
\end{aligned}
$$

Mathematica [A]

time = 0.24, size = 98, normalized size = 0.83

$$
-\frac{\sqrt{a+bx^2}\,(8b^2cx^4-2abx^2(2c+5dx^2)+a^2(3c+5dx^2+15ex^4))}{15a^3x^5}-\frac{f\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{\sqrt{b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/15*(Sqrt[a + b*x^2]*(8*b^2*c*x^4 - 2*a*b*x^2*(2*c + 5*d*x^2) + a^2*(3*c
+ 5*d*x^2 + 15*e*x^4)))/(a^3*x^5) - (f*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])
/Sqrt[b]
```

Maple [A]

time = 0.12, size = 141, normalized size = 1.19

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,(15a^2e\,x^4-10abd\,x^4+8b^2c\,x^4+5a^2d\,x^2-4abc\,x^2+3a^2c)}{15a^3x^5}+\frac{f\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$
default	$\frac{f\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+c\left(-\frac{\sqrt{b\,x^2+a}}{5a\,x^5}-\frac{4b\left(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}\right)}{5a}\right)+d\left(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)+c*(-1/5/a/x^5*(b*x^2+a)^(1/2)-4/5*b
/a*(-1/3/a/x^3*(b*x^2+a)^(1/2)+2/3*b/a^2*(b*x^2+a)^(1/2)/x))+d*(-1/3/a/x^3*
(b*x^2+a)^(1/2)+2/3*b/a^2*(b*x^2+a)^(1/2)/x)-e*(b*x^2+a)^(1/2)/a/x
```

Maxima [A]
time = 0.26, size = 129, normalized size = 1.09

$$\frac{f\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{8\sqrt{bx^2+a}\,b^2c}{15\,a^3x}+\frac{2\sqrt{bx^2+a}\,bd}{3\,a^2x}-\frac{\sqrt{bx^2+a}\,e}{ax}+\frac{4\sqrt{bx^2+a}\,bc}{15\,a^2x^3}-\frac{\sqrt{bx^2+a}\,d}{3\,ax^3}-\frac{\sqrt{bx^2+a}\,c}{5\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] f*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 8/15*sqrt(b*x^2 + a)*b^2*c/(a^3*x) + 2/3
*sqrt(b*x^2 + a)*b*d/(a^2*x) - sqrt(b*x^2 + a)*e/(a*x) + 4/15*sqrt(b*x^2 +
a)*b*c/(a^2*x^3) - 1/3*sqrt(b*x^2 + a)*d/(a*x^3) - 1/5*sqrt(b*x^2 + a)*c/(a
*x^5)
```

Fricas [A]
time = 2.53, size = 231, normalized size = 1.96

$$\left[\frac{15\,a^3\sqrt{b}\,fx^5\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(15\,a^2bx^4e+2\,(4\,b^3c-5\,ab^2d)x^4+3\,a^2bc-(4\,ab^2c-5\,a^2bd)x^2)\sqrt{bx^2+a}}{30\,a^3bx^5},\,-\frac{15\,a^3\sqrt{-b}\,fx^5\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(15\,a^2bx^4e+2\,(4\,b^3c-5\,ab^2d)x^4+3\,a^2bc-(4\,ab^2c-5\,a^2bd)x^2)\sqrt{bx^2+a}}{15\,a^3bx^5}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/30*(15*a^3*sqrt(b)*f*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 - 2*(15*a^2*b*x^4*e + 2*(4*b^3*c - 5*a*b^2*d)*x^4 + 3*a^2*b*c - (4*a*b^2*c
 - 5*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^5), -1/15*(15*a^3*sqrt(-b)*f*x
^5*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (15*a^2*b*x^4*e + 2*(4*b^3*c - 5*a*
b^2*d)*x^4 + 3*a^2*b*c - (4*a*b^2*c - 5*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3
*b*x^5)]
```

Sympy [A]
time = 1.70, size = 456, normalized size = 3.86

$$-\frac{3a^4b^{\frac{9}{2}}c\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{2a^3b^{\frac{11}{2}}cx^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{3a^2b^{\frac{13}{2}}cx^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{12ab^{\frac{15}{2}}cx^6\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{8b^{\frac{17}{2}}cx^8\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}+f\left(\begin{cases}\frac{\sqrt{-\frac{a}{b}}\operatorname{asin}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\: a>0\wedge b<0\\\frac{\sqrt{\frac{a}{b}}\operatorname{asinh}\left(x\sqrt{\frac{b}{a}}\right)}{\sqrt{a}} & \text{for}\: a>0\wedge b>0\\\frac{\sqrt{-\frac{a}{b}}\operatorname{acosh}\left(x\sqrt{-\frac{b}{a}}\right)}{\sqrt{-a}} & \text{for}\: b>0\wedge a<0\end{cases}\right)-\frac{\sqrt{b}\,d\sqrt{\frac{a}{bx^2}+1}}{3ax^2}-\frac{\sqrt{b}\,e\sqrt{\frac{a}{bx^2}+1}}{a}+\frac{2b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**(1/2),x)
```

```
[Out] -3*a**4*b**(9/2)*c*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x
**6 + 15*a**3*b**6*x**8) - 2*a**3*b**(11/2)*c*x**2*sqrt(a/(b*x**2) + 1)/(15
*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 3*a**2*b**(13/2)
*c*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a*
*3*b**6*x**8) - 12*a*b**(15/2)*c*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x*
*4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 8*b**(17/2)*c*x**8*sqrt(a/(b*
x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + f*
Piecewise((sqrt(-a/b)*asin(x*sqrt(-b/a))/sqrt(a), (a > 0) & (b < 0)), (sqrt
(a/b)*asinh(x*sqrt(b/a))/sqrt(a), (a > 0) & (b > 0)), (sqrt(-a/b)*acosh(x*s
qrt(-b/a))/sqrt(-a), (b > 0) & (a < 0))) - sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(
3*a*x**2) - sqrt(b)*e*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/2)*d*sqrt(a/(b*x**2)
 + 1)/(3*a**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(101) = 202$.
time = 1.11, size = 324, normalized size = 2.75

$$-\frac{f\log\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2\right)}{2\sqrt{b}}+\frac{2\left(15\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^8\sqrt{b}\,e+30\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6b^{\frac{3}{2}}d-60\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^6a\sqrt{b}\,e+80\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4b^{\frac{5}{2}}c-70\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4ab^{\frac{3}{2}}d+90\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^4a^2\sqrt{b}\,e-40\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2ab^{\frac{5}{2}}c+50\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^2b^{\frac{3}{2}}d-60\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2a^3\sqrt{b}\,e+8\,a^2b^{\frac{5}{2}}c-10\,a^3b^{\frac{3}{2}}d+15\,a^4\sqrt{b}\,e\right)}{15\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*f*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/sqrt(b) + 2/15*(15*(sqrt(b)*x -
 sqrt(b*x^2 + a))^8*sqrt(b)*e + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(3/2)*
d - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*sqrt(b)*e + 80*(sqrt(b)*x - sqrt(b
*x^2 + a))^4*b^(5/2)*c - 70*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(3/2)*d + 9
0*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*sqrt(b)*e - 40*(sqrt(b)*x - sqrt(b*x^
2 + a))^2*a*b^(5/2)*c + 50*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(3/2)*d -
60*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*sqrt(b)*e + 8*a^2*b^(5/2)*c - 10*a^3
*b^(3/2)*d + 15*a^4*sqrt(b)*e)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5
```

Mupad [B]
time = 1.72, size = 105, normalized size = 0.89

$$\frac{f\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{e\,\sqrt{b\,x^2+a}}{a\,x}-\frac{d\,\sqrt{b\,x^2+a}\,\left(a-2\,b\,x^2\right)}{3\,a^2\,x^3}-\frac{c\,\sqrt{b\,x^2+a}\,\left(3\,a^2-4\,a\,b\,x^2+8\,b^2\,x^4\right)}{15\,a^3\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^(1/2)),x)
```

```
[Out] (f*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (e*(a + b*x^2)^(1/2))/(a*x
) - (d*(a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3) - (c*(a + b*x^2)^(1/2)*
(3*a^2 + 8*b^2*x^4 - 4*a*b*x^2))/(15*a^3*x^5)
```

3.157 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=140

$$-\frac{c\sqrt{a+bx^2}}{7ax^7}+\frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5}-\frac{(24b^2c-28abd+35a^2e)\sqrt{a+bx^2}}{105a^3x^3}+\frac{(48b^3c-56ab^2d+70a^2be-1}{105a^4x}$$

```
[Out] -1/7*c*(b*x^2+a)^(1/2)/a/x^7+1/35*(-7*a*d+6*b*c)*(b*x^2+a)^(1/2)/a^2/x^5-1/
105*(35*a^2*e-28*a*b*d+24*b^2*c)*(b*x^2+a)^(1/2)/a^3/x^3+1/105*(-105*a^3*f+
70*a^2*b*e-56*a*b^2*d+48*b^3*c)*(b*x^2+a)^(1/2)/a^4/x
```

Rubi [A]
time = 0.13, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1817, 12, 270}

$$\frac{\sqrt{a+bx^2}\,(6bc-7ad)}{35a^2x^5}-\frac{\sqrt{a+bx^2}\,(35a^2e-28abd+24b^2c)}{105a^3x^3}+\frac{\sqrt{a+bx^2}\,(-105a^3f+70a^2be-56ab^2d+48b^3c)}{105a^4x}-\frac{c\sqrt{a+bx^2}}{7ax^7}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/7*(c*Sqrt[a + b*x^2])/(a*x^7) + ((6*b*c - 7*a*d)*Sqrt[a + b*x^2])/(35*a^
2*x^5) - ((24*b^2*c - 28*a*b*d + 35*a^2*e)*Sqrt[a + b*x^2])/(105*a^3*x^3) +
 ((48*b^3*c - 56*a*b^2*d + 70*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2])/(105*a^
4*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx &= -\frac{c\sqrt{a+bx^2}}{7ax^7}-\frac{\int \frac{6bc-7a\left(d+ex^2+fx^4\right)}{x^6\sqrt{a+bx^2}}\,dx}{7a}\\
&= -\frac{c\sqrt{a+bx^2}}{7ax^7}+\frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5}+\frac{\int \frac{4b(6bc-7ad)-5a\left(-7ae-7afx^2\right)}{x^4\sqrt{a+bx^2}}\,dx}{35a^2}\\
&= -\frac{c\sqrt{a+bx^2}}{7ax^7}+\frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5}-\frac{\left(24b^2c-28abd+35a^2e\right)\sqrt{a+bx^2}}{105a^3x^3}\\
&= -\frac{c\sqrt{a+bx^2}}{7ax^7}+\frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5}-\frac{\left(24b^2c-28abd+35a^2e\right)\sqrt{a+bx^2}}{105a^3x^3}\\
&= -\frac{c\sqrt{a+bx^2}}{7ax^7}+\frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5}-\frac{\left(24b^2c-28abd+35a^2e\right)\sqrt{a+bx^2}}{105a^3x^3}
\end{aligned}
$$

Mathematica [A]
time = 0.22, size = 103, normalized size = 0.74

$$
\frac{\sqrt{a+bx^2}\left(48b^3cx^6-8ab^2x^4\left(3c+7dx^2\right)+2a^2bx^2\left(9c+14dx^2+35ex^4\right)-a^3\left(15c+21dx^2+35x^4\left(e+3fx^2\right)\right)\right)}{105a^4x^7}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*Sqrt[a + b*x^2]),x]
```

```
[Out] (Sqrt[a + b*x^2]*(48*b^3*c*x^6 - 8*a*b^2*x^4*(3*c + 7*d*x^2) + 2*a^2*b*x^2*
(9*c + 14*d*x^2 + 35*e*x^4) - a^3*(15*c + 21*d*x^2 + 35*x^4*(e + 3*f*x^2)))
)/(105*a^4*x^7)
```

Maple [A]
time = 0.13, size = 206, normalized size = 1.47

method	result
gosper	$-\frac{\sqrt{b\,x^2+a}\,\left(105a^3f\,x^6-70a^2be\,x^6+56a\,b^2d\,x^6-48b^3c\,x^6+35a^3e\,x^4-28a^2bd\,x^4+24a\,b^2c\,x^4+21a^3d\,x^2-18a^2bc\,x^2+15a^3c\right)}{105x^7a^4}$
trager	$-\frac{\sqrt{b\,x^2+a}\,\left(105a^3f\,x^6-70a^2be\,x^6+56a\,b^2d\,x^6-48b^3c\,x^6+35a^3e\,x^4-28a^2bd\,x^4+24a\,b^2c\,x^4+21a^3d\,x^2-18a^2bc\,x^2+15a^3c\right)}{105x^7a^4}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(105a^3f\,x^6-70a^2be\,x^6+56a\,b^2d\,x^6-48b^3c\,x^6+35a^3e\,x^4-28a^2bd\,x^4+24a\,b^2c\,x^4+21a^3d\,x^2-18a^2bc\,x^2+15a^3c\right)}{105x^7a^4}$

default	$d\left(-\frac{\sqrt{b\,x^{2}+a}}{5a\,x^{5}}-\frac{4b\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)}{5a}\right)+c\left(-\frac{\sqrt{b\,x^{2}+a}}{7a\,x^{7}}-\frac{6b\left(-\frac{\sqrt{b\,x^{2}+a}}{5a\,x^{5}}-\frac{4b\left(-\right.}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] d*(-1/5/a/x^5*(b*x^2+a)^(1/2)-4/5*b/a*(-1/3/a/x^3*(b*x^2+a)^(1/2)+2/3*b/a^2
*(b*x^2+a)^(1/2)/x))+c*(-1/7/a/x^7*(b*x^2+a)^(1/2)-6/7*b/a*(-1/5/a/x^5*(b*x
^2+a)^(1/2)-4/5*b/a*(-1/3/a/x^3*(b*x^2+a)^(1/2)+2/3*b/a^2*(b*x^2+a)^(1/2)/x
)))+e*(-1/3/a/x^3*(b*x^2+a)^(1/2)+2/3*b/a^2*(b*x^2+a)^(1/2)/x)-f*(b*x^2+a)^
(1/2)/a/x
```

Maxima [**A**]
time = 0.28, size = 195, normalized size = 1.39

$$\frac{16\sqrt{bx^2+a}\,b^3c}{35\,a^4x}-\frac{8\sqrt{bx^2+a}\,b^2d}{15\,a^3x}-\frac{\sqrt{bx^2+a}\,f}{ax}+\frac{2\sqrt{bx^2+a}\,be}{3\,a^2x}-\frac{8\sqrt{bx^2+a}\,b^2c}{35\,a^3x^3}+\frac{4\sqrt{bx^2+a}\,bd}{15\,a^2x^3}-\frac{\sqrt{bx^2+a}\,e}{3\,ax^3}+\frac{6\sqrt{bx^2+a}\,bc}{35\,a^2x^5}-\frac{\sqrt{bx^2+a}\,d}{5\,ax^5}-\frac{\sqrt{bx^2+a}\,c}{7\,ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 16/35*sqrt(b*x^2 + a)*b^3*c/(a^4*x) - 8/15*sqrt(b*x^2 + a)*b^2*d/(a^3*x) -
sqrt(b*x^2 + a)*f/(a*x) + 2/3*sqrt(b*x^2 + a)*b*e/(a^2*x) - 8/35*sqrt(b*x^2
 + a)*b^2*c/(a^3*x^3) + 4/15*sqrt(b*x^2 + a)*b*d/(a^2*x^3) - 1/3*sqrt(b*x^2
 + a)*e/(a*x^3) + 6/35*sqrt(b*x^2 + a)*b*c/(a^2*x^5) - 1/5*sqrt(b*x^2 + a)*
d/(a*x^5) - 1/7*sqrt(b*x^2 + a)*c/(a*x^7)
```

Fricas [**A**]
time = 3.30, size = 109, normalized size = 0.78

$$\frac{\left((48\,b^3c-56\,ab^2d-105\,a^3f)x^6-4\,(6\,ab^2c-7\,a^2bd)x^4-15\,a^3c+3\,(6\,a^2bc-7\,a^3d)x^2+35\,(2\,a^2bx^6-a^3x^4)e\right)\sqrt{bx^2+a}}{105\,a^4x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*((48*b^3*c - 56*a*b^2*d - 105*a^3*f)*x^6 - 4*(6*a*b^2*c - 7*a^2*b*d)*
x^4 - 15*a^3*c + 3*(6*a^2*b*c - 7*a^3*d)*x^2 + 35*(2*a^2*b*x^6 - a^3*x^4)*e
)*sqrt(b*x^2 + a)/(a^4*x^7)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 891 vs. $2(136) = 272$.
time = 2.31, size = 891, normalized size = 6.36

$$-\frac{5a^6b^{\frac{19}{2}}c\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}-\frac{9a^5b^{\frac{21}{2}}cx^2\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}-\frac{5a^4b^{\frac{23}{2}}cx^4\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}-\frac{3a^4b^{\frac{9}{2}}d\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}+\frac{5a^3b^{\frac{25}{2}}cx^6\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}-\frac{2a^3b^{\frac{11}{2}}dx^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}+\frac{30a^2b^{\frac{27}{2}}cx^8\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}-\frac{3a^2b^{\frac{13}{2}}dx^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}+\frac{40ab^{\frac{29}{2}}cx^{10}\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}-\frac{12ab^{\frac{15}{2}}dx^6\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}+\frac{16b^{\frac{31}{2}}cx^{12}\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}}-\frac{8b^{\frac{17}{2}}dx^8\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8}-\frac{\sqrt{b}\,e\sqrt{\frac{a}{bx^2}+1}}{3ax^2}-\frac{\sqrt{b}\,f\sqrt{\frac{a}{bx^2}+1}}{a}+\frac{2b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**(1/2),x)
```

```
[Out] -5*a**6*b**(19/2)*c*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**1
0*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 9*a**5*b**(21/2)*c*x
**2*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**
5*b**11*x**10 + 35*a**4*b**12*x**12) - 5*a**4*b**(23/2)*c*x**4*sqrt(a/(b*x*
*2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 +
35*a**4*b**12*x**12) - 3*a**4*b**(9/2)*d*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4
*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + 5*a**3*b**(25/2)*c*x**6*sq
rt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**1
1*x**10 + 35*a**4*b**12*x**12) - 2*a**3*b**(11/2)*d*x**2*sqrt(a/(b*x**2) +
1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + 30*a**2*b*
*(27/2)*c*x**8*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**
8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 3*a**2*b**(13/2)*d*x**4*s
qrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x
**8) + 40*a*b**(29/2)*c*x**10*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105
*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 12*a*b**(1
5/2)*d*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 1
5*a**3*b**6*x**8) + 16*b**(31/2)*c*x**12*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9
*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) -
 8*b**(17/2)*d*x**8*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*
x**6 + 15*a**3*b**6*x**8) - sqrt(b)*e*sqrt(a/(b*x**2) + 1)/(3*a*x**2) - sqr
t(b)*f*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/2)*e*sqrt(a/(b*x**2) + 1)/(3*a**2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 554 vs. 2(126) = 252.
time = 1.00, size = 554, normalized size = 3.96

$$\frac{2\left(105\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}\sqrt{b}f-630\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a\sqrt{b}f+210\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{3}{2}}e+560\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}b^{\frac{5}{2}}d+1575\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^2\sqrt{b}f-910\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}ab^{\frac{3}{2}}e+1680\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}b^{\frac{7}{2}}c-1400\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}ab^{\frac{5}{2}}d-2100\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^3\sqrt{b}f+1540\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^2b^{\frac{3}{2}}e-1008\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}ab^{\frac{7}{2}}c+1176\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^2b^{\frac{5}{2}}d+1575\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^4\sqrt{b}f-1260\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^3b^{\frac{3}{2}}e+336\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^2b^{\frac{7}{2}}c-392\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^3b^{\frac{5}{2}}d-630\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^5\sqrt{b}f+490\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^4b^{\frac{3}{2}}e-48a^3b^{\frac{7}{2}}c+56a^4b^{\frac{5}{2}}d+105a^6\sqrt{b}f-70a^5b^{\frac{3}{2}}e\right)}{105\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*sqrt(b)*f - 630*(sqrt(b)*x - sq
rt(b*x^2 + a))^10*a*sqrt(b)*f + 210*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(3/2
)*e + 560*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(5/2)*d + 1575*(sqrt(b)*x - sqr
t(b*x^2 + a))^8*a^2*sqrt(b)*f - 910*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a*b^(3/
2)*e + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(7/2)*c - 1400*(sqrt(b)*x - s
qrt(b*x^2 + a))^6*a*b^(5/2)*d - 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*sq
rt(b)*f + 1540*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^2*b^(3/2)*e - 1008*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*a*b^(7/2)*c + 1176*(sqrt(b)*x - sqrt(b*x^2 + a))^4
*a^2*b^(5/2)*d + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*sqrt(b)*f - 1260*
(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(3/2)*e + 336*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*a^2*b^(7/2)*c - 392*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*b^(5/2)*d
```

```
- 630*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*sqrt(b)*f + 490*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*a^4*b^(3/2)*e - 48*a^3*b^(7/2)*c + 56*a^4*b^(5/2)*d + 105*a^
6*sqrt(b)*f - 70*a^5*b^(3/2)*e)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7
```

Mupad [B]
time = 1.28, size = 124, normalized size = 0.89

$$\frac{\sqrt{b\,x^2+a}\,\left(-105\,f\,a^3+70\,e\,a^2\,b-56\,d\,a\,b^2+48\,c\,b^3\right)}{105\,a^4\,x}-\frac{\sqrt{b\,x^2+a}\,\left(7\,a\,d-6\,b\,c\right)}{35\,a^2\,x^5}-\frac{\sqrt{b\,x^2+a}\,\left(35\,e\,a^2-28\,d\,a\,b+24\,c\,b^2\right)}{105\,a^3\,x^3}-\frac{c\,\sqrt{b\,x^2+a}}{7\,a\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*(48*b^3*c - 105*a^3*f - 56*a*b^2*d + 70*a^2*b*e))/(105*a
^4*x) - ((a + b*x^2)^(1/2)*(7*a*d - 6*b*c))/(35*a^2*x^5) - ((a + b*x^2)^(1/
2)*(24*b^2*c + 35*a^2*e - 28*a*b*d))/(105*a^3*x^3) - (c*(a + b*x^2)^(1/2))/
(7*a*x^7)
```

3.158 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=189

$$-\frac{c\sqrt{a+bx^2}}{9ax^9}+\frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7}-\frac{(16b^2c-18abd+21a^2e)\sqrt{a+bx^2}}{105a^3x^5}+\frac{(64b^3c-72ab^2d+84a^2be-10}{315a^4x^3}$$

```
[Out] -1/9*c*(b*x^2+a)^(1/2)/a/x^9+1/63*(-9*a*d+8*b*c)*(b*x^2+a)^(1/2)/a^2/x^7-1/
105*(21*a^2*e-18*a*b*d+16*b^2*c)*(b*x^2+a)^(1/2)/a^3/x^5+1/315*(-105*a^3*f+
84*a^2*b*e-72*a*b^2*d+64*b^3*c)*(b*x^2+a)^(1/2)/a^4/x^3-2/315*b*(-105*a^3*f
+84*a^2*b*e-72*a*b^2*d+64*b^3*c)*(b*x^2+a)^(1/2)/a^5/x
```

Rubi [A]

time = 0.18, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1817, 12, 277, 270}

$$\frac{\sqrt{a+bx^2}\,(8bc-9ad)}{63a^2x^7}-\frac{\sqrt{a+bx^2}\,(21a^2e-18abd+16b^2c)}{105a^3x^5}-\frac{2b\sqrt{a+bx^2}\,(-105a^3f+84a^2be-72ab^2d+64b^3c)}{315a^5x}+\frac{\sqrt{a+bx^2}\,(-105a^3f+84a^2be-72ab^2d+64b^3c)}{315a^4x^3}-\frac{c\sqrt{a+bx^2}}{9ax^9}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/9*(c*Sqrt[a + b*x^2])/(a*x^9) + ((8*b*c - 9*a*d)*Sqrt[a + b*x^2])/(63*a^
2*x^7) - ((16*b^2*c - 18*a*b*d + 21*a^2*e)*Sqrt[a + b*x^2])/(105*a^3*x^5) +
 ((64*b^3*c - 72*a*b^2*d + 84*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2])/(315*a^
4*x^3) - (2*b*(64*b^3*c - 72*a*b^2*d + 84*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x
^2])/(315*a^5*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx &= -\frac{c\sqrt{a+bx^2}}{9ax^9}-\frac{\int \frac{8bc-9a(d+ex^2+fx^4)}{x^8\sqrt{a+bx^2}}\,dx}{9a}\\
&= -\frac{c\sqrt{a+bx^2}}{9ax^9}+\frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7}+\frac{\int \frac{6b(8bc-9ad)-7a(-9ae-9afx^2)}{x^6\sqrt{a+bx^2}}\,dx}{63a^2}\\
&= -\frac{c\sqrt{a+bx^2}}{9ax^9}+\frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7}-\frac{(16b^2c-18abd+21a^2e)\sqrt{a+bx^2}}{105a^3x^5}\\
&= -\frac{c\sqrt{a+bx^2}}{9ax^9}+\frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7}-\frac{(16b^2c-18abd+21a^2e)\sqrt{a+bx^2}}{105a^3x^5}\\
&= -\frac{c\sqrt{a+bx^2}}{9ax^9}+\frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7}-\frac{(16b^2c-18abd+21a^2e)\sqrt{a+bx^2}}{105a^3x^5}\\
&= -\frac{c\sqrt{a+bx^2}}{9ax^9}+\frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7}-\frac{(16b^2c-18abd+21a^2e)\sqrt{a+bx^2}}{105a^3x^5}
\end{aligned}$$

Mathematica [A]
time = 0.30, size = 134, normalized size = 0.71

$$-\frac{\sqrt{a+bx^2}\,(128b^4cx^8-16ab^3x^6(4c+9dx^2)+24a^2b^2x^4(2c+3dx^2+7ex^4)-2a^3bx^2(20c+27dx^2+42ex^4+105fx^6)+a^4(35c+45dx^2+63ex^4+105fx^6))}{315a^5x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*Sqrt[a + b*x^2]),x]
```

```
[Out] -1/315*(Sqrt[a + b*x^2]*(128*b^4*c*x^8 - 16*a*b^3*x^6*(4*c + 9*d*x^2) + 24*
a^2*b^2*x^4*(2*c + 3*d*x^2 + 7*e*x^4) - 2*a^3*b*x^2*(20*c + 27*d*x^2 + 42*e
*x^4 + 105*f*x^6) + a^4*(35*c + 45*d*x^2 + 63*e*x^4 + 105*f*x^6)))/(a^5*x^9
)
```

Maple [A]
time = 0.12, size = 298, normalized size = 1.58

method	result
gosper	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-210a^{3}bf\,x^{8}+168a^{2}b^{2}e\,x^{8}-144a\,b^{3}d\,x^{8}+128b^{4}c\,x^{8}+105a^{4}f\,x^{6}-84a^{3}be\,x^{6}+72a^{2}b^{2}d\,x^{6}-64a\,b^{3}c\,x^{6}+63a^{4}e\,x^{4}-54}{315a^{5}x^{9}}$
trager	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-210a^{3}bf\,x^{8}+168a^{2}b^{2}e\,x^{8}-144a\,b^{3}d\,x^{8}+128b^{4}c\,x^{8}+105a^{4}f\,x^{6}-84a^{3}be\,x^{6}+72a^{2}b^{2}d\,x^{6}-64a\,b^{3}c\,x^{6}+63a^{4}e\,x^{4}-54}{315a^{5}x^{9}}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-210a^{3}bf\,x^{8}+168a^{2}b^{2}e\,x^{8}-144a\,b^{3}d\,x^{8}+128b^{4}c\,x^{8}+105a^{4}f\,x^{6}-84a^{3}be\,x^{6}+72a^{2}b^{2}d\,x^{6}-64a\,b^{3}c\,x^{6}+63a^{4}e\,x^{4}-54}{315a^{5}x^{9}}$
default	$e\left(-\frac{\sqrt{b\,x^{2}+a}}{5a\,x^{5}}-\frac{4b\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)}{5a}\right)+c\left(-\frac{\sqrt{b\,x^{2}+a}}{9a\,x^{9}}-\frac{8b\left(-\frac{\sqrt{b\,x^{2}+a}}{7a\,x^{7}}-\frac{6b\left(-\sqrt{}\right.}{}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] e*(-1/5/a/x^5*(b*x^2+a)^(1/2)-4/5*b/a*(-1/3/a/x^3*(b*x^2+a)^(1/2)+2/3*b/a^2
*(b*x^2+a)^(1/2)/x))+c*(-1/9/a/x^9*(b*x^2+a)^(1/2)-8/9*b/a*(-1/7/a/x^7*(b*x
^2+a)^(1/2)-6/7*b/a*(-1/5/a/x^5*(b*x^2+a)^(1/2)-4/5*b/a*(-1/3/a/x^3*(b*x^2+
a)^(1/2)+2/3*b/a^2*(b*x^2+a)^(1/2)/x))))+d*(-1/7/a/x^7*(b*x^2+a)^(1/2)-6/7*
b/a*(-1/5/a/x^5*(b*x^2+a)^(1/2)-4/5*b/a*(-1/3/a/x^3*(b*x^2+a)^(1/2)+2/3*b/a
^2*(b*x^2+a)^(1/2)/x)))+f*(-1/3/a/x^3*(b*x^2+a)^(1/2)+2/3*b/a^2*(b*x^2+a)^(
1/2)/x)
```

Maxima [A]

time = 0.27, size = 278, normalized size = 1.47

$$-\frac{128\,\sqrt{bx^{2}+a}\,b^{4}c}{315\,a^{5}x}+\frac{16\,\sqrt{bx^{2}+a}\,b^{3}d}{35\,a^{4}x}+\frac{2\,\sqrt{bx^{2}+a}\,bf}{3\,a^{2}x}-\frac{8\,\sqrt{bx^{2}+a}\,b^{2}e}{15\,a^{3}x}+\frac{64\,\sqrt{bx^{2}+a}\,b^{3}c}{315\,a^{4}x^{3}}-\frac{8\,\sqrt{bx^{2}+a}\,b^{2}d}{35\,a^{3}x^{3}}-\frac{\sqrt{bx^{2}+a}\,f}{3\,ax^{3}}+\frac{4\,\sqrt{bx^{2}+a}\,be}{15\,a^{2}x^{3}}-\frac{16\,\sqrt{bx^{2}+a}\,b^{2}c}{105\,a^{3}x^{5}}+\frac{6\,\sqrt{bx^{2}+a}\,bd}{35\,a^{2}x^{5}}-\frac{\sqrt{bx^{2}+a}\,e}{5\,ax^{5}}+\frac{8\,\sqrt{bx^{2}+a}\,bc}{63\,a^{2}x^{7}}-\frac{\sqrt{bx^{2}+a}\,d}{7\,ax^{7}}-\frac{\sqrt{bx^{2}+a}\,c}{9\,ax^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -128/315*sqrt(b*x^2 + a)*b^4*c/(a^5*x) + 16/35*sqrt(b*x^2 + a)*b^3*d/(a^4*x
) + 2/3*sqrt(b*x^2 + a)*b*f/(a^2*x) - 8/15*sqrt(b*x^2 + a)*b^2*e/(a^3*x) +
64/315*sqrt(b*x^2 + a)*b^3*c/(a^4*x^3) - 8/35*sqrt(b*x^2 + a)*b^2*d/(a^3*x^
```

```
3) - 1/3*sqrt(b*x^2 + a)*f/(a*x^3) + 4/15*sqrt(b*x^2 + a)*b*e/(a^2*x^3) - 1
6/105*sqrt(b*x^2 + a)*b^2*c/(a^3*x^5) + 6/35*sqrt(b*x^2 + a)*b*d/(a^2*x^5)
- 1/5*sqrt(b*x^2 + a)*e/(a*x^5) + 8/63*sqrt(b*x^2 + a)*b*c/(a^2*x^7) - 1/7*
sqrt(b*x^2 + a)*d/(a*x^7) - 1/9*sqrt(b*x^2 + a)*c/(a*x^9)
```

Fricas [A]
time = 7.51, size = 152, normalized size = 0.80

$$-\frac{(2\,(64\,b^4c-72\,ab^3d-105\,a^3bf)x^8-(64\,ab^3c-72\,a^2b^2d-105\,a^4f)x^6+35\,a^4c+6\,(8\,a^2b^2c-9\,a^3bd)x^4-5\,(8\,a^3bc-9\,a^4d)x^2+21\,(8\,a^2b^2x^8-4\,a^3bx^6+3\,a^4x^4)e)\sqrt{bx^2+a}}{315\,a^5x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/315*(2*(64*b^4*c - 72*a*b^3*d - 105*a^3*b*f)*x^8 - (64*a*b^3*c - 72*a^2*
b^2*d - 105*a^4*f)*x^6 + 35*a^4*c + 6*(8*a^2*b^2*c - 9*a^3*b*d)*x^4 - 5*(8*
a^3*b*c - 9*a^4*d)*x^2 + 21*(8*a^2*b^2*x^8 - 4*a^3*b*x^6 + 3*a^4*x^4)*e)*sq
rt(b*x^2 + a)/(a^5*x^9)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1642 vs. 2(190) = 380.
time = 3.07, size = 1642, normalized size = 8.69

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**(1/2),x)
```

```
[Out] -35*a**8*b**(33/2)*c*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*
b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**2
0*x**16) - 100*a**7*b**(35/2)*c*x**2*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x
**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14
 + 315*a**5*b**20*x**16) - 98*a**6*b**(37/2)*c*x**4*sqrt(a/(b*x**2) + 1)/(3
15*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a
**6*b**19*x**14 + 315*a**5*b**20*x**16) - 5*a**6*b**(19/2)*d*sqrt(a/(b*x**2
) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35
*a**4*b**12*x**12) - 28*a**5*b**(39/2)*c*x**6*sqrt(a/(b*x**2) + 1)/(315*a**
9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b*
*19*x**14 + 315*a**5*b**20*x**16) - 9*a**5*b**(21/2)*d*x**2*sqrt(a/(b*x**2)
 + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*
a**4*b**12*x**12) - 35*a**4*b**(41/2)*c*x**8*sqrt(a/(b*x**2) + 1)/(315*a**9
*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**
19*x**14 + 315*a**5*b**20*x**16) - 5*a**4*b**(23/2)*d*x**4*sqrt(a/(b*x**2)
+ 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a
**4*b**12*x**12) - 3*a**4*b**(9/2)*e*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**
4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 280*a**3*b**(43/2)*c*x**10*sqr
t(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*
```

```
b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) + 5*a**3*b**(25
/2)*d*x**6*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 +
105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 2*a**3*b**(11/2)*e*x**2*sqrt(
a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8)
 - 560*a**2*b**(45/2)*c*x**12*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1
260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*
a**5*b**20*x**16) + 30*a**2*b**(27/2)*d*x**8*sqrt(a/(b*x**2) + 1)/(35*a**7*
b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**1
2) - 3*a**2*b**(13/2)*e*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a
**4*b**5*x**6 + 15*a**3*b**6*x**8) - 448*a*b**(47/2)*c*x**14*sqrt(a/(b*x**2
) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12
 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) + 40*a*b**(29/2)*d*x**10*s
qrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**
11*x**10 + 35*a**4*b**12*x**12) - 12*a*b**(15/2)*e*x**6*sqrt(a/(b*x**2) + 1
)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 128*b**(49/
2)*c*x**16*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**1
0 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) +
 16*b**(31/2)*d*x**12*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b*
*10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 8*b**(17/2)*e*x**8
*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6
*x**8) - sqrt(b)*f*sqrt(a/(b*x**2) + 1)/(3*a*x**2) + 2*b**(3/2)*f*sqrt(a/(b
*x**2) + 1)/(3*a**2)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 667 vs. 2(172) = 344.
time = 1.22, size = 667, normalized size = 3.53

$$\frac{4\left(315\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{14}b^{\frac{3}{2}}f-1995\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}ab^{\frac{3}{2}}f+840\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{12}b^{\frac{5}{2}}e+2520\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}b^{\frac{7}{2}}d+5355\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}a^2b^{\frac{3}{2}}f-3780\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{10}ab^{\frac{5}{2}}e+8064\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}b^{\frac{9}{2}}c-6552\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}ab^{\frac{7}{2}}d-7875\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^3b^{\frac{3}{2}}f+6804\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{8}a^2b^{\frac{5}{2}}e-5376\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}ab^{\frac{9}{2}}c+6048\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^2b^{\frac{7}{2}}d+6825\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^4b^{\frac{3}{2}}f-6216\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{6}a^3b^{\frac{5}{2}}e+2304\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^2b^{\frac{9}{2}}c-2592\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^3b^{\frac{7}{2}}d-3465\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^5b^{\frac{3}{2}}f+3024\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{4}a^4b^{\frac{5}{2}}e-576\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^3b^{\frac{9}{2}}c+648\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^4b^{\frac{7}{2}}d+945\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^6b^{\frac{3}{2}}f-756\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^{2}a^5b^{\frac{5}{2}}e+64a^4b^{\frac{9}{2}}c-72a^5b^{\frac{7}{2}}d-105a^7b^{\frac{3}{2}}f+84a^6b^{\frac{5}{2}}e\right)}{315\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 4/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(3/2)*f - 1995*(sqrt(b)*x - s
qrt(b*x^2 + a))^12*a*b^(3/2)*f + 840*(sqrt(b)*x - sqrt(b*x^2 + a))^12*b^(5/
2)*e + 2520*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2)*d + 5355*(sqrt(b)*x -
sqrt(b*x^2 + a))^10*a^2*b^(3/2)*f - 3780*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a
*b^(5/2)*e + 8064*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c - 6552*(sqrt(b)
*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*d - 7875*(sqrt(b)*x - sqrt(b*x^2 + a))^8*
a^3*b^(3/2)*f + 6804*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*b^(5/2)*e - 5376*(
sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c + 6048*(sqrt(b)*x - sqrt(b*x^2 +
 a))^6*a^2*b^(7/2)*d + 6825*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(3/2)*f -
 6216*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^(5/2)*e + 2304*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*a^2*b^(9/2)*c - 2592*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^
(7/2)*d - 3465*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*f + 3024*(sqrt(b
)*x - sqrt(b*x^2 + a))^4*a^4*b^(5/2)*e - 576*(sqrt(b)*x - sqrt(b*x^2 + a))^
```

```
2*a^3*b^(9/2)*c + 648*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(7/2)*d + 945*(
sqrt(b)*x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*f - 756*(sqrt(b)*x - sqrt(b*x^2
+ a))^2*a^5*b^(5/2)*e + 64*a^4*b^(9/2)*c - 72*a^5*b^(7/2)*d - 105*a^7*b^(3/
2)*f + 84*a^6*b^(5/2)*e)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^9
```

Mupad [B]
time = 1.28, size = 171, normalized size = 0.90

$$\frac{\sqrt{b\,x^2+a}\,\left(-105\,f\,a^3+84\,e\,a^2\,b-72\,d\,a\,b^2+64\,c\,b^3\right)}{315\,a^4\,x^3}-\frac{\sqrt{b\,x^2+a}\,\left(9\,a\,d-8\,b\,c\right)}{63\,a^2\,x^7}-\frac{\sqrt{b\,x^2+a}\,\left(21\,e\,a^2-18\,d\,a\,b+16\,c\,b^2\right)}{105\,a^3\,x^5}-\frac{\sqrt{b\,x^2+a}\,\left(-210\,f\,a^3\,b+168\,e\,a^2\,b^2-144\,d\,a\,b^3+128\,c\,b^4\right)}{315\,a^5\,x}-\frac{c\,\sqrt{b\,x^2+a}}{9\,a\,x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^(1/2)),x)
```

```
[Out] ((a + b*x^2)^(1/2)*(64*b^3*c - 105*a^3*f - 72*a*b^2*d + 84*a^2*b*e))/(315*a
^4*x^3) - ((a + b*x^2)^(1/2)*(9*a*d - 8*b*c))/(63*a^2*x^7) - ((a + b*x^2)^(
1/2)*(16*b^2*c + 21*a^2*e - 18*a*b*d))/(105*a^3*x^5) - ((a + b*x^2)^(1/2)*(
128*b^4*c + 168*a^2*b^2*e - 144*a*b^3*d - 210*a^3*b*f))/(315*a^5*x) - (c*(a
 + b*x^2)^(1/2))/(9*a*x^9)
```

3.159 $\int \frac{x^8\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=381

$$\frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}}-\frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}}-\frac{(16Ab^3-3a(24b^2B-66abC+143a^2D}{210a^2b^4\left(a+bx^2\right)^{3/2}}$$

```
[Out] 1/7*(A-a*(B*b^2-C*a*b+D*a^2)/b^3)*x^9/a/(b*x^2+a)^(7/2)-1/35*(2*A*b^3-a*(9*
B*b^2-16*C*a*b+23*D*a^2))*x^9/a^2/b^3/(b*x^2+a)^(5/2)-1/210*(16*A*b^3-3*a*(
24*B*b^2-66*C*a*b+143*D*a^2))*x^7/a^2/b^4/(b*x^2+a)^(3/2)+1/6*D*x^9/b^3/(b*
x^2+a)^(3/2)+1/16*(16*A*b^3-72*B*a*b^2+198*C*a^2*b-429*D*a^3)*arctanh(x*b^(
1/2)/(b*x^2+a)^(1/2))/b^(15/2)-1/30*(16*A*b^3-3*a*(24*B*b^2-66*C*a*b+143*D*
a^2))*x^5/a^2/b^5/(b*x^2+a)^(1/2)-1/16*(16*A*b^3-3*a*(24*B*b^2-66*C*a*b+143
*D*a^2))*x*(b*x^2+a)^(1/2)/a/b^7+1/24*(16*A*b^3-3*a*(24*B*b^2-66*C*a*b+143*
D*a^2))*x^3*(b*x^2+a)^(1/2)/a^2/b^6
```

Rubi [A]

time = 0.46, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {1818, 1599, 1277, 1598, 470, 294, 327, 223, 212}

$$-\frac{x^9\left(2Ab^3-a\left(23a^2D-16abC+9b^2B\right)\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{x\sqrt{a+bx^2}\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{16ab^7}+\frac{x^3\sqrt{a+bx^2}\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{24a^2b^6}-\frac{x^5\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{30a^2b^5\sqrt{a+bx^2}}-\frac{x^7\left(16Ab^3-3a\left(143a^2D-66abC+24b^2B\right)\right)}{210a^2b^4\left(a+bx^2\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\left(-429a^3D+198a^2bC-72ab^2B+16Ab^3\right)}{16b^{15/2}}+\frac{Dx^9}{6b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^9)/(7*a*(a + b*x^2)^(7/2)) - ((2*A
*b^3 - a*(9*b^2*B - 16*a*b*C + 23*a^2*D))*x^9)/(35*a^2*b^3*(a + b*x^2)^(5/2
)) - ((16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x^7)/(210*a^2*b^4*
(a + b*x^2)^(3/2)) + (D*x^9)/(6*b^3*(a + b*x^2)^(3/2)) - ((16*A*b^3 - 3*a*(
24*b^2*B - 66*a*b*C + 143*a^2*D))*x^5)/(30*a^2*b^5*Sqrt[a + b*x^2]) - ((16*
A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x*Sqrt[a + b*x^2])/(16*a*b^7
) + ((16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*x^3*Sqrt[a + b*x^2]
)/(24*a^2*b^6) + ((16*A*b^3 - 72*a*b^2*B + 198*a^2*b*C - 429*a^3*D)*ArcTanh
[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(16*b^(15/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &
& GtQ[m, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^8\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^7\Bigg(\bigg(2Ab-\frac{9a\left(b^2B-abC+a^2D\right)}{b^2}\bigg)x-7a\left(C-\frac{aD}{b}\right)x^3-7}{\left(a+bx^2\right)^{7/2}}}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^8\Bigg(2Ab-\frac{9a\left(b^2B-abC+a^2D\right)}{b^2}-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4}{\left(a+bx^2\right)^{7/2}}}{7ab}$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} +$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} +$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} +$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} -$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} -$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} -$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} -$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} -$$

$$= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^9}{7a\left(a+bx^2\right)^{7/2}} - \frac{\left(2Ab^3-a\left(9b^2B-16abC+23a^2D\right)\right)x^9}{35a^2b^3\left(a+bx^2\right)^{5/2}} -$$

Mathematica [A]

time = 1.08, size = 251, normalized size = 0.66

$$\frac{x\left(45045a^6D-2310a^5b\left(9C-65Dx^2\right)+42a^4b^2\left(180B-1650Cx^2+4147Dx^4\right)-12a^3b^3\left(140A-2100Bx^2+6699Cx^4-6292Dx^6\right)-2ab^5x^4\left(3248A-6336Bx^2+1155Cx^4+455Dx^6\right)+a^2b^4x^2\left(-5600A+29232Bx^2-34848Cx^4+5005Dx^6\right)+4b^6x^6\left(-704A+35\left(6Bx^2+3Cx^4+2Dx^6\right)\right)\right)}{1680b^7\left(a+bx^2\right)^{7/2}}+\frac{\left(-16Ab^3+3a\left(24b^2B-66abC+143a^2D\right)\right)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{16b^{15/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(45045*a^6*D - 2310*a^5*b*(9*C - 65*D*x^2) + 42*a^4*b^2*(180*B - 1650*C*
x^2 + 4147*D*x^4) - 12*a^3*b^3*(140*A - 2100*B*x^2 + 6699*C*x^4 - 6292*D*x^
6) - 2*a*b^5*x^4*(3248*A - 6336*B*x^2 + 1155*C*x^4 + 455*D*x^6) + a^2*b^4*x
^2*(-5600*A + 29232*B*x^2 - 34848*C*x^4 + 5005*D*x^6) + 4*b^6*x^6*(-704*A +
 35*(6*B*x^2 + 3*C*x^4 + 2*D*x^6))))/(1680*b^7*(a + b*x^2)^(7/2)) + ((-16*A
*b^3 + 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*Log[-(Sqrt[b]*x) + Sqrt[a + b
*x^2]])/(16*b^(15/2))
```

Maple [**A**]
time = 0.39, size = 562, normalized size = 1.48

method	result

default	D	$\left(\frac{x^{13}}{6b(b\,x^2+a)^{\frac{7}{2}}}-\frac{13a\left(\frac{x^{11}}{4b\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{11a\left(\frac{x^{9}}{2b\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{9a\left(-\frac{x^{7}}{7b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{-\frac{x^{5}}{5b\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^{3}}{3b\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+}}}{}}{}}{b}\right.}{2b}\right.}{4b}\right.}{6b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] D*(1/6*x^13/b/(b*x^2+a)^(7/2)-13/6*a/b*(1/4*x^11/b/(b*x^2+a)^(7/2)-11/4*a/b
*(1/2*x^9/b/(b*x^2+a)^(7/2)-9/2*a/b*(-1/7*x^7/b/(b*x^2+a)^(7/2)+1/b*(-1/5*x
^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1
/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))))))+C*(1/4*x^11/b/(b*x^2+a)^
(7/2)-11/4*a/b*(1/2*x^9/b/(b*x^2+a)^(7/2)-9/2*a/b*(-1/7*x^7/b/(b*x^2+a)^(7/
2)+1/b*(-1/5*x^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/
b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))))+B*(1/2*x^9/
b/(b*x^2+a)^(7/2)-9/2*a/b*(-1/7*x^7/b/(b*x^2+a)^(7/2)+1/b*(-1/5*x^5/b/(b*x^
2+a)^(5/2)+1/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3
/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))))+A*(-1/7*x^7/b/(b*x^2+a)^(7/2)+1/b*(-
1/5*x^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+
a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1221 vs. $2(345) = 690$.
time = 0.32, size = 1221, normalized size = 3.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/6*D*x^13/((b*x^2 + a)^(7/2)*b) - 13/24*D*a*x^11/((b*x^2 + a)^(7/2)*b^2) +
 1/4*C*x^11/((b*x^2 + a)^(7/2)*b) + 143/48*D*a^2*x^9/((b*x^2 + a)^(7/2)*b^3
) - 11/8*C*a*x^9/((b*x^2 + a)^(7/2)*b^2) + 1/2*B*x^9/((b*x^2 + a)^(7/2)*b)
- 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 5
6*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*A*x + 4
29/560*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 5
6*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*D*a^3*x
/b^3 - 99/280*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b
^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*
C*a^2*x/b^2 + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7
/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b
^4))*B*a*x/b + 143/80*D*a^3*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*
x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^4 - 33/40*C*a^2*x*(1
5*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*
x^2 + a)^(5/2)*b^3))/b^3 + 3/10*B*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*
x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/15*A*x
*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/(
(b*x^2 + a)^(5/2)*b^3))/b + 143/16*D*a^3*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2
```

```
*a/((b*x^2 + a)^(3/2)*b^2))/b^5 - 33/8*C*a^2*x*(3*x^2/((b*x^2 + a)^(3/2)*b)
 + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^4 + 3/2*B*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b
) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*A*x*(3*x^2/((b*x^2 + a)^(3/2)*b)
 + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 429/16*D*a^4*x^3/((b*x^2 + a)^(5/2)*b
^6) - 99/8*C*a^3*x^3/((b*x^2 + a)^(5/2)*b^5) + 9/2*B*a^2*x^3/((b*x^2 + a)^(
5/2)*b^4) - A*a*x^3/((b*x^2 + a)^(5/2)*b^3) - 19877/560*D*a^3*x/(sqrt(b*x^2
 + a)*b^7) - 2431/560*D*a^4*x/((b*x^2 + a)^(3/2)*b^7) + 12441/560*D*a^5*x/(
(b*x^2 + a)^(5/2)*b^7) + 4587/280*C*a^2*x/(sqrt(b*x^2 + a)*b^6) + 561/280*C
*a^3*x/((b*x^2 + a)^(3/2)*b^6) - 2871/280*C*a^4*x/((b*x^2 + a)^(5/2)*b^6) -
 417/70*B*a*x/(sqrt(b*x^2 + a)*b^5) - 51/70*B*a^2*x/((b*x^2 + a)^(3/2)*b^5)
 + 261/70*B*a^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*A*x/(sqrt(b*x^2 + a)*b^
4) + 17/105*A*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*A*a^2*x/((b*x^2 + a)^(5/2
)*b^4) - 429/16*D*a^3*arcsinh(b*x/sqrt(a*b))/b^(15/2) + 99/8*C*a^2*arcsinh(
b*x/sqrt(a*b))/b^(13/2) - 9/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(11/2) + A*arcsi
nh(b*x/sqrt(a*b))/b^(9/2)
```

Fricas [**A**]
time = 6.52, size = 987, normalized size = 2.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/3360*(105*((429*D*a^3*b^4 - 198*C*a^2*b^5 + 72*B*a*b^6 - 16*A*b^7)*x^8 +
 429*D*a^7 - 198*C*a^6*b + 72*B*a^5*b^2 - 16*A*a^4*b^3 + 4*(429*D*a^4*b^3 -
 198*C*a^3*b^4 + 72*B*a^2*b^5 - 16*A*a*b^6)*x^6 + 6*(429*D*a^5*b^2 - 198*C*
a^4*b^3 + 72*B*a^3*b^4 - 16*A*a^2*b^5)*x^4 + 4*(429*D*a^6*b - 198*C*a^5*b^2
 + 72*B*a^4*b^3 - 16*A*a^3*b^4)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 +
a)*sqrt(b)*x - a) + 2*(280*D*b^7*x^13 - 70*(13*D*a*b^6 - 6*C*b^7)*x^11 + 35
*(143*D*a^2*b^5 - 66*C*a*b^6 + 24*B*b^7)*x^9 + 176*(429*D*a^3*b^4 - 198*C*a
^2*b^5 + 72*B*a*b^6 - 16*A*b^7)*x^7 + 406*(429*D*a^4*b^3 - 198*C*a^3*b^4 +
72*B*a^2*b^5 - 16*A*a*b^6)*x^5 + 350*(429*D*a^5*b^2 - 198*C*a^4*b^3 + 72*B*
a^3*b^4 - 16*A*a^2*b^5)*x^3 + 105*(429*D*a^6*b - 198*C*a^5*b^2 + 72*B*a^4*b
^3 - 16*A*a^3*b^4)*x)*sqrt(b*x^2 + a))/(b^12*x^8 + 4*a*b^11*x^6 + 6*a^2*b^1
0*x^4 + 4*a^3*b^9*x^2 + a^4*b^8), 1/1680*(105*((429*D*a^3*b^4 - 198*C*a^2*b
^5 + 72*B*a*b^6 - 16*A*b^7)*x^8 + 429*D*a^7 - 198*C*a^6*b + 72*B*a^5*b^2 -
16*A*a^4*b^3 + 4*(429*D*a^4*b^3 - 198*C*a^3*b^4 + 72*B*a^2*b^5 - 16*A*a*b^6
)*x^6 + 6*(429*D*a^5*b^2 - 198*C*a^4*b^3 + 72*B*a^3*b^4 - 16*A*a^2*b^5)*x^4
 + 4*(429*D*a^6*b - 198*C*a^5*b^2 + 72*B*a^4*b^3 - 16*A*a^3*b^4)*x^2)*sqrt(
-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (280*D*b^7*x^13 - 70*(13*D*a*b^6 -
 6*C*b^7)*x^11 + 35*(143*D*a^2*b^5 - 66*C*a*b^6 + 24*B*b^7)*x^9 + 176*(429*
D*a^3*b^4 - 198*C*a^2*b^5 + 72*B*a*b^6 - 16*A*b^7)*x^7 + 406*(429*D*a^4*b^3
 - 198*C*a^3*b^4 + 72*B*a^2*b^5 - 16*A*a*b^6)*x^5 + 350*(429*D*a^5*b^2 - 19
8*C*a^4*b^3 + 72*B*a^3*b^4 - 16*A*a^2*b^5)*x^3 + 105*(429*D*a^6*b - 198*C*a
```

```
^5*b^2 + 72*B*a^4*b^3 - 16*A*a^3*b^4)*x)*sqrt(b*x^2 + a))/(b^12*x^8 + 4*a*b
^11*x^6 + 6*a^2*b^10*x^4 + 4*a^3*b^9*x^2 + a^4*b^8)]
```

Sympy **[F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac **[A]**
time = 1.17, size = 342, normalized size = 0.90

$$\frac{\left(\left(\left(\left(35\left(2\left(\frac{4Dx^2}{b}-\frac{13Da^4b^{11}-6Ca^3b^{12}}{a^3b^{13}}\right)x^2+\frac{143Da^5b^{10}-66Ca^4b^{11}+24Ba^3b^{12}}{a^3b^{13}}\right)x^2+\frac{176\left(429Da^6b^9-198Ca^5b^{10}+72Ba^4b^{11}-16Aa^3b^{12}\right)}{a^3b^{13}}\right)x^2+\frac{406\left(429Da^7b^8-198Ca^6b^9+72Ba^5b^{10}-16Aa^4b^{11}\right)}{a^3b^{13}}\right)x^2+\frac{350\left(429Da^8b^7-198Ca^7b^8+72Ba^6b^9-16Aa^5b^{10}\right)}{a^3b^{13}}\right)x^2+\frac{105\left(429Da^9b^6-198Ca^8b^7+72Ba^7b^8-16Aa^6b^9\right)}{a^3b^{13}}\right)x}{1680\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{\left(429Da^3-198Ca^2b+72Bab^2-16Ab^3\right)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16b^{\frac{15}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/1680*((((35*(2*(4*D*x^2/b - (13*D*a^4*b^11 - 6*C*a^3*b^12)/(a^3*b^13))*x^
2 + (143*D*a^5*b^10 - 66*C*a^4*b^11 + 24*B*a^3*b^12)/(a^3*b^13))*x^2 + 176*
(429*D*a^6*b^9 - 198*C*a^5*b^10 + 72*B*a^4*b^11 - 16*A*a^3*b^12)/(a^3*b^13)
)*x^2 + 406*(429*D*a^7*b^8 - 198*C*a^6*b^9 + 72*B*a^5*b^10 - 16*A*a^4*b^11)
/(a^3*b^13))*x^2 + 350*(429*D*a^8*b^7 - 198*C*a^7*b^8 + 72*B*a^6*b^9 - 16*A
*a^5*b^10)/(a^3*b^13))*x^2 + 105*(429*D*a^9*b^6 - 198*C*a^8*b^7 + 72*B*a^7*
b^8 - 16*A*a^6*b^9)/(a^3*b^13))*x/(b*x^2 + a)^(7/2) + 1/16*(429*D*a^3 - 198
*C*a^2*b + 72*B*a*b^2 - 16*A*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^
(15/2)
```

Mupad **[F]**
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^8\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^8*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

3.160 $\int \frac{x^6\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=279

$$\frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\left(a+bx^2\right)^{7/2}}+\frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\left(a+bx^2\right)^{5/2}}+\frac{\left(8b^2B-36abC+99a^2D\right)x^5}{60ab^4\left(a+bx^2\right)^{3/2}}+\frac{Dx^7}{4b^3\left(a+bx^2\right)^{3/2}}+\frac{(8b^2}{}$$

```
[Out] 1/7*(A-a*(B*b^2-C*a*b+D*a^2)/b^3)*x^7/a/(b*x^2+a)^(7/2)+1/5*(B*b^2-2*C*a*b+
3*D*a^2)*x^7/a/b^3/(b*x^2+a)^(5/2)+1/60*(8*B*b^2-36*C*a*b+99*D*a^2)*x^5/a/b
^4/(b*x^2+a)^(3/2)+1/4*D*x^7/b^3/(b*x^2+a)^(3/2)+1/8*(8*B*b^2-36*C*a*b+99*D
*a^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(13/2)+1/12*(8*B*b^2-36*C*a*b+99
*D*a^2)*x^3/a/b^5/(b*x^2+a)^(1/2)-1/8*(8*B*b^2-36*C*a*b+99*D*a^2)*x*(b*x^2+
a)^(1/2)/a/b^6
```

Rubi [A]

time = 0.31, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {1818, 1599, 1277, 1598, 470, 294, 327, 223, 212}

$$\frac{x^7\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)\left(99a^2D-36abC+8b^2B\right)}{8b^{13/2}}-\frac{x\sqrt{a+bx^2}\left(99a^2D-36abC+8b^2B\right)}{8ab^6}+\frac{x^3\left(99a^2D-36abC+8b^2B\right)}{12ab^5\sqrt{a+bx^2}}+\frac{x^5\left(99a^2D-36abC+8b^2B\right)}{60ab^4\left(a+bx^2\right)^{3/2}}+\frac{x^7\left(3a^2D-2abC+b^2B\right)}{5ab^3\left(a+bx^2\right)^{5/2}}+\frac{Dx^7}{4b^3\left(a+bx^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^7)/(7*a*(a + b*x^2)^(7/2)) + ((b^2
*B - 2*a*b*C + 3*a^2*D)*x^7)/(5*a*b^3*(a + b*x^2)^(5/2)) + ((8*b^2*B - 36*a
*b*C + 99*a^2*D)*x^5)/(60*a*b^4*(a + b*x^2)^(3/2)) + (D*x^7)/(4*b^3*(a + b*
x^2)^(3/2)) + ((8*b^2*B - 36*a*b*C + 99*a^2*D)*x^3)/(12*a*b^5*Sqrt[a + b*x^
2]) - ((8*b^2*B - 36*a*b*C + 99*a^2*D)*x*Sqrt[a + b*x^2])/(8*a*b^6) + ((8*b
^2*B - 36*a*b*C + 99*a^2*D)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(8*b^(13/
2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &
& GtQ[m, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx &= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\,(a+bx^2)^{7/2}} - \frac{\int \frac{x^5\left(-7a\left(B-\frac{a(bC-aD)}{b^2}\right)x-7a\left(C-\frac{aD}{b}\right)x^3-7aDx^5\right)}{(a+bx^2)^{7/2}}\,dx}{7ab} \\
&= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\,(a+bx^2)^{7/2}} - \frac{\int \frac{x^6\left(-7a\left(B-\frac{a(bC-aD)}{b^2}\right)-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4\right)}{(a+bx^2)^{7/2}}\,dx}{7ab} \\
&= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\,(a+bx^2)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\,(a+bx^2)^{5/2}} + \int \frac{x^5\left(-7a\left(2B-\frac{a(9\right.\right.}{(a} \\
&= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\,(a+bx^2)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\,(a+bx^2)^{5/2}} + \int \frac{x^6\left(-7a\left(2B-\frac{a(9\right.\right.}{(a-} \\
&= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\,(a+bx^2)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\,(a+bx^2)^{5/2}} + \frac{Dx^7}{4b^3\,(a+bx^2)^{3/2}} \\
&= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\,(a+bx^2)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\,(a+bx^2)^{5/2}} + \frac{(8b^2B-36abC}{60ab^4\,(a+} \\
&= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\,(a+bx^2)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\,(a+bx^2)^{5/2}} + \frac{(8b^2B-36abC}{60ab^4\,(a+} \\
&= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\,(a+bx^2)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\,(a+bx^2)^{5/2}} + \frac{(8b^2B-36abC}{60ab^4\,(a+} \\
&= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\,(a+bx^2)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\,(a+bx^2)^{5/2}} + \frac{(8b^2B-36abC}{60ab^4\,(a+} \\
&= \frac{\left(A-\frac{a\left(b^2B-abC+a^2D\right)}{b^3}\right)x^7}{7a\,(a+bx^2)^{7/2}} + \frac{\left(b^2B-2abC+3a^2D\right)x^7}{5ab^3\,(a+bx^2)^{5/2}} + \frac{(8b^2B-36abC}{60ab^4\,(a+}
\end{aligned}
$$

Mathematica [A]
time = 0.89, size = 207, normalized size = 0.74

$$
\frac{x(-10395a^6D+120Ab^6x^6+630a^5b(6C-55Dx^2)+a^2b^4x^4(-3248B+6336Cx^2-1155Dx^4)-42a^4b^2(20B-300Cx^2+957Dx^4)-8a^3b^3x^2(350B-1827Cx^2+2178Dx^4)+2ab^5x^6(-704B+105(2Cx^2+Dx^4)))}{840ab^6\,(a+bx^2)^{7/2}}-\frac{(8b^2B-36abC+99a^2D)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{8b^{13/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(-10395*a^6*D + 120*A*b^6*x^6 + 630*a^5*b*(6*C - 55*D*x^2) + a^2*b^4*x^4
*(-3248*B + 6336*C*x^2 - 1155*D*x^4) - 42*a^4*b^2*(20*B - 300*C*x^2 + 957*D
*x^4) - 8*a^3*b^3*x^2*(350*B - 1827*C*x^2 + 2178*D*x^4) + 2*a*b^5*x^6*(-704
*B + 105*(2*C*x^2 + D*x^4))))/(840*a*b^6*(a + b*x^2)^(7/2)) - ((8*b^2*B - 3
6*a*b*C + 99*a^2*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(13/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(247) = 494$.
time = 0.12, size = 531, normalized size = 1.90

method	result

default	$D(\frac{x^{11}}{4b(b\,x^2+a)^{\frac{7}{2}}}-\frac{11a(\frac{x^9}{2b(b\,x^2+a)^{\frac{7}{2}}}-\frac{9a(-\frac{x^7}{7b(b\,x^2+a)^{\frac{7}{2}}}+\frac{-\frac{x^5}{5b(b\,x^2+a)^{\frac{5}{2}}}+\frac{-\frac{x^3}{3b(b\,x^2+a)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln(x\sqrt{b}+\sqrt{b\,x}}{b^{\frac{3}{2}}}}{b}}{b}}{b}}{2b}}{4b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] D*(1/4*x^11/b/(b*x^2+a)^(7/2)-11/4*a/b*(1/2*x^9/b/(b*x^2+a)^(7/2)-9/2*a/b*(
-1/7*x^7/b/(b*x^2+a)^(7/2)+1/b*(-1/5*x^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^3/b/
(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^
(1/2))))))))+C*(1/2*x^9/b/(b*x^2+a)^(7/2)-9/2*a/b*(-1/7*x^7/b/(b*x^2+a)^(7/
2)+1/b*(-1/5*x^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/
b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))))+B*(-1/7*x^7/
b/(b*x^2+a)^(7/2)+1/b*(-1/5*x^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^3/b/(b*x^2+a)
^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))
)+A*(-1/2*x^5/b/(b*x^2+a)^(7/2)+5/2*a/b*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b
*(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/
(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))
)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 986 vs. $2(247) = 494$.

time = 0.31, size = 986, normalized size = 3.53

$$\frac{Dx^{11}}{4\,(bx^2+a)^{\frac{7}{2}}b}-\frac{11\,Dax^9}{8\,(bx^2+a)^{\frac{7}{2}}b^2}+\frac{Cx^9}{2\,(bx^2+a)^{\frac{7}{2}}b}-\frac{1}{35}\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b}+\frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)Bx-\frac{99\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b}+\frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)Da^2x}{280\,b^2}+\frac{9\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b}+\frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)Cax}{70\,b}-\frac{33\,Da^2x\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b}+\frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)}{40\,b^3}+\frac{3\,Cax\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b}+\frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)}{10\,b^2}-\frac{Bx\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b}+\frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)}{15\,b}-\frac{Ax^5}{2\,(bx^2+a)^{\frac{7}{2}}b}-\frac{33\,Da^2x\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{8\,b^4}+\frac{3\,Cax\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{2\,b^3}-\frac{Bx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b^2}-\frac{99\,Da^3x^3}{8\,(bx^2+a)^{\frac{5}{2}}b^5}+\frac{9\,Ca^2x^3}{2\,(bx^2+a)^{\frac{5}{2}}b^4}-\frac{Bax^3}{(bx^2+a)^{\frac{5}{2}}b^3}-\frac{5\,Aax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2}+\frac{4587\,Da^2x}{280\,\sqrt{bx^2+a}\,b^6}+\frac{561\,Da^3x}{280\,(bx^2+a)^{\frac{3}{2}}b^6}-\frac{2871\,Da^4x}{280\,(bx^2+a)^{\frac{5}{2}}b^6}-\frac{417\,Cax}{70\,\sqrt{bx^2+a}\,b^5}-\frac{51\,Ca^2x}{70\,(bx^2+a)^{\frac{3}{2}}b^5}+\frac{261\,Ca^3x}{70\,(bx^2+a)^{\frac{5}{2}}b^5}+\frac{139\,Bx}{105\,\sqrt{bx^2+a}\,b^4}+\frac{17\,Bax}{105\,(bx^2+a)^{\frac{3}{2}}b^4}-\frac{29\,Ba^2x}{35\,(bx^2+a)^{\frac{5}{2}}b^4}+\frac{Ax}{14\,(bx^2+a)^{\frac{3}{2}}b^3}+\frac{Ax}{7\,\sqrt{bx^2+a}\,ab^3}+\frac{3\,Aax}{56\,(bx^2+a)^{\frac{3}{2}}b^3}-\frac{15\,Aa^2x}{56\,(bx^2+a)^{\frac{5}{2}}b^3}+\frac{99\,Da^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{13}{2}}}-\frac{9\,Ca\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{11}{2}}}+\frac{B\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}$$

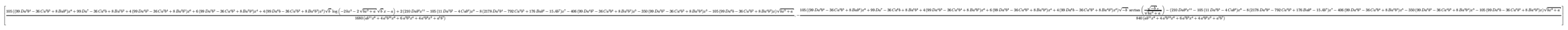
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/4*D*x^11/((b*x^2 + a)^(7/2)*b) - 11/8*D*a*x^9/((b*x^2 + a)^(7/2)*b^2) + 1
/2*C*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*
x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/(
(b*x^2 + a)^(7/2)*b^4))*B*x - 99/280*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x
^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((
b*x^2 + a)^(7/2)*b^4))*D*a^2*x/b^2 + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 7
0*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a
^3/((b*x^2 + a)^(7/2)*b^4))*C*a*x/b - 33/40*D*a^2*x*(15*x^4/((b*x^2 + a)^(5
/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/
b^3 + 3/10*C*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2
)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/15*B*x*(15*x^4/((b*x^2 + a)
^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3
))/b - 1/2*A*x^5/((b*x^2 + a)^(7/2)*b) - 33/8*D*a^2*x*(3*x^2/((b*x^2 + a)^(
3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^4 + 3/2*C*a*x*(3*x^2/((b*x^2 + a)^
(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*B*x*(3*x^2/((b*x^2 + a)^(
3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - 99/8*D*a^3*x^3/((b*x^2 + a)^(5
/2)*b^5) + 9/2*C*a^2*x^3/((b*x^2 + a)^(5/2)*b^4) - B*a*x^3/((b*x^2 + a)^(5/
2)*b^3) - 5/8*A*a*x^3/((b*x^2 + a)^(7/2)*b^2) + 4587/280*D*a^2*x/(sqrt(b*x^
```

```
2 + a)*b^6) + 561/280*D*a^3*x/((b*x^2 + a)^(3/2)*b^6) - 2871/280*D*a^4*x/((
b*x^2 + a)^(5/2)*b^6) - 417/70*C*a*x/(sqrt(b*x^2 + a)*b^5) - 51/70*C*a^2*x/
((b*x^2 + a)^(3/2)*b^5) + 261/70*C*a^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*
B*x/(sqrt(b*x^2 + a)*b^4) + 17/105*B*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*B*
a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*A*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*A*x/
(sqrt(b*x^2 + a)*a*b^3) + 3/56*A*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*A*a^2*
x/((b*x^2 + a)^(7/2)*b^3) + 99/8*D*a^2*arcsinh(b*x/sqrt(a*b))/b^(13/2) - 9/
2*C*a*arcsinh(b*x/sqrt(a*b))/b^(11/2) + B*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

Fricas [**A**]
time = 5.32, size = 816, normalized size = 2.92



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/1680*(105*((99*D*a^3*b^4 - 36*C*a^2*b^5 + 8*B*a*b^6)*x^8 + 99*D*a^7 - 36
*C*a^6*b + 8*B*a^5*b^2 + 4*(99*D*a^4*b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^5)*x^6
+ 6*(99*D*a^5*b^2 - 36*C*a^4*b^3 + 8*B*a^3*b^4)*x^4 + 4*(99*D*a^6*b - 36*C*
a^5*b^2 + 8*B*a^4*b^3)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b
)*x - a) + 2*(210*D*a*b^6*x^11 - 105*(11*D*a^2*b^5 - 4*C*a*b^6)*x^9 - 8*(21
78*D*a^3*b^4 - 792*C*a^2*b^5 + 176*B*a*b^6 - 15*A*b^7)*x^7 - 406*(99*D*a^4*
b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^5)*x^5 - 350*(99*D*a^5*b^2 - 36*C*a^4*b^3 +
8*B*a^3*b^4)*x^3 - 105*(99*D*a^6*b - 36*C*a^5*b^2 + 8*B*a^4*b^3)*x)*sqrt(b*
x^2 + a))/(a*b^11*x^8 + 4*a^2*b^10*x^6 + 6*a^3*b^9*x^4 + 4*a^4*b^8*x^2 + a^
5*b^7), -1/840*(105*((99*D*a^3*b^4 - 36*C*a^2*b^5 + 8*B*a*b^6)*x^8 + 99*D*a
^7 - 36*C*a^6*b + 8*B*a^5*b^2 + 4*(99*D*a^4*b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^
5)*x^6 + 6*(99*D*a^5*b^2 - 36*C*a^4*b^3 + 8*B*a^3*b^4)*x^4 + 4*(99*D*a^6*b
- 36*C*a^5*b^2 + 8*B*a^4*b^3)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) - (210*D*a*b^6*x^11 - 105*(11*D*a^2*b^5 - 4*C*a*b^6)*x^9 - 8*(2178*D*a^
3*b^4 - 792*C*a^2*b^5 + 176*B*a*b^6 - 15*A*b^7)*x^7 - 406*(99*D*a^4*b^3 - 3
6*C*a^3*b^4 + 8*B*a^2*b^5)*x^5 - 350*(99*D*a^5*b^2 - 36*C*a^4*b^3 + 8*B*a^3
*b^4)*x^3 - 105*(99*D*a^6*b - 36*C*a^5*b^2 + 8*B*a^4*b^3)*x)*sqrt(b*x^2 + a
))/(a*b^11*x^8 + 4*a^2*b^10*x^6 + 6*a^3*b^9*x^4 + 4*a^4*b^8*x^2 + a^5*b^7)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 9649 vs. $2(274) = 548$.
time = 211.75, size = 9649, normalized size = 34.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/
a) + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1
 + b*x**2/a)) + B*(105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/
sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(
101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b
*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(19
7/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*s
qrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 6
30*a**(203/2)*b**46*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a
**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqr
t(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100
*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)
*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/
a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(201/2)*
b**47*x**4*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(
99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a
) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b*
*(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 +
 b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(1
93/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 2100*a**(199/2)*b**48*x**6*sqr
t(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 +
 b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(2
01/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*
sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 6
30*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/
2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(197/2)*b**49*x**8*sqrt(1 + b*x**2/a
)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 6
30*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/
2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**
2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*
b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(
1 + b*x**2/a)) + 630*a**(195/2)*b**50*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b
)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*
b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1
 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a*
*(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**
10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a))
 + 105*a**(193/2)*b**51*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(
105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**
2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) +
 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(1
07/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*
x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 105*a**102*
b**(91/2)*x/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b
**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1
```

```
+ b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**
(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**1
0*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a))
- 665*a**101*b**(93/2)*x**3/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) +
630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103
/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x*
*2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)
*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt
(1 + b*x**2/a)) - 1771*a**100*b**(95/2)*x**5/(105*a**(205/2)*b**(99/2)*sqrt
(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a
**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x
**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a)
 + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(
111/2)*x**12*sqrt(1 + b*x**2/a)) - 2549*a**99*b**(97/2)*x**7/(105*a**(205/2
)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*
x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199
/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sq
rt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105
*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 2096*a**98*b**(99/2)*x**
9/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*
x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a
) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*...
```

Giac [A]
time = 1.20, size = 265, normalized size = 0.95

$$\frac{\left(\left(\left(\left(105\left(\frac{2\,Dx^2}{b}-\frac{11\,Da^4b^9-4\,Ca^3b^{10}}{a^3b^{11}}\right)x^2-\frac{8\left(2178\,Da^5b^8-792\,Ca^4b^9+176\,Ba^3b^{10}-15\,Aa^2b^{11}\right)}{a^3b^{11}}\right)x^2-\frac{406\left(99\,Da^6b^7-36\,Ca^5b^8+8\,Ba^4b^9\right)}{a^3b^{11}}\right)x^2-\frac{350\left(99\,Da^7b^6-36\,Ca^6b^7+8\,Ba^5b^8\right)}{a^3b^{11}}\right)x^2-\frac{105\left(99\,Da^8b^5-36\,Ca^7b^6+8\,Ba^6b^7\right)}{a^3b^{11}}\right)x}{840\,(bx^2+a)^{\frac{7}{2}}}-\frac{\left(99\,Da^2-36\,Cab+8\,Bb^2\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{13}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/840*((((105*(2*D*x^2/b - (11*D*a^4*b^9 - 4*C*a^3*b^10)/(a^3*b^11))*x^2 -
8*(2178*D*a^5*b^8 - 792*C*a^4*b^9 + 176*B*a^3*b^10 - 15*A*a^2*b^11)/(a^3*b^
11))*x^2 - 406*(99*D*a^6*b^7 - 36*C*a^5*b^8 + 8*B*a^4*b^9)/(a^3*b^11))*x^2
- 350*(99*D*a^7*b^6 - 36*C*a^6*b^7 + 8*B*a^5*b^8)/(a^3*b^11))*x^2 - 105*(99
*D*a^8*b^5 - 36*C*a^7*b^6 + 8*B*a^6*b^7)/(a^3*b^11))*x/(b*x^2 + a)^(7/2) -
1/8*(99*D*a^2 - 36*C*a*b + 8*B*b^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/
b^(13/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
[Out] int((x^6*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

3.161 $\int \frac{x^4\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=210

$$\frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a\left(a+bx^2\right)^{7/2}}+\frac{\left(2Ab^3+a(5b^2B-12abC+19a^2D)\right)x^5}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{a(bC-3aD)x}{3b^5\left(a+bx^2\right)^{3/2}}-\frac{(4bC-15aD)x}{3b^5\sqrt{a+bx^2}}+Dx$$

```
[Out] 1/7*(A-a*(B*b^2-C*a*b+D*a^2)/b^3)*x^5/a/(b*x^2+a)^(7/2)+1/35*(2*A*b^3+a*(5*
B*b^2-12*C*a*b+19*D*a^2))*x^5/a^2/b^3/(b*x^2+a)^(5/2)+1/3*a*(C*b-3*D*a)*x/b
^5/(b*x^2+a)^(3/2)+1/2*(2*C*b-9*D*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(
11/2)-1/3*(4*C*b-15*D*a)*x/b^5/(b*x^2+a)^(1/2)+1/2*D*x*(b*x^2+a)^(1/2)/b^5
```

Rubi [A]
time = 0.27, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = $\{1818, 1599, 1277, 1598, 466, 1171, 396, 223, 212\}$

$$\frac{x^5\left(a(19a^2D-12abC+5b^2B)+2Ab^3\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}+\frac{(2bC-9aD)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}-\frac{x(4bC-15aD)}{3b^5\sqrt{a+bx^2}}+\frac{ax(bC-3aD)}{3b^5\left(a+bx^2\right)^{3/2}}+\frac{Dx\sqrt{a+bx^2}}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^5)/(7*a*(a + b*x^2)^(7/2)) + ((2*A
*b^3 + a*(5*b^2*B - 12*a*b*C + 19*a^2*D))*x^5)/(35*a^2*b^3*(a + b*x^2)^(5/2
)) + (a*(b*C - 3*a*D)*x)/(3*b^5*(a + b*x^2)^(3/2)) - ((4*b*C - 15*a*D)*x)/(
3*b^5*Sqrt[a + b*x^2]) + (D*x*Sqrt[a + b*x^2])/(2*b^5) + ((2*b*C - 9*a*D)*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &
& GtQ[m, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
```

```
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a\,(a+bx^2)^{7/2}} - \frac{\int \frac{x^3\left(-\left(2Ab+\frac{5a\left(b^2B-abC+a^2D\right)}{b^2}\right)x-7a\left(C-\frac{aD}{b}\right)x^3-}{(a+bx^2)^{7/2}}}{7ab}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a\,(a+bx^2)^{7/2}} - \frac{\int \frac{x^4\left(-2Ab-\frac{5a\left(b^2B-abC+a^2D\right)}{b^2}-7a\left(C-\frac{aD}{b}\right)x^2-7aD\right.}{(a+bx^2)^{7/2}}}{7ab}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a\,(a+bx^2)^{7/2}} + \frac{(2Ab^3+a(5b^2B-12abC+19a^2D))\,x^5}{35a^2b^3\,(a+bx^2)^{5/2}} +$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a\,(a+bx^2)^{7/2}} + \frac{(2Ab^3+a(5b^2B-12abC+19a^2D))\,x^5}{35a^2b^3\,(a+bx^2)^{5/2}} +$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a\,(a+bx^2)^{7/2}} + \frac{(2Ab^3+a(5b^2B-12abC+19a^2D))\,x^5}{35a^2b^3\,(a+bx^2)^{5/2}} +$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a\,(a+bx^2)^{7/2}} + \frac{(2Ab^3+a(5b^2B-12abC+19a^2D))\,x^5}{35a^2b^3\,(a+bx^2)^{5/2}} +$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a\,(a+bx^2)^{7/2}} + \frac{(2Ab^3+a(5b^2B-12abC+19a^2D))\,x^5}{35a^2b^3\,(a+bx^2)^{5/2}} +$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a\,(a+bx^2)^{7/2}} + \frac{(2Ab^3+a(5b^2B-12abC+19a^2D))\,x^5}{35a^2b^3\,(a+bx^2)^{5/2}} +$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a\,(a+bx^2)^{7/2}} + \frac{(2Ab^3+a(5b^2B-12abC+19a^2D))\,x^5}{35a^2b^3\,(a+bx^2)^{5/2}} +$$

Mathematica [A]

time = 0.66, size = 173, normalized size = 0.82

$$\frac{x(945a^6D+12Ab^6x^6+6ab^5x^4(7A+5Bx^2)-210a^5b(C-15Dx^2)+a^2b^4x^6(-352C+105Dx^2)+14a^4b^2x^2(-50C+261Dx^2)+4a^3b^3x^4(-203C+396Dx^2))}{210a^2b^5\,(a+bx^2)^{7/2}}+\frac{(-2bC+9aD)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(945*a^6*D + 12*A*b^6*x^6 + 6*a*b^5*x^4*(7*A + 5*B*x^2) - 210*a^5*b*(C -
 15*D*x^2) + a^2*b^4*x^6*(-352*C + 105*D*x^2) + 14*a^4*b^2*x^2*(-50*C + 261
```

```
*D*x^2) + 4*a^3*b^3*x^4*(-203*C + 396*D*x^2)))/(210*a^2*b^5*(a + b*x^2)^(7/
2)) + ((-2*b*C + 9*a*D)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(182) = 364$.
time = 0.11, size = 500, normalized size = 2.38

method	result

default	$D\left(\frac{x^{9}}{2 b \left(b \,x^{2}+a \right)^{\frac{7}{2}}}-\frac{9 a \left(-\frac{x^{7}}{7 b \left(b \,x^{2}+a \right)^{\frac{7}{2}}}+\frac{-\frac{x^{5}}{5 b \left(b \,x^{2}+a \right)^{\frac{5}{2}}}+\frac{-\frac{x^{3}}{3 b \left(b \,x^{2}+a \right)^{\frac{3}{2}}}+\frac{-\frac{x}{b \sqrt{b \,x^{2}+a}}+\frac{\ln \left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)}{2 b}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] D*(1/2*x^9/b/(b*x^2+a)^(7/2)-9/2*a/b*(-1/7*x^7/b/(b*x^2+a)^(7/2)+1/b*(-1/5*
x^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(
1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))))+C*(-1/7*x^7/b/(b*x^2+a)^(
7/2)+1/b*(-1/5*x^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-
x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))+B*(-1/2*x^5
/b/(b*x^2+a)^(7/2)+5/2*a/b*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/6*x/b/(b
*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/
2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))))+A*(-1/4*x
^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*
x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+
2/3*x/a^2/(b*x^2+a)^(1/2))))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 753 vs. $2(183) = 366$.
time = 0.32, size = 753, normalized size = 3.59

$$\frac{Dx^{9}}{2\,(bx^{2}+a)^{\frac{7}{2}}b}-\frac{1}{35}\left(\frac{35\,x^{6}}{(bx^{2}+a)^{\frac{7}{2}}b}+\frac{70\,ax^{4}}{(bx^{2}+a)^{\frac{7}{2}}b^{2}}+\frac{56\,a^{2}x^{2}}{(bx^{2}+a)^{\frac{7}{2}}b^{3}}+\frac{16\,a^{3}}{(bx^{2}+a)^{\frac{7}{2}}b^{4}}\right)Cx+\frac{9\left(\frac{35\,x^{6}}{(bx^{2}+a)^{\frac{7}{2}}b}+\frac{70\,ax^{4}}{(bx^{2}+a)^{\frac{7}{2}}b^{2}}+\frac{56\,a^{2}x^{2}}{(bx^{2}+a)^{\frac{7}{2}}b^{3}}+\frac{16\,a^{3}}{(bx^{2}+a)^{\frac{7}{2}}b^{4}}\right)Dax}{70\,b}+\frac{3\,Dax\left(\frac{15\,x^{4}}{(bx^{2}+a)^{\frac{5}{2}}b}+\frac{20\,ax^{2}}{(bx^{2}+a)^{\frac{5}{2}}b^{2}}+\frac{8\,a^{2}}{(bx^{2}+a)^{\frac{5}{2}}b^{3}}\right)}{10\,b^{2}}-\frac{Cx\left(\frac{15\,x^{4}}{(bx^{2}+a)^{\frac{5}{2}}b}+\frac{20\,ax^{2}}{(bx^{2}+a)^{\frac{5}{2}}b^{2}}+\frac{8\,a^{2}}{(bx^{2}+a)^{\frac{5}{2}}b^{3}}\right)}{15\,b}-\frac{Bx^{5}}{2\,(bx^{2}+a)^{\frac{7}{2}}b}+\frac{3\,Dax\left(\frac{3\,x^{2}}{(bx^{2}+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^{2}+a)^{\frac{3}{2}}b^{2}}\right)}{2\,b^{3}}-\frac{Cx\left(\frac{3\,x^{2}}{(bx^{2}+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^{2}+a)^{\frac{3}{2}}b^{2}}\right)}{3\,b^{2}}+\frac{9\,Da^{2}x^{3}}{2\,(bx^{2}+a)^{\frac{5}{2}}b^{4}}-\frac{Cax^{3}}{(bx^{2}+a)^{\frac{5}{2}}b^{3}}-\frac{5\,Bax^{3}}{8\,(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{Ax^{3}}{4\,(bx^{2}+a)^{\frac{7}{2}}b}-\frac{417\,Dax}{70\,\sqrt{bx^{2}+a}\,b^{5}}-\frac{51\,Da^{2}x}{70\,(bx^{2}+a)^{\frac{3}{2}}b^{5}}+\frac{261\,Da^{3}x}{70\,(bx^{2}+a)^{\frac{5}{2}}b^{5}}+\frac{139\,Cx}{105\,\sqrt{bx^{2}+a}\,b^{4}}+\frac{17\,Cax}{105\,(bx^{2}+a)^{\frac{3}{2}}b^{4}}-\frac{29\,Ca^{2}x}{35\,(bx^{2}+a)^{\frac{5}{2}}b^{4}}+\frac{Bx}{14\,(bx^{2}+a)^{\frac{3}{2}}b^{3}}+\frac{Bx}{7\,\sqrt{bx^{2}+a}\,ab^{3}}+\frac{3\,Bax}{56\,(bx^{2}+a)^{\frac{5}{2}}b^{3}}-\frac{15\,Ba^{2}x}{56\,(bx^{2}+a)^{\frac{7}{2}}b^{3}}+\frac{3\,Ax}{140\,(bx^{2}+a)^{\frac{5}{2}}b^{2}}+\frac{2\,Ax}{35\,\sqrt{bx^{2}+a}\,a^{2}b^{2}}+\frac{Ax}{35\,(bx^{2}+a)^{\frac{3}{2}}ab^{2}}-\frac{3\,Aax}{28\,(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{9\,Da\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{11}{2}}}+\frac{C\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/2*D*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a
*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/
((b*x^2 + a)^(7/2)*b^4))*C*x + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^
4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b
*x^2 + a)^(7/2)*b^4))*D*a*x/b + 3/10*D*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) +
20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/1
5*C*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*
a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*B*x^5/((b*x^2 + a)^(7/2)*b) + 3/2*D*a*
x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*C*x
*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 9/2*D*a^
2*x^3/((b*x^2 + a)^(5/2)*b^4) - C*a*x^3/((b*x^2 + a)^(5/2)*b^3) - 5/8*B*a*x
^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*A*x^3/((b*x^2 + a)^(7/2)*b) - 417/70*D*a*x
/(sqrt(b*x^2 + a)*b^5) - 51/70*D*a^2*x/((b*x^2 + a)^(3/2)*b^5) + 261/70*D*a
^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*C*x/(sqrt(b*x^2 + a)*b^4) + 17/105*C
*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*C*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14
*B*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*B*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*B*a*x
/((b*x^2 + a)^(5/2)*b^3) - 15/56*B*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*A*
x/((b*x^2 + a)^(5/2)*b^2) + 2/35*A*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*A*x/(
```

```
(b*x^2 + a)^(3/2)*a*b^2) - 3/28*A*a*x/((b*x^2 + a)^(7/2)*b^2) - 9/2*D*a*arc
sinh(b*x/sqrt(a*b))/b^(11/2) + C*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

Fricas [**A**]
time = 5.55, size = 653, normalized size = 3.11

$$\left[\frac{105\left((9\,Da^3b^4-2\,Ca^2b^5)x^8+9\,Da^7-2\,Ca^6b+4\,(9\,Da^4b^3-2\,Ca^3b^4)x^6+6\,(9\,Da^5b^2-2\,Ca^4b^3)x^4+4\,(9\,Da^6b-2\,Ca^5b^2)x^2\right)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)+2\,(105\,Da^2b^5x^9+2\,(792\,Da^3b^4-176\,Ca^2b^5+15\,Bab^6+6\,Ab^7)x^7+14\,(261\,Da^4b^3-58\,Ca^3b^4+3\,Aab^6)x^5+350\,(9\,Da^5b^2-2\,Ca^4b^3)x^3+105\,(9\,Da^6b-2\,Ca^5b^2)x)\sqrt{bx^2+a}}{420\,(a^2b^{10}x^8+4\,a^3b^9x^6+6\,a^4b^8x^4+4\,a^5b^7x^2+a^6b^6)}, \frac{105\left((9\,Da^3b^4-2\,Ca^2b^5)x^8+9\,Da^7-2\,Ca^6b+4\,(9\,Da^4b^3-2\,Ca^3b^4)x^6+6\,(9\,Da^5b^2-2\,Ca^4b^3)x^4+4\,(9\,Da^6b-2\,Ca^5b^2)x^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(105\,Da^2b^5x^9+2\,(792\,Da^3b^4-176\,Ca^2b^5+15\,Bab^6+6\,Ab^7)x^7+14\,(261\,Da^4b^3-58\,Ca^3b^4+3\,Aab^6)x^5+350\,(9\,Da^5b^2-2\,Ca^4b^3)x^3+105\,(9\,Da^6b-2\,Ca^5b^2)x)\sqrt{bx^2+a}}{210\,(a^2b^{10}x^8+4\,a^3b^9x^6+6\,a^4b^8x^4+4\,a^5b^7x^2+a^6b^6)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/420*(105*((9*D*a^3*b^4 - 2*C*a^2*b^5)*x^8 + 9*D*a^7 - 2*C*a^6*b + 4*(9*D
*a^4*b^3 - 2*C*a^3*b^4)*x^6 + 6*(9*D*a^5*b^2 - 2*C*a^4*b^3)*x^4 + 4*(9*D*a^
6*b - 2*C*a^5*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x
- a) + 2*(105*D*a^2*b^5*x^9 + 2*(792*D*a^3*b^4 - 176*C*a^2*b^5 + 15*B*a*b^6
 + 6*A*b^7)*x^7 + 14*(261*D*a^4*b^3 - 58*C*a^3*b^4 + 3*A*a*b^6)*x^5 + 350*(
9*D*a^5*b^2 - 2*C*a^4*b^3)*x^3 + 105*(9*D*a^6*b - 2*C*a^5*b^2)*x)*sqrt(b*x^
2 + a))/(a^2*b^10*x^8 + 4*a^3*b^9*x^6 + 6*a^4*b^8*x^4 + 4*a^5*b^7*x^2 + a^6
*b^6), 1/210*(105*((9*D*a^3*b^4 - 2*C*a^2*b^5)*x^8 + 9*D*a^7 - 2*C*a^6*b +
4*(9*D*a^4*b^3 - 2*C*a^3*b^4)*x^6 + 6*(9*D*a^5*b^2 - 2*C*a^4*b^3)*x^4 + 4*(
9*D*a^6*b - 2*C*a^5*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) +
 (105*D*a^2*b^5*x^9 + 2*(792*D*a^3*b^4 - 176*C*a^2*b^5 + 15*B*a*b^6 + 6*A*b
^7)*x^7 + 14*(261*D*a^4*b^3 - 58*C*a^3*b^4 + 3*A*a*b^6)*x^5 + 350*(9*D*a^5*
b^2 - 2*C*a^4*b^3)*x^3 + 105*(9*D*a^6*b - 2*C*a^5*b^2)*x)*sqrt(b*x^2 + a))/
(a^2*b^10*x^8 + 4*a^3*b^9*x^6 + 6*a^4*b^8*x^4 + 4*a^5*b^7*x^2 + a^6*b^6)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6467 vs. $2(199) = 398$.
time = 98.69, size = 6467, normalized size = 30.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 +
 b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x
**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a
**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/
a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + B*x**7/(7*a**(9/2)*sqrt(1
 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 21*a**(5/2)*b**2*x**
4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + C*(105*a*
*(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*
b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x*
*2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2
)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt
```

```
(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a
**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(203/2)*b**46*x**2*
sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(
1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a*
*(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x*
*6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a)
+ 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(1
11/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(201/2)*b**47*x**4*sqrt(1 + b*x**
2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a)
+ 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(1
03/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*
x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/
2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sq
rt(1 + b*x**2/a)) + 2100*a**(199/2)*b**48*x**6*sqrt(1 + b*x**2/a)*asinh(sqr
t(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/
2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqr
t(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575
*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*
x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/
a)) + 1575*a**(197/2)*b**49*x**8*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a)
)/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*
x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a
) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b*
*(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 +
 b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(
195/2)*b**50*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205
/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 +
b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(1
99/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*
sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 1
05*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 105*a**(193/2)*b**51*x
**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*
sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 15
75*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/
2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**
2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*
b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 105*a**102*b**(91/2)*x/(105*a**(205/
2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b
*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(19
9/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*s
qrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 10
5*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 665*a**101*b**(93/2)*x*
*3/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)
*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/
a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b
```

```
**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1
+ b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 1771*a*
*100*b**(95/2)*x**5/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(
203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4
*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) +
1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109
/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x
**2/a)) - 2549*a**99*b**(97/2)*x**7/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x*
*2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)
*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(
1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8...
```

Giac [A]
time = 2.45, size = 203, normalized size = 0.97

$$\frac{\left(\left(\left(\left(\frac{105\,Dx^{2}}{b}+\frac{2\,(792\,Da^{4}b^{7}-176\,Ca^{3}b^{8}+15\,Ba^{2}b^{9}+6\,Aab^{10})}{a^{3}b^{9}}\right)x^{2}+\frac{14\,(261\,Da^{5}b^{6}-58\,Ca^{4}b^{7}+3\,Aa^{2}b^{9})}{a^{3}b^{9}}\right)x^{2}+\frac{350\,(9\,Da^{6}b^{5}-2\,Ca^{5}b^{6})}{a^{3}b^{9}}\right)x^{2}+\frac{105\,(9\,Da^{7}b^{4}-2\,Ca^{6}b^{5})}{a^{3}b^{9}}\right)x}{210\,(bx^{2}+a)^{\frac{7}{2}}}+\frac{(9\,Da-2\,Cb)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/210*((((105*D*x^2/b + 2*(792*D*a^4*b^7 - 176*C*a^3*b^8 + 15*B*a^2*b^9 + 6
*A*a*b^10)/(a^3*b^9))*x^2 + 14*(261*D*a^5*b^6 - 58*C*a^4*b^7 + 3*A*a^2*b^9)
/(a^3*b^9))*x^2 + 350*(9*D*a^6*b^5 - 2*C*a^5*b^6)/(a^3*b^9))*x^2 + 105*(9*D
*a^7*b^4 - 2*C*a^6*b^5)/(a^3*b^9))*x/(b*x^2 + a)^(7/2) + 1/2*(9*D*a - 2*C*b
)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

3.162 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

Optimal. Leaf size=179

$$-\frac{a^3Dx}{b^4\left(a+bx^2\right)^{7/2}}+\frac{\left(Ab^3-10a^3D\right)x^3}{3ab^3\left(a+bx^2\right)^{7/2}}+\frac{\left(4Ab^3+3ab^2B-58a^3D\right)x^5}{15a^2b^2\left(a+bx^2\right)^{7/2}}+\frac{\left(8Ab^3+6ab^2B+15a^2bC-176a^3D\right)x^7}{105a^3b\left(a+bx^2\right)^{7/2}}$$

```
[Out] -a^3*D*x/b^4/(b*x^2+a)^(7/2)+1/3*(A*b^3-10*D*a^3)*x^3/a/b^3/(b*x^2+a)^(7/2)
+1/15*(4*A*b^3+3*B*a*b^2-58*D*a^3)*x^5/a^2/b^2/(b*x^2+a)^(7/2)+1/105*(8*A*b
^3+6*B*a*b^2+15*C*a^2*b-176*D*a^3)*x^7/a^3/b/(b*x^2+a)^(7/2)+D*arctanh(x*b^
(1/2)/(b*x^2+a)^(1/2))/b^(9/2)
```

Rubi [A]
time = 0.22, antiderivative size = 192, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1818, 1599, 1277, 1598, 463, 294, 223, 212}

$$\frac{x^3\left(a\left(17a^2D-10abC+3b^2B\right)+4Ab^3\right)}{35a^2b^3\left(a+bx^2\right)^{5/2}}+\frac{x^3\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}+\frac{x^3\left(a\left(-71a^2D+15abC+6b^2B\right)+8Ab^3\right)}{105a^3b^3\left(a+bx^2\right)^{3/2}}+\frac{D\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}-\frac{Dx}{b^4\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^3)/(7*a*(a + b*x^2)^(7/2)) + ((4*A
*b^3 + a*(3*b^2*B - 10*a*b*C + 17*a^2*D))*x^3)/(35*a^2*b^3*(a + b*x^2)^(5/2
)) + ((8*A*b^3 + a*(6*b^2*B + 15*a*b*C - 71*a^2*D))*x^3)/(105*a^3*b^3*(a +
b*x^2)^(3/2)) - (D*x)/(b^4*Sqrt[a + b*x^2]) + (D*ArcTanh[(Sqrt[b]*x)/Sqrt[a
 + b*x^2]])/b^(9/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
```

```
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*b*
e*(m + 1))), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &
& GtQ[m, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^3}{7a\,(a+bx^2)^{7/2}} - \frac{\int \frac{x\left(-\left(4Ab+\frac{3a\left(b^2B-abC+a^2D\right)}{b^2}\right)x-7a\left(C-\frac{aD}{b}\right)x^3-7a}{(a+bx^2)^{7/2}}}{7ab}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^3}{7a\,(a+bx^2)^{7/2}} - \frac{\int \frac{x^2\left(-4Ab-\frac{3a\left(b^2B-abC+a^2D\right)}{b^2}-7a\left(C-\frac{aD}{b}\right)x^2-7aDx^4}{(a+bx^2)^{7/2}}}{7ab}$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^3}{7a\,(a+bx^2)^{7/2}} + \frac{(4Ab^3+a(3b^2B-10abC+17a^2D))\,x^3}{35a^2b^3\,(a+bx^2)^{5/2}} +$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^3}{7a\,(a+bx^2)^{7/2}} + \frac{(4Ab^3+a(3b^2B-10abC+17a^2D))\,x^3}{35a^2b^3\,(a+bx^2)^{5/2}} +$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^3}{7a\,(a+bx^2)^{7/2}} + \frac{(4Ab^3+a(3b^2B-10abC+17a^2D))\,x^3}{35a^2b^3\,(a+bx^2)^{5/2}} +$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^3}{7a\,(a+bx^2)^{7/2}} + \frac{(4Ab^3+a(3b^2B-10abC+17a^2D))\,x^3}{35a^2b^3\,(a+bx^2)^{5/2}} +$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^3}{7a\,(a+bx^2)^{7/2}} + \frac{(4Ab^3+a(3b^2B-10abC+17a^2D))\,x^3}{35a^2b^3\,(a+bx^2)^{5/2}} +$$

$$= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^3}{7a\,(a+bx^2)^{7/2}} + \frac{(4Ab^3+a(3b^2B-10abC+17a^2D))\,x^3}{35a^2b^3\,(a+bx^2)^{5/2}} +$$

Mathematica [A]
time = 0.61, size = 147, normalized size = 0.82

$$\frac{-105a^6Dx-350a^5bDx^3-406a^4b^2Dx^5+8Ab^6x^7-176a^3b^3Dx^7+2ab^5x^5(14A+3Bx^2)+a^2b^4x^3(35A+21Bx^2+15Cx^4)}{105a^3b^4\,(a+bx^2)^{7/2}}-\frac{D\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

```
[Out] (-105*a^6*D*x - 350*a^5*b*D*x^3 - 406*a^4*b^2*D*x^5 + 8*A*b^6*x^7 - 176*a^3
*b^3*D*x^7 + 2*a*b^5*x^5*(14*A + 3*B*x^2) + a^2*b^4*x^3*(35*A + 21*B*x^2 +
```

```
15*C*x^4))/(105*a^3*b^4*(a + b*x^2)^(7/2)) - (D*Log[-(Sqrt[b]*x) + Sqrt[a +
 b*x^2]])/b^(9/2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(159) = 318$.
time = 0.11, size = 469, normalized size = 2.62

method	result

default	$D\left(-\frac{x^{7}}{7b(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{-\frac{x^{5}}{5b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^{3}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)+C\left(-\frac{x^{5}}{2b(b\,x^{2}+a)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] D*(-1/7*x^7/b/(b*x^2+a)^(7/2)+1/b*(-1/5*x^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^3
/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+
a)^(1/2))))))+C*(-1/2*x^5/b/(b*x^2+a)^(7/2)+5/2*a/b*(-1/4*x^3/b/(b*x^2+a)^(
7/2)+3/4*a/b*(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7
/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2
+a)^(1/2)))))))+B*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/6*x/b/(b*x^2+a)^(
7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*
(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))))+A*(-1/6*x/b/(b*x^2+
a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/
5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(160) = 320$.
time = 0.31, size = 533, normalized size = 2.98

$$-\frac{1}{35}\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b}+\frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)Dx-\frac{Dx\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b}+\frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)}{15\,b}-\frac{Cx^5}{2\,(bx^2+a)^{\frac{7}{2}}b}-\frac{Dx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b^2}-\frac{Dax^3}{(bx^2+a)^{\frac{5}{2}}b^3}-\frac{5\,Cax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{Bx^3}{4\,(bx^2+a)^{\frac{7}{2}}b}+\frac{139\,Dx}{105\,\sqrt{bx^2+a}\,b^4}+\frac{17\,Dax}{105\,(bx^2+a)^{\frac{3}{2}}b^4}-\frac{29\,Da^2x}{35\,(bx^2+a)^{\frac{5}{2}}b^4}+\frac{Cx}{14\,(bx^2+a)^{\frac{3}{2}}b^3}+\frac{Cx}{7\,\sqrt{bx^2+a}\,ab^3}+\frac{3\,Cax}{56\,(bx^2+a)^{\frac{5}{2}}b^3}-\frac{15\,Ca^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3}+\frac{3\,Bx}{140\,(bx^2+a)^{\frac{5}{2}}b^2}+\frac{2\,Bx}{35\,\sqrt{bx^2+a}\,a^2b^2}+\frac{Bx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2}-\frac{3\,Bax}{28\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}b}+\frac{8\,Ax}{105\,\sqrt{bx^2+a}\,a^3b}+\frac{4\,Ax}{105\,(bx^2+a)^{\frac{3}{2}}a^2b}+\frac{Ax}{35\,(bx^2+a)^{\frac{5}{2}}ab}+\frac{D\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56
*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*D*x - 1/
15*D*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8
*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*C*x^5/((b*x^2 + a)^(7/2)*b) - 1/3*D*x
*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - D*a*x^3/
((b*x^2 + a)^(5/2)*b^3) - 5/8*C*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*B*x^3/(
(b*x^2 + a)^(7/2)*b) + 139/105*D*x/(sqrt(b*x^2 + a)*b^4) + 17/105*D*a*x/((b
*x^2 + a)^(3/2)*b^4) - 29/35*D*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*C*x/((b
*x^2 + a)^(3/2)*b^3) + 1/7*C*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*C*a*x/((b*x^2
 + a)^(5/2)*b^3) - 15/56*C*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*B*x/((b*x^
2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*B*x/((b*x^2 +
 a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*A*x/((b*x^2 + a
)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((b*x^2 + a)^(3/
2)*a^2*b) + 1/35*A*x/((b*x^2 + a)^(5/2)*a*b) + D*arcsinh(b*x/sqrt(a*b))/b^(
9/2)
```

Fricas [**A**]
time = 4.31, size = 491, normalized size = 2.74

$$\left[\frac{105\,(Da^3b^4x^8+4\,Da^4b^3x^6+6\,Da^5b^2x^4+4\,Da^6bx^2+Da^7)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,(105\,Da^6bx+(176\,Da^3b^4-15\,Ca^2b^5-6\,Bab^6-8\,Ab^7)x^7+7\,(58\,Da^4b^3-3\,Ba^2b^5-4\,Aab^6)x^5+35\,(10\,Da^5b^2-Aa^2b^5)x^3)\sqrt{bx^2+a}}{210\,(a^3b^9x^8+4\,a^4b^8x^6+6\,a^5b^7x^4+4\,a^6b^6x^2+a^7b^5)},-\frac{105\,(Da^3b^4x^8+4\,Da^4b^3x^6+6\,Da^5b^2x^4+4\,Da^6bx^2+Da^7)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+(105\,Da^6bx+(176\,Da^3b^4-15\,Ca^2b^5-6\,Bab^6-8\,Ab^7)x^7+7\,(58\,Da^4b^3-3\,Ba^2b^5-4\,Aab^6)x^5+35\,(10\,Da^5b^2-Aa^2b^5)x^3)\sqrt{bx^2+a}}{105\,(a^3b^9x^8+4\,a^4b^8x^6+6\,a^5b^7x^4+4\,a^6b^6x^2+a^7b^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/210*(105*(D*a^3*b^4*x^8 + 4*D*a^4*b^3*x^6 + 6*D*a^5*b^2*x^4 + 4*D*a^6*b*
x^2 + D*a^7)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(1
05*D*a^6*b*x + (176*D*a^3*b^4 - 15*C*a^2*b^5 - 6*B*a*b^6 - 8*A*b^7)*x^7 + 7
*(58*D*a^4*b^3 - 3*B*a^2*b^5 - 4*A*a*b^6)*x^5 + 35*(10*D*a^5*b^2 - A*a^2*b^
5)*x^3)*sqrt(b*x^2 + a))/(a^3*b^9*x^8 + 4*a^4*b^8*x^6 + 6*a^5*b^7*x^4 + 4*a
^6*b^6*x^2 + a^7*b^5), -1/105*(105*(D*a^3*b^4*x^8 + 4*D*a^4*b^3*x^6 + 6*D*a
^5*b^2*x^4 + 4*D*a^6*b*x^2 + D*a^7)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
 a)) + (105*D*a^6*b*x + (176*D*a^3*b^4 - 15*C*a^2*b^5 - 6*B*a*b^6 - 8*A*b^7
)*x^7 + 7*(58*D*a^4*b^3 - 3*B*a^2*b^5 - 4*A*a*b^6)*x^5 + 35*(10*D*a^5*b^2 -
 A*a^2*b^5)*x^3)*sqrt(b*x^2 + a))/(a^3*b^9*x^8 + 4*a^4*b^8*x^6 + 6*a^5*b^7*
x^4 + 4*a^6*b^6*x^2 + a^7*b^5)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3803 vs. $2(178) = 356$.

time = 69.64, size = 3803, normalized size = 21.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sq
rt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/
2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a
)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**
2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**
(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x*
*2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2
)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a*
*(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2
/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*
sqrt(1 + b*x**2/a))) + B*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a
**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/
a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqr
t(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + C*
x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a)
 + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 +
 b*x**2/a)) + D*(105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sq
rt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(10
```

```
1/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x
**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/
2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqr
t(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630
*a**(203/2)*b**46*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**
(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(
1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a
**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x
**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a)
 + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(201/2)*b*
*47*x**4*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99
/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a)
+ 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(
105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b
*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193
/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 2100*a**(199/2)*b**48*x**6*sqrt(
1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b
*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201
/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sq
rt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630
*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)
*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(197/2)*b**49*x**8*sqrt(1 + b*x**2/a)*
asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630
*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)
*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/
a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b*
*(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1
+ b*x**2/a)) + 630*a**(195/2)*b**50*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*
x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b*
*(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 +
 b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(
197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10
*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) +
 105*a**(193/2)*b**51*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(10
5*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*
sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2
100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107
/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x*
*2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 105*a**102*b*
*(91/2)*x/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**
(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 +
b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(1
97/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*
sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) -
665*a**101*b**(93/2)*x**3/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 63
```

```
0*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a)...
```

Giac [A]

time = 1.46, size = 160, normalized size = 0.89

$$-\frac{\left(\left(x^{2}\left(\frac{(176\,Da^{3}b^{6}-15\,Ca^{2}b^{7}-6\,Bab^{8}-8\,Ab^{9})x^{2}}{a^{3}b^{7}}+\frac{7\,(58\,Da^{4}b^{5}-3\,Ba^{2}b^{7}-4\,Aab^{8})}{a^{3}b^{7}}\right)+\frac{35\,(10\,Da^{5}b^{4}-Aa^{2}b^{7})}{a^{3}b^{7}}\right)x^{2}+\frac{105\,Da^{3}}{b^{4}}\right)x}{105\,(bx^{2}+a)^{\frac{7}{2}}}-\frac{D\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((x^2*((176*D*a^3*b^6 - 15*C*a^2*b^7 - 6*B*a*b^8 - 8*A*b^9)*x^2/(a^3
*b^7) + 7*(58*D*a^4*b^5 - 3*B*a^2*b^7 - 4*A*a*b^8)/(a^3*b^7)) + 35*(10*D*a^
5*b^4 - A*a^2*b^7)/(a^3*b^7))*x^2 + 105*D*a^3/b^4)*x/(b*x^2 + a)^(7/2) - D*
log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

3.163 $\int \frac{A+Bx^2+Cx^4+Dx^6}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=134

$$\frac{Ax}{a\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}}+\frac{\left(24Ab^2+a(4bB+3aC)\right)x^5}{15a^3\left(a+bx^2\right)^{7/2}}+\frac{\left(48Ab^3+a\left(8b^2B+6abC+15a^2D\right)\right)x^7}{105a^4\left(a+bx^2\right)^{7/2}}$$

```
[Out] A*x/a/(b*x^2+a)^(7/2)+1/3*(6*A*b+B*a)*x^3/a^2/(b*x^2+a)^(7/2)+1/15*(24*A*b^
2+a*(4*B*b+3*C*a))*x^5/a^3/(b*x^2+a)^(7/2)+1/105*(48*A*b^3+a*(8*B*b^2+6*C*a
*b+15*D*a^2))*x^7/a^4/(b*x^2+a)^(7/2)
```

Rubi [A]

time = 0.15, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1827, 1817, 12, 270}

$$\frac{x^5\left(a(3aC+4bB)+24Ab^2\right)}{15a^3\left(a+bx^2\right)^{7/2}}+\frac{x^3(aB+6Ab)}{3a^2\left(a+bx^2\right)^{7/2}}+\frac{x^7\left(a\left(15a^2D+6abC+8b^2B\right)+48Ab^3\right)}{105a^4\left(a+bx^2\right)^{7/2}}+\frac{Ax}{a\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/2),x]
```

```
[Out] (A*x)/(a*(a + b*x^2)^(7/2)) + ((6*A*b + a*B)*x^3)/(3*a^2*(a + b*x^2)^(7/2))
 + ((24*A*b^2 + a*(4*b*B + 3*a*C))*x^5)/(15*a^3*(a + b*x^2)^(7/2)) + ((48*A
*b^3 + a*(8*b^2*B + 6*a*b*C + 15*a^2*D))*x^7)/(105*a^4*(a + b*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1827

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*x*((a + b*
x^2)^(p + 1)/a), x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx &= \frac{Ax}{a\left(a+bx^2\right)^{7/2}}+\frac{\int \frac{x^2\left(6Ab+a\left(B+Cx^2+Dx^4\right)\right)}{(a+bx^2)^{9/2}}\,dx}{a}\\
&= \frac{Ax}{a\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}}+\frac{\int \frac{x^4\left(4b(6Ab+aB)+3a\left(aC+aDx^2\right)\right)}{(a+bx^2)^{9/2}}\,dx}{3a^2}\\
&= \frac{Ax}{a\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}}+\frac{\left(24Ab^2+a(4bB+3aC)\right)x^5}{15a^3\left(a+bx^2\right)^{7/2}}+\int (2b(24A\\
&= \frac{Ax}{a\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}}+\frac{\left(24Ab^2+a(4bB+3aC)\right)x^5}{15a^3\left(a+bx^2\right)^{7/2}}+(48Ab^3+\\
&= \frac{Ax}{a\left(a+bx^2\right)^{7/2}}+\frac{(6Ab+aB)x^3}{3a^2\left(a+bx^2\right)^{7/2}}+\frac{\left(24Ab^2+a(4bB+3aC)\right)x^5}{15a^3\left(a+bx^2\right)^{7/2}}+(48Ab^3+
\end{aligned}$$

Mathematica [**A**]
time = 0.43, size = 98, normalized size = 0.73

$$\frac{48Ab^3x^7+8ab^2x^5(21A+Bx^2)+2a^2bx^3(105A+14Bx^2+3Cx^4)+a^3(105Ax+35Bx^3+21Cx^5+15Dx^7)}{105a^4\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/2),x]
```

```
[Out] (48*A*b^3*x^7 + 8*a*b^2*x^5*(21*A + B*x^2) + 2*a^2*b*x^3*(105*A + 14*B*x^2
+ 3*C*x^4) + a^3*(105*A*x + 35*B*x^3 + 21*C*x^5 + 15*D*x^7))/(105*a^4*(a +
b*x^2)^(7/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(120) = 240$.
time = 0.11, size = 440, normalized size = 3.28

method	result

gosper	$\frac{x\left(48A\,b^{3}x^{6}+8Ba\,b^{2}x^{6}+6a^{2}bC\,x^{6}+15Da^{3}x^{6}+168aA\,b^{2}x^{4}+28B\,a^{2}b\,x^{4}+21a^{3}C\,x^{4}+210a^{2}Ab\,x^{2}+35B\,a^{3}x^{2}+105a^{3}A\right)}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$
trager	$\frac{x\left(48A\,b^{3}x^{6}+8Ba\,b^{2}x^{6}+6a^{2}bC\,x^{6}+15Da^{3}x^{6}+168aA\,b^{2}x^{4}+28B\,a^{2}b\,x^{4}+21a^{3}C\,x^{4}+210a^{2}Ab\,x^{2}+35B\,a^{3}x^{2}+105a^{3}A\right)}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{4}}$

default	D	$\left(-\frac{x^{5}}{2b(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^{3}}{4b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}}}\right.}{7a}}{a}\right.}{6b}\right.}{4b}\right.}{2b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] D*(-1/2*x^5/b/(b*x^2+a)^(7/2)+5/2*a/b*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(
-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b
*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))))
)+C*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(
1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^
2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))))+B*(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a
/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(
b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))+A*(1/7*x/a/(b*x^2+a)^(7/2)+6/7
/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2
+a)^(1/2))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(120) = 240$.
time = 0.31, size = 335, normalized size = 2.50

$$-\frac{Dx^5}{2\,(bx^2+a)^{\frac{7}{2}}b}-\frac{5\,Dax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{Cx^3}{4\,(bx^2+a)^{\frac{7}{2}}b}+\frac{16\,Ax}{35\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Ax}{35\,(bx^2+a)^{\frac{3}{2}}a^3}+\frac{6\,Ax}{35\,(bx^2+a)^{\frac{5}{2}}a^2}+\frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}a}+\frac{Dx}{14\,(bx^2+a)^{\frac{3}{2}}b^3}+\frac{Dx}{7\,\sqrt{bx^2+a}\,ab^3}+\frac{3\,Dax}{56\,(bx^2+a)^{\frac{5}{2}}b^3}-\frac{15\,Da^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3}+\frac{3\,Cx}{140\,(bx^2+a)^{\frac{5}{2}}b^2}+\frac{2\,Cx}{35\,\sqrt{bx^2+a}\,a^2b^2}+\frac{Cx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2}-\frac{3\,Cax}{28\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}b}+\frac{8\,Bx}{105\,\sqrt{bx^2+a}\,a^3b}+\frac{4\,Bx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b}+\frac{Bx}{35\,(bx^2+a)^{\frac{5}{2}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/2*D*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*D*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/
4*C*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/
((b*x^2 + a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^
2 + a)^(7/2)*a) + 1/14*D*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*D*x/(sqrt(b*x^2 +
a)*a*b^3) + 3/56*D*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*D*a^2*x/((b*x^2 + a)
^(7/2)*b^3) + 3/140*C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^2 + a)
*a^2*b^2) + 1/35*C*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2 + a)^(7
/2)*b^2) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8/105*B*x/(sqrt(b*x^2 + a)*a^3*b
) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*B*x/((b*x^2 + a)^(5/2)*a*b)
```

Fricas [**A**]
time = 4.10, size = 141, normalized size = 1.05

$$\frac{((15\,Da^3+6\,Ca^2b+8\,Bab^2+48\,Ab^3)x^7+7\,(3\,Ca^3+4\,Ba^2b+24\,Aab^2)x^5+105\,Aa^3x+35\,(Ba^3+6\,Aa^2b)x^3)\sqrt{bx^2+a}}{105\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7bx^2+a^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*((15*D*a^3 + 6*C*a^2*b + 8*B*a*b^2 + 48*A*b^3)*x^7 + 7*(3*C*a^3 + 4*B
*a^2*b + 24*A*a*b^2)*x^5 + 105*A*a^3*x + 35*(B*a^3 + 6*A*a^2*b)*x^3)*sqrt(b
*x^2 + a)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a^8)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2088 vs. $2(129) = 258$.
time = 51.26, size = 2088, normalized size = 15.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)

[Out] A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(
1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*
b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) +
 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt
(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a
**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**
2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8
*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(
25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sq
rt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*
b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt
(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b*
*3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*
x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x
**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a
**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b
*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(
35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a
) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sq
rt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/
2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(
1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 52
5*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1
+ b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x*
*13/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2
/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*
sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(2
7/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**
2/a))) + B*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*
b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 42
0*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 +
 b*x**2/a)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17
/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a)
+ 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt
```

```
(1 + b*x**2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420
*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x
**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x*
*8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a)
 + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1
 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b
**4*x**8*sqrt(1 + b*x**2/a))) + C*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a
) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1
+ b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(
11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**
(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/
a))) + D*x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 +
b*x**2/a) + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6
*sqrt(1 + b*x**2/a))
```

Giac [A]

time = 0.90, size = 131, normalized size = 0.98

$$\frac{\left(\left(x^{2}\left(\frac{(15\,Da^{3}b^{3}+6\,Ca^{2}b^{4}+8\,Bab^{5}+48\,Ab^{6})x^{2}}{a^{4}b^{3}}+\frac{7\,(3\,Ca^{3}b^{3}+4\,Ba^{2}b^{4}+24\,Aab^{5})}{a^{4}b^{3}}\right)+\frac{35\,(Ba^{3}b^{3}+6\,Aa^{2}b^{4})}{a^{4}b^{3}}\right)x^{2}+\frac{105\,A}{a}\right)x}{105\,(bx^{2}+a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((15*D*a^3*b^3 + 6*C*a^2*b^4 + 8*B*a*b^5 + 48*A*b^6)*x^2/(a^4*b
^3) + 7*(3*C*a^3*b^3 + 4*B*a^2*b^4 + 24*A*a*b^5)/(a^4*b^3)) + 35*(B*a^3*b^3
 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*x/(b*x^2 + a)^(7/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,x^{2}+C\,x^{4}+x^{6}\,D}{\left(b\,x^{2}+a\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(9/2),x)
```

```
[Out] int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(9/2), x)
```

3.164 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=185

$$-\frac{A}{ax\left(a+bx^2\right)^{7/2}}-\frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}}-\frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{\left(4b(48Ab^2-a(6bB+aC))-3a^3D\right)x^5}{15a^4\left(a+bx^2\right)^{7/2}}-2b$$

```
[Out] -A/a/x/(b*x^2+a)^(7/2)-(8*A*b-B*a)*x/a^2/(b*x^2+a)^(7/2)-1/3*(48*A*b^2-a*(6
*B*b+C*a))*x^3/a^3/(b*x^2+a)^(7/2)-1/15*(4*b*(48*A*b^2-a*(6*B*b+C*a))-3*a^3
*D)*x^5/a^4/(b*x^2+a)^(7/2)-2/105*b*(4*b*(48*A*b^2-a*(6*B*b+C*a))-3*a^3*D)*
x^7/a^5/(b*x^2+a)^(7/2)
```

Rubi [A]
time = 0.17, antiderivative size = 179, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1817, 1827, 12, 277, 270}

$$-\frac{x^3(48Ab^2-a(aC+6bB))}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{x(8Ab-aB)}{a^2\left(a+bx^2\right)^{7/2}}-\frac{2bx^7(-3a^3D-4ab(aC+6bB)+192Ab^3)}{105a^5\left(a+bx^2\right)^{7/2}}-\frac{x^5(-3a^3D-4ab(aC+6bB)+192Ab^3)}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^2)^(7/2))) - ((8*A*b - a*B)*x)/(a^2*(a + b*x^2)^(7/2)) -
((48*A*b^2 - a*(6*b*B + a*C))*x^3)/(3*a^3*(a + b*x^2)^(7/2)) - ((192*A*b^3
- 4*a*b*(6*b*B + a*C) - 3*a^3*D)*x^5)/(15*a^4*(a + b*x^2)^(7/2)) - (2*b*(19
2*A*b^3 - 4*a*b*(6*b*B + a*C) - 3*a^3*D)*x^7)/(105*a^5*(a + b*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1827

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*x*((a + b*
x^2)^(p + 1)/a), x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx &= -\frac{A}{ax\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{8Ab-a\left(B+Cx^2+Dx^4\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{a}\\
&= -\frac{A}{ax\left(a+bx^2\right)^{7/2}}-\frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{x^2\left(6b(8Ab-aB)+a\left(-aC-aDx^2\right)\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{a^2}\\
&= -\frac{A}{ax\left(a+bx^2\right)^{7/2}}-\frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}}-\frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{(4b(4}{}}{}\\
&= -\frac{A}{ax\left(a+bx^2\right)^{7/2}}-\frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}}-\frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{(192A}{}\\
&= -\frac{A}{ax\left(a+bx^2\right)^{7/2}}-\frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}}-\frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{(192A}{}\\
&= -\frac{A}{ax\left(a+bx^2\right)^{7/2}}-\frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}}-\frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{(192A}{}
\end{aligned}$$

Mathematica [A]
time = 0.49, size = 133, normalized size = 0.72

$$\frac{-384Ab^4x^8+48ab^3x^6(-28A+Bx^2)+8a^2b^2x^4(-210A+21Bx^2+Cx^4)-7a^4(15A-15Bx^2-5Cx^4-3Dx^6)+2a^3bx^2(-420A+105Bx^2+14Cx^4+3Dx^6)}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-384*A*b^4*x^8 + 48*a*b^3*x^6*(-28*A + B*x^2) + 8*a^2*b^2*x^4*(-210*A + 21
*B*x^2 + C*x^4) - 7*a^4*(15*A - 15*B*x^2 - 5*C*x^4 - 3*D*x^6) + 2*a^3*b*x^2
*(-420*A + 105*B*x^2 + 14*C*x^4 + 3*D*x^6))/(105*a^5*x*(a + b*x^2)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(169) = 338$.
time = 0.11, size = 394, normalized size = 2.13

method	result
gosper	$-\frac{384 A\, b^4 x^8-48 B a\, b^3 x^8-8 C\, a^2 b^2 x^8-6 D a^3 b\, x^8+1344 a\, b^3 A\, x^6-168 B\, a^2 b^2 x^6-28 C\, a^3 b\, x^6-21 D a^4 x^6+1680 A\, a^2 b^2 x^4-210 B\, a^3 b\, x^4-}{105 x (b\, x^2+a)^{\frac{7}{2}} a^5}$
trager	$-\frac{384 A\, b^4 x^8-48 B a\, b^3 x^8-8 C\, a^2 b^2 x^8-6 D a^3 b\, x^8+1344 a\, b^3 A\, x^6-168 B\, a^2 b^2 x^6-28 C\, a^3 b\, x^6-21 D a^4 x^6+1680 A\, a^2 b^2 x^4-210 B\, a^3 b\, x^4-}{105 x (b\, x^2+a)^{\frac{7}{2}} a^5}$
default	$D \left(-\frac{x^3}{4 b (b\, x^2+a)^{\frac{7}{2}}}+\frac{3 a \left(-\frac{x}{6 b \left(b\, x^2+a\right)^{\frac{7}{2}}}+\frac{a \left(\frac{x}{7 a \left(b\, x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6 x}{35 a \left(b\, x^2+a\right)^{\frac{5}{2}}}+\frac{6 \left(\frac{4 x}{15 a \left(b\, x^2+a\right)^{\frac{3}{2}}}+\frac{8 x}{15 a^2 \sqrt{b\, x^2+a}}\right)}{7 a}}{a}\right)}{6 b}\right)}{4 b}\right)+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] D*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/
7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+
```

```
a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))))+C*(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b
*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*
x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))+B*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a
*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a
)^(1/2))))+A*(-1/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(
1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^
(1/2)))))
```

Maxima [**A**]
time = 0.31, size = 313, normalized size = 1.69

$$-\frac{Dx^{3}}{4\,(bx^{2}+a)^{\frac{7}{2}}b}+\frac{16\,Bx}{35\,\sqrt{bx^{2}+a}\,a^{4}}+\frac{8\,Bx}{35\,(bx^{2}+a)^{\frac{3}{2}}a^{3}}+\frac{6\,Bx}{35\,(bx^{2}+a)^{\frac{5}{2}}a^{2}}+\frac{Bx}{7\,(bx^{2}+a)^{\frac{7}{2}}a}+\frac{3\,Dx}{140\,(bx^{2}+a)^{\frac{5}{2}}b^{2}}+\frac{2\,Dx}{35\,\sqrt{bx^{2}+a}\,a^{2}b^{2}}+\frac{Dx}{35\,(bx^{2}+a)^{\frac{3}{2}}ab^{2}}-\frac{3\,Dax}{28\,(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{Cx}{7\,(bx^{2}+a)^{\frac{7}{2}}b}+\frac{8\,Cx}{105\,\sqrt{bx^{2}+a}\,a^{3}b}+\frac{4\,Cx}{105\,(bx^{2}+a)^{\frac{3}{2}}a^{2}b}+\frac{Cx}{35\,(bx^{2}+a)^{\frac{5}{2}}ab}-\frac{128\,Abx}{35\,\sqrt{bx^{2}+a}\,a^{5}}-\frac{64\,Abx}{35\,(bx^{2}+a)^{\frac{3}{2}}a^{4}}-\frac{48\,Abx}{35\,(bx^{2}+a)^{\frac{5}{2}}a^{3}}-\frac{8\,Abx}{7\,(bx^{2}+a)^{\frac{7}{2}}a^{2}}-\frac{A}{(bx^{2}+a)^{\frac{7}{2}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/4*D*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*B
*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*B*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/((b
*x^2 + a)^(7/2)*a) + 3/140*D*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*D*x/(sqrt(b*x
^2 + a)*a^2*b^2) + 1/35*D*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*D*a*x/((b*x^2
+ a)^(7/2)*b^2) - 1/7*C*x/((b*x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 + a
)*a^3*b) + 4/105*C*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5/2
)*a*b) - 128/35*A*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 + a)^(3/2
)*a^4) - 48/35*A*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 + a)^(7/2)
*a^2) - A/((b*x^2 + a)^(7/2)*a*x)
```

Fricas [**A**]
time = 4.23, size = 182, normalized size = 0.98

$$\frac{\left(2\,(3\,Da^{3}b+4\,Ca^{2}b^{2}+24\,Bab^{3}-192\,Ab^{4})x^{8}+7\,(3\,Da^{4}+4\,Ca^{3}b+24\,Ba^{2}b^{2}-192\,Aab^{3})x^{6}-105\,Aa^{4}+35\,(Ca^{4}+6\,Ba^{3}b-48\,Aa^{2}b^{2})x^{4}+105\,(Ba^{4}-8\,Aa^{3}b)x^{2}\right)\sqrt{bx^{2}+a}}{105\,(a^{5}b^{4}x^{9}+4\,a^{6}b^{3}x^{7}+6\,a^{7}b^{2}x^{5}+4\,a^{8}bx^{3}+a^{9}x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(2*(3*D*a^3*b + 4*C*a^2*b^2 + 24*B*a*b^3 - 192*A*b^4)*x^8 + 7*(3*D*a^
4 + 4*C*a^3*b + 24*B*a^2*b^2 - 192*A*a*b^3)*x^6 - 105*A*a^4 + 35*(C*a^4 + 6
*B*a^3*b - 48*A*a^2*b^2)*x^4 + 105*(B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a)
/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2392 vs. $2(170) = 340$.
time = 82.95, size = 2392, normalized size = 12.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)

[Out] A*(-35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 28
0*a**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 56
0*a**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 44
8*a*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 128*b
**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 2
10*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)) + B*(35*a**
14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**
2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6
*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(
27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x*
*2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*
b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 70
0*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 +
 b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**
6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sqrt(1 + b*
x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4
*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(
29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x*
*2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b**3*x**7/(
35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) +
 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(
1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*
b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a))
 + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x
**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a
**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*
x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x
**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(1 + b*x**
2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sq
rt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/
2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/
a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x**13/(35*a
**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525
*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 +
b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5
*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a))) +
C*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sq
rt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/
2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a
```

```
)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**
2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**
(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x*
*2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2
)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a*
*(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2
/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*
sqrt(1 + b*x**2/a))) + D*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a
**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/
a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqr
t(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)))
```

Giac [**A**]

time = 1.07, size = 211, normalized size = 1.14

$$\frac{\left(\left(x^2\left(\frac{\left(6\,Da^{12}b^4+8\,Ca^{11}b^5+48\,Ba^{10}b^6-279\,Aa^9b^7\right)x^2}{a^{14}b^3}+\frac{7\,\left(3\,Da^{13}b^3+4\,Ca^{12}b^4+24\,Ba^{11}b^5-132\,Aa^{10}b^6\right)}{a^{14}b^3}\right)+\frac{35\,\left(Ca^{13}b^3+6\,Ba^{12}b^4-30\,Aa^{11}b^5\right)}{a^{14}b^3}\right)x^2+\frac{105\,\left(Ba^{13}b^3-4\,Aa^{12}b^4\right)}{a^{14}b^3}\right)x}{105\,\left(bx^2+a\right)^{\frac{7}{2}}}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((6*D*a^12*b^4 + 8*C*a^11*b^5 + 48*B*a^10*b^6 - 279*A*a^9*b^7)*
x^2/(a^14*b^3) + 7*(3*D*a^13*b^3 + 4*C*a^12*b^4 + 24*B*a^11*b^5 - 132*A*a^1
0*b^6)/(a^14*b^3)) + 35*(C*a^13*b^3 + 6*B*a^12*b^4 - 30*A*a^11*b^5)/(a^14*b
^3))*x^2 + 105*(B*a^13*b^3 - 4*A*a^12*b^4)/(a^14*b^3))*x/(b*x^2 + a)^(7/2)
+ 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^2\,{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a + b*x^2)^(9/2)),x)
```

```
[Out] int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a + b*x^2)^(9/2)), x)
```

3.165 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=242

$$-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}+\frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}}+\frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}}+\frac{\left(160Ab^3-a(48b^2B-6abC-a^2D)\right)x}{3a^4\left(a+bx^2\right)^{7/2}}$$

```
[Out] -1/3*A/a/x^3/(b*x^2+a)^(7/2)+1/3*(10*A*b-3*B*a)/a^2/x/(b*x^2+a)^(7/2)+1/3*(
80*A*b^2-3*a*(8*B*b-C*a))*x/a^3/(b*x^2+a)^(7/2)+1/3*(160*A*b^3-a*(48*B*b^2-
6*C*a*b-D*a^2))*x^3/a^4/(b*x^2+a)^(7/2)+4/15*b*(160*A*b^3-a*(48*B*b^2-6*C*a
*b-D*a^2))*x^5/a^5/(b*x^2+a)^(7/2)+8/105*b^2*(160*A*b^3-a*(48*B*b^2-6*C*a*b
-D*a^2))*x^7/a^6/(b*x^2+a)^(7/2)
```

Rubi [A]

time = 0.22, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1817, 1827, 12, 277, 270}

$$\frac{x\left(80Ab^2-3a(8bB-aC)\right)}{3a^3\left(a+bx^2\right)^{7/2}}+\frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}}+\frac{8b^2x^7\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{105a^6\left(a+bx^2\right)^{7/2}}+\frac{4bx^5\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{15a^5\left(a+bx^2\right)^{7/2}}+\frac{x^3\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{3a^4\left(a+bx^2\right)^{7/2}}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] -1/3*A/(a*x^3*(a + b*x^2)^(7/2)) + (10*A*b - 3*a*B)/(3*a^2*x*(a + b*x^2)^(7
/2)) + ((80*A*b^2 - 3*a*(8*b*B - a*C))*x)/(3*a^3*(a + b*x^2)^(7/2)) + ((160
*A*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^3)/(3*a^4*(a + b*x^2)^(7/2)) + (
4*b*(160*A*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^5)/(15*a^5*(a + b*x^2)^(
7/2)) + (8*b^2*(160*A*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*x^7)/(105*a^6*(
a + b*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
```

```
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1827

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*x*((a + b*
x^2)^(p + 1)/a), x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{10Ab-3a\left(B+Cx^2+Dx^4\right)}{x^2\left(a+bx^2\right)^{9/2}}\,dx}{3a}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{8b(10Ab-3aB)-a\left(-3aC-3aDx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{3a^2}$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} +$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} +$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} +$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} +$$

$$= -\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}} + \frac{10Ab-3aB}{3a^2x\left(a+bx^2\right)^{7/2}} + \frac{\left(80Ab^2-3a(8bB-aC)\right)x}{3a^3\left(a+bx^2\right)^{7/2}} +$$

Mathematica [A]
time = 0.53, size = 165, normalized size = 0.68

$$\frac{1280Ab^5x^{10}+128ab^4x^8(35A-3Bx^2)+16a^2b^3x^6(350A-84Bx^2+3Cx^4)-35a^5(A+3Bx^2-3Cx^4-Dx^6)+8a^3b^2x^4(350A-210Bx^2+21Cx^4+Dx^6)+14a^4bx^2(25A-60Bx^2+15Cx^4+2Dx^6)}{105a^6x^3\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^(9/2)),x]
```

```
[Out] (1280*A*b^5*x^10 + 128*a*b^4*x^8*(35*A - 3*B*x^2) + 16*a^2*b^3*x^6*(350*A -
 84*B*x^2 + 3*C*x^4) - 35*a^5*(A + 3*B*x^2 - 3*C*x^4 - D*x^6) + 8*a^3*b^2*x
^4*(350*A - 210*B*x^2 + 21*C*x^4 + D*x^6) + 14*a^4*b*x^2*(25*A - 60*B*x^2 +
 15*C*x^4 + 2*D*x^6))/(105*a^6*x^3*(a + b*x^2)^(7/2))
```

Maple [A]
time = 0.19, size = 396, normalized size = 1.64

method	result
gosper	$-\frac{-1280A\,b^5x^{10}+384Ba\,b^4x^{10}-48C\,a^2b^3x^{10}-8Da^3b^2x^{10}-4480aA\,b^4x^8+1344B\,a^2b^3x^8-168C\,a^3b^2x^8-28Da^4b\,x^8-5600a^2A\,b^3x^6+}{105x^3(b\,x^2+a)^{\frac{7}{2}}}$
trager	$-\frac{-1280A\,b^5x^{10}+384Ba\,b^4x^{10}-48C\,a^2b^3x^{10}-8Da^3b^2x^{10}-4480aA\,b^4x^8+1344B\,a^2b^3x^8-168C\,a^3b^2x^8-28Da^4b\,x^8-5600a^2A\,b^3x^6+}{105x^3(b\,x^2+a)^{\frac{7}{2}}}$

default	$D\left(-\frac{x}{6b(b\,x^2+a)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)}{6b}\right)+C\left(\frac{x}{7a(b\,x^2+a)^{\frac{7}{2}}}+\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] D*(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a
/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))
))+C*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a
/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))+A*(-1/3/a/x^3/(b*x^2+a)^(7/2)
-10/3*b/a*(-1/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5
*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/
2))))))+B*(-1/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5
*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/
2)))))
```

Maxima [A]
time = 0.29, size = 337, normalized size = 1.39

$$\frac{16\,Cx}{35\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Cx}{35\,(bx^2+a)^{\frac{3}{2}}a^3}+\frac{6\,Cx}{35\,(bx^2+a)^{\frac{5}{2}}a^2}+\frac{Cx}{7\,(bx^2+a)^{\frac{7}{2}}a}-\frac{Dx}{7\,(bx^2+a)^{\frac{7}{2}}b}+\frac{8\,Dx}{105\,\sqrt{bx^2+a}\,a^3b}+\frac{4\,Dx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b}+\frac{Dx}{35\,(bx^2+a)^{\frac{5}{2}}ab}-\frac{128\,Bbx}{35\,\sqrt{bx^2+a}\,a^5}-\frac{64\,Bbx}{35\,(bx^2+a)^{\frac{3}{2}}a^4}-\frac{48\,Bbx}{35\,(bx^2+a)^{\frac{5}{2}}a^3}-\frac{8\,Bbx}{7\,(bx^2+a)^{\frac{7}{2}}a^2}+\frac{256\,Ab^2x}{21\,\sqrt{bx^2+a}\,a^6}+\frac{128\,Ab^2x}{21\,(bx^2+a)^{\frac{3}{2}}a^5}+\frac{32\,Ab^2x}{7\,(bx^2+a)^{\frac{5}{2}}a^4}+\frac{80\,Ab^2x}{21\,(bx^2+a)^{\frac{7}{2}}a^3}-\frac{B}{(bx^2+a)^{\frac{7}{2}}ax}+\frac{10\,Ab}{3\,(bx^2+a)^{\frac{7}{2}}a^2x}-\frac{A}{3\,(bx^2+a)^{\frac{7}{2}}ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*C*x/(sqrt(b*x^2 + a)*a^4) + 8/35*C*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*C
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*C*x/((b*x^2 + a)^(7/2)*a) - 1/7*D*x/((b*x^
2 + a)^(7/2)*b) + 8/105*D*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*D*x/((b*x^2 + a
)^(3/2)*a^2*b) + 1/35*D*x/((b*x^2 + a)^(5/2)*a*b) - 128/35*B*b*x/(sqrt(b*x^
2 + a)*a^5) - 64/35*B*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*B*b*x/((b*x^2 + a
)^(5/2)*a^3) - 8/7*B*b*x/((b*x^2 + a)^(7/2)*a^2) + 256/21*A*b^2*x/(sqrt(b*x
^2 + a)*a^6) + 128/21*A*b^2*x/((b*x^2 + a)^(3/2)*a^5) + 32/7*A*b^2*x/((b*x^
2 + a)^(5/2)*a^4) + 80/21*A*b^2*x/((b*x^2 + a)^(7/2)*a^3) - B/((b*x^2 + a)^
(7/2)*a*x) + 10/3*A*b/((b*x^2 + a)^(7/2)*a^2*x) - 1/3*A/((b*x^2 + a)^(7/2)*
a*x^3)
```

Fricas [**A**]
time = 4.31, size = 225, normalized size = 0.93

$$\frac{\left(8\left(Da^{3}b^{2}+6\,Ca^{2}b^{3}-48\,Bab^{4}+160\,Ab^{5}\right)x^{10}+28\left(Da^{4}b+6\,Ca^{3}b^{2}-48\,Ba^{2}b^{3}+160\,Aab^{4}\right)x^{8}+35\left(Da^{5}+6\,Ca^{4}b-48\,Ba^{3}b^{2}+160\,Aa^{2}b^{3}\right)x^{6}-35\,Aa^{5}+35\left(3\,Ca^{5}-24\,Ba^{4}b+80\,Aa^{3}b^{2}\right)x^{4}-35\left(3\,Ba^{5}-10\,Aa^{4}b\right)x^{2}\right)\sqrt{bx^{2}+a}}{105\left(a^{6}b^{4}x^{11}+4\,a^{7}b^{3}x^{9}+6\,a^{8}b^{2}x^{7}+4\,a^{9}bx^{5}+a^{10}x^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(D*a^3*b^2 + 6*C*a^2*b^3 - 48*B*a*b^4 + 160*A*b^5)*x^10 + 28*(D*a^
4*b + 6*C*a^3*b^2 - 48*B*a^2*b^3 + 160*A*a*b^4)*x^8 + 35*(D*a^5 + 6*C*a^4*b
 - 48*B*a^3*b^2 + 160*A*a^2*b^3)*x^6 - 35*A*a^5 + 35*(3*C*a^5 - 24*B*a^4*b
+ 80*A*a^3*b^2)*x^4 - 35*(3*B*a^5 - 10*A*a^4*b)*x^2)*sqrt(b*x^2 + a)/(a^6*b
^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2861 vs. 2(224) = 448.
time = 128.93, size = 2861, normalized size = 11.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**4/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(-7*a**6*b**(51/2)*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*
b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**
10 + 21*a**6*b**30*x**12) + 63*a**5*b**(53/2)*x**2*sqrt(a/(b*x**2) + 1)/(21
*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b
**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 630*a**4*b**(55/2
)*x**4*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 2
10*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b
**30*x**12) + 1680*a**3*b**(57/2)*x**6*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25
*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 +
105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 2016*a**2*b**(59/2)*x**8*sqrt
```

```
(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**
27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12)
 + 1152*a*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a
**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**2
9*x**10 + 21*a**6*b**30*x**12) + 256*b**(63/2)*x**12*sqrt(a/(b*x**2) + 1)/(
21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8
*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12)) + B*(-35*a**4*b*
*(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*a**
7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 280*a**3*b**(35/
2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*a**
7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 560*a**2*b**(37/
2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*a**
7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 448*a*b**(39/2)*
x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*a**7*b
**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 128*b**(41/2)*x**8*
sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*a**7*b**18*
x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)) + C*(35*a**14*x/(35*a**(3
7/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**
(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x*
*2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**
10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 175*a
**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1
+ b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b*
*3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 2
10*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1
 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*
a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x*
*2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**
8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**
(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b**3*x**7/(35*a**(37/2)*s
qrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)
*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a)
+ 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqr
t(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 286*a**10*b
**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b
*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*
x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*
a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 +
b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**
(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/
a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*s
qrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25
/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x**13/(35*a**(37/2)*sqrt(
1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 52
```

```
5*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1
+ b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a))) + D*(35*a**5*x**
3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/
a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*s
qrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 63*a**4*b
*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x
**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x*
*6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 36*a*
*3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(
1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*
b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a))
+ 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x*...
```

Giac [A]
time = 1.41, size = 349, normalized size = 1.44

$$\frac{\left(\left(x^{2}\left(\frac{\left(8\,Da^{15}b^{5}+48\,Ca^{14}b^{6}-279\,Ba^{13}b^{7}+790\,Aa^{12}b^{8}\right)x^{2}}{a^{18}b^{3}}+\frac{7\left(4\,Da^{16}b^{4}+24\,Ca^{15}b^{5}-132\,Ba^{14}b^{6}+365\,Aa^{13}b^{7}\right)}{a^{18}b^{3}}\right)+\frac{35\left(Da^{17}b^{3}+6\,Ca^{16}b^{4}-30\,Ba^{15}b^{5}+80\,Aa^{14}b^{6}\right)}{a^{18}b^{3}}\right)x^{2}+\frac{105\left(Ca^{17}b^{3}-4\,Ba^{16}b^{4}+10\,Aa^{15}b^{5}\right)}{a^{18}b^{3}}\right)x}{105\left(bx^{2}+a\right)^{\frac{7}{2}}}+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Ba\sqrt{b}-12\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Ab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Ba^{2}\sqrt{b}+30\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Aab^{\frac{3}{2}}+3\,Ba^{3}\sqrt{b}-14\,Aa^{2}b^{\frac{3}{2}}\right)}{3\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{3}a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((8*D*a^15*b^5 + 48*C*a^14*b^6 - 279*B*a^13*b^7 + 790*A*a^12*b^
8)*x^2/(a^18*b^3) + 7*(4*D*a^16*b^4 + 24*C*a^15*b^5 - 132*B*a^14*b^6 + 365*
A*a^13*b^7)/(a^18*b^3)) + 35*(D*a^17*b^3 + 6*C*a^16*b^4 - 30*B*a^15*b^5 + 8
0*A*a^14*b^6)/(a^18*b^3))*x^2 + 105*(C*a^17*b^3 - 4*B*a^16*b^4 + 10*A*a^15*
b^5)/(a^18*b^3))*x/(b*x^2 + a)^(7/2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))
^4*B*a*sqrt(b) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 6*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A
*a*b^(3/2) + 3*B*a^3*sqrt(b) - 14*A*a^2*b^(3/2))/(((sqrt(b)*x - sqrt(b*x^2
+ a))^2 - a)^3*a^5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^4\,{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a + b*x^2)^(9/2)),x)
```

```
[Out] int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a + b*x^2)^(9/2)), x)
```

3.166 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=281

$$-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}+\frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}}-\frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}}-\frac{(192Ab^3-a(80b^2B-24abC+3a^2D}{21a^4\left(a+bx^2\right)^{7/2}}$$

```
[Out] -1/5*A/a/x^5/(b*x^2+a)^(7/2)+1/15*(12*A*b-5*B*a)/a^2/x^3/(b*x^2+a)^(7/2)+1/
3*(-24*A*b^2+a*(10*B*b-3*C*a))/a^3/x/(b*x^2+a)^(7/2)-1/21*(192*A*b^3-a*(80*
B*b^2-24*C*a*b+3*D*a^2))*x/a^4/(b*x^2+a)^(7/2)-2/35*(192*A*b^3-a*(80*B*b^2-
24*C*a*b+3*D*a^2))*x/a^5/(b*x^2+a)^(5/2)-8/105*(192*A*b^3-a*(80*B*b^2-24*C*
a*b+3*D*a^2))*x/a^6/(b*x^2+a)^(3/2)-16/105*(192*A*b^3-a*(80*B*b^2-24*C*a*b+
3*D*a^2))*x/a^7/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 275, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1817, 12, 198, 197}

$$-\frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}}+\frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}}-\frac{16x\left(-3a^3D-8ab(10bB-3aC)+192Ab^3\right)}{105a^7\sqrt{a+bx^2}}-\frac{8x\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)}{105a^6\left(a+bx^2\right)^{3/2}}-\frac{2x\left(-3a^3D-8ab(10bB-3aC)+192Ab^3\right)}{35a^5\left(a+bx^2\right)^{5/2}}-\frac{x\left(-3a^3D-8ab(10bB-3aC)+192Ab^3\right)}{21a^4\left(a+bx^2\right)^{7/2}}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^(9/2)),x]
```

```
[Out] -1/5*A/(a*x^5*(a + b*x^2)^(7/2)) + (12*A*b - 5*a*B)/(15*a^2*x^3*(a + b*x^2)
^(7/2)) - (24*A*b^2 - a*(10*b*B - 3*a*C))/(3*a^3*x*(a + b*x^2)^(7/2)) - ((1
92*A*b^3 - 8*a*b*(10*b*B - 3*a*C) - 3*a^3*D)*x)/(21*a^4*(a + b*x^2)^(7/2))
- (2*(192*A*b^3 - 8*a*b*(10*b*B - 3*a*C) - 3*a^3*D)*x)/(35*a^5*(a + b*x^2)^
(5/2)) - (8*(192*A*b^3 - a*(80*b^2*B - 24*a*b*C + 3*a^2*D))*x)/(105*a^6*(a
+ b*x^2)^(3/2)) - (16*(192*A*b^3 - 8*a*b*(10*b*B - 3*a*C) - 3*a^3*D)*x)/(10
5*a^7*Sqrt[a + b*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx &= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{12Ab-5a\left(B+Cx^2+Dx^4\right)}{x^4\left(a+bx^2\right)^{9/2}}\,dx}{5a} \\
&= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{10b(12Ab-5aB)-3a\left(-5aC-5aDx^2\right)}{x^2\left(a+bx^2\right)^{9/2}}}{15a^2} \\
&= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \\
&= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \\
&= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \\
&= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \\
&= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} - \\
&= -\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}} + \frac{12Ab-5aB}{15a^2x^3\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(10bB-3aC)}{3a^3x\left(a+bx^2\right)^{7/2}} -
\end{aligned}
$$

Mathematica [A]

time = 0.59, size = 202, normalized size = 0.72

$$\frac{-3072Ab^6x^{12}+256ab^5x^{10}(-42A+5Bx^2)-128a^2b^4x^8(105A-35Bx^2+3Cx^4)+16a^3b^3x^6(-420A+350Bx^2-84Cx^4+3Dx^6)+56a^4b^2x^4(-15A+50Bx^2-30Cx^4+3Dx^6)+14a^5bx^2(6A+25Bx^2-60Cx^4+15Dx^6)-7a^6(3A+5x^2(B+3Cx^2-3Dx^4))}{105a^7x^5\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-3072*A*b^6*x^12 + 256*a*b^5*x^10*(-42*A + 5*B*x^2) - 128*a^2*b^4*x^8*(105
*A - 35*B*x^2 + 3*C*x^4) + 16*a^3*b^3*x^6*(-420*A + 350*B*x^2 - 84*C*x^4 +
3*D*x^6) + 56*a^4*b^2*x^4*(-15*A + 50*B*x^2 - 30*C*x^4 + 3*D*x^6) + 14*a^5*
b*x^2*(6*A + 25*B*x^2 - 60*C*x^4 + 15*D*x^6) - 7*a^6*(3*A + 5*x^2*(B + 3*C*
x^2 - 3*D*x^4)))/(105*a^7*x^5*(a + b*x^2)^(7/2))
```

Maple [A]
time = 0.27, size = 446, normalized size = 1.59

method	result
gosper	$-\frac{3072A b^{6} x^{12}-1280B a b^{5} x^{12}+384C a^{2} b^{4} x^{12}-48D a^{3} b^{3} x^{12}+10752A a b^{5} x^{10}-4480B a^{2} b^{4} x^{10}+1344C a^{3} b^{3} x^{10}-168D a^{4} b^{2} x^{10}+134}{}$
trager	$-\frac{3072A b^{6} x^{12}-1280B a b^{5} x^{12}+384C a^{2} b^{4} x^{12}-48D a^{3} b^{3} x^{12}+10752A a b^{5} x^{10}-4480B a^{2} b^{4} x^{10}+1344C a^{3} b^{3} x^{10}-168D a^{4} b^{2} x^{10}+134}{}$

default	$D\left(\frac{x}{7a(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}\right)}{7a}}{a}\right)+A\left(-\frac{1}{5a\,x^{5}(b\,x^{2}+a)^{\frac{7}{2}}}-\frac{12b\left(-\frac{1}{3a\,x^{3}\left(b\,x^{2}-\right.}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] D*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b
*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))+A*(-1/5/a/x^5/(b*x^2+a)^(7/2)-12
/5*b/a*(-1/3/a/x^3/(b*x^2+a)^(7/2)-10/3*b/a*(-1/a/x/(b*x^2+a)^(7/2)-8*b/a*(
1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^
2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))))+B*(-1/3/a/x^3/(b*x^2+a)^(7/2)-10
/3*b/a*(-1/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/
a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))
))))+C*(-1/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/
a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))
)))
```

Maxima [**A**]
time = 0.29, size = 398, normalized size = 1.42

$$\frac{16\,Dx}{35\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Dx}{35\,(bx^2+a)^{\frac{3}{2}}a^3}+\frac{6\,Dx}{35\,(bx^2+a)^{\frac{5}{2}}a^2}+\frac{Dx}{7\,(bx^2+a)^{\frac{7}{2}}a}-\frac{128\,Cbx}{35\,\sqrt{bx^2+a}\,a^5}-\frac{64\,Cbx}{35\,(bx^2+a)^{\frac{3}{2}}a^4}-\frac{48\,Cbx}{35\,(bx^2+a)^{\frac{5}{2}}a^3}-\frac{8\,Cbx}{7\,(bx^2+a)^{\frac{7}{2}}a^2}+\frac{256\,Bb^2x}{21\,\sqrt{bx^2+a}\,a^6}+\frac{128\,Bb^2x}{21\,(bx^2+a)^{\frac{3}{2}}a^5}+\frac{32\,Bb^2x}{7\,(bx^2+a)^{\frac{5}{2}}a^4}+\frac{80\,Bb^2x}{21\,(bx^2+a)^{\frac{7}{2}}a^3}-\frac{1024\,Ab^3x}{35\,\sqrt{bx^2+a}\,a^7}-\frac{512\,Ab^3x}{35\,(bx^2+a)^{\frac{3}{2}}a^6}-\frac{384\,Ab^3x}{35\,(bx^2+a)^{\frac{5}{2}}a^5}-\frac{64\,Ab^3x}{7\,(bx^2+a)^{\frac{7}{2}}a^4}-\frac{C}{(bx^2+a)^{\frac{7}{2}}ax}+\frac{10\,Bb}{3\,(bx^2+a)^{\frac{7}{2}}a^2x}-\frac{8\,Ab^2}{(bx^2+a)^{\frac{7}{2}}a^3x}-\frac{B}{3\,(bx^2+a)^{\frac{7}{2}}ax^3}+\frac{4\,Ab}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^3}-\frac{A}{5\,(bx^2+a)^{\frac{7}{2}}ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 16/35*D*x/(sqrt(b*x^2 + a)*a^4) + 8/35*D*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*D
*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*D*x/((b*x^2 + a)^(7/2)*a) - 128/35*C*b*x/(
sqrt(b*x^2 + a)*a^5) - 64/35*C*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*C*b*x/((
b*x^2 + a)^(5/2)*a^3) - 8/7*C*b*x/((b*x^2 + a)^(7/2)*a^2) + 256/21*B*b^2*x/
(sqrt(b*x^2 + a)*a^6) + 128/21*B*b^2*x/((b*x^2 + a)^(3/2)*a^5) + 32/7*B*b^2
*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*B*b^2*x/((b*x^2 + a)^(7/2)*a^3) - 1024/3
5*A*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*A*b^3*x/((b*x^2 + a)^(3/2)*a^6) -
384/35*A*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*A*b^3*x/((b*x^2 + a)^(7/2)*a^
4) - C/((b*x^2 + a)^(7/2)*a*x) + 10/3*B*b/((b*x^2 + a)^(7/2)*a^2*x) - 8*A*b
^2/((b*x^2 + a)^(7/2)*a^3*x) - 1/3*B/((b*x^2 + a)^(7/2)*a*x^3) + 4/5*A*b/((
b*x^2 + a)^(7/2)*a^2*x^3) - 1/5*A/((b*x^2 + a)^(7/2)*a*x^5)
```

Fricas [**A**]
time = 5.53, size = 270, normalized size = 0.96

$$\frac{(16\,(3\,Da^3b^3-24\,Ca^2b^4+80\,Bab^5-192\,Ab^6)x^{12}+56\,(3\,Da^4b^2-24\,Ca^3b^3+80\,Ba^2b^4-192\,Aab^5)x^{10}+70\,(3\,Da^5b-24\,Ca^4b^2+80\,Ba^3b^3-192\,Aa^2b^4)x^8-21\,Aa^6+35\,(3\,Da^6-24\,Ca^5b+80\,Ba^4b^2-192\,Aa^3b^3)x^6-35\,(3\,Ca^6-10\,Ba^5b+24\,Aa^4b^2)x^4-7\,(5\,Ba^6-12\,Aa^5b)x^2)\sqrt{bx^2+a}}{105\,(a^7b^4x^{13}+4\,a^8b^3x^{11}+6\,a^9b^2x^9+4\,a^{10}bx^7+a^{11}x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/105*(16*(3*D*a^3*b^3 - 24*C*a^2*b^4 + 80*B*a*b^5 - 192*A*b^6)*x^12 + 56*(
3*D*a^4*b^2 - 24*C*a^3*b^3 + 80*B*a^2*b^4 - 192*A*a*b^5)*x^10 + 70*(3*D*a^5
```

```
*b - 24*C*a^4*b^2 + 80*B*a^3*b^3 - 192*A*a^2*b^4)*x^8 - 21*A*a^6 + 35*(3*D*
a^6 - 24*C*a^5*b + 80*B*a^4*b^2 - 192*A*a^3*b^3)*x^6 - 35*(3*C*a^6 - 10*B*a
^5*b + 24*A*a^4*b^2)*x^4 - 7*(5*B*a^6 - 12*A*a^5*b)*x^2)*sqrt(b*x^2 + a)/(a
^7*b^4*x^13 + 4*a^8*b^3*x^11 + 6*a^9*b^2*x^9 + 4*a^10*b*x^7 + a^11*x^5)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3313 vs. $2(277) = 554$.
time = 202.10, size = 3313, normalized size = 11.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**6/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(-7*a**8*b**(73/2)*sqrt(a/(b*x**2) + 1)/(35*a**13*b**36*x**4 + 210*a**12*
b**37*x**6 + 525*a**11*b**38*x**8 + 700*a**10*b**39*x**10 + 525*a**9*b**40*
x**12 + 210*a**8*b**41*x**14 + 35*a**7*b**42*x**16) + 14*a**7*b**(75/2)*x**
2*sqrt(a/(b*x**2) + 1)/(35*a**13*b**36*x**4 + 210*a**12*b**37*x**6 + 525*a*
*11*b**38*x**8 + 700*a**10*b**39*x**10 + 525*a**9*b**40*x**12 + 210*a**8*b*
*41*x**14 + 35*a**7*b**42*x**16) - 231*a**6*b**(77/2)*x**4*sqrt(a/(b*x**2)
+ 1)/(35*a**13*b**36*x**4 + 210*a**12*b**37*x**6 + 525*a**11*b**38*x**8 + 7
00*a**10*b**39*x**10 + 525*a**9*b**40*x**12 + 210*a**8*b**41*x**14 + 35*a**
7*b**42*x**16) - 2772*a**5*b**(79/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**13*b*
*36*x**4 + 210*a**12*b**37*x**6 + 525*a**11*b**38*x**8 + 700*a**10*b**39*x*
*10 + 525*a**9*b**40*x**12 + 210*a**8*b**41*x**14 + 35*a**7*b**42*x**16) -
9240*a**4*b**(81/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**13*b**36*x**4 + 210*a*
*12*b**37*x**6 + 525*a**11*b**38*x**8 + 700*a**10*b**39*x**10 + 525*a**9*b*
*40*x**12 + 210*a**8*b**41*x**14 + 35*a**7*b**42*x**16) - 14784*a**3*b**(83
/2)*x**10*sqrt(a/(b*x**2) + 1)/(35*a**13*b**36*x**4 + 210*a**12*b**37*x**6
+ 525*a**11*b**38*x**8 + 700*a**10*b**39*x**10 + 525*a**9*b**40*x**12 + 210
*a**8*b**41*x**14 + 35*a**7*b**42*x**16) - 12672*a**2*b**(85/2)*x**12*sqrt(
a/(b*x**2) + 1)/(35*a**13*b**36*x**4 + 210*a**12*b**37*x**6 + 525*a**11*b**
38*x**8 + 700*a**10*b**39*x**10 + 525*a**9*b**40*x**12 + 210*a**8*b**41*x**
14 + 35*a**7*b**42*x**16) - 5632*a*b**(87/2)*x**14*sqrt(a/(b*x**2) + 1)/(35
*a**13*b**36*x**4 + 210*a**12*b**37*x**6 + 525*a**11*b**38*x**8 + 700*a**10
*b**39*x**10 + 525*a**9*b**40*x**12 + 210*a**8*b**41*x**14 + 35*a**7*b**42*
x**16) - 1024*b**(89/2)*x**16*sqrt(a/(b*x**2) + 1)/(35*a**13*b**36*x**4 + 2
10*a**12*b**37*x**6 + 525*a**11*b**38*x**8 + 700*a**10*b**39*x**10 + 525*a*
*9*b**40*x**12 + 210*a**8*b**41*x**14 + 35*a**7*b**42*x**16)) + B*(-7*a**6*
b**(51/2)*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4
+ 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**
6*b**30*x**12) + 63*a**5*b**(53/2)*x**2*sqrt(a/(b*x**2) + 1)/(21*a**11*b**2
5*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 +
 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 630*a**4*b**(55/2)*x**4*sqrt
(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**
```

```
27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12)
 + 1680*a**3*b**(57/2)*x**6*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105
*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b*
*29*x**10 + 21*a**6*b**30*x**12) + 2016*a**2*b**(59/2)*x**8*sqrt(a/(b*x**2)
 + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 2
10*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 1152*a*b
**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*
x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 2
1*a**6*b**30*x**12) + 256*b**(63/2)*x**12*sqrt(a/(b*x**2) + 1)/(21*a**11*b*
*25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8
 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12)) + C*(-35*a**4*b**(33/2)*sqr
t(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*a**7*b**18*x**
4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 280*a**3*b**(35/2)*x**2*sqr
t(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*a**7*b**18*x**
4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 560*a**2*b**(37/2)*x**4*sqr
t(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*a**7*b**18*x**
4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 448*a*b**(39/2)*x**6*sqrt(a
/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*a**7*b**18*x**4 +
 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 128*b**(41/2)*x**8*sqrt(a/(b*x
**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 210*a**7*b**18*x**4 + 140*
a**6*b**19*x**6 + 35*a**5*b**20*x**8)) + D*(35*a**14*x/(35*a**(37/2)*sqrt(1
 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2
*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525
*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 +
 b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 175*a**13*b*x**3
/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a)
 + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqr
t(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2
)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a
)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b
*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700
*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 +
b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6
*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b**3*x**7/(35*a**(37/2)*sqrt(1 + b*x
**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*
sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sq...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. 2(252) = 504.

time = 2.56, size = 592, normalized size = 2.11

$$\frac{\left(\left(x^{2}\left(\frac{\left(48\,Da^{18}b^{6}-279\,Ca^{17}b^{7}+790\,Ba^{16}b^{8}-1686\,Aa^{15}b^{9}\right)x^{2}}{a^{22}b^{3}}+\frac{7\,\left(24\,Da^{19}b^{5}-132\,Ca^{18}b^{6}+365\,Ba^{17}b^{7}-768\,Aa^{16}b^{8}\right)}{a^{22}b^{3}}\right)+\frac{35\,\left(6\,Da^{20}b^{4}-30\,Ca^{19}b^{5}+80\,Ba^{18}b^{6}-165\,Aa^{17}b^{7}\right)}{a^{22}b^{3}}\right)x^{2}+\frac{105\,\left(Da^{21}b^{3}-4\,Ca^{20}b^{4}+10\,Ba^{19}b^{5}-20\,Aa^{18}b^{6}\right)}{a^{22}b^{3}}\right)x}{105\,\left(bx^{2}+a\right)^{\frac{7}{2}}}+\frac{2\,\left(15\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{8}Ca^{2}\sqrt{b}-60\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{8}Bab^{\frac{3}{2}}+150\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{8}Ab^{\frac{5}{2}}-60\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}Ca^{3}\sqrt{b}+270\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}Ba^{2}b^{\frac{3}{2}}-720\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{6}Aab^{\frac{5}{2}}+90\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Ca^{4}\sqrt{b}-430\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Ba^{3}b^{\frac{3}{2}}+1260\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{4}Aa^{2}b^{\frac{5}{2}}-60\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Ca^{5}\sqrt{b}+290\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Ba^{4}b^{\frac{3}{2}}-840\,\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}Aa^{3}b^{\frac{5}{2}}+15\,Ca^{6}\sqrt{b}-70\,Ba^{5}b^{\frac{3}{2}}+198\,Aa^{4}b^{\frac{5}{2}}\right)}{15\,\left(\left(\sqrt{b}x-\sqrt{bx^{2}+a}\right)^{2}-a\right)^{5}a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((48*D*a^18*b^6 - 279*C*a^17*b^7 + 790*B*a^16*b^8 - 1686*A*a^15
*b^9)*x^2/(a^22*b^3) + 7*(24*D*a^19*b^5 - 132*C*a^18*b^6 + 365*B*a^17*b^7 -
 768*A*a^16*b^8)/(a^22*b^3)) + 35*(6*D*a^20*b^4 - 30*C*a^19*b^5 + 80*B*a^18
*b^6 - 165*A*a^17*b^7)/(a^22*b^3))*x^2 + 105*(D*a^21*b^3 - 4*C*a^20*b^4 + 1
0*B*a^19*b^5 - 20*A*a^18*b^6)/(a^22*b^3))*x/(b*x^2 + a)^(7/2) + 2/15*(15*(s
qrt(b)*x - sqrt(b*x^2 + a))^8*C*a^2*sqrt(b) - 60*(sqrt(b)*x - sqrt(b*x^2 +
a))^8*B*a*b^(3/2) + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) - 60*(sqr
t(b)*x - sqrt(b*x^2 + a))^6*C*a^3*sqrt(b) + 270*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*B*a^2*b^(3/2) - 720*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(5/2) + 90*(
sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^4*sqrt(b) - 430*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*B*a^3*b^(3/2) + 1260*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^2*b^(5/2)
- 60*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^5*sqrt(b) + 290*(sqrt(b)*x - sqrt(
b*x^2 + a))^2*B*a^4*b^(3/2) - 840*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^3*b^(
5/2) + 15*C*a^6*sqrt(b) - 70*B*a^5*b^(3/2) + 198*A*a^4*b^(5/2))/(((sqrt(b)*
x - sqrt(b*x^2 + a))^2 - a)^5*a^6)
```

Mupad [**B**]
time = 2.40, size = 405, normalized size = 1.44

$$\frac{\frac{61\,A\,b}{35\,a^3}+\frac{78\,A\,b^2\,x^2}{35\,a^4}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{\frac{128\,B\,b}{21\,a^5}+\frac{256\,B\,b^2\,x^2}{21\,a^6}}{x\,\sqrt{b\,x^2+a}}+\frac{x\,D}{{\left(b\,x^2+a\right)}^{9/2}}-\frac{\frac{B}{3\,a^2}+\frac{19\,B\,b\,x^2}{21\,a^3}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{\frac{C}{a^4}+\frac{128\,C\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}}-\frac{\frac{512\,A\,b^2}{35\,a^6}+\frac{1024\,A\,b^3\,x^2}{35\,a^7}}{x\,\sqrt{b\,x^2+a}}-\frac{A\,\sqrt{b\,x^2+a}}{5\,a^5\,x^5}+\frac{18\,b^2\,x^5\,D}{5\,a^2\,{\left(b\,x^2+a\right)}^{9/2}}+\frac{72\,b^3\,x^7\,D}{35\,a^3\,{\left(b\,x^2+a\right)}^{9/2}}+\frac{16\,b^4\,x^9\,D}{35\,a^4\,{\left(b\,x^2+a\right)}^{9/2}}-\frac{A\,b}{7\,a^2\,x^3\,{\left(b\,x^2+a\right)}^{7/2}}-\frac{32\,B\,b}{21\,a^4\,x\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{B\,b^2\,x}{7\,a^3\,{\left(b\,x^2+a\right)}^{7/2}}+\frac{27\,A\,b^2}{7\,a^5\,x\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{3\,b\,x^3\,D}{a\,{\left(b\,x^2+a\right)}^{9/2}}-\frac{29\,C\,b\,x}{35\,a^4\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{13\,C\,b\,x}{35\,a^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{C\,b\,x}{7\,a^2\,{\left(b\,x^2+a\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^6*(a + b*x^2)^(9/2)),x)
```

```
[Out] ((61*A*b)/(35*a^3) + (78*A*b^2*x^2)/(35*a^4))/(x^3*(a + b*x^2)^(5/2)) + ((1
28*B*b)/(21*a^5) + (256*B*b^2*x^2)/(21*a^6))/(x*(a + b*x^2)^(1/2)) + (x*D)/
(a + b*x^2)^(9/2) - (B/(3*a^2) + (19*B*b*x^2)/(21*a^3))/(x^3*(a + b*x^2)^(5
/2)) - (C/a^4 + (128*C*b*x^2)/(35*a^5))/(x*(a + b*x^2)^(1/2)) - ((512*A*b^2
)/(35*a^6) + (1024*A*b^3*x^2)/(35*a^7))/(x*(a + b*x^2)^(1/2)) - (A*(a + b*x
^2)^(1/2))/(5*a^5*x^5) + (18*b^2*x^5*D)/(5*a^2*(a + b*x^2)^(9/2)) + (72*b^3
*x^7*D)/(35*a^3*(a + b*x^2)^(9/2)) + (16*b^4*x^9*D)/(35*a^4*(a + b*x^2)^(9/
2)) - (A*b)/(7*a^2*x^3*(a + b*x^2)^(7/2)) - (32*B*b)/(21*a^4*x*(a + b*x^2)^
(3/2)) + (B*b^2*x)/(7*a^3*(a + b*x^2)^(7/2)) + (27*A*b^2)/(7*a^5*x*(a + b*x
^2)^(3/2)) + (3*b*x^3*D)/(a*(a + b*x^2)^(9/2)) - (29*C*b*x)/(35*a^4*(a + b*
x^2)^(3/2)) - (13*C*b*x)/(35*a^3*(a + b*x^2)^(5/2)) - (C*b*x)/(7*a^2*(a + b
*x^2)^(7/2))
```

3.167 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=334

$$-\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}+\frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}}-\frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}}+\frac{48Ab^3-a\left(24b^2B-10abC+3a^2D\right)}{3a^4x\left(a+bx^2\right)^{7/2}}+$$

```
[Out] -1/7*A/a/x^7/(b*x^2+a)^(7/2)+1/5*(2*A*b-B*a)/a^2/x^5/(b*x^2+a)^(7/2)+1/15*(
-24*A*b^2+a*(12*B*b-5*C*a))/a^3/x^3/(b*x^2+a)^(7/2)+1/3*(48*A*b^3-a*(24*B*b
^2-10*C*a*b+3*D*a^2))/a^4/x/(b*x^2+a)^(7/2)+8/21*b*(48*A*b^3-a*(24*B*b^2-10
*C*a*b+3*D*a^2))*x/a^5/(b*x^2+a)^(7/2)+16/35*b*(48*A*b^3-a*(24*B*b^2-10*C*a
*b+3*D*a^2))*x/a^6/(b*x^2+a)^(5/2)+64/105*b*(48*A*b^3-a*(24*B*b^2-10*C*a*b+
3*D*a^2))*x/a^7/(b*x^2+a)^(3/2)+128/105*b*(48*A*b^3-a*(24*B*b^2-10*C*a*b+3*
D*a^2))*x/a^8/(b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1817, 12, 277, 198, 197}

$$-\frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}}+\frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}}+\frac{128bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{105a^8\sqrt{a+bx^2}}+\frac{64bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{105a^7\left(a+bx^2\right)^{3/2}}+\frac{16bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{35a^6\left(a+bx^2\right)^{5/2}}+\frac{8bx\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{21a^5\left(a+bx^2\right)^{7/2}}+\frac{48Ab^3-a\left(3a^2D-10abC+24b^2B\right)}{3a^4x\left(a+bx^2\right)^{7/2}}-\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^(9/2)),x]
```

```
[Out] -1/7*A/(a*x^7*(a + b*x^2)^(7/2)) + (2*A*b - a*B)/(5*a^2*x^5*(a + b*x^2)^(7/
2)) - (24*A*b^2 - a*(12*b*B - 5*a*C))/(15*a^3*x^3*(a + b*x^2)^(7/2)) + (48*
A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))/(3*a^4*x*(a + b*x^2)^(7/2)) + (8
*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(21*a^5*(a + b*x^2)^(7
/2)) + (16*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(35*a^6*(a +
 b*x^2)^(5/2)) + (64*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*x)/(1
05*a^7*(a + b*x^2)^(3/2)) + (128*b*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a
^2*D))*x)/(105*a^8*Sqrt[a + b*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{14Ab-7a\left(B+Cx^2+Dx^4\right)}{x^6\left(a+bx^2\right)^{9/2}}\,dx}{7a}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} + \frac{\int \frac{12b(14Ab-7aB)-5a\left(-7aC-7aDx^2\right)}{x^4\left(a+bx^2\right)^{9/2}}}{35a^2}$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} -$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} - (48$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} + 48$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} + 48$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} + 48$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} + 48$$

$$= -\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}} + \frac{2Ab-aB}{5a^2x^5\left(a+bx^2\right)^{7/2}} - \frac{24Ab^2-a(12bB-5aC)}{15a^3x^3\left(a+bx^2\right)^{7/2}} + 48$$

Mathematica [A]
time = 0.65, size = 234, normalized size = 0.70

$$\frac{6144Ab^7x^{14}-3072ab^6x^{12}\left(-7A+Bx^2\right)+256a^2b^5x^{10}\left(105A-42Bx^2+5Cx^4\right)+14a^6bx^2\left(3A+6Bx^2+25Cx^4-60Dx^6\right)+112a^4b^3x^6\left(15A-60Bx^2+50Cx^4-12Dx^6\right)+128a^3b^4x^8\left(105A-105Bx^2+35Cx^4-3Dx^6\right)-56a^5b^2x^4\left(3A+15Bx^2-50Cx^4+30Dx^6\right)-a^7\left(15A+21Bx^2+35x^4\left(C+3Dx^2\right)\right)}{105a^8x^7\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^(9/2)),x]
```

```
[Out] (6144*A*b^7*x^14 - 3072*a*b^6*x^12*(-7*A + B*x^2) + 256*a^2*b^5*x^10*(105*A
 - 42*B*x^2 + 5*C*x^4) + 14*a^6*b*x^2*(3*A + 6*B*x^2 + 25*C*x^4 - 60*D*x^6)
 + 112*a^4*b^3*x^6*(15*A - 60*B*x^2 + 50*C*x^4 - 12*D*x^6) + 128*a^3*b^4*x^
8*(105*A - 105*B*x^2 + 35*C*x^4 - 3*D*x^6) - 56*a^5*b^2*x^4*(3*A + 15*B*x^2
 - 50*C*x^4 + 30*D*x^6) - a^7*(15*A + 21*B*x^2 + 35*x^4*(C + 3*D*x^2)))/(10
5*a^8*x^7*(a + b*x^2)^(7/2))
```

Maple [A]
time = 0.29, size = 542, normalized size = 1.62

method	result
gosper	$-\frac{-6144A\,b^{7}x^{14}+3072Ba\,b^{6}x^{14}-1280C\,a^{2}b^{5}x^{14}+384Da^{3}b^{4}x^{14}-21504Aa\,b^{6}x^{12}+10752B\,a^{2}b^{5}x^{12}-4480C\,a^{3}b^{4}x^{12}+1344Da^{4}b^{3}x^{12}}{}$
trager	$-\frac{-6144A\,b^{7}x^{14}+3072Ba\,b^{6}x^{14}-1280C\,a^{2}b^{5}x^{14}+384Da^{3}b^{4}x^{14}-21504Aa\,b^{6}x^{12}+10752B\,a^{2}b^{5}x^{12}-4480C\,a^{3}b^{4}x^{12}+1344Da^{4}b^{3}x^{12}}{}$

default	$B\left(-\frac{1}{5a\,x^5(b\,x^2+a)^{\frac{7}{2}}}-\frac{12b\left(-\frac{1}{3a\,x^3\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{10b\left(-\frac{1}{ax\left(b\,x^2+a\right)^{\frac{7}{2}}}-\frac{8b\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{}{15a}\right.}{7a}}{a}\right.}{a}\right.}{3a}\right.}{5a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/5/a/x^5/(b*x^2+a)^(7/2)-12/5*b/a*(-1/3/a/x^3/(b*x^2+a)^(7/2)-10/3*b/a
*(-1/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x
^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))))+
A*(-1/7/a/x^7/(b*x^2+a)^(7/2)-2*b/a*(-1/5/a/x^5/(b*x^2+a)^(7/2)-12/5*b/a*(-
1/3/a/x^3/(b*x^2+a)^(7/2)-10/3*b/a*(-1/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/(
b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2
)+2/3*x/a^2/(b*x^2+a)^(1/2))))))))+C*(-1/3/a/x^3/(b*x^2+a)^(7/2)-10/3*b/a*(
-1/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2
+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))))+D*(
-1/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2
+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))
```

Maxima [A]
time = 0.29, size = 489, normalized size = 1.46

$$-\frac{128\,Dbx}{35\,\sqrt{bx^2+a}\,a^5}-\frac{64\,Dbx}{35\,(bx^2+a)^{\frac{3}{2}}a^4}-\frac{48\,Dbx}{35\,(bx^2+a)^{\frac{5}{2}}a^3}-\frac{8\,Dbx}{7\,(bx^2+a)^{\frac{7}{2}}a^2}+\frac{256\,Cb^2x}{21\,\sqrt{bx^2+a}\,a^6}+\frac{128\,Cb^2x}{21\,(bx^2+a)^{\frac{3}{2}}a^5}+\frac{32\,Cb^2x}{7\,(bx^2+a)^{\frac{5}{2}}a^4}+\frac{80\,Cb^2x}{21\,(bx^2+a)^{\frac{7}{2}}a^3}-\frac{1024\,Bb^3x}{35\,\sqrt{bx^2+a}\,a^7}-\frac{512\,Bb^3x}{35\,(bx^2+a)^{\frac{3}{2}}a^6}-\frac{384\,Bb^3x}{35\,(bx^2+a)^{\frac{5}{2}}a^5}-\frac{64\,Bb^3x}{7\,(bx^2+a)^{\frac{7}{2}}a^4}+\frac{2048\,Ab^4x}{35\,\sqrt{bx^2+a}\,a^8}+\frac{1024\,Ab^4x}{35\,(bx^2+a)^{\frac{3}{2}}a^7}+\frac{768\,Ab^4x}{35\,(bx^2+a)^{\frac{5}{2}}a^6}+\frac{128\,Ab^4x}{7\,(bx^2+a)^{\frac{7}{2}}a^5}-\frac{D}{(bx^2+a)^{\frac{7}{2}}ax}+\frac{10\,Cb}{3\,(bx^2+a)^{\frac{7}{2}}a^2x}-\frac{8\,Bb^2}{(bx^2+a)^{\frac{7}{2}}a^3x}+\frac{16\,Ab^3}{(bx^2+a)^{\frac{7}{2}}a^4x}-\frac{C}{3\,(bx^2+a)^{\frac{7}{2}}ax^3}+\frac{4\,Bb}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^3}-\frac{8\,Ab^2}{5\,(bx^2+a)^{\frac{7}{2}}a^3x^3}-\frac{B}{5\,(bx^2+a)^{\frac{7}{2}}ax^5}+\frac{2\,Ab}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^5}-\frac{A}{7\,(bx^2+a)^{\frac{7}{2}}ax^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] -128/35*D*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*D*b*x/((b*x^2 + a)^(3/2)*a^4) -
 48/35*D*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*D*b*x/((b*x^2 + a)^(7/2)*a^2) +
256/21*C*b^2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*C*b^2*x/((b*x^2 + a)^(3/2)*a^
5) + 32/7*C*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*C*b^2*x/((b*x^2 + a)^(7/2
)*a^3) - 1024/35*B*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*B*b^3*x/((b*x^2 + a
)^(3/2)*a^6) - 384/35*B*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*B*b^3*x/((b*x^
2 + a)^(7/2)*a^4) + 2048/35*A*b^4*x/(sqrt(b*x^2 + a)*a^8) + 1024/35*A*b^4*x
/((b*x^2 + a)^(3/2)*a^7) + 768/35*A*b^4*x/((b*x^2 + a)^(5/2)*a^6) + 128/7*A
*b^4*x/((b*x^2 + a)^(7/2)*a^5) - D/((b*x^2 + a)^(7/2)*a*x) + 10/3*C*b/((b*x
^2 + a)^(7/2)*a^2*x) - 8*B*b^2/((b*x^2 + a)^(7/2)*a^3*x) + 16*A*b^3/((b*x^2
 + a)^(7/2)*a^4*x) - 1/3*C/((b*x^2 + a)^(7/2)*a*x^3) + 4/5*B*b/((b*x^2 + a)
^(7/2)*a^2*x^3) - 8/5*A*b^2/((b*x^2 + a)^(7/2)*a^3*x^3) - 1/5*B/((b*x^2 + a
)^(7/2)*a*x^5) + 2/5*A*b/((b*x^2 + a)^(7/2)*a^2*x^5) - 1/7*A/((b*x^2 + a)^(
7/2)*a*x^7)
```

Fricas [A]
time = 4.65, size = 311, normalized size = 0.93

$$-\frac{(128\,(3\,Da^3b^4-10\,Ca^2b^5+24\,Bab^6-48\,Ab^7)x^{14}+448\,(3\,Da^4b^3-10\,Ca^3b^4+24\,Ba^2b^5-48\,Aab^6)x^{12}+560\,(3\,Da^5b^2-10\,Ca^4b^3+24\,Ba^3b^4-48\,Aa^2b^5)x^{10}+280\,(3\,Da^6b-10\,Ca^5b^2+24\,Ba^4b^3-48\,Aa^3b^4)x^8+15\,Aa^7+35\,(3\,Da^7-10\,Ca^6b+24\,Ba^5b^2-48\,Aa^4b^3)x^6+7\,(5\,Ca^7-12\,Ba^6b+24\,Aa^5b^2)x^4+21\,(Ba^7-2\,Aa^6b)x^2)\sqrt{bx^2+a}}{105\,(a^8b^4x^{15}+4\,a^9b^3x^{13}+6\,a^{10}b^2x^{11}+4\,a^{11}bx^9+a^{12}x^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(128*(3*D*a^3*b^4 - 10*C*a^2*b^5 + 24*B*a*b^6 - 48*A*b^7)*x^14 + 448
*(3*D*a^4*b^3 - 10*C*a^3*b^4 + 24*B*a^2*b^5 - 48*A*a*b^6)*x^12 + 560*(3*D*a
^5*b^2 - 10*C*a^4*b^3 + 24*B*a^3*b^4 - 48*A*a^2*b^5)*x^10 + 280*(3*D*a^6*b
- 10*C*a^5*b^2 + 24*B*a^4*b^3 - 48*A*a^3*b^4)*x^8 + 15*A*a^7 + 35*(3*D*a^7
- 10*C*a^6*b + 24*B*a^5*b^2 - 48*A*a^4*b^3)*x^6 + 7*(5*C*a^7 - 12*B*a^6*b +
 24*A*a^5*b^2)*x^4 + 21*(B*a^7 - 2*A*a^6*b)*x^2)*sqrt(b*x^2 + a)/(a^8*b^4*x
^15 + 4*a^9*b^3*x^13 + 6*a^10*b^2*x^11 + 4*a^11*b*x^9 + a^12*x^7)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**8/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 938 vs. 2(300) = 600.
time = 0.98, size = 938, normalized size = 2.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((x^2*((279*D*a^21*b^7 - 790*C*a^20*b^8 + 1686*B*a^19*b^9 - 3072*A*a
^18*b^10)*x^2/(a^26*b^3) + 7*(132*D*a^22*b^6 - 365*C*a^21*b^7 + 768*B*a^20*
b^8 - 1386*A*a^19*b^9)/(a^26*b^3)) + 35*(30*D*a^23*b^5 - 80*C*a^22*b^6 + 16
5*B*a^21*b^7 - 294*A*a^20*b^8)/(a^26*b^3))*x^2 + 105*(4*D*a^24*b^4 - 10*C*a
^23*b^5 + 20*B*a^22*b^6 - 35*A*a^21*b^7)/(a^26*b^3))*x/(b*x^2 + a)^(7/2) +
2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*a^3*sqrt(b) - 420*(sqrt(b)*x
- sqrt(b*x^2 + a))^12*C*a^2*b^(3/2) + 1050*(sqrt(b)*x - sqrt(b*x^2 + a))^12
*B*a*b^(5/2) - 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*b^(7/2) - 630*(sqrt(
b)*x - sqrt(b*x^2 + a))^10*D*a^4*sqrt(b) + 2730*(sqrt(b)*x - sqrt(b*x^2 + a
))^10*C*a^3*b^(3/2) - 7140*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2*b^(5/2) +
 14700*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a*b^(7/2) + 1575*(sqrt(b)*x - sqr
t(b*x^2 + a))^8*D*a^5*sqrt(b) - 7210*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^4*
b^(3/2) + 19950*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(5/2) - 42840*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(7/2) - 2100*(sqrt(b)*x - sqrt(b*x^2 + a
))^6*D*a^6*sqrt(b) + 9940*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^5*b^(3/2) - 2
8560*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(5/2) + 64680*(sqrt(b)*x - sqr
t(b*x^2 + a))^6*A*a^3*b^(7/2) + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^4*D*a^7*
```

```
sqrt(b) - 7560*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^6*b^(3/2) + 21966*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) - 49812*(sqrt(b)*x - sqrt(b*x^2 + a
))^4*A*a^4*b^(7/2) - 630*(sqrt(b)*x - sqrt(b*x^2 + a))^2*D*a^8*sqrt(b) + 30
10*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^7*b^(3/2) - 8652*(sqrt(b)*x - sqrt(b
*x^2 + a))^2*B*a^6*b^(5/2) + 19404*(sqrt(b)*x - sqrt(b*x^2 + a))^2*A*a^5*b^
(7/2) + 105*D*a^9*sqrt(b) - 490*C*a^8*b^(3/2) + 1386*B*a^7*b^(5/2) - 3072*A
*a^6*b^(7/2))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^7*a^7)
```

Mupad [B]

time = 2.84, size = 421, normalized size = 1.26

$$\frac{\frac{61\,B\,b}{35\,a^3}+\frac{78\,B\,b^2\,x^2}{35\,a^4}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{\frac{128\,C\,b}{21\,a^5}+\frac{256\,C\,b^2\,x^2}{21\,a^6}}{x\,\sqrt{b\,x^2+a}}-\frac{\frac{C}{3\,a^2}+\frac{19\,C\,b\,x^2}{21\,a^3}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{\frac{167\,A\,b^2}{35\,a^4}+\frac{191\,A\,b^3\,x^2}{35\,a^5}}{x^3\,{\left(b\,x^2+a\right)}^{5/2}}+\frac{\frac{1024\,A\,b^3}{35\,a^7}+\frac{2048\,A\,b^4\,x^2}{35\,a^8}}{x\,\sqrt{b\,x^2+a}}-\frac{\frac{512\,B\,b^2}{35\,a^6}+\frac{1024\,B\,b^3\,x^2}{35\,a^7}}{x\,\sqrt{b\,x^2+a}}-\frac{A\,\sqrt{b\,x^2+a}}{7\,a^5\,x^7}-\frac{B\,\sqrt{b\,x^2+a}}{5\,a^5\,x^5}-\frac{{\left(\frac{a}{b\,x^2}+1\right)}^{9/2}\,D\,{{}}_2{\mathrm{F}}_1\left(\frac{9}{2},5;\ 6;\ -\frac{a}{b\,x^2}\right)}{10\,x\,{\left(b\,x^2+a\right)}^{9/2}}+\frac{34\,A\,b\,\sqrt{b\,x^2+a}}{35\,a^6\,x^5}-\frac{B\,b}{7\,a^2\,x^3\,{\left(b\,x^2+a\right)}^{7/2}}-\frac{32\,C\,b}{21\,a^4\,x\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{C\,b^2\,x}{7\,a^3\,{\left(b\,x^2+a\right)}^{7/2}}-\frac{58\,A\,b^3}{7\,a^6\,x\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{A\,b^2}{7\,a^3\,x^3\,{\left(b\,x^2+a\right)}^{7/2}}+\frac{27\,B\,b^2}{7\,a^5\,x\,{\left(b\,x^2+a\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^8*(a + b*x^2)^(9/2)),x)
```

```
[Out] ((61*B*b)/(35*a^3) + (78*B*b^2*x^2)/(35*a^4))/(x^3*(a + b*x^2)^(5/2)) + ((1
28*C*b)/(21*a^5) + (256*C*b^2*x^2)/(21*a^6))/(x*(a + b*x^2)^(1/2)) - (C/(3*
a^2) + (19*C*b*x^2)/(21*a^3))/(x^3*(a + b*x^2)^(5/2)) - ((167*A*b^2)/(35*a^
4) + (191*A*b^3*x^2)/(35*a^5))/(x^3*(a + b*x^2)^(5/2)) + ((1024*A*b^3)/(35*
a^7) + (2048*A*b^4*x^2)/(35*a^8))/(x*(a + b*x^2)^(1/2)) - ((512*B*b^2)/(35*
a^6) + (1024*B*b^3*x^2)/(35*a^7))/(x*(a + b*x^2)^(1/2)) - (A*(a + b*x^2)^(1
/2))/(7*a^5*x^7) - (B*(a + b*x^2)^(1/2))/(5*a^5*x^5) - ((a/(b*x^2) + 1)^(9/
2)*D*hypergeom([9/2, 5], 6, -a/(b*x^2)))/(10*x*(a + b*x^2)^(9/2)) + (34*A*b
*(a + b*x^2)^(1/2))/(35*a^6*x^5) - (B*b)/(7*a^2*x^3*(a + b*x^2)^(7/2)) - (3
2*C*b)/(21*a^4*x*(a + b*x^2)^(3/2)) + (C*b^2*x)/(7*a^3*(a + b*x^2)^(7/2)) -
 (58*A*b^3)/(7*a^6*x*(a + b*x^2)^(3/2)) + (A*b^2)/(7*a^3*x^3*(a + b*x^2)^(7
/2)) + (27*B*b^2)/(7*a^5*x*(a + b*x^2)^(3/2))
```

3.168 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=392

$$-\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}+\frac{128Ab^3-3a(24b^2B-12abC+5a^2D)}{45a^4x^3\left(a+bx^2\right)^{7/2}}$$

```
[Out] -1/9*A/a/x^9/(b*x^2+a)^(7/2)+1/63*(16*A*b-9*B*a)/a^2/x^7/(b*x^2+a)^(7/2)+1/
45*(-32*A*b^2+9*a*(2*B*b-C*a))/a^3/x^5/(b*x^2+a)^(7/2)+1/45*(128*A*b^3-3*a*
(24*B*b^2-12*C*a*b+5*D*a^2))/a^4/x^3/(b*x^2+a)^(7/2)-2/9*b*(128*A*b^3-3*a*(
24*B*b^2-12*C*a*b+5*D*a^2))/a^5/x/(b*x^2+a)^(7/2)-16/63*b^2*(128*A*b^3-3*a*
(24*B*b^2-12*C*a*b+5*D*a^2))*x/a^6/(b*x^2+a)^(7/2)-32/105*b^2*(128*A*b^3-3*
a*(24*B*b^2-12*C*a*b+5*D*a^2))*x/a^7/(b*x^2+a)^(5/2)-128/315*b^2*(128*A*b^3
-3*a*(24*B*b^2-12*C*a*b+5*D*a^2))*x/a^8/(b*x^2+a)^(3/2)-256/315*b^2*(128*A*
b^3-3*a*(24*B*b^2-12*C*a*b+5*D*a^2))*x/a^9/(b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 380, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {1817, 12, 277, 198, 197}

$$-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}-\frac{256b^2x(-15a^3D-36ab(2bB-aC)+128Ab^3)}{315a^9\sqrt{a+bx^2}}-\frac{128b^2x(-15a^3D-36ab(2bB-aC)+128Ab^3)}{315a^8\left(a+bx^2\right)^{3/2}}-\frac{32b^2x(-15a^3D-36ab(2bB-aC)+128Ab^3)}{105a^7\left(a+bx^2\right)^{5/2}}-\frac{16b^2x(-15a^3D-36ab(2bB-aC)+128Ab^3)}{63a^6\left(a+bx^2\right)^{7/2}}-\frac{2b(-15a^3D-36ab(2bB-aC)+128Ab^3)}{9a^5x\left(a+bx^2\right)^{7/2}}+\frac{-15a^3D-36ab(2bB-aC)+128Ab^3}{45a^4x^3\left(a+bx^2\right)^{7/2}}-\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^(9/2)),x]
```

```
[Out] -1/9*A/(a*x^9*(a + b*x^2)^(7/2)) + (16*A*b - 9*a*B)/(63*a^2*x^7*(a + b*x^2)
^(7/2)) - (32*A*b^2 - 9*a*(2*b*B - a*C))/(45*a^3*x^5*(a + b*x^2)^(7/2)) + (
128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)/(45*a^4*x^3*(a + b*x^2)^(7/2))
 - (2*b*(128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D))/(9*a^5*x*(a + b*x^2)
^(7/2)) - (16*b^2*(128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)*x)/(63*a^6*
(a + b*x^2)^(7/2)) - (32*b^2*(128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)*
x)/(105*a^7*(a + b*x^2)^(5/2)) - (128*b^2*(128*A*b^3 - 36*a*b*(2*b*B - a*C)
 - 15*a^3*D)*x)/(315*a^8*(a + b*x^2)^(3/2)) - (256*b^2*(128*A*b^3 - 36*a*b*
(2*b*B - a*C) - 15*a^3*D)*x)/(315*a^9*Sqrt[a + b*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx &= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{16Ab-9a\left(B+Cx^2+Dx^4\right)}{x^8\left(a+bx^2\right)^{9/2}}\,dx}{9a} \\
&= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}+\frac{\int \frac{14b(16Ab-9aB)-7a\left(-9aC-9aDx^2\right)}{x^6\left(a+bx^2\right)^{9/2}}}{63a^2} \\
&= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}-\frac{\int 1}{} \\
&= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}-\frac{(12}{} \\
&= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}+\frac{128}{} \\
&= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}+\frac{128}{} \\
&= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}+\frac{128}{} \\
&= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}+\frac{128}{} \\
&= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}+\frac{128}{} \\
&= -\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7\left(a+bx^2\right)^{7/2}}-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5\left(a+bx^2\right)^{7/2}}+\frac{128}{}
\end{aligned}$$

Mathematica [A]
time = 0.74, size = 270, normalized size = 0.69

$$\frac{-32768Ab^8x^{16}+2048ab^7x^{14}(-56A+9Bx^2)-1024a^2b^6x^{12}(140A-63Bx^2+9Cx^4)-56a^6b^2x^4(4A+9Bx^2+45Cx^4-150Dx^6)+4480a^4b^4x^8(-2A+9Bx^2-9Cx^4+3Dx^6)+256a^3b^5x^{10}(-280A+315Bx^2-126Cx^4+15Dx^6)-a^8(35A+45Bx^2+63Cx^4+105Dx^6)+112a^5b^3x^6(8A+45Bx^2-180Cx^4+150Dx^6)+2a^7bx^2(40A+21(3Bx^2+6Cx^4+25Dx^6))}{315a^9x^9\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-32768*A*b^8*x^16 + 2048*a*b^7*x^14*(-56*A + 9*B*x^2) - 1024*a^2*b^6*x^12*
(140*A - 63*B*x^2 + 9*C*x^4) - 56*a^6*b^2*x^4*(4*A + 9*B*x^2 + 45*C*x^4 - 1
50*D*x^6) + 4480*a^4*b^4*x^8*(-2*A + 9*B*x^2 - 9*C*x^4 + 3*D*x^6) + 256*a^3
*b^5*x^10*(-280*A + 315*B*x^2 - 126*C*x^4 + 15*D*x^6) - a^8*(35*A + 45*B*x^
2 + 63*C*x^4 + 105*D*x^6) + 112*a^5*b^3*x^6*(8*A + 45*B*x^2 - 180*C*x^4 + 1
```

```
50*D*x^6) + 2*a^7*b*x^2*(40*A + 21*(3*B*x^2 + 6*C*x^4 + 25*D*x^6)))/(315*a^
9*x^9*(a + b*x^2)^(7/2))
```

Maple [A]
time = 0.36, size = 638, normalized size = 1.63

method	result
gosper	$-\frac{32768 A\,b^{8} x^{16}-18432 B a\,b^{7} x^{16}+9216 C\,a^{2} b^{6} x^{16}-3840 D a^{3} b^{5} x^{16}+114688 A a\,b^{7} x^{14}-64512 B\,a^{2} b^{6} x^{14}+32256 C\,a^{3} b^{5} x^{14}-13440 D a^{4} b}{}$
trager	$-\frac{32768 A\,b^{8} x^{16}-18432 B a\,b^{7} x^{16}+9216 C\,a^{2} b^{6} x^{16}-3840 D a^{3} b^{5} x^{16}+114688 A a\,b^{7} x^{14}-64512 B\,a^{2} b^{6} x^{14}+32256 C\,a^{3} b^{5} x^{14}-13440 D a^{4} b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] C*(-1/5/a/x^5/(b*x^2+a)^(7/2)-12/5*b/a*(-1/3/a/x^3/(b*x^2+a)^(7/2)-10/3*b/a
*(-1/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x
^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))))+
A*(-1/9/a/x^9/(b*x^2+a)^(7/2)-16/9*b/a*(-1/7/a/x^7/(b*x^2+a)^(7/2)-2*b/a*(-
1/5/a/x^5/(b*x^2+a)^(7/2)-12/5*b/a*(-1/3/a/x^3/(b*x^2+a)^(7/2)-10/3*b/a*(-1
/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a
)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))))))+B*
(-1/7/a/x^7/(b*x^2+a)^(7/2)-2*b/a*(-1/5/a/x^5/(b*x^2+a)^(7/2)-12/5*b/a*(-1/
3/a/x^3/(b*x^2+a)^(7/2)-10/3*b/a*(-1/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/(b*
x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+
2/3*x/a^2/(b*x^2+a)^(1/2))))))))+D*(-1/3/a/x^3/(b*x^2+a)^(7/2)-10/3*b/a*(-1
/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a
)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))))
```

Maxima [A]
time = 0.30, size = 579, normalized size = 1.48

$$\frac{256\,Db^2x}{21\,\sqrt{bx^2+a}\,a^6}+\frac{128\,Db^2x}{21\,(bx^2+a)^{\frac{3}{2}}a^5}+\frac{32\,Db^2x}{7\,(bx^2+a)^{\frac{5}{2}}a^4}+\frac{80\,Db^2x}{21\,(bx^2+a)^{\frac{7}{2}}a^3}-\frac{1024\,Cb^3x}{35\,\sqrt{bx^2+a}\,a^7}-\frac{512\,Cb^3x}{35\,(bx^2+a)^{\frac{3}{2}}a^6}-\frac{384\,Cb^3x}{35\,(bx^2+a)^{\frac{5}{2}}a^5}-\frac{64\,Cb^3x}{7\,(bx^2+a)^{\frac{7}{2}}a^4}+\frac{2048\,Bb^4x}{35\,\sqrt{bx^2+a}\,a^8}+\frac{1024\,Bb^4x}{35\,(bx^2+a)^{\frac{3}{2}}a^7}+\frac{768\,Bb^4x}{35\,(bx^2+a)^{\frac{5}{2}}a^6}+\frac{128\,Bb^4x}{7\,(bx^2+a)^{\frac{7}{2}}a^5}-\frac{32768\,Ab^5x}{315\,\sqrt{bx^2+a}\,a^9}-\frac{16384\,Ab^5x}{315\,(bx^2+a)^{\frac{3}{2}}a^8}-\frac{4096\,Ab^5x}{105\,(bx^2+a)^{\frac{5}{2}}a^7}-\frac{2048\,Ab^5x}{63\,(bx^2+a)^{\frac{7}{2}}a^6}+\frac{10\,Db}{3\,(bx^2+a)^{\frac{7}{2}}a^2x}-\frac{8\,Cb^2}{(bx^2+a)^{\frac{7}{2}}a^3x}+\frac{16\,Bb^3}{(bx^2+a)^{\frac{7}{2}}a^4x}-\frac{256\,Ab^4}{9\,(bx^2+a)^{\frac{7}{2}}a^5x}-\frac{D}{3\,(bx^2+a)^{\frac{7}{2}}ax^3}+\frac{4\,Cb}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^3}-\frac{8\,Bb^2}{5\,(bx^2+a)^{\frac{7}{2}}a^3x^3}+\frac{128\,Ab^3}{45\,(bx^2+a)^{\frac{7}{2}}a^4x^3}-\frac{C}{5\,(bx^2+a)^{\frac{7}{2}}ax^5}+\frac{2\,Bb}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^5}-\frac{32\,Ab^2}{45\,(bx^2+a)^{\frac{7}{2}}a^3x^5}-\frac{B}{7\,(bx^2+a)^{\frac{7}{2}}ax^7}+\frac{16\,Ab}{63\,(bx^2+a)^{\frac{7}{2}}a^2x^7}-\frac{A}{9\,(bx^2+a)^{\frac{7}{2}}ax^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

```
[Out] 256/21*D*b^2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*D*b^2*x/((b*x^2 + a)^(3/2)*a^
5) + 32/7*D*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*D*b^2*x/((b*x^2 + a)^(7/2
)*a^3) - 1024/35*C*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*C*b^3*x/((b*x^2 + a
)^(3/2)*a^6) - 384/35*C*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*C*b^3*x/((b*x^
2 + a)^(7/2)*a^4) + 2048/35*B*b^4*x/(sqrt(b*x^2 + a)*a^8) + 1024/35*B*b^4*x
/((b*x^2 + a)^(3/2)*a^7) + 768/35*B*b^4*x/((b*x^2 + a)^(5/2)*a^6) + 128/7*B
*b^4*x/((b*x^2 + a)^(7/2)*a^5) - 32768/315*A*b^5*x/(sqrt(b*x^2 + a)*a^9) -
16384/315*A*b^5*x/((b*x^2 + a)^(3/2)*a^8) - 4096/105*A*b^5*x/((b*x^2 + a)^(
5/2)*a^7) - 2048/63*A*b^5*x/((b*x^2 + a)^(7/2)*a^6) + 10/3*D*b/((b*x^2 + a)
^(7/2)*a^2*x) - 8*C*b^2/((b*x^2 + a)^(7/2)*a^3*x) + 16*B*b^3/((b*x^2 + a)^(
7/2)*a^4*x) - 256/9*A*b^4/((b*x^2 + a)^(7/2)*a^5*x) - 1/3*D/((b*x^2 + a)^(7
/2)*a*x^3) + 4/5*C*b/((b*x^2 + a)^(7/2)*a^2*x^3) - 8/5*B*b^2/((b*x^2 + a)^(
7/2)*a^3*x^3) + 128/45*A*b^3/((b*x^2 + a)^(7/2)*a^4*x^3) - 1/5*C/((b*x^2 +
a)^(7/2)*a*x^5) + 2/5*B*b/((b*x^2 + a)^(7/2)*a^2*x^5) - 32/45*A*b^2/((b*x^2
 + a)^(7/2)*a^3*x^5) - 1/7*B/((b*x^2 + a)^(7/2)*a*x^7) + 16/63*A*b/((b*x^2
+ a)^(7/2)*a^2*x^7) - 1/9*A/((b*x^2 + a)^(7/2)*a*x^9)
```

Fricas [A]
time = 3.53, size = 354, normalized size = 0.90

$$\frac{\left(256\,(15\,Da^3b^5-36\,Ca^2b^6+72\,Bab^7-128\,Ab^8)x^{16}+896\,(15\,Da^4b^4-36\,Ca^3b^5+72\,Ba^2b^6-128\,Aab^7)x^{14}+1120\,(15\,Da^5b^3-36\,Ca^4b^4+72\,Ba^3b^5-128\,Aa^2b^6)x^{12}+560\,(15\,Da^6b^2-36\,Ca^5b^3+72\,Ba^4b^4-128\,Aa^3b^5)x^{10}-35\,Aa^8+70\,(15\,Da^7b-36\,Ca^6b^2+72\,Ba^5b^3-128\,Aa^4b^4)x^8-7\,(15\,Da^8-36\,Ca^7b+72\,Ba^6b^2-128\,Aa^5b^3)x^6-7\,(9\,Ca^8-18\,Ba^7b+32\,Aa^6b^2)x^4-5\,(9\,Ba^8-16\,Aa^7b)x^2\right)\sqrt{bx^2+a}}{315\,(a^9b^4x^{17}+4\,a^{10}b^3x^{15}+6\,a^{11}b^2x^{13}+4\,a^{12}bx^{11}+a^{13}x^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/315*(256*(15*D*a^3*b^5 - 36*C*a^2*b^6 + 72*B*a*b^7 - 128*A*b^8)*x^16 + 89
6*(15*D*a^4*b^4 - 36*C*a^3*b^5 + 72*B*a^2*b^6 - 128*A*a*b^7)*x^14 + 1120*(1
5*D*a^5*b^3 - 36*C*a^4*b^4 + 72*B*a^3*b^5 - 128*A*a^2*b^6)*x^12 + 560*(15*D
*a^6*b^2 - 36*C*a^5*b^3 + 72*B*a^4*b^4 - 128*A*a^3*b^5)*x^10 - 35*A*a^8 + 7
0*(15*D*a^7*b - 36*C*a^6*b^2 + 72*B*a^5*b^3 - 128*A*a^4*b^4)*x^8 - 7*(15*D*
a^8 - 36*C*a^7*b + 72*B*a^6*b^2 - 128*A*a^5*b^3)*x^6 - 7*(9*C*a^8 - 18*B*a^
7*b + 32*A*a^6*b^2)*x^4 - 5*(9*B*a^8 - 16*A*a^7*b)*x^2)*sqrt(b*x^2 + a)/(a^
9*b^4*x^17 + 4*a^10*b^3*x^15 + 6*a^11*b^2*x^13 + 4*a^12*b*x^11 + a^13*x^9)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x**6+C*x**4+B*x**2+A)/x**10/(b*x**2+a)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. 2(355) = 710.
time = 0.99, size = 1162, normalized size = 2.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] 1/105*((x^2*((790*D*a^24*b^8 - 1686*C*a^23*b^9 + 3072*B*a^22*b^10 - 5053*A*
a^21*b^11)*x^2/(a^30*b^3) + 7*(365*D*a^25*b^7 - 768*C*a^24*b^8 + 1386*B*a^2
3*b^9 - 2264*A*a^22*b^10)/(a^30*b^3)) + 35*(80*D*a^26*b^6 - 165*C*a^25*b^7
+ 294*B*a^24*b^8 - 476*A*a^23*b^9)/(a^30*b^3))*x^2 + 105*(10*D*a^27*b^5 - 2
0*C*a^26*b^6 + 35*B*a^25*b^7 - 56*A*a^24*b^8)/(a^30*b^3))*x/(b*x^2 + a)^(7/
2) - 2/315*(1260*(sqrt(b)*x - sqrt(b*x^2 + a))^16*D*a^3*b^(3/2) - 3150*(sqr
t(b)*x - sqrt(b*x^2 + a))^16*C*a^2*b^(5/2) + 6300*(sqrt(b)*x - sqrt(b*x^2 +
 a))^16*B*a*b^(7/2) - 11025*(sqrt(b)*x - sqrt(b*x^2 + a))^16*A*b^(9/2) - 10
710*(sqrt(b)*x - sqrt(b*x^2 + a))^14*D*a^4*b^(3/2) + 27720*(sqrt(b)*x - sqr
t(b*x^2 + a))^14*C*a^3*b^(5/2) - 56700*(sqrt(b)*x - sqrt(b*x^2 + a))^14*B*a
^2*b^(7/2) + 100800*(sqrt(b)*x - sqrt(b*x^2 + a))^14*A*a*b^(9/2) + 39270*(s
qrt(b)*x - sqrt(b*x^2 + a))^12*D*a^5*b^(3/2) - 105840*(sqrt(b)*x - sqrt(b*x
^2 + a))^12*C*a^4*b^(5/2) + 223020*(sqrt(b)*x - sqrt(b*x^2 + a))^12*B*a^3*b
^(7/2) - 405300*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*a^2*b^(9/2) - 81270*(sqr
```

```
t(b)*x - sqrt(b*x^2 + a))^10*D*a^6*b^(3/2) + 226800*(sqrt(b)*x - sqrt(b*x^2
 + a))^10*C*a^5*b^(5/2) - 495180*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^4*b^(
7/2) + 927360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a^3*b^(9/2) + 103950*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*D*a^7*b^(3/2) - 297108*(sqrt(b)*x - sqrt(b*x^2 +
 a))^8*C*a^6*b^(5/2) + 666036*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^5*b^(7/2)
 - 1291374*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^4*b^(9/2) - 84210*(sqrt(b)*x
 - sqrt(b*x^2 + a))^6*D*a^8*b^(3/2) + 243432*(sqrt(b)*x - sqrt(b*x^2 + a))^
6*C*a^7*b^(5/2) - 551124*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^6*b^(7/2) + 10
73856*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^5*b^(9/2) + 42210*(sqrt(b)*x - sq
rt(b*x^2 + a))^4*D*a^9*b^(3/2) - 121968*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a
^8*b^(5/2) + 275076*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^7*b^(7/2) - 533124*
(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a^6*b^(9/2) - 11970*(sqrt(b)*x - sqrt(b*x
^2 + a))^2*D*a^10*b^(3/2) + 34272*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^9*b^(
5/2) - 76644*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^8*b^(7/2) + 147456*(sqrt(b
)*x - sqrt(b*x^2 + a))^2*A*a^7*b^(9/2) + 1470*D*a^11*b^(3/2) - 4158*C*a^10*
b^(5/2) + 9216*B*a^9*b^(7/2) - 17609*A*a^8*b^(9/2))/(((sqrt(b)*x - sqrt(b*x
^2 + a))^2 - a)^9*a^8)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^{10}\,{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^(9/2)),x)
```

```
[Out] int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^(9/2)), x)
```

3.169 $\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=214

$$\frac{a^2(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^6}-\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\left(a+bx^2\right)^{3/2}}{3b^6}+\frac{(b^3c-3ab^2d+6a^2$$

```
[Out] -1/3*a*(-5*a^3*f+4*a^2*b*e-3*a*b^2*d+2*b^3*c)*(b*x^2+a)^(3/2)/b^6+1/5*(-10*
a^3*f+6*a^2*b*e-3*a*b^2*d+b^3*c)*(b*x^2+a)^(5/2)/b^6+1/7*(10*a^2*f-4*a*b*e+
b^2*d)*(b*x^2+a)^(7/2)/b^6+1/9*(-5*a*f+b*e)*(b*x^2+a)^(9/2)/b^6+1/11*f*(b*x
^2+a)^(11/2)/b^6+a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a)^(1/2)/b^6
```

Rubi [A]

time = 0.15, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1825, 1813, 1864}

$$\frac{\left(a+bx^2\right)^{7/2}\left(10a^2f-4abe+b^2d\right)}{7b^6}+\frac{\left(a+bx^2\right)^{5/2}\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{5b^6}-\frac{a\left(a+bx^2\right)^{3/2}\left(-5a^3f+4a^2be-3ab^2d+2b^3c\right)}{3b^6}+\frac{a^2\sqrt{a+bx^2}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^6}+\frac{\left(a+bx^2\right)^{9/2}(be-5af)}{9b^6}+\frac{f\left(a+bx^2\right)^{11/2}}{11b^6}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^5 + d*x^7 + e*x^9 + f*x^11)/Sqrt[a + b*x^2],x]
```

```
[Out] (a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (a*(2*b^3*c
 - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/(3*b^6) + ((b^3*c -
3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)^(5/2))/(5*b^6) + ((b^2*d - 4*
a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2))/(7*b^6) + ((b*e - 5*a*f)*(a + b*x^2)^(
9/2))/(9*b^6) + (f*(a + b*x^2)^(11/2))/(11*b^6)
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1825

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx &= \int \frac{x(cx^4+dx^6+ex^8+fx^{10})}{\sqrt{a+bx^2}}\,dx\\ &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{cx^2+dx^3+ex^4+fx^5}{\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\mathrm{Subst}\Bigg(\int \Bigg(-\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)}{b^5\sqrt{a+bx}}+\frac{a(-2b^3c+3ab^2d-4a^2be}{b^5}\\ &= \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^6}-\frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\,(}{3b^6}\end{aligned}$$

Mathematica [A]

time = 0.03, size = 158, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\,(-1280a^5f+128a^4b(11e+5fx^2)-16a^3b^2(99d+44ex^2+30fx^4)+8a^2b^3(231c+99dx^2+66ex^4+50fx^6)-2ab^4x^2(462c+297dx^2+220ex^4+175fx^6)+b^5x^4(693c+5(99dx^2+77ex^4+63fx^6)))}{3465b^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^5 + d*x^7 + e*x^9 + f*x^11)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-1280*a^5*f + 128*a^4*b*(11*e + 5*f*x^2) - 16*a^3*b^2*(99
*d + 44*e*x^2 + 30*f*x^4) + 8*a^2*b^3*(231*c + 99*d*x^2 + 66*e*x^4 + 50*f*x
^6) - 2*a*b^4*x^2*(462*c + 297*d*x^2 + 220*e*x^4 + 175*f*x^6) + b^5*x^4*(69
3*c + 5*(99*d*x^2 + 77*e*x^4 + 63*f*x^6))))/(3465*b^6)
```

Maple [A]

time = 0.14, size = 382, normalized size = 1.79

method	result
gosper	$-\frac{\sqrt{b\,x^2+a}\,(-315f\,x^{10}b^5+350a\,b^4f\,x^8-385b^5e\,x^8-400a^2b^3f\,x^6+440a\,b^4e\,x^6-495b^5d\,x^6+480a^3b^2f\,x^4-528a^2b^3e\,x^4+594a\,b^4d}{3465b^6}$
trager	$-\frac{\sqrt{b\,x^2+a}\,(-315f\,x^{10}b^5+350a\,b^4f\,x^8-385b^5e\,x^8-400a^2b^3f\,x^6+440a\,b^4e\,x^6-495b^5d\,x^6+480a^3b^2f\,x^4-528a^2b^3e\,x^4+594a\,b^4d}{3465b^6}$
risch	$-\frac{\sqrt{b\,x^2+a}\,(-315f\,x^{10}b^5+350a\,b^4f\,x^8-385b^5e\,x^8-400a^2b^3f\,x^6+440a\,b^4e\,x^6-495b^5d\,x^6+480a^3b^2f\,x^4-528a^2b^3e\,x^4+594a\,b^4d}{3465b^6}$

default	$f\left(\frac{x^{10}\sqrt{b\,x^{2}+a}}{11b}-\frac{10a\left(\frac{x^{8}\sqrt{b\,x^{2}+a}}{9b}-\frac{8a\left(\frac{x^{6}\sqrt{b\,x^{2}+a}}{7b}-\frac{6a\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5b}-\frac{4a\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x}}{3}\right.}{5b}\right.}{7b}\right.}{9b}\right.}{11b}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/11*x^10/b*(b*x^2+a)^(1/2)-10/11*a/b*(1/9*x^8/b*(b*x^2+a)^(1/2)-8/9*a/b
*(1/7*x^6/b*(b*x^2+a)^(1/2)-6/7*a/b*(1/5*x^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/3
*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))))))+e*(1/9*x^8/b*(b*x^2+a
)^(1/2)-8/9*a/b*(1/7*x^6/b*(b*x^2+a)^(1/2)-6/7*a/b*(1/5*x^4/b*(b*x^2+a)^(1/
2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2)))))+d*(1/7*
x^6/b*(b*x^2+a)^(1/2)-6/7*a/b*(1/5*x^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/3*x^2/b
*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))))+c*(1/5*x^4/b*(b*x^2+a)^(1/2)-
4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2)))
```

Maxima [A]

time = 0.38, size = 352, normalized size = 1.64

$$\frac{\sqrt{bx^2+a}\,fx^{10}}{11\,b}-\frac{10\,\sqrt{bx^2+a}\,afx^8}{99\,b^2}+\frac{\sqrt{bx^2+a}\,x^8e}{9\,b}+\frac{\sqrt{bx^2+a}\,dx^6}{7\,b}+\frac{80\,\sqrt{bx^2+a}\,a^2fx^6}{693\,b^3}-\frac{8\,\sqrt{bx^2+a}\,ax^6e}{63\,b^2}+\frac{\sqrt{bx^2+a}\,cx^4}{5\,b}-\frac{6\,\sqrt{bx^2+a}\,adx^4}{35\,b^2}-\frac{32\,\sqrt{bx^2+a}\,a^3fx^4}{231\,b^4}+\frac{16\,\sqrt{bx^2+a}\,a^2x^4e}{105\,b^3}-\frac{4\,\sqrt{bx^2+a}\,acx^2}{15\,b^2}+\frac{8\,\sqrt{bx^2+a}\,a^2dx^2}{35\,b^3}+\frac{128\,\sqrt{bx^2+a}\,a^4fx^2}{693\,b^5}-\frac{64\,\sqrt{bx^2+a}\,a^3x^2e}{315\,b^4}+\frac{8\,\sqrt{bx^2+a}\,a^2c}{15\,b^3}-\frac{16\,\sqrt{bx^2+a}\,a^3d}{35\,b^4}-\frac{256\,\sqrt{bx^2+a}\,a^5f}{693\,b^6}+\frac{128\,\sqrt{bx^2+a}\,a^4e}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/11*sqrt(b*x^2 + a)*f*x^10/b - 10/99*sqrt(b*x^2 + a)*a*f*x^8/b^2 + 1/9*sqr
t(b*x^2 + a)*x^8*e/b + 1/7*sqrt(b*x^2 + a)*d*x^6/b + 80/693*sqrt(b*x^2 + a)
*a^2*f*x^6/b^3 - 8/63*sqrt(b*x^2 + a)*a*x^6*e/b^2 + 1/5*sqrt(b*x^2 + a)*c*x
^4/b - 6/35*sqrt(b*x^2 + a)*a*d*x^4/b^2 - 32/231*sqrt(b*x^2 + a)*a^3*f*x^4/
b^4 + 16/105*sqrt(b*x^2 + a)*a^2*x^4*e/b^3 - 4/15*sqrt(b*x^2 + a)*a*c*x^2/b
^2 + 8/35*sqrt(b*x^2 + a)*a^2*d*x^2/b^3 + 128/693*sqrt(b*x^2 + a)*a^4*f*x^2
/b^5 - 64/315*sqrt(b*x^2 + a)*a^3*x^2*e/b^4 + 8/15*sqrt(b*x^2 + a)*a^2*c/b^
3 - 16/35*sqrt(b*x^2 + a)*a^3*d/b^4 - 256/693*sqrt(b*x^2 + a)*a^5*f/b^6 + 1
28/315*sqrt(b*x^2 + a)*a^4*e/b^5
```

Fricas [**A**]
time = 1.30, size = 186, normalized size = 0.87

$$\frac{(315\,b^5fx^{10} - 350\,ab^4fx^8 + 5\,(99\,b^5d + 80\,a^2b^3f)x^6 + 1848\,a^2b^3c - 1584\,a^3b^2d - 1280\,a^5f + 3\,(231\,b^5c - 198\,ab^4d - 160\,a^3b^2f)x^4 - 4\,(231\,ab^4c - 198\,a^2b^3d - 160\,a^4bf)x^2 + 11\,(35\,b^5x^8 - 40\,ab^4x^6 + 48\,a^2b^3x^4 - 64\,a^3b^2x^2 + 128\,a^4b)e)\sqrt{bx^2 + a}}{3465\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3465*(315*b^5*f*x^10 - 350*a*b^4*f*x^8 + 5*(99*b^5*d + 80*a^2*b^3*f)*x^6
+ 1848*a^2*b^3*c - 1584*a^3*b^2*d - 1280*a^5*f + 3*(231*b^5*c - 198*a*b^4*d
 - 160*a^3*b^2*f)*x^4 - 4*(231*a*b^4*c - 198*a^2*b^3*d - 160*a^4*b*f)*x^2 +
 11*(35*b^5*x^8 - 40*a*b^4*x^6 + 48*a^2*b^3*x^4 - 64*a^3*b^2*x^2 + 128*a^4*
b)*e)*sqrt(b*x^2 + a)/b^6
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(214) = 428.
time = 0.56, size = 442, normalized size = 2.07

$$\begin{cases} - \frac{256a^5f\sqrt{a + bx^2}}{693b^6} + \frac{128a^4e\sqrt{a + bx^2}}{315b^5} + \frac{128a^4fx^2\sqrt{a + bx^2}}{693b^5} - \frac{16a^3d\sqrt{a + bx^2}}{35b^4} - \frac{64a^3ex^2\sqrt{a + bx^2}}{315b^4} - \frac{32a^3fx^4\sqrt{a + bx^2}}{231b^4} + \frac{8a^2c\sqrt{a + bx^2}}{15b^3} + \frac{8a^2dx^2\sqrt{a + bx^2}}{35b^3} + \frac{16a^2ex^4\sqrt{a + bx^2}}{105b^3} + \frac{80a^2fx^6\sqrt{a + bx^2}}{693b^3} - \frac{4acx^2\sqrt{a + bx^2}}{15b^2} - \frac{6adx^4\sqrt{a + bx^2}}{35b^2} - \frac{8aex^6\sqrt{a + bx^2}}{63b^2} - \frac{10afx^8\sqrt{a + bx^2}}{99b^2} + \frac{cx^4\sqrt{a + bx^2}}{5b} + \frac{dx^6\sqrt{a + bx^2}}{7b} + \frac{ex^8\sqrt{a + bx^2}}{9b} + \frac{fx^{10}\sqrt{a + bx^2}}{11b} & \text{for}\: b \neq 0 \\ \frac{\frac{cx^6}{6} + \frac{dx^8}{8} + \frac{ex^{10}}{10} + \frac{fx^{12}}{12}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**11+e*x**9+d*x**7+c*x**5)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-256*a**5*f*sqrt(a + b*x**2)/(693*b**6) + 128*a**4*e*sqrt(a + b*
x**2)/(315*b**5) + 128*a**4*f*x**2*sqrt(a + b*x**2)/(693*b**5) - 16*a**3*d*
sqrt(a + b*x**2)/(35*b**4) - 64*a**3*e*x**2*sqrt(a + b*x**2)/(315*b**4) - 3
2*a**3*f*x**4*sqrt(a + b*x**2)/(231*b**4) + 8*a**2*c*sqrt(a + b*x**2)/(15*b
**3) + 8*a**2*d*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*e*x**4*sqrt(a + b
*x**2)/(105*b**3) + 80*a**2*f*x**6*sqrt(a + b*x**2)/(693*b**3) - 4*a*c*x**2
*sqrt(a + b*x**2)/(15*b**2) - 6*a*d*x**4*sqrt(a + b*x**2)/(35*b**2) - 8*a*e
*x**6*sqrt(a + b*x**2)/(63*b**2) - 10*a*f*x**8*sqrt(a + b*x**2)/(99*b**2) +
 c*x**4*sqrt(a + b*x**2)/(5*b) + d*x**6*sqrt(a + b*x**2)/(7*b) + e*x**8*sqr
```

```
t(a + b*x**2)/(9*b) + f*x**10*sqrt(a + b*x**2)/(11*b), Ne(b, 0)), ((c*x**6/
6 + d*x**8/8 + e*x**10/10 + f*x**12/12)/sqrt(a), True))
```

Giac [A]
time = 0.94, size = 264, normalized size = 1.23

$$\frac{(a^2b^3c-a^3b^2d-a^5f+a^4be)\sqrt{bx^2+a}}{b^6}+\frac{693\,(bx^2+a)^{\frac{5}{2}}b^3c-2310\,(bx^2+a)^{\frac{3}{2}}ab^3c+495\,(bx^2+a)^{\frac{7}{2}}b^2d-2079\,(bx^2+a)^{\frac{5}{2}}ab^2d+3465\,(bx^2+a)^{\frac{3}{2}}a^2b^2d+315\,(bx^2+a)^{\frac{11}{2}}f-1925\,(bx^2+a)^{\frac{9}{2}}af+4950\,(bx^2+a)^{\frac{7}{2}}a^2f-6930\,(bx^2+a)^{\frac{5}{2}}a^3f+5775\,(bx^2+a)^{\frac{3}{2}}a^4f+385\,(bx^2+a)^{\frac{9}{2}}be-1980\,(bx^2+a)^{\frac{7}{2}}abe+4158\,(bx^2+a)^{\frac{5}{2}}a^2be-4620\,(bx^2+a)^{\frac{3}{2}}a^3be}{3465\,b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (a^2*b^3*c - a^3*b^2*d - a^5*f + a^4*b*e)*sqrt(b*x^2 + a)/b^6 + 1/3465*(693
*(b*x^2 + a)^(5/2)*b^3*c - 2310*(b*x^2 + a)^(3/2)*a*b^3*c + 495*(b*x^2 + a)
^(7/2)*b^2*d - 2079*(b*x^2 + a)^(5/2)*a*b^2*d + 3465*(b*x^2 + a)^(3/2)*a^2*
b^2*d + 315*(b*x^2 + a)^(11/2)*f - 1925*(b*x^2 + a)^(9/2)*a*f + 4950*(b*x^2
 + a)^(7/2)*a^2*f - 6930*(b*x^2 + a)^(5/2)*a^3*f + 5775*(b*x^2 + a)^(3/2)*a
^4*f + 385*(b*x^2 + a)^(9/2)*b*e - 1980*(b*x^2 + a)^(7/2)*a*b*e + 4158*(b*x
^2 + a)^(5/2)*a^2*b*e - 4620*(b*x^2 + a)^(3/2)*a^3*b*e)/b^6
```

Mupad [B]
time = 1.20, size = 186, normalized size = 0.87

$$\sqrt{b\,x^2+a}\,\left(\frac{x^6\,\left(400\,f\,a^2\,b^3-440\,e\,a\,b^4+495\,d\,b^5\right)}{3465\,b^6}-\frac{1280\,f\,a^5-1408\,e\,a^4\,b+1584\,d\,a^3\,b^2-1848\,c\,a^2\,b^3}{3465\,b^6}+\frac{x^4\,\left(-480\,f\,a^3\,b^2+528\,e\,a^2\,b^3-594\,d\,a\,b^4+693\,c\,b^5\right)}{3465\,b^6}+\frac{f\,x^{10}}{11\,b}+\frac{x^8\,\left(385\,b^5\,e-350\,a\,b^4\,f\right)}{3465\,b^6}-\frac{4\,a\,x^2\,\left(-160\,f\,a^3+176\,e\,a^2\,b-198\,d\,a\,b^2+231\,c\,b^3\right)}{3465\,b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^5 + d*x^7 + e*x^9 + f*x^11)/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((x^6*(495*b^5*d + 400*a^2*b^3*f - 440*a*b^4*e))/(3465*b^
6) - (1280*a^5*f - 1848*a^2*b^3*c + 1584*a^3*b^2*d - 1408*a^4*b*e)/(3465*b^
6) + (x^4*(693*b^5*c + 528*a^2*b^3*e - 480*a^3*b^2*f - 594*a*b^4*d))/(3465*
b^6) + (f*x^10)/(11*b) + (x^8*(385*b^5*e - 350*a*b^4*f))/(3465*b^6) - (4*a*
x^2*(231*b^3*c - 160*a^3*f - 198*a*b^2*d + 176*a^2*b*e))/(3465*b^5))
```

3.170 $\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=167

$$-\frac{a(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^5}+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,(a+bx^2)^{3/2}}{3b^5}+\frac{(b^2d-3abe+6a^2f)\,(a}{5b^5}$$

```
[Out] 1/3*(-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^3*c)*(b*x^2+a)^(3/2)/b^5+1/5*(6*a^2*f-3
*a*b*e+b^2*d)*(b*x^2+a)^(5/2)/b^5+1/7*(-4*a*f+b*e)*(b*x^2+a)^(7/2)/b^5+1/9*
f*(b*x^2+a)^(9/2)/b^5-a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a)^(1/2)/b^5
```

Rubi [A]
time = 0.12, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1825, 1813, 1864}

$$\frac{(a+bx^2)^{5/2}\,(6a^2f-3abe+b^2d)}{5b^5}+\frac{(a+bx^2)^{3/2}\,(-4a^3f+3a^2be-2ab^2d+b^3c)}{3b^5}-\frac{a\sqrt{a+bx^2}\,(a^3(-f)+a^2be-ab^2d+b^3c)}{b^5}+\frac{(a+bx^2)^{7/2}\,(be-4af)}{7b^5}+\frac{f(a+bx^2)^{9/2}}{9b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x^3 + d*x^5 + e*x^7 + f*x^9)/Sqrt[a + b*x^2],x]
```

```
[Out] -((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5) + ((b^3*c -
2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*b^5) + ((b^2*d - 3*a
*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) + ((b*e - 4*a*f)*(a + b*x^2)^(7/
2))/(7*b^5) + (f*(a + b*x^2)^(9/2))/(9*b^5)
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1825

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx &= \int \frac{x(cx^2+dx^4+ex^6+fx^8)}{\sqrt{a+bx^2}}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{cx+dx^2+ex^3+fx^4}{\sqrt{a+bx}}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\Bigg(\int \Bigg(\frac{a(-b^3c+ab^2d-a^2be+a^3f)}{b^4\sqrt{a+bx}} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f}{b^4} \\
&= -\frac{a(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^5} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,(a}{3b^5}
\end{aligned}
$$

Mathematica [A]

time = 0.02, size = 122, normalized size = 0.73

$$\frac{\sqrt{a+bx^2}\,(128a^4f-16a^3b(9e+4fx^2)+24a^2b^2(7d+3ex^2+2fx^4)-2ab^3(105c+42dx^2+27ex^4+20fx^6)+b^4x^2(105c+63dx^2+45ex^4+35fx^6))}{315b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x^3 + d*x^5 + e*x^7 + f*x^9)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(128*a^4*f - 16*a^3*b*(9*e + 4*f*x^2) + 24*a^2*b^2*(7*d +
3*e*x^2 + 2*f*x^4) - 2*a*b^3*(105*c + 42*d*x^2 + 27*e*x^4 + 20*f*x^6) + b^4
*x^2*(105*c + 63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5)
```

Maple [A]

time = 0.11, size = 286, normalized size = 1.71

method	result
gosper	$\frac{\sqrt{b\,x^2+a}\,(35f\,x^8b^4-40a\,b^3f\,x^6+45b^4e\,x^6+48a^2b^2f\,x^4-54a\,b^3e\,x^4+63b^4d\,x^4-64a^3bf\,x^2+72a^2b^2e\,x^2-84a\,b^3d\,x^2+105b^4c\,x^2-}{315b^5}$
trager	$\frac{\sqrt{b\,x^2+a}\,(35f\,x^8b^4-40a\,b^3f\,x^6+45b^4e\,x^6+48a^2b^2f\,x^4-54a\,b^3e\,x^4+63b^4d\,x^4-64a^3bf\,x^2+72a^2b^2e\,x^2-84a\,b^3d\,x^2+105b^4c\,x^2-}{315b^5}$
risch	$\frac{\sqrt{b\,x^2+a}\,(35f\,x^8b^4-40a\,b^3f\,x^6+45b^4e\,x^6+48a^2b^2f\,x^4-54a\,b^3e\,x^4+63b^4d\,x^4-64a^3bf\,x^2+72a^2b^2e\,x^2-84a\,b^3d\,x^2+105b^4c\,x^2-}{315b^5}$

default	$f\left(\frac{x^{8}\sqrt{b\,x^{2}+a}}{9b}-\frac{8a\left(\frac{x^{6}\sqrt{b\,x^{2}+a}}{7b}-\frac{6a\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5b}-\frac{4a\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)}{5b}\right)}{7b}\right)}{9b}\right)+e\left(\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/9*x^8/b*(b*x^2+a)^(1/2)-8/9*a/b*(1/7*x^6/b*(b*x^2+a)^(1/2)-6/7*a/b*(1/
5*x^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2
+a)^(1/2)))))+e*(1/7*x^6/b*(b*x^2+a)^(1/2)-6/7*a/b*(1/5*x^4/b*(b*x^2+a)^(1/
2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))))+d*(1/5*x
^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)
^(1/2)))+c*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))
```

Maxima [A]
time = 0.38, size = 267, normalized size = 1.60

$$\frac{\sqrt{bx^{2}+a}\,fx^{8}}{9\,b}-\frac{8\,\sqrt{bx^{2}+a}\,afx^{6}}{63\,b^{2}}+\frac{\sqrt{bx^{2}+a}\,x^{6}e}{7\,b}+\frac{\sqrt{bx^{2}+a}\,dx^{4}}{5\,b}+\frac{16\,\sqrt{bx^{2}+a}\,a^{2}fx^{4}}{105\,b^{3}}-\frac{6\,\sqrt{bx^{2}+a}\,ax^{4}e}{35\,b^{2}}+\frac{\sqrt{bx^{2}+a}\,cx^{2}}{3\,b}-\frac{4\,\sqrt{bx^{2}+a}\,adx^{2}}{15\,b^{2}}-\frac{64\,\sqrt{bx^{2}+a}\,a^{3}fx^{2}}{315\,b^{4}}+\frac{8\,\sqrt{bx^{2}+a}\,a^{2}x^{2}e}{35\,b^{3}}-\frac{2\,\sqrt{bx^{2}+a}\,ac}{3\,b^{2}}+\frac{8\,\sqrt{bx^{2}+a}\,a^{2}d}{15\,b^{3}}+\frac{128\,\sqrt{bx^{2}+a}\,a^{4}f}{315\,b^{5}}-\frac{16\,\sqrt{bx^{2}+a}\,a^{3}e}{35\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(b*x^2 + a)*f*x^8/b - 8/63*sqrt(b*x^2 + a)*a*f*x^6/b^2 + 1/7*sqrt(b
*x^2 + a)*x^6*e/b + 1/5*sqrt(b*x^2 + a)*d*x^4/b + 16/105*sqrt(b*x^2 + a)*a^
2*f*x^4/b^3 - 6/35*sqrt(b*x^2 + a)*a*x^4*e/b^2 + 1/3*sqrt(b*x^2 + a)*c*x^2/
b - 4/15*sqrt(b*x^2 + a)*a*d*x^2/b^2 - 64/315*sqrt(b*x^2 + a)*a^3*f*x^2/b^4
 + 8/35*sqrt(b*x^2 + a)*a^2*x^2*e/b^3 - 2/3*sqrt(b*x^2 + a)*a*c/b^2 + 8/15*
sqrt(b*x^2 + a)*a^2*d/b^3 + 128/315*sqrt(b*x^2 + a)*a^4*f/b^5 - 16/35*sqrt(
b*x^2 + a)*a^3*e/b^4
```

Fricas [A]
time = 1.76, size = 141, normalized size = 0.84

$$\frac{(35\,b^{4}fx^{8}-40\,ab^{3}fx^{6}-210\,ab^{3}c+168\,a^{2}b^{2}d+128\,a^{4}f+3\,(21\,b^{4}d+16\,a^{2}b^{2}f)x^{4}+(105\,b^{4}c-84\,ab^{3}d-64\,a^{3}bf)x^{2}+9\,(5\,b^{4}x^{6}-6\,ab^{3}x^{4}+8\,a^{2}b^{2}x^{2}-16\,a^{3}b)e)\sqrt{bx^{2}+a}}{315\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/315*(35*b^4*f*x^8 - 40*a*b^3*f*x^6 - 210*a*b^3*c + 168*a^2*b^2*d + 128*a^
4*f + 3*(21*b^4*d + 16*a^2*b^2*f)*x^4 + (105*b^4*c - 84*a*b^3*d - 64*a^3*b*
f)*x^2 + 9*(5*b^4*x^6 - 6*a*b^3*x^4 + 8*a^2*b^2*x^2 - 16*a^3*b)*e)*sqrt(b*x
^2 + a)/b^5
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(163) = 326$.
time = 0.44, size = 340, normalized size = 2.04

$$\begin{cases} \frac{128a^4f\sqrt{a+bx^2}}{315b^5} - \frac{16a^3e\sqrt{a+bx^2}}{35b^4} - \frac{64a^3fx^2\sqrt{a+bx^2}}{315b^4} + \frac{8a^2d\sqrt{a+bx^2}}{15b^3} + \frac{8a^2ex^2\sqrt{a+bx^2}}{35b^3} + \frac{16a^2fx^4\sqrt{a+bx^2}}{105b^3} - \frac{2ac\sqrt{a+bx^2}}{3b^2} - \frac{4adx^2\sqrt{a+bx^2}}{15b^2} - \frac{6aex^4\sqrt{a+bx^2}}{35b^2} - \frac{8afx^6\sqrt{a+bx^2}}{63b^2} + \frac{cx^2\sqrt{a+bx^2}}{3b} + \frac{dx^4\sqrt{a+bx^2}}{5b} + \frac{ex^6\sqrt{a+bx^2}}{7b} + \frac{fx^8\sqrt{a+bx^2}}{9b} & \text{for}\: b \neq 0 \\ \frac{\frac{cx^4}{4} + \frac{dx^6}{6} + \frac{ex^8}{8} + \frac{fx^{10}}{10}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**9+e*x**7+d*x**5+c*x**3)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((128*a**4*f*sqrt(a + b*x**2)/(315*b**5) - 16*a**3*e*sqrt(a + b*x*
*2)/(35*b**4) - 64*a**3*f*x**2*sqrt(a + b*x**2)/(315*b**4) + 8*a**2*d*sqrt(
a + b*x**2)/(15*b**3) + 8*a**2*e*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*
f*x**4*sqrt(a + b*x**2)/(105*b**3) - 2*a*c*sqrt(a + b*x**2)/(3*b**2) - 4*a*
d*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*e*x**4*sqrt(a + b*x**2)/(35*b**2) -
 8*a*f*x**6*sqrt(a + b*x**2)/(63*b**2) + c*x**2*sqrt(a + b*x**2)/(3*b) + d*
x**4*sqrt(a + b*x**2)/(5*b) + e*x**6*sqrt(a + b*x**2)/(7*b) + f*x**8*sqrt(a
 + b*x**2)/(9*b), Ne(b, 0)), ((c*x**4/4 + d*x**6/6 + e*x**8/8 + f*x**10/10)
/sqrt(a), True))
```

Giac [**A**]
time = 1.36, size = 197, normalized size = 1.18

$$-\frac{(ab^3c - a^2b^2d - a^4f + a^3be)\sqrt{bx^2+a}}{b^5} + \frac{105\,(bx^2+a)^{\frac{3}{2}}b^3c + 63\,(bx^2+a)^{\frac{5}{2}}b^2d - 210\,(bx^2+a)^{\frac{3}{2}}ab^2d + 35\,(bx^2+a)^{\frac{9}{2}}f - 180\,(bx^2+a)^{\frac{7}{2}}af + 378\,(bx^2+a)^{\frac{5}{2}}a^2f - 420\,(bx^2+a)^{\frac{3}{2}}a^3f + 45\,(bx^2+a)^{\frac{7}{2}}be - 189\,(bx^2+a)^{\frac{5}{2}}abe + 315\,(bx^2+a)^{\frac{3}{2}}a^2be}{315\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*b^3*c - a^2*b^2*d - a^4*f + a^3*b*e)*sqrt(b*x^2 + a)/b^5 + 1/315*(105*(
b*x^2 + a)^(3/2)*b^3*c + 63*(b*x^2 + a)^(5/2)*b^2*d - 210*(b*x^2 + a)^(3/2)
*a*b^2*d + 35*(b*x^2 + a)^(9/2)*f - 180*(b*x^2 + a)^(7/2)*a*f + 378*(b*x^2
+ a)^(5/2)*a^2*f - 420*(b*x^2 + a)^(3/2)*a^3*f + 45*(b*x^2 + a)^(7/2)*b*e -
 189*(b*x^2 + a)^(5/2)*a*b*e + 315*(b*x^2 + a)^(3/2)*a^2*b*e)/b^5
```

Mupad [**B**]
time = 1.14, size = 146, normalized size = 0.87

$$\sqrt{b\,x^2+a}\,\left(\frac{128\,f\,a^4-144\,e\,a^3\,b+168\,d\,a^2\,b^2-210\,c\,a\,b^3}{315\,b^5}+\frac{x^4\,\left(48\,f\,a^2\,b^2-54\,e\,a\,b^3+63\,d\,b^4\right)}{315\,b^5}+\frac{f\,x^8}{9\,b}+\frac{x^6\,\left(45\,b^4\,e-40\,a\,b^3\,f\right)}{315\,b^5}+\frac{x^2\,\left(-64\,f\,a^3\,b+72\,e\,a^2\,b^2-84\,d\,a\,b^3+105\,c\,b^4\right)}{315\,b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^3 + d*x^5 + e*x^7 + f*x^9)/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((128*a^4*f + 168*a^2*b^2*d - 210*a*b^3*c - 144*a^3*b*e)/
(315*b^5) + (x^4*(63*b^4*d + 48*a^2*b^2*f - 54*a*b^3*e))/(315*b^5) + (f*x^8
)/(9*b) + (x^6*(45*b^4*e - 40*a*b^3*f))/(315*b^5) + (x^2*(105*b^4*c + 72*a^
2*b^2*e - 84*a*b^3*d - 64*a^3*b*f))/(315*b^5))
```

3.171 $\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx$

Optimal. Leaf size=121

$$\frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{a+bx^2}}{b^4}+\frac{\left(b^2d-2abe+3a^2f\right)\left(a+bx^2\right)^{3/2}}{3b^4}+\frac{\left(be-3af\right)\left(a+bx^2\right)^{5/2}}{5b^4}+\frac{f(a+b}{7b^4}$$

```
[Out] 1/3*(3*a^2*f-2*a*b*e+b^2*d)*(b*x^2+a)^(3/2)/b^4+1/5*(-3*a*f+b*e)*(b*x^2+a)^
(5/2)/b^4+1/7*f*(b*x^2+a)^(7/2)/b^4+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a
)^(1/2)/b^4
```

Rubi [A]

time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1825, 1813, 1864}

$$\frac{\left(a+bx^2\right)^{3/2}\left(3a^2f-2abe+b^2d\right)}{3b^4}+\frac{\sqrt{a+bx^2}\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^4}+\frac{\left(a+bx^2\right)^{5/2}\left(be-3af\right)}{5b^4}+\frac{f\left(a+bx^2\right)^{7/2}}{7b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c*x + d*x^3 + e*x^5 + f*x^7)/Sqrt[a + b*x^2],x]
```

```
[Out] ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + ((b^2*d - 2*a*b
*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + ((b*e - 3*a*f)*(a + b*x^2)^(5/2)
)/(5*b^4) + (f*(a + b*x^2)^(7/2))/(7*b^4)
```

Rule 1813

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1825

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[x*PolynomialQuotient
[Pq, x, x]*(a + b*x^2)^p, x] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && EqQ[
Coeff[Pq, x, 0], 0] &&  !MatchQ[Pq, x^(m_.)*(u_.) /; IntegerQ[m]]
```

Rule 1864

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx &= \int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{c+dx+ex^2+fx^3}{\sqrt{a+bx}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{b^3c-ab^2d+a^2be-a^3f}{b^3\sqrt{a+bx}}+\frac{(b^2d-2abe+3a^2f)\sqrt{a+bx}}{b^3}+(be\right.\right.\\ &= \frac{(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^4}+\frac{(b^2d-2abe+3a^2f)\left(a+bx^2\right)^{3/2}}{3b^4}+(b\end{aligned}$$

Mathematica [A]
time = 0.02, size = 89, normalized size = 0.74

$$\frac{\sqrt{a+bx^2}\,(-48a^3f+8a^2b(7e+3fx^2)-2ab^2(35d+14ex^2+9fx^4)+b^3(105c+35dx^2+21ex^4+15fx^6))}{105b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*x + d*x^3 + e*x^5 + f*x^7)/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x^2) - 2*a*b^2*(35*d + 14*
e*x^2 + 9*f*x^4) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^4)
```

Maple [A]
time = 0.12, size = 193, normalized size = 1.60

method	result
gosper	$-\frac{\sqrt{b\,x^2+a}\,(-15f\,x^6b^3+18a\,b^2f\,x^4-21b^3e\,x^4-24a^2bf\,x^2+28a\,b^2e\,x^2-35b^3d\,x^2+48a^3f-56a^2be+70a\,b^2d-105b^3c)}{105b^4}$
trager	$-\frac{\sqrt{b\,x^2+a}\,(-15f\,x^6b^3+18a\,b^2f\,x^4-21b^3e\,x^4-24a^2bf\,x^2+28a\,b^2e\,x^2-35b^3d\,x^2+48a^3f-56a^2be+70a\,b^2d-105b^3c)}{105b^4}$
risch	$-\frac{\sqrt{b\,x^2+a}\,(-15f\,x^6b^3+18a\,b^2f\,x^4-21b^3e\,x^4-24a^2bf\,x^2+28a\,b^2e\,x^2-35b^3d\,x^2+48a^3f-56a^2be+70a\,b^2d-105b^3c)}{105b^4}$
default	$f\left(\frac{x^6\sqrt{b\,x^2+a}}{7b}-\frac{6a\left(\frac{x^4\sqrt{b\,x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)}{5b}\right)}{7b}\right)+e\left(\frac{x^4\sqrt{b\,x^2+a}}{5b}-4a(\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] f*(1/7*x^6/b*(b*x^2+a)^(1/2)-6/7*a/b*(1/5*x^4/b*(b*x^2+a)^(1/2)-4/5*a/b*(1/
3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))))+e*(1/5*x^4/b*(b*x^2+a)
^(1/2)-4/5*a/b*(1/3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2)))+d*(1/
3*x^2/b*(b*x^2+a)^(1/2)-2/3*a/b^2*(b*x^2+a)^(1/2))+c*(b*x^2+a)^(1/2)/b
```

Maxima [A]
time = 0.39, size = 183, normalized size = 1.51

$$\frac{\sqrt{bx^2+a}\,fx^6}{7\,b}-\frac{6\,\sqrt{bx^2+a}\,afx^4}{35\,b^2}+\frac{\sqrt{bx^2+a}\,x^4e}{5\,b}+\frac{\sqrt{bx^2+a}\,dx^2}{3\,b}+\frac{8\,\sqrt{bx^2+a}\,a^2fx^2}{35\,b^3}-\frac{4\,\sqrt{bx^2+a}\,ax^2e}{15\,b^2}+\frac{\sqrt{bx^2+a}\,c}{b}-\frac{2\,\sqrt{bx^2+a}\,ad}{3\,b^2}-\frac{16\,\sqrt{bx^2+a}\,a^3f}{35\,b^4}+\frac{8\,\sqrt{bx^2+a}\,a^2e}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*sqrt(b*x^2 + a)*f*x^6/b - 6/35*sqrt(b*x^2 + a)*a*f*x^4/b^2 + 1/5*sqrt(b
*x^2 + a)*x^4*e/b + 1/3*sqrt(b*x^2 + a)*d*x^2/b + 8/35*sqrt(b*x^2 + a)*a^2*
f*x^2/b^3 - 4/15*sqrt(b*x^2 + a)*a*x^2*e/b^2 + sqrt(b*x^2 + a)*c/b - 2/3*sq
rt(b*x^2 + a)*a*d/b^2 - 16/35*sqrt(b*x^2 + a)*a^3*f/b^4 + 8/15*sqrt(b*x^2 +
 a)*a^2*e/b^3
```

Fricas [A]
time = 0.80, size = 99, normalized size = 0.82

$$\frac{\left(15\,b^3fx^6-18\,ab^2fx^4+105\,b^3c-70\,ab^2d-48\,a^3f+\left(35\,b^3d+24\,a^2bf\right)x^2+7\,\left(3\,b^3x^4-4\,ab^2x^2+8\,a^2b\right)e\right)\sqrt{bx^2+a}}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*b^3*f*x^6 - 18*a*b^2*f*x^4 + 105*b^3*c - 70*a*b^2*d - 48*a^3*f +
(35*b^3*d + 24*a^2*b*f)*x^2 + 7*(3*b^3*x^4 - 4*a*b^2*x^2 + 8*a^2*b)*e)*sqrt
(b*x^2 + a)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(112) = 224$.
time = 0.36, size = 238, normalized size = 1.97

$$\begin{cases} -\frac{16a^3f\sqrt{a+bx^2}}{35b^4}+\frac{8a^2e\sqrt{a+bx^2}}{15b^3}+\frac{8a^2fx^2\sqrt{a+bx^2}}{35b^3}-\frac{2ad\sqrt{a+bx^2}}{3b^2}-\frac{4aex^2\sqrt{a+bx^2}}{15b^2}-\frac{6afx^4\sqrt{a+bx^2}}{35b^2}+\frac{c\sqrt{a+bx^2}}{b}+\frac{dx^2\sqrt{a+bx^2}}{3b}+\frac{ex^4\sqrt{a+bx^2}}{5b}+\frac{fx^6\sqrt{a+bx^2}}{7b} & \text{for}\: b \neq 0 \\ \frac{\frac{cx^2}{2}+\frac{dx^4}{4}+\frac{ex^6}{6}+\frac{fx^8}{8}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**7+e*x**5+d*x**3+c*x)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-16*a**3*f*sqrt(a + b*x**2)/(35*b**4) + 8*a**2*e*sqrt(a + b*x**2
)/(15*b**3) + 8*a**2*f*x**2*sqrt(a + b*x**2)/(35*b**3) - 2*a*d*sqrt(a + b*x
**2)/(3*b**2) - 4*a*e*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*f*x**4*sqrt(a +
 b*x**2)/(35*b**2) + c*sqrt(a + b*x**2)/b + d*x**2*sqrt(a + b*x**2)/(3*b) +
```

```
 e*x**4*sqrt(a + b*x**2)/(5*b) + f*x**6*sqrt(a + b*x**2)/(7*b), Ne(b, 0)),
((c*x**2/2 + d*x**4/4 + e*x**6/6 + f*x**8/8)/sqrt(a), True))
```

Giac [A]
time = 1.33, size = 130, normalized size = 1.07

$$\frac{(b^3c-ab^2d-a^3f+a^2be)\sqrt{bx^2+a}}{b^4}+\frac{35\,(bx^2+a)^{\frac{3}{2}}b^2d+15\,(bx^2+a)^{\frac{7}{2}}f-63\,(bx^2+a)^{\frac{5}{2}}af+105\,(bx^2+a)^{\frac{3}{2}}a^2f+21\,(bx^2+a)^{\frac{5}{2}}be-70\,(bx^2+a)^{\frac{3}{2}}abe}{105\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] (b^3*c - a*b^2*d - a^3*f + a^2*b*e)*sqrt(b*x^2 + a)/b^4 + 1/105*(35*(b*x^2
+ a)^(3/2)*b^2*d + 15*(b*x^2 + a)^(7/2)*f - 63*(b*x^2 + a)^(5/2)*a*f + 105*
(b*x^2 + a)^(3/2)*a^2*f + 21*(b*x^2 + a)^(5/2)*b*e - 70*(b*x^2 + a)^(3/2)*a
*b*e)/b^4
```

Mupad [B]
time = 1.08, size = 103, normalized size = 0.85

$$\sqrt{b\,x^2+a}\,\left(\frac{-48\,f\,a^3+56\,e\,a^2\,b-70\,d\,a\,b^2+105\,c\,b^3}{105\,b^4}+\frac{f\,x^6}{7\,b}+\frac{x^2\,\left(24\,f\,a^2\,b-28\,e\,a\,b^2+35\,d\,b^3\right)}{105\,b^4}+\frac{x^4\,\left(21\,b^3\,e-18\,a\,b^2\,f\right)}{105\,b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x + d*x^3 + e*x^5 + f*x^7)/(a + b*x^2)^(1/2),x)
```

```
[Out] (a + b*x^2)^(1/2)*((105*b^3*c - 48*a^3*f - 70*a*b^2*d + 56*a^2*b*e)/(105*b^
4) + (f*x^6)/(7*b) + (x^2*(35*b^3*d - 28*a*b^2*e + 24*a^2*b*f))/(105*b^4) +
 (x^4*(21*b^3*e - 18*a*b^2*f))/(105*b^4))
```

3.172 $\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6+Fx^8\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=261

$$\frac{\left(Ab^4-a(b^3B-ab^2C+a^2bD-a^3F)\right)x^3}{7ab^4\left(a+bx^2\right)^{7/2}}+\frac{\left(4Ab^4+a(3b^3B-10ab^2C+17a^2bD-24a^3F)\right)x^3}{35a^2b^4\left(a+bx^2\right)^{5/2}}+(8Ab^4+a($$

```
[Out] 1/7*(A*b^4-a*(B*b^3-C*a*b^2+D*a^2*b-F*a^3))*x^3/a/b^4/(b*x^2+a)^(7/2)+1/35*
(4*A*b^4+a*(3*B*b^3-10*C*a*b^2+17*D*a^2*b-24*F*a^3))*x^3/a^2/b^4/(b*x^2+a)^
(5/2)+1/105*(8*A*b^4+a*(6*B*b^3+15*C*a*b^2-71*D*a^2*b+162*F*a^3))*x^3/a^3/b
^4/(b*x^2+a)^(3/2)+1/2*(2*D*b-9*F*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(
11/2)-(D*b-4*F*a)*x/b^5/(b*x^2+a)^(1/2)+1/2*F*x*(b*x^2+a)^(1/2)/b^5
```

Rubi [A]
time = 0.50, antiderivative size = 257, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.243, Rules used = {1818, 1814, 1599, 1277, 1598, 466, 396, 223, 212}

$$\frac{x^3\left(a\left(162a^3F-71a^2bD+15ab^2C+6b^3B\right)+8Ab^4\right)}{105a^3b^4\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)+4Ab^4\right)}{35a^2b^4\left(a+bx^2\right)^{5/2}}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}+\frac{(2bD-9aF)\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}-\frac{x(bD-4aF)}{b^5\sqrt{a+bx^2}}+\frac{Fx\sqrt{a+bx^2}}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6 + F*x^8))/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A/a - (b^3*B - a*b^2*C + a^2*b*D - a^3*F)/b^4)*x^3)/(7*(a + b*x^2)^(7/2))
 + ((4*A*b^4 + a*(3*b^3*B - 10*a*b^2*C + 17*a^2*b*D - 24*a^3*F))*x^3)/(35*a
^2*b^4*(a + b*x^2)^(5/2)) + ((8*A*b^4 + a*(6*b^3*B + 15*a*b^2*C - 71*a^2*b*
D + 162*a^3*F))*x^3)/(105*a^3*b^4*(a + b*x^2)^(3/2)) - ((b*D - 4*a*F)*x)/(b
^5*Sqrt[a + b*x^2]) + (F*x*Sqrt[a + b*x^2])/(2*b^5) + ((2*b*D - 9*a*F)*ArcT
anh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Dist[f/(2*d*(q + 1)), Int[(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1] &
& GtQ[m, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1599

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1814

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/c, Int[(c*x)^(m + 1)*PolynomialQuotient[Pq, x, x]*(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && EqQ[Coeff[Pq, x, 0], 0]
```

Rule 1818

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
```

```
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum
[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\,(a+bx^2)^{7/2}} - \frac{\int \frac{x\left(-\left(4Ab+\frac{3a\left(b^3B-ab^2C+a^2bD-a^3F\right)}{b^3}\right)x}{(a}}{}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\,(a+bx^2)^{7/2}} - \frac{\int \frac{x^2\left(-4Ab-\frac{3a\left(b^3B-ab^2C+a^2bD-a^3F\right)}{b^3}-\frac{7}{}\right.}{(a+}}{}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\,(a+bx^2)^{7/2}} + \frac{(4Ab^4+a(3b^3B-10ab^2C+17a}{35a^2b^4\,(a+bx^2)^5}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\,(a+bx^2)^{7/2}} + \frac{(4Ab^4+a(3b^3B-10ab^2C+17a}{35a^2b^4\,(a+bx^2)^5}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\,(a+bx^2)^{7/2}} + \frac{(4Ab^4+a(3b^3B-10ab^2C+17a}{35a^2b^4\,(a+bx^2)^5}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\,(a+bx^2)^{7/2}} + \frac{(4Ab^4+a(3b^3B-10ab^2C+17a}{35a^2b^4\,(a+bx^2)^5}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\,(a+bx^2)^{7/2}} + \frac{(4Ab^4+a(3b^3B-10ab^2C+17a}{35a^2b^4\,(a+bx^2)^5}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\,(a+bx^2)^{7/2}} + \frac{(4Ab^4+a(3b^3B-10ab^2C+17a}{35a^2b^4\,(a+bx^2)^5}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\,(a+bx^2)^{7/2}} + \frac{(4Ab^4+a(3b^3B-10ab^2C+17a}{35a^2b^4\,(a+bx^2)^5}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x^3}{7\,(a+bx^2)^{7/2}} + \frac{(4Ab^4+a(3b^3B-10ab^2C+17a}{35a^2b^4\,(a+bx^2)^5}$$

Mathematica [A]
time = 0.71, size = 200, normalized size = 0.77

$$\frac{x(945a^7F+16Ab^7x^6+4ab^6x^4(14A+3Bx^2)-210a^6b(D-15Fx^2)+a^3b^4x^6(-352D+105Fx^2)+14a^5b^2x^2(-50D+261Fx^2)+4a^4b^3x^4(-203D+396Fx^2)+2a^2b^5x^2(35A+21Bx^2+15Cx^4))}{210a^3b^5\,(a+bx^2)^{7/2}}+\frac{(-2bD+9aF)\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{2b^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6 + F*x^8))/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(945*a^7*F + 16*A*b^7*x^6 + 4*a*b^6*x^4*(14*A + 3*B*x^2) - 210*a^6*b*(D
- 15*F*x^2) + a^3*b^4*x^6*(-352*D + 105*F*x^2) + 14*a^5*b^2*x^2*(-50*D + 26
1*F*x^2) + 4*a^4*b^3*x^4*(-203*D + 396*F*x^2) + 2*a^2*b^5*x^2*(35*A + 21*B*
x^2 + 15*C*x^4)))/(210*a^3*b^5*(a + b*x^2)^(7/2)) + ((-2*b*D + 9*a*F)*Log[-
(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(11/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(235) = 470$.
time = 0.10, size = 597, normalized size = 2.29

method	result

default	$F\left(\frac{x^{9}}{2b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{9a\left(-\frac{x^{7}}{7b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{-\frac{x^{5}}{5b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^{3}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)}{2b}\right)+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE
)
```

```
[Out] F*(1/2*x^9/b/(b*x^2+a)^(7/2)-9/2*a/b*(-1/7*x^7/b/(b*x^2+a)^(7/2)+1/b*(-1/5*
x^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(
1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))))+D*(-1/7*x^7/b/(b*x^2+a)^(
7/2)+1/b*(-1/5*x^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-
x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))+C*(-1/2*x^5
/b/(b*x^2+a)^(7/2)+5/2*a/b*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/6*x/b/(b
*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/
2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))))+B*(-1/4*x
^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*
x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+
2/3*x/a^2/(b*x^2+a)^(1/2))))))+A*(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a
/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3
/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(235) = 470$.
time = 0.47, size = 826, normalized size = 3.16

$$\frac{Fx^{9}}{2\,(bx^{2}+a)^{\frac{7}{2}}b}-\frac{1}{35}\left(\frac{35\,x^{6}}{(bx^{2}+a)^{\frac{7}{2}}b}+\frac{70\,ax^{4}}{(bx^{2}+a)^{\frac{7}{2}}b^{2}}+\frac{56\,a^{2}x^{2}}{(bx^{2}+a)^{\frac{7}{2}}b^{3}}+\frac{16\,a^{3}}{(bx^{2}+a)^{\frac{7}{2}}b^{4}}\right)Dx+\frac{9\left(\frac{35\,x^{6}}{(bx^{2}+a)^{\frac{7}{2}}b}+\frac{70\,ax^{4}}{(bx^{2}+a)^{\frac{7}{2}}b^{2}}+\frac{56\,a^{2}x^{2}}{(bx^{2}+a)^{\frac{7}{2}}b^{3}}+\frac{16\,a^{3}}{(bx^{2}+a)^{\frac{7}{2}}b^{4}}\right)Fax}{70\,b}+\frac{3\,Fax\left(\frac{15\,x^{4}}{(bx^{2}+a)^{\frac{5}{2}}b}+\frac{20\,ax^{2}}{(bx^{2}+a)^{\frac{5}{2}}b^{2}}+\frac{8\,a^{2}}{(bx^{2}+a)^{\frac{5}{2}}b^{3}}\right)}{10\,b^{2}}-\frac{Dx\left(\frac{15\,x^{4}}{(bx^{2}+a)^{\frac{5}{2}}b}+\frac{20\,ax^{2}}{(bx^{2}+a)^{\frac{5}{2}}b^{2}}+\frac{8\,a^{2}}{(bx^{2}+a)^{\frac{5}{2}}b^{3}}\right)}{15\,b}-\frac{Cx^{5}}{2\,(bx^{2}+a)^{\frac{7}{2}}b}+\frac{3\,Fax\left(\frac{3\,x^{2}}{(bx^{2}+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^{2}+a)^{\frac{3}{2}}b^{2}}\right)}{2\,b^{3}}-\frac{Dx\left(\frac{3\,x^{2}}{(bx^{2}+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^{2}+a)^{\frac{3}{2}}b^{2}}\right)}{3\,b^{2}}+\frac{9\,Fa^{2}x^{3}}{2\,(bx^{2}+a)^{\frac{5}{2}}b^{4}}-\frac{Dax^{3}}{(bx^{2}+a)^{\frac{5}{2}}b^{3}}-\frac{5\,Cax^{3}}{8\,(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{Bx^{3}}{4\,(bx^{2}+a)^{\frac{7}{2}}b}-\frac{417\,Fax}{70\,\sqrt{bx^{2}+a}\,b^{5}}-\frac{51\,Fa^{2}x}{70\,(bx^{2}+a)^{\frac{3}{2}}b^{5}}+\frac{261\,Fa^{3}x}{70\,(bx^{2}+a)^{\frac{5}{2}}b^{5}}+\frac{139\,Dx}{105\,\sqrt{bx^{2}+a}\,b^{4}}+\frac{17\,Dax}{105\,(bx^{2}+a)^{\frac{3}{2}}b^{4}}-\frac{29\,Da^{2}x}{35\,(bx^{2}+a)^{\frac{5}{2}}b^{4}}+\frac{Cx}{14\,(bx^{2}+a)^{\frac{3}{2}}b^{3}}+\frac{Cx}{7\,\sqrt{bx^{2}+a}\,ab^{3}}+\frac{3\,Cax}{56\,(bx^{2}+a)^{\frac{5}{2}}b^{3}}-\frac{15\,Ca^{2}x}{56\,(bx^{2}+a)^{\frac{7}{2}}b^{3}}+\frac{3\,Bx}{140\,(bx^{2}+a)^{\frac{5}{2}}b^{2}}+\frac{2\,Bx}{35\,\sqrt{bx^{2}+a}\,a^{2}b^{2}}+\frac{Bx}{35\,(bx^{2}+a)^{\frac{3}{2}}ab^{2}}-\frac{3\,Bax}{28\,(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{Ax}{7\,(bx^{2}+a)^{\frac{7}{2}}b}+\frac{8\,Ax}{105\,\sqrt{bx^{2}+a}\,a^{3}b}+\frac{4\,Ax}{105\,(bx^{2}+a)^{\frac{3}{2}}a^{2}b}+\frac{Ax}{35\,(bx^{2}+a)^{\frac{5}{2}}ab}-\frac{9\,Fa\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{11}{2}}}+\frac{D\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="max
ima")
```

```
[Out] 1/2*F*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a
*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/
((b*x^2 + a)^(7/2)*b^4))*D*x + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^
4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b
*x^2 + a)^(7/2)*b^4))*F*a*x/b + 3/10*F*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) +
20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/1
5*D*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*
a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*C*x^5/((b*x^2 + a)^(7/2)*b) + 3/2*F*a*
x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*D*x
*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 9/2*F*a^
2*x^3/((b*x^2 + a)^(5/2)*b^4) - D*a*x^3/((b*x^2 + a)^(5/2)*b^3) - 5/8*C*a*x
^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*B*x^3/((b*x^2 + a)^(7/2)*b) - 417/70*F*a*x
/(sqrt(b*x^2 + a)*b^5) - 51/70*F*a^2*x/((b*x^2 + a)^(3/2)*b^5) + 261/70*F*a
^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*D*x/(sqrt(b*x^2 + a)*b^4) + 17/105*D
*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*D*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14
```

*C*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*C*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*C*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*C*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*B*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*B*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*A*x/((b*x^2 + a)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*A*x/((b*x^2 + a)^(5/2)*a*b) - 9/2*F*a*arcsinh(b*x/sqrt(a*b))/b^(11/2) + D*arcsinh(b*x/sqrt(a*b))/b^(9/2)

Fricas [**A**]
time = 2.81, size = 705, normalized size = 2.70

$$\left[-\frac{105\,(9\,Fa^{8}-2\,Da^{7}b+(9\,Fa^{4}b^{4}-2\,Da^{3}b^{5})x^{8}+4\,(9\,Fa^{5}b^{3}-2\,Da^{4}b^{4})x^{6}+6\,(9\,Fa^{6}b^{2}-2\,Da^{5}b^{3})x^{4}+4\,(9\,Fa^{7}b-2\,Da^{6}b^{2})x^{2})\sqrt{b}\log\left(-2\,bx^{2}-2\,\sqrt{bx^{2}+a}\sqrt{b}x-a\right)-2\,(105\,Fa^{3}b^{5}x^{9}+2\,(792\,Fa^{4}b^{4}-176\,Da^{3}b^{5}+15\,Ca^{2}b^{6}+6\,Bab^{7}+8\,Ab^{8})x^{7}+14\,(261\,Fa^{5}b^{3}-58\,Da^{4}b^{4}+3\,Ba^{2}b^{6}+4\,Aab^{7})x^{5}+70\,(45\,Fa^{6}b^{2}-10\,Da^{5}b^{3}+Aa^{2}b^{6})x^{3}+105\,(9\,Fa^{7}b-2\,Da^{6}b^{2})x)\sqrt{bx^{2}+a}}{420\,(a^{3}b^{10}x^{8}+4\,a^{4}b^{9}x^{6}+6\,a^{5}b^{8}x^{4}+4\,a^{6}b^{7}x^{2}+a^{7}b^{6})}, \frac{105\,(9\,Fa^{8}-2\,Da^{7}b+(9\,Fa^{4}b^{4}-2\,Da^{3}b^{5})x^{8}+4\,(9\,Fa^{5}b^{3}-2\,Da^{4}b^{4})x^{6}+6\,(9\,Fa^{6}b^{2}-2\,Da^{5}b^{3})x^{4}+4\,(9\,Fa^{7}b-2\,Da^{6}b^{2})x^{2})\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^{2}+a}}\right)+(105\,Fa^{3}b^{5}x^{9}+2\,(792\,Fa^{4}b^{4}-176\,Da^{3}b^{5}+15\,Ca^{2}b^{6}+6\,Bab^{7}+8\,Ab^{8})x^{7}+14\,(261\,Fa^{5}b^{3}-58\,Da^{4}b^{4}+3\,Ba^{2}b^{6}+4\,Aab^{7})x^{5}+70\,(45\,Fa^{6}b^{2}-10\,Da^{5}b^{3}+Aa^{2}b^{6})x^{3}+105\,(9\,Fa^{7}b-2\,Da^{6}b^{2})x)\sqrt{bx^{2}+a}}{210\,(a^{3}b^{10}x^{8}+4\,a^{4}b^{9}x^{6}+6\,a^{5}b^{8}x^{4}+4\,a^{6}b^{7}x^{2}+a^{7}b^{6})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")

[Out] [-1/420*(105*(9*F*a^8 - 2*D*a^7*b + (9*F*a^4*b^4 - 2*D*a^3*b^5)*x^8 + 4*(9*F*a^5*b^3 - 2*D*a^4*b^4)*x^6 + 6*(9*F*a^6*b^2 - 2*D*a^5*b^3)*x^4 + 4*(9*F*a^7*b - 2*D*a^6*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(105*F*a^3*b^5*x^9 + 2*(792*F*a^4*b^4 - 176*D*a^3*b^5 + 15*C*a^2*b^6 + 6*B*a*b^7 + 8*A*b^8)*x^7 + 14*(261*F*a^5*b^3 - 58*D*a^4*b^4 + 3*B*a^2*b^6 + 4*A*a*b^7)*x^5 + 70*(45*F*a^6*b^2 - 10*D*a^5*b^3 + A*a^2*b^6)*x^3 + 105*(9*F*a^7*b - 2*D*a^6*b^2)*x)*sqrt(b*x^2 + a))/(a^3*b^10*x^8 + 4*a^4*b^9*x^6 + 6*a^5*b^8*x^4 + 4*a^6*b^7*x^2 + a^7*b^6), 1/210*(105*(9*F*a^8 - 2*D*a^7*b + (9*F*a^4*b^4 - 2*D*a^3*b^5)*x^8 + 4*(9*F*a^5*b^3 - 2*D*a^4*b^4)*x^6 + 6*(9*F*a^6*b^2 - 2*D*a^5*b^3)*x^4 + 4*(9*F*a^7*b - 2*D*a^6*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (105*F*a^3*b^5*x^9 + 2*(792*F*a^4*b^4 - 176*D*a^3*b^5 + 15*C*a^2*b^6 + 6*B*a*b^7 + 8*A*b^8)*x^7 + 14*(261*F*a^5*b^3 - 58*D*a^4*b^4 + 3*B*a^2*b^6 + 4*A*a*b^7)*x^5 + 70*(45*F*a^6*b^2 - 10*D*a^5*b^3 + A*a^2*b^6)*x^3 + 105*(9*F*a^7*b - 2*D*a^6*b^2)*x)*sqrt(b*x^2 + a))/(a^3*b^10*x^8 + 4*a^4*b^9*x^6 + 6*a^5*b^8*x^4 + 4*a^6*b^7*x^2 + a^7*b^6)]

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6987 vs. 2(253) = 506.
time = 113.64, size = 6987, normalized size = 26.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)

[Out] A*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a

```
)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**
2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**
(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x*
*2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2
)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a*
*(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2
/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*
sqrt(1 + b*x**2/a))) + B*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a
**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/
a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqr
t(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + C*
x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a)
 + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 +
 b*x**2/a)) + D*(105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sq
rt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(10
1/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x
**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/
2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqr
t(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630
*a**(203/2)*b**46*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**
(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(
1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a
**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x
**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a)
 + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(201/2)*b*
*47*x**4*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99
/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a)
+ 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(
105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b
*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193
/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 2100*a**(199/2)*b**48*x**6*sqrt(
1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b
*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201
/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sq
rt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630
*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)
*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(197/2)*b**49*x**8*sqrt(1 + b*x**2/a)*
asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630
*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)
*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/
a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b*
*(109/2)*x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1
```

```
+ b*x**2/a)) + 630*a**(195/2)*b**50*x**10*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*
x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b*
*(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 +
 b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(
197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10
*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) +
 105*a**(193/2)*b**51*x**12*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(10
5*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*
sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2
100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107
/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x*
*2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) - 105*a**102*b*
*(91/2)*x/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**
(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 +
b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(1
97/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*
sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) -
665*a**101*b**(93/2)*x**3/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 63
0*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a)...
```

Giac [A]

time = 1.40, size = 224, normalized size = 0.86

$$\frac{\left(\left(\left(\left(\frac{105\,Fx^{2}}{b}+\frac{2\,\left(792\,Fa^{4}b^{7}-176\,Da^{3}b^{8}+15\,Ca^{2}b^{9}+6\,Bab^{10}+8\,Ab^{11}\right)}{a^{3}b^{9}}\right)x^{2}+\frac{14\,\left(261\,Fa^{5}b^{6}-58\,Da^{4}b^{7}+3\,Ba^{2}b^{9}+4\,Aab^{10}\right)}{a^{3}b^{9}}\right)x^{2}+\frac{70\,\left(45\,Fa^{6}b^{5}-10\,Da^{5}b^{6}+Aa^{2}b^{9}\right)}{a^{3}b^{9}}\right)x^{2}+\frac{105\,\left(9\,Fa^{7}b^{4}-2\,Da^{6}b^{5}\right)}{a^{3}b^{9}}\right)x}{210\,\left(bx^{2}+a\right)^{\frac{7}{2}}}+\frac{\left(9\,Fa-2\,Db\right)\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="gia
c")
```

```
[Out] 1/210*((((105*F*x^2/b + 2*(792*F*a^4*b^7 - 176*D*a^3*b^8 + 15*C*a^2*b^9 + 6
*B*a*b^10 + 8*A*b^11)/(a^3*b^9))*x^2 + 14*(261*F*a^5*b^6 - 58*D*a^4*b^7 + 3
*B*a^2*b^9 + 4*A*a*b^10)/(a^3*b^9))*x^2 + 70*(45*F*a^6*b^5 - 10*D*a^5*b^6 +
 A*a^2*b^9)/(a^3*b^9))*x^2 + 105*(9*F*a^7*b^4 - 2*D*a^6*b^5)/(a^3*b^9))*x/(
b*x^2 + a)^(7/2) + 1/2*(9*F*a - 2*D*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)
))/b^(11/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{2}\,\left(A+B\,x^{2}+C\,x^{4}+F\,x^{8}+x^{6}\,D\right)}{{\left(b\,x^{2}+a\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2 + C*x^4 + F*x^8 + x^6*D))/(a + b*x^2)^(9/2),x)
```

```
[Out] int((x^2*(A + B*x^2 + C*x^4 + F*x^8 + x^6*D))/(a + b*x^2)^(9/2), x)
```

3.173 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=214

$$\frac{\left(Ab^4-a^4F\right)x}{ab^4\left(a+bx^2\right)^{7/2}}+\frac{\left(6Ab^4+ab^3B-10a^4F\right)x^3}{3a^2b^3\left(a+bx^2\right)^{7/2}}+\frac{\left(24Ab^4+a(4b^3B+3ab^2C-58a^3F)\right)x^5}{15a^3b^2\left(a+bx^2\right)^{7/2}}+\frac{(48Ab^4+a(8b^3B}{1}$$

```
[Out] (A*b^4-F*a^4)*x/a/b^4/(b*x^2+a)^(7/2)+1/3*(6*A*b^4+B*a*b^3-10*F*a^4)*x^3/a^
2/b^3/(b*x^2+a)^(7/2)+1/15*(24*A*b^4+a*(4*B*b^3+3*C*a*b^2-58*F*a^3))*x^5/a^
3/b^2/(b*x^2+a)^(7/2)+1/105*(48*A*b^4+a*(8*B*b^3+6*C*a*b^2+15*D*a^2*b-176*F
*a^3))*x^7/a^4/b/(b*x^2+a)^(7/2)+F*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(9/
2)
```

Rubi [A]
time = 0.29, antiderivative size = 250, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1828, 1171, 393, 223, 212}

$$\frac{x\left(\frac{-176a^3F+15a^2bD+6ab^2C+8b^3B}{b^4}+\frac{48A}{a}\right)}{105a^3\sqrt{a+bx^2}}+\frac{x\left(a\left(122a^3F-45a^2bD+3ab^2C+4b^3B\right)+24Ab^4\right)}{105a^3b^4\left(a+bx^2\right)^{3/2}}+\frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2C+b^3B}{b^4}+\frac{6A}{a}\right)}{35a\left(a+bx^2\right)^{5/2}}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}+\frac{F\tanh^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(a + b*x^2)^(9/2),x]
```

```
[Out] ((A/a - (b^3*B - a*b^2*C + a^2*b*D - a^3*F)/b^4)*x)/(7*(a + b*x^2)^(7/2)) +
 (((6*A)/a + (b^3*B - 8*a*b^2*C + 15*a^2*b*D - 22*a^3*F)/b^4)*x)/(35*a*(a +
 b*x^2)^(5/2)) + ((24*A*b^4 + a*(4*b^3*B + 3*a*b^2*C - 45*a^2*b*D + 122*a^3
*F))*x)/(105*a^3*b^4*(a + b*x^2)^(3/2)) + (((48*A)/a + (8*b^3*B + 6*a*b^2*C
 + 15*a^2*b*D - 176*a^3*F)/b^4)*x)/(105*a^3*Sqrt[a + b*x^2]) + (F*ArcTanh[(
Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(9/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\,(a+bx^2)^{7/2}} - \frac{\int \frac{-6A-\frac{a\left(b^3B-ab^2C+a^2bD-a^3F\right)}{b^4}-\frac{7a\left(b^2C-abD}{b^3}}{(a+bx^2)^{7/2}}}{7a}$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\,(a+bx^2)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\,(a+bx^2)^{5/2}} +$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\,(a+bx^2)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\,(a+bx^2)^{5/2}} +$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\,(a+bx^2)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\,(a+bx^2)^{5/2}} +$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\,(a+bx^2)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\,(a+bx^2)^{5/2}} +$$

$$= \frac{\left(\frac{A}{a}-\frac{b^3B-ab^2C+a^2bD-a^3F}{b^4}\right)x}{7\,(a+bx^2)^{7/2}} + \frac{\left(\frac{6A}{a}+\frac{b^3B-8ab^2C+15a^2bD-22a^3F}{b^4}\right)x}{35a\,(a+bx^2)^{5/2}} +$$

Mathematica [A]
time = 0.67, size = 176, normalized size = 0.82

$$\frac{x(-105a^7F-350a^6bFx^2-406a^5b^2Fx^4+48Ab^7x^6-176a^4b^3Fx^6+8ab^6x^4(21A+Bx^2)+2a^2b^5x^2(105A+14Bx^2+3Cx^4)+a^3b^4(105A+35Bx^2+21Cx^4+15Dx^6))}{105a^4b^4\,(a+bx^2)^{7/2}}-\frac{F\log\left(-\sqrt{b}\,x+\sqrt{a+bx^2}\right)}{b^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(a + b*x^2)^(9/2),x]
```

```
[Out] (x*(-105*a^7*F - 350*a^6*b*F*x^2 - 406*a^5*b^2*F*x^4 + 48*A*b^7*x^6 - 176*a
^4*b^3*F*x^6 + 8*a*b^6*x^4*(21*A + B*x^2) + 2*a^2*b^5*x^2*(105*A + 14*B*x^2
 + 3*C*x^4) + a^3*b^4*(105*A + 35*B*x^2 + 21*C*x^4 + 15*D*x^6)))/(105*a^4*b
^4*(a + b*x^2)^(7/2)) - (F*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(9/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(194) = 388.
time = 0.11, size = 544, normalized size = 2.54

method	result

default	$F\left(-\frac{x^{7}}{7b(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{-\frac{x^{5}}{5b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^{3}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)+D\left(-\frac{x^{5}}{2b(b\,x^{2}+a)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] F*(-1/7*x^7/b/(b*x^2+a)^(7/2)+1/b*(-1/5*x^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^3
/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+
a)^(1/2))))))+D*(-1/2*x^5/b/(b*x^2+a)^(7/2)+5/2*a/b*(-1/4*x^3/b/(b*x^2+a)^(
7/2)+3/4*a/b*(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7
/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2
+a)^(1/2)))))))+C*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/6*x/b/(b*x^2+a)^(
7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*
(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))))+B*(-1/6*x/b/(b*x^2+
a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/
5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))+A*(1/7*x/a/(b*x^
2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/
3*x/a^2/(b*x^2+a)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(198) = 396$.
time = 0.45, size = 597, normalized size = 2.79

$$-\frac{1}{35}\left(\frac{35\,x^{6}}{(bx^{2}+a)^{\frac{7}{2}}b}+\frac{70\,ax^{4}}{(bx^{2}+a)^{\frac{7}{2}}b^{2}}+\frac{56\,a^{2}x^{2}}{(bx^{2}+a)^{\frac{7}{2}}b^{3}}+\frac{16\,a^{3}}{(bx^{2}+a)^{\frac{7}{2}}b^{4}}\right)Fx-\frac{Fx\left(\frac{15\,x^{4}}{(bx^{2}+a)^{\frac{5}{2}}b}+\frac{20\,ax^{2}}{(bx^{2}+a)^{\frac{5}{2}}b^{2}}+\frac{8\,a^{2}}{(bx^{2}+a)^{\frac{5}{2}}b^{3}}\right)}{15\,b}-\frac{Dx^{5}}{2\,(bx^{2}+a)^{\frac{7}{2}}b}-\frac{Fx\left(\frac{3\,x^{2}}{(bx^{2}+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^{2}+a)^{\frac{3}{2}}b^{2}}\right)}{3\,b^{2}}-\frac{Fax^{3}}{(bx^{2}+a)^{\frac{5}{2}}b^{3}}-\frac{5\,Dax^{3}}{8\,(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{Cx^{3}}{4\,(bx^{2}+a)^{\frac{7}{2}}b}+\frac{16\,Ax}{35\,\sqrt{bx^{2}+a}\,a^{4}}+\frac{8\,Ax}{35\,(bx^{2}+a)^{\frac{3}{2}}a^{3}}+\frac{6\,Ax}{35\,(bx^{2}+a)^{\frac{5}{2}}a^{2}}+\frac{Ax}{7\,(bx^{2}+a)^{\frac{7}{2}}a}+\frac{139\,Fx}{105\,\sqrt{bx^{2}+a}\,b^{4}}+\frac{17\,Fax}{105\,(bx^{2}+a)^{\frac{3}{2}}b^{4}}-\frac{29\,Fa^{2}x}{35\,(bx^{2}+a)^{\frac{5}{2}}b^{4}}+\frac{Dx}{14\,(bx^{2}+a)^{\frac{3}{2}}b^{3}}+\frac{Dx}{7\,\sqrt{bx^{2}+a}\,ab^{3}}+\frac{3\,Dax}{56\,(bx^{2}+a)^{\frac{5}{2}}b^{3}}-\frac{15\,Da^{2}x}{56\,(bx^{2}+a)^{\frac{7}{2}}b^{3}}+\frac{3\,Cx}{140\,(bx^{2}+a)^{\frac{5}{2}}b^{2}}+\frac{2\,Cx}{35\,\sqrt{bx^{2}+a}\,a^{2}b^{2}}+\frac{Cx}{35\,(bx^{2}+a)^{\frac{3}{2}}ab^{2}}-\frac{3\,Cax}{28\,(bx^{2}+a)^{\frac{7}{2}}b^{2}}-\frac{Bx}{7\,(bx^{2}+a)^{\frac{7}{2}}b}+\frac{8\,Bx}{105\,\sqrt{bx^{2}+a}\,a^{3}b}+\frac{4\,Bx}{105\,(bx^{2}+a)^{\frac{3}{2}}a^{2}b}+\frac{Bx}{35\,(bx^{2}+a)^{\frac{5}{2}}ab}+\frac{F\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima"
)
```

```
[Out] -1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56
*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*F*x - 1/
15*F*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8
*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*D*x^5/((b*x^2 + a)^(7/2)*b) - 1/3*F*x
*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - F*a*x^3/
((b*x^2 + a)^(5/2)*b^3) - 5/8*D*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*C*x^3/(
(b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/((b*x^2 +
 a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^2 + a)^(7
/2)*a) + 139/105*F*x/(sqrt(b*x^2 + a)*b^4) + 17/105*F*a*x/((b*x^2 + a)^(3/2
)*b^4) - 29/35*F*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*D*x/((b*x^2 + a)^(3/2
)*b^3) + 1/7*D*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*D*a*x/((b*x^2 + a)^(5/2)*b^
3) - 15/56*D*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*C*x/((b*x^2 + a)^(5/2)*b
^2) + 2/35*C*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*C*x/((b*x^2 + a)^(3/2)*a*b^
2) - 3/28*C*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8
/105*B*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/
35*B*x/((b*x^2 + a)^(5/2)*a*b) + F*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

Fricas [**A**]
time = 3.58, size = 567, normalized size = 2.65

$$\left[\frac{105\,(Fa^4b^4x^8+4\,Fa^5b^3x^6+6\,Fa^6b^2x^4+4\,Fa^7bx^2+Fa^8)\sqrt{b}\,\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\,\sqrt{b}\,x-a\right)-2\,((176\,Fa^4b^4-15\,Da^3b^5-6\,Ca^2b^6-8\,Bab^7-48\,Ab^8)x^7+7\,(58\,Fa^5b^3-3\,Ca^3b^5-4\,Ba^2b^6-24\,Aab^7)x^5+35\,(10\,Fa^6b^2-Ba^3b^5-6\,Aa^2b^6)x^3+105\,(Fa^7b-Aa^3b^5)x)\sqrt{bx^2+a}}{210\,(a^4b^9x^8+4\,a^5b^8x^6+6\,a^6b^7x^4+4\,a^7b^6x^2+a^8b^5)},-\frac{105\,(Fa^4b^4x^8+4\,Fa^5b^3x^6+6\,Fa^6b^2x^4+4\,Fa^7bx^2+Fa^8)\sqrt{-b}\,\arctan\left(\frac{\sqrt{-b}\,x}{\sqrt{bx^2+a}}\right)+((176\,Fa^4b^4-15\,Da^3b^5-6\,Ca^2b^6-8\,Bab^7-48\,Ab^8)x^7+7\,(58\,Fa^5b^3-3\,Ca^3b^5-4\,Ba^2b^6-24\,Aab^7)x^5+35\,(10\,Fa^6b^2-Ba^3b^5-6\,Aa^2b^6)x^3+105\,(Fa^7b-Aa^3b^5)x)\sqrt{bx^2+a}}{105\,(a^4b^9x^8+4\,a^5b^8x^6+6\,a^6b^7x^4+4\,a^7b^6x^2+a^8b^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas"
)
```

```
[Out] [1/210*(105*(F*a^4*b^4*x^8 + 4*F*a^5*b^3*x^6 + 6*F*a^6*b^2*x^4 + 4*F*a^7*b*
x^2 + F*a^8)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*((
176*F*a^4*b^4 - 15*D*a^3*b^5 - 6*C*a^2*b^6 - 8*B*a*b^7 - 48*A*b^8)*x^7 + 7*
(58*F*a^5*b^3 - 3*C*a^3*b^5 - 4*B*a^2*b^6 - 24*A*a*b^7)*x^5 + 35*(10*F*a^6*
b^2 - B*a^3*b^5 - 6*A*a^2*b^6)*x^3 + 105*(F*a^7*b - A*a^3*b^5)*x)*sqrt(b*x^
2 + a))/(a^4*b^9*x^8 + 4*a^5*b^8*x^6 + 6*a^6*b^7*x^4 + 4*a^7*b^6*x^2 + a^8*
b^5), -1/105*(105*(F*a^4*b^4*x^8 + 4*F*a^5*b^3*x^6 + 6*F*a^6*b^2*x^4 + 4*F*
a^7*b*x^2 + F*a^8)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((176*F*a^
4*b^4 - 15*D*a^3*b^5 - 6*C*a^2*b^6 - 8*B*a*b^7 - 48*A*b^8)*x^7 + 7*(58*F*a^
5*b^3 - 3*C*a^3*b^5 - 4*B*a^2*b^6 - 24*A*a*b^7)*x^5 + 35*(10*F*a^6*b^2 - B*
a^3*b^5 - 6*A*a^2*b^6)*x^3 + 105*(F*a^7*b - A*a^3*b^5)*x)*sqrt(b*x^2 + a))/
(a^4*b^9*x^8 + 4*a^5*b^8*x^6 + 6*a^6*b^7*x^4 + 4*a^7*b^6*x^2 + a^8*b^5)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5071 vs. $2(211) = 422$.
time = 78.31, size = 5071, normalized size = 23.70

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(
1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*
b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) +
 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt
(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a
**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**
2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8
*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(
25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sq
rt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*
b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt
(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b*
*3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*
```

```
x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x
**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a
**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b
*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(
35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a
) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sq
rt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/
2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(
1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 52
5*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1
+ b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x*
*13/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2
/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*
sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(2
7/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**
2/a))) + B*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*
b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 42
0*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 +
 b*x**2/a)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17
/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a)
+ 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt
(1 + b*x**2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420
*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x
**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x*
*8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a)
 + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1
 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b
**4*x**8*sqrt(1 + b*x**2/a))) + C*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a
) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1
+ b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(
11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**
(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/
a))) + D*x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 +
b*x**2/a) + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6
*sqrt(1 + b*x**2/a)) + F*(105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqr
t(b)*x/sqrt(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/
2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqr
t(1 + b*x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575
*a**(197/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*
x**10*sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/
a)) + 630*a**(203/2)*b**46*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))
/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x
**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a)
 + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**
```

```
(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 +
b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(
201/2)*b**47*x**4*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/
2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b
*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(19
9/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*s
qrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 10
5*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a...
```

Giac [A]

time = 1.74, size = 204, normalized size = 0.95

$$-\frac{\left(\left(x^{2}\left(\frac{(176\,Fa^{4}b^{6}-15\,Da^{3}b^{7}-6\,Ca^{2}b^{8}-8\,Bab^{9}-48\,Ab^{10})x^{2}}{a^{4}b^{7}}+\frac{7\,(58\,Fa^{5}b^{5}-3\,Ca^{3}b^{7}-4\,Ba^{2}b^{8}-24\,Aab^{9})}{a^{4}b^{7}}\right)+\frac{35\,(10\,Fa^{6}b^{4}-Ba^{3}b^{7}-6\,Aa^{2}b^{8})}{a^{4}b^{7}}\right)x^{2}+\frac{105\,(Fa^{7}b^{3}-Aa^{3}b^{7})}{a^{4}b^{7}}\right)x}{105\,(bx^{2}+a)^{\frac{7}{2}}}-\frac{F\log\left(\left|-\sqrt{b}\,x+\sqrt{bx^{2}+a}\right|\right)}{b^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

```
[Out] -1/105*((x^2*((176*F*a^4*b^6 - 15*D*a^3*b^7 - 6*C*a^2*b^8 - 8*B*a*b^9 - 48*
A*b^10)*x^2/(a^4*b^7) + 7*(58*F*a^5*b^5 - 3*C*a^3*b^7 - 4*B*a^2*b^8 - 24*A*
a*b^9)/(a^4*b^7)) + 35*(10*F*a^6*b^4 - B*a^3*b^7 - 6*A*a^2*b^8)/(a^4*b^7))*
x^2 + 105*(F*a^7*b^3 - A*a^3*b^7)/(a^4*b^7))*x/(b*x^2 + a)^(7/2) - F*log(ab
s(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+B\,x^2+C\,x^4+F\,x^8+x^6\,D}{{\left(b\,x^2+a\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(a + b*x^2)^(9/2),x)
```

```
[Out] int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(a + b*x^2)^(9/2), x)
```

3.174 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2\left(a+bx^2\right)^{9/2}}\,dx$

Optimal. Leaf size=193

$$-\frac{A}{ax\left(a+bx^2\right)^{7/2}}-\frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}}-\frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{\left(192Ab^3-a\left(24b^2B+4abC+3a^2D\right)\right)x^5}{15a^4\left(a+bx^2\right)^{7/2}}-$$

```
[Out] -A/a/x/(b*x^2+a)^(7/2)-(8*A*b-B*a)*x/a^2/(b*x^2+a)^(7/2)-1/3*(48*A*b^2-a*(6
*B*b+C*a))*x^3/a^3/(b*x^2+a)^(7/2)-1/15*(192*A*b^3-a*(24*B*b^2+4*C*a*b+3*D*
a^2))*x^5/a^4/(b*x^2+a)^(7/2)-1/105*(384*A*b^4-a*(48*B*b^3+8*C*a*b^2+6*D*a^
2*b+15*F*a^3))*x^7/a^5/(b*x^2+a)^(7/2)
```

Rubi [A]

time = 0.24, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {1817, 1827, 12, 270}

$$-\frac{x^3\left(48Ab^2-a(aC+6bB)\right)}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{x(8Ab-aB)}{a^2\left(a+bx^2\right)^{7/2}}-\frac{x^5\left(192Ab^3-a\left(3a^2D+4abC+24b^2B\right)\right)}{15a^4\left(a+bx^2\right)^{7/2}}-\frac{x^7\left(384Ab^4-a\left(15a^3F+6a^2bD+8ab^2C+48b^3B\right)\right)}{105a^5\left(a+bx^2\right)^{7/2}}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] -(A/(a*x*(a + b*x^2)^(7/2))) - ((8*A*b - a*B)*x)/(a^2*(a + b*x^2)^(7/2)) -
((48*A*b^2 - a*(6*b*B + a*C))*x^3)/(3*a^3*(a + b*x^2)^(7/2)) - ((192*A*b^3
- a*(24*b^2*B + 4*a*b*C + 3*a^2*D))*x^5)/(15*a^4*(a + b*x^2)^(7/2)) - ((384
*A*b^4 - a*(48*b^3*B + 8*a*b^2*C + 6*a^2*b*D + 15*a^3*F))*x^7)/(105*a^5*(a
+ b*x^2)^(7/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 1817

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Dist[1/(a*(m + 1)), Int[x
^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
```

```
FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

Rule 1827

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*x*((a + b*
x^2)^(p + 1)/a), x] + Dist[1/a, Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3))
, x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ[
Expon[Pq, x] + 2*p + 1, 0]
```

Rubi steps

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{8Ab-a\left(B+Cx^2+Dx^4+Fx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx}{a}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\int \frac{x^2 (6b(8Ab-aB)+a(-aC-aDx^2-aFx}{\left(a+bx^2\right)^{9/2}}}{a^2}$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} -$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} -$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} -$$

$$= -\frac{A}{ax\left(a+bx^2\right)^{7/2}} - \frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} - \frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}} -$$

Mathematica [A]

time = 0.52, size = 138, normalized size = 0.72

$$\frac{-384Ab^4x^8+48ab^3x^6(-28A+Bx^2)+8a^2b^2x^4(-210A+21Bx^2+Cx^4)+2a^3bx^2(-420A+105Bx^2+14Cx^4+3Dx^6)+a^4(-105A+105Bx^2+35Cx^4+21Dx^6+15Fx^8)}{105a^5x\left(a+bx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^2*(a + b*x^2)^(9/2)),x]
```

```
[Out] (-384*A*b^4*x^8 + 48*a*b^3*x^6*(-28*A + B*x^2) + 8*a^2*b^2*x^4*(-210*A + 21
*B*x^2 + C*x^4) + 2*a^3*b*x^2*(-420*A + 105*B*x^2 + 14*C*x^4 + 3*D*x^6) + a
```

```
^4*(-105*A + 105*B*x^2 + 35*C*x^4 + 21*D*x^6 + 15*F*x^8))/(105*a^5*x*(a + b
*x^2)^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 538 vs. $2(177) = 354$.
time = 0.10, size = 539, normalized size = 2.79

method	result
gosper	$-\frac{384A\,b^4x^8-48Ba\,b^3x^8-8C\,a^2b^2x^8-6Da^3b\,x^8-15F\,a^4x^8+1344a\,b^3A\,x^6-168B\,a^2b^2x^6-28C\,a^3b\,x^6-21Da^4x^6+1680A\,a^2b^2x^4-}{105x(b\,x^2+a)^{\frac{7}{2}}a^5}$
trager	$-\frac{384A\,b^4x^8-48Ba\,b^3x^8-8C\,a^2b^2x^8-6Da^3b\,x^8-15F\,a^4x^8+1344a\,b^3A\,x^6-168B\,a^2b^2x^6-28C\,a^3b\,x^6-21Da^4x^6+1680A\,a^2b^2x^4-}{105x(b\,x^2+a)^{\frac{7}{2}}a^5}$

default	F	$-\frac{x^{5}}{2b(b x^{2}+a)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^{3}}{4b\left(b x^{2}+a\right)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b x^{2}+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b x^{2}}}\right)}{7a}}{a}\right)}{6b}\right)}{4b}\right)}{2b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE
)
```

```
[Out] F*(-1/2*x^5/b/(b*x^2+a)^(7/2)+5/2*a/b*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(
-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b
*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))))
)+D*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a/b*(
1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^
2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))))+C*(-1/6*x/b/(b*x^2+a)^(7/2)+1/6*a
/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(
b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))+B*(1/7*x/a/(b*x^2+a)^(7/2)+6/7
/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2
+a)^(1/2))))+A*(-1/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a
*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a
)^(1/2)))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(175) = 350$.
time = 0.43, size = 421, normalized size = 2.18

$$-\frac{Fx^5}{2\,(bx^2+a)^{\frac{7}{2}}b}-\frac{5\,Fax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{Dx^3}{4\,(bx^2+a)^{\frac{7}{2}}b}+\frac{16\,Bx}{35\,\sqrt{bx^2+a}\,a^4}+\frac{8\,Bx}{35\,(bx^2+a)^{\frac{3}{2}}a^3}+\frac{6\,Bx}{35\,(bx^2+a)^{\frac{5}{2}}a^2}+\frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}a}+\frac{Fx}{14\,(bx^2+a)^{\frac{3}{2}}b^3}+\frac{Fx}{7\,\sqrt{bx^2+a}\,ab^3}+\frac{3\,Fax}{56\,(bx^2+a)^{\frac{5}{2}}b^3}-\frac{15\,Fa^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3}+\frac{3\,Dx}{140\,(bx^2+a)^{\frac{5}{2}}b^2}+\frac{2\,Dx}{35\,\sqrt{bx^2+a}\,a^2b^2}+\frac{Dx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2}-\frac{3\,Dax}{28\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{Cx}{7\,(bx^2+a)^{\frac{7}{2}}b}+\frac{8\,Cx}{105\,\sqrt{bx^2+a}\,a^3b}+\frac{4\,Cx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b}+\frac{Cx}{35\,(bx^2+a)^{\frac{5}{2}}ab}-\frac{128\,Abx}{35\,\sqrt{bx^2+a}\,a^5}-\frac{64\,Abx}{35\,(bx^2+a)^{\frac{3}{2}}a^4}-\frac{48\,Abx}{35\,(bx^2+a)^{\frac{5}{2}}a^3}-\frac{8\,Abx}{7\,(bx^2+a)^{\frac{7}{2}}a^2}-\frac{A}{(bx^2+a)^{\frac{7}{2}}ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="max
ima")
```

```
[Out] -1/2*F*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*F*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/
4*D*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*B*x/
((b*x^2 + a)^(3/2)*a^3) + 6/35*B*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/((b*x^
2 + a)^(7/2)*a) + 1/14*F*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*F*x/(sqrt(b*x^2 +
a)*a*b^3) + 3/56*F*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*F*a^2*x/((b*x^2 + a)
^(7/2)*b^3) + 3/140*D*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*D*x/(sqrt(b*x^2 + a)
*a^2*b^2) + 1/35*D*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*D*a*x/((b*x^2 + a)^(7
/2)*b^2) - 1/7*C*x/((b*x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 + a)*a^3*b
) + 4/105*C*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5/2)*a*b)
- 128/35*A*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 + a)^(3/2)*a^4)
- 48/35*A*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 + a)^(7/2)*a^2) -
 A/((b*x^2 + a)^(7/2)*a*x)
```

Fricas [**A**]
time = 3.32, size = 187, normalized size = 0.97

$$\frac{((15\,Fa^4+6\,Da^3b+8\,Ca^2b^2+48\,Bab^3-384\,Ab^4)x^8+7\,(3\,Da^4+4\,Ca^3b+24\,Ba^2b^2-192\,Aab^3)x^6-105\,Aa^4+35\,(Ca^4+6\,Ba^3b-48\,Aa^2b^2)x^4+105\,(Ba^4-8\,Aa^3b)x^2)\sqrt{bx^2+a}}{105\,(a^5b^4x^9+4\,a^6b^3x^7+6\,a^7b^2x^5+4\,a^8bx^3+a^9x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fri
cas")
```

```
[Out] 1/105*((15*F*a^4 + 6*D*a^3*b + 8*C*a^2*b^2 + 48*B*a*b^3 - 384*A*b^4)*x^8 +
7*(3*D*a^4 + 4*C*a^3*b + 24*B*a^2*b^2 - 192*A*a*b^3)*x^6 - 105*A*a^4 + 35*(
C*a^4 + 6*B*a^3*b - 48*A*a^2*b^2)*x^4 + 105*(B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b
*x^2 + a)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*
x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 2490 vs. $2(184) = 368$.
time = 106.66, size = 2490, normalized size = 12.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)
```

```
[Out] A*(-35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 28
0*a**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 56
0*a**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*
x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 44
8*a*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**
2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) - 128*b
**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17*x**2 + 2
10*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)) + B*(35*a**
14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**
2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6
*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(
27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x*
*2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*
b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 70
0*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 +
 b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**
6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**(37/2)*sqrt(1 + b*
x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4
*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(
29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x*
*2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 429*a**11*b**3*x**7/(
35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) +
 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(
1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*
```

```
b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a))
 + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x
**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a
**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*
x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x
**12*sqrt(1 + b*x**2/a)) + 104*a**9*b**5*x**11/(35*a**(37/2)*sqrt(1 + b*x**
2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sq
rt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/
2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/
a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 16*a**8*b**6*x**13/(35*a
**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525
*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 +
b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5
*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a))) +
C*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sq
rt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/
2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a
)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**
2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**
(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x*
*2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2
)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) +
420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a*
*(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2
/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*
sqrt(1 + b*x**2/a))) + D*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a
**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/
a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqr
t(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**
2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + F*
x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a)
 + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 +
 b*x**2/a))
```

Giac [A]

time = 1.90, size = 220, normalized size = 1.14

$$\frac{\left(\left(x^{2}\left(\frac{\left(15\,Fa^{13}b^{3}+6\,Da^{12}b^{4}+8\,Ca^{11}b^{5}+48\,Ba^{10}b^{6}-279\,Aa^{9}b^{7}\right)x^{2}}{a^{14}b^{3}}+\frac{7\,\left(3\,Da^{13}b^{3}+4\,Ca^{12}b^{4}+24\,Ba^{11}b^{5}-132\,Aa^{10}b^{6}\right)}{a^{14}b^{3}}\right)+\frac{35\,\left(Ca^{13}b^{3}+6\,Ba^{12}b^{4}-30\,Aa^{11}b^{5}\right)}{a^{14}b^{3}}\right)x^{2}+\frac{105\,\left(Ba^{13}b^{3}-4\,Aa^{12}b^{4}\right)}{a^{14}b^{3}}\right)x}{105\,\left(bx^{2}+a\right)^{\frac{7}{2}}}+\frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}\,x-\sqrt{bx^{2}+a}\right)^{2}-a\right)a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="gia
c")
```

```
[Out] 1/105*((x^2*((15*F*a^13*b^3 + 6*D*a^12*b^4 + 8*C*a^11*b^5 + 48*B*a^10*b^6 -
 279*A*a^9*b^7)*x^2/(a^14*b^3) + 7*(3*D*a^13*b^3 + 4*C*a^12*b^4 + 24*B*a^11
```

```
*b^5 - 132*A*a^10*b^6)/(a^14*b^3)) + 35*(C*a^13*b^3 + 6*B*a^12*b^4 - 30*A*a
^11*b^5)/(a^14*b^3))*x^2 + 105*(B*a^13*b^3 - 4*A*a^12*b^4)/(a^14*b^3))*x/(b
*x^2 + a)^(7/2) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + B\,x^2 + C\,x^4 + F\,x^8 + x^6\,D}{x^2\,{\left(b\,x^2 + a\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(x^2*(a + b*x^2)^(9/2)),x)
```

```
[Out] int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(x^2*(a + b*x^2)^(9/2)), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
       if m:
          print ("func ", func , " is elementary_function")
       else:
          print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
       print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
       print ("m=",m)
       if m:
          print ("func ", func ," is special_function")
       else:
          print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != −1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```